

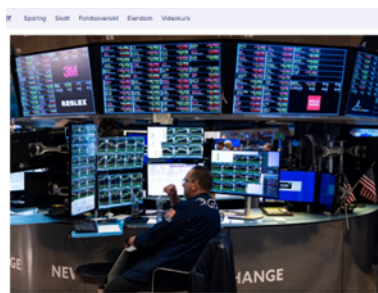
1 - Introduksjon og case

Espen Sirnes

2025-08-10

Table of contents

1	Introduksjon	2
1.1	Caset	2
1.2	I denne forelesningen	2
2	Forelesning 2: Forventning, nytte og risiko	2
3	Forelesning 3: Porteføljeteori og matriser	4
3.1	Porteføljefronten	5
3.2	Utrekninger	5
3.3	Porteføljefronten med optimal portefølje og tangeringslinje .	10
4	Forelesning 4: Faktorer	11
5	Forelesning 5: VaR	13



Disse fondene får terningkast seks akkurat nå

Disse Penger sammenligner nesten 200 fond. 30 fond får terningkast 6.

Andreas Wolden Fredriksen og Emma Funderud Øvrete

potensiell avkastning blir blåst opp, mens de reelle kostnadene og den faktiske **risikoadjusterte** avkastningen blir tilsørt.

Han forklarer at det finnes to aktivklasser: Aksjer og rentepapirer. Alt annet er tilsøring for å selge dyre produkter pakket inn som noe «spennende» og «sofistikert».

– Det koster nesten ingenting å eie passive fond og rentefond. Forskningen er tydelig på at gang på gang gjør fond med lave kostnader det bedre enn dyre fond på sikt.

– Alternative aktivklasser, som private equity, er kjempedyrt. Private equity selger seg inn som noe annet og alternativt, men det er bare aksjer i ny innpakning til 10, 20 eller 30 ganger kostnaden som et indeksfond, sier Riksen.



1 Introduksjon

1.1 Caset

Hver gruppe får tildelt [tre aksjer](#)

1. Bruk historisk avkastning frem til 1. januar 2025 og tilgjengelig offentlig informasjon da, slik som års-/kvartalrapporter, tiog lage en portefølje for to typer investorer:
 - a) Risikoavers
 - b) Risikosøkende
2. Tegn opp i samme diagram
 - a) porteføljefronten
 - b) hver av de tre aksjene
 - c) faktorene
 - d) den optimale tilpasningen.
3. Regn ut den optimale porteføljen, og forklar hvorfor du bruker eller ikke bruker disse vektene.
4. Velg en VaR-modell basert på de historiske datane og kjør en tilbaketest på den (backtesting) og gi din vurdering av modellen.
5. Hent nye data på aksjene og bruk den til å:
 - a) Evaluer faktisk avkastning
 - b) Tegn opp ny porteføljefront med aksjer og faktorer
 - c) Evaluer VaR'en.

1.2 I denne forelesningen

Korte om kursets tema: * Forelesning 2: Forventning, nytte og risiko * Forelesning 3: Porteføljeteori og matriser * Forelesning 4: Faktorer * Forelesning 5: Value at Risk (VaR)

2 Forelesning 2: Forventning, nytte og risiko

Nyttefunksjonen:

```
import numpy as np
import matplotlib.pyplot as plt

# Logarithmic utility function
def u_func(x):
```

```

    return np.log(x)

def x_func(u):
    return np.exp(u)

# Values for wealth and utility
x_vals = np.linspace(0.1, 2, 100)
u_x = u_func(x_vals)

# Gamble outcomes
x_gamble = [0.5, 1.5] # Outcomes of the gamble
p_gamble = [0.5, 0.5] # Probabilities

# Certain outcome
x_certain = 1

# Expected utility of the gamble
expected_utility = np.sum(np.array(p_gamble) * u_func(np.array(x_gamble)))

# Plotting the utility function
plt.figure(figsize=(10, 6))
plt.plot(x_vals, u_x, label=r'Utility Function:  $u(x) = \ln(x)$ ', color='green')

# Plotting the certain outcome
plt.axvline(x=x_certain, color='black', linestyle='--')
plt.text(x_certain, u_func(x_certain) + 0.1, "$1$", horizontalalignment='center')

# Plotting the gamble outcomes
plt.axvline(x=x_gamble[0], color='black', linestyle='--')
plt.axvline(x=x_gamble[1], color='black', linestyle='--')
plt.text(x_gamble[0], u_func(x_gamble[0]) - 0.3, "$0.5$", horizontalalignment='center')
plt.text(x_gamble[1], u_func(x_gamble[1]) - 0.3, "$1.5$", horizontalalignment='center')

# Plotting the expected utility
plt.axhline(y=expected_utility, color='gray', linestyle='--', label='EU(X)')
plt.text(0.15, expected_utility, '$EU(X)$', verticalalignment='center')

# Risk premium - distance between expected utility and utility of ce

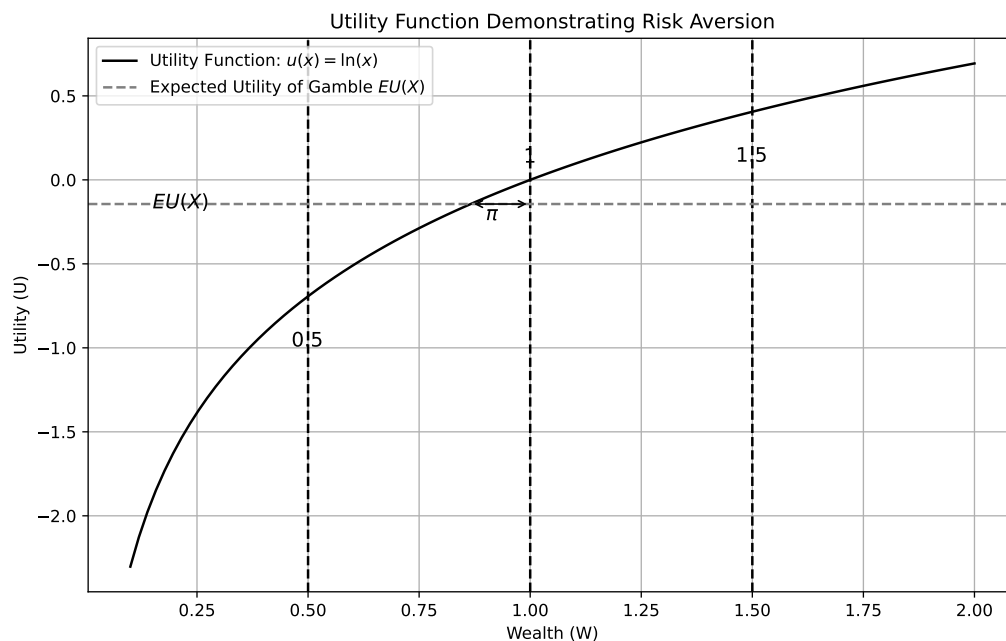
```

```

risk_premium = u_func(x_certain) - expected_utility
certainty_equivalence = x_func(expected_utility)
plt.annotate(' ', xy=(1, expected_utility), xytext=(certainty_equivalence,
    arrowprops=dict(facecolor='black', arrowstyle='<->'))
# Separate annotation for the label ( $\pi$ ) without the arrow
plt.annotate(r'$\pi$', xy=(0.9, expected_utility - 0.1), fontsize=12)

# Labels and title
plt.title('Utility Function Demonstrating Risk Aversion')
plt.xlabel('Wealth (W)')
plt.ylabel('Utility (U)')
plt.legend()
plt.grid(True)

```



3 Forelesning 3: Porteføljeteori og matriser

Her bruker vi titlondatabasen:

```

from IPython.display import IFram
# Embed the web page using an iframe

```

```
IFrame("https://titlon.uit.no/", width=700, height=200)
```

```
<IPython.lib.display.IFrame at 0x1242f80f770>
```

Vi bruker scriptmuligheten i Titlon for å hente data

3.1 Porteføljefronten

3.2 Utregninger

Reduserer utvalget:

```
import numpy as np
import pandas as pd

df = pd.read_pickle('data/stocks.df')
# Defining annual risk free rate.
rf = df['NOWA_DayLnrate'].mean()*7

isin_with_first_date = df[df['Date'] == df['Date'].min()][['ISIN']].unique()
isin_with_last_date = df[df['Date'] == df['Date'].max()][['ISIN']].unique()
valid_isins = set(isin_with_first_date).intersection(isin_with_last_date)
df = df[df['ISIN'].isin(valid_isins)]

df['Name (ISIN)'] =df['Name'].str.upper().str.strip() + '(' + df['ISIN']

# keeping only the most traded shares
res = (
    df.groupby(['Name (ISIN)'])
      .agg({'Turnover': 'sum'})
      .sort_values(by='Turnover', ascending=False)
)
df = df.merge(res.head(4), on=['Name (ISIN)'],
              how='inner')
res.head(4)
```

		Turnover
Name (ISIN)		
EQUINOR(NO0010096985)		1.789492e+12

Name (ISIN)	Turnover
NORSK HYDRO(NO0005052605)	6.394193e+11
TELENOR(NO0010063308)	5.545262e+11
YARA INTERNATIONAL(NO0010208051)	5.404874e+11

Lager avkastningsmatrisen:

```
def get_matrix(df, field):
    """Converts the df to a matrix df that can
    be used to calculate the covariance matrix"""

    import pandas as pd
    df['Date'] = pd.to_datetime(df['Date'])
    df_unique = df.drop_duplicates(
        subset=['Date', 'ISIN'])
    pivot_df = df_unique.pivot(index='Date',
                                columns='Symbol',
                                values=field)

    pivot_df = pivot_df.dropna()

    # Annualized weekly returns
    df_weekly = pivot_df.resample('W').sum()

    return df_weekly

#X is a matrix with e
X_df = get_matrix(df, 'lnDeltaP')
X_df = X_df.sort_index()

X_df
```

Symbol	EQNR	NHY	TEL	YAR
Date				
2016-01-10	-0.118288	-0.137636	-0.008125	-0.058065
2016-01-17	-0.060966	-0.054818	-0.085838	-0.047905

Symbol Date	EQNR	NHY	TEL	YAR
2016-01-24	0.060966	0.023505	0.049143	0.001741
2016-01-31	0.074498	0.024710	-0.007077	-0.053584
2016-02-07	0.027490	0.065780	-0.029552	0.024170
...
2025-07-06	0.026730	0.021321	0.010887	0.021778
2025-07-13	0.042289	0.041652	-0.004469	0.023654
2025-07-20	-0.027114	-0.007206	0.040745	-0.006776
2025-07-27	-0.030583	0.047819	-0.027399	0.014796
2025-08-03	0.038459	-0.036054	0.001892	-0.010360

Finner gjennomsnittsvektoren og varians-kovarians-matrisen:

```
# Converting X to a numpy array:
X = np.array(X_df)

# Calculating the covariance
cov_matrix = np.cov(X, rowvar=False)

# Calculating the means vector, and reshaping it to a
# column vector.

means = np.mean(X, axis=0).reshape((X.shape[1],1))
```

Definerer porteføljefrontfunksjonen:

```
ones = np.ones((len(means),1))

A = (ones.T @ np.linalg.inv(cov_matrix) @ ones)[0][0]

B = (ones.T @ np.linalg.inv(cov_matrix)
      @ (means-rf))[0][0]

C = ((means.T-rf) @ np.linalg.inv(cov_matrix)
      @ (means-rf))[0][0]

def portfolio_front(expected_excess_return, a, b, c):
    r = expected_excess_return
```

```

minimum_variance = (1/a
                    + ((r - abs(b)/a)**2) / (c - b**2/a))
minimum_volatility = minimum_variance**0.5
return minimum_volatility

```

Plotter porteføljefronten:

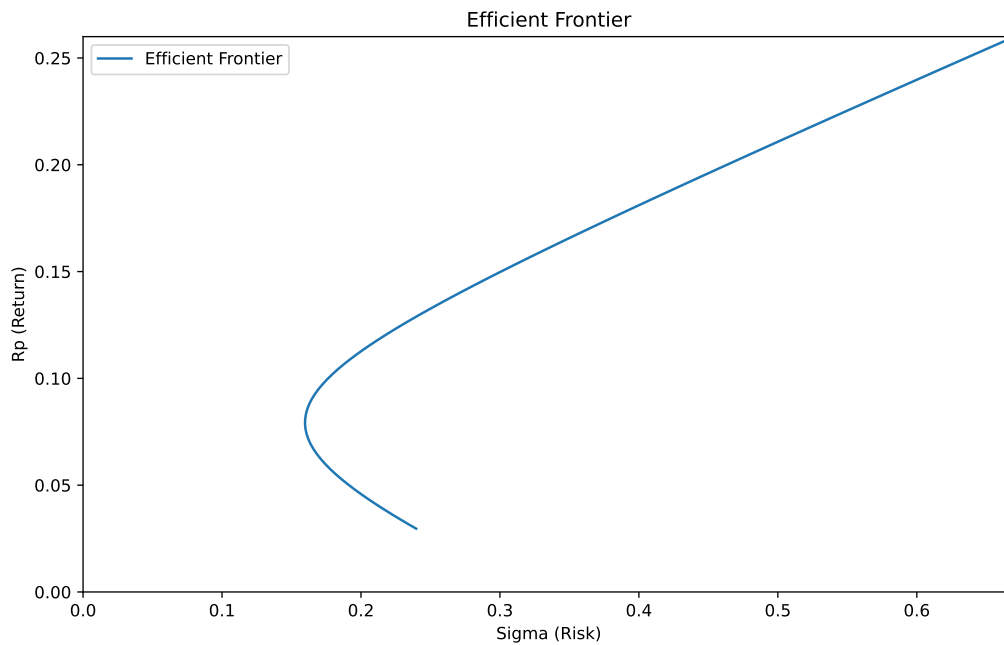
```

from matplotlib import pyplot as plt
#Creating plot
fig, ax = plt.subplots(figsize=(10, 6))
plot_scale = 52
MAX_AXIS = 0.005
#applying the function
rp_values = np.linspace(0, MAX_AXIS-rf, 100)
sigma_values = portfolio_front(rp_values, A, B, C)

#plotting, after annualizing the weekly data
ax.plot(plot_scale**0.5*(sigma_values), plot_scale*(rp_values+rf),
        label='Efficient Frontier')

#plot settings:
ax.set_xlim([0, np.max(sigma_values*plot_scale**0.5)])
ax.set_ylim([0, (np.max(rp_values)+rf)*plot_scale])
ax.set_xlabel('Sigma (Risk)')
ax.set_ylabel('Rp (Return)')
ax.set_title('Efficient Frontier')
ax.legend()

```

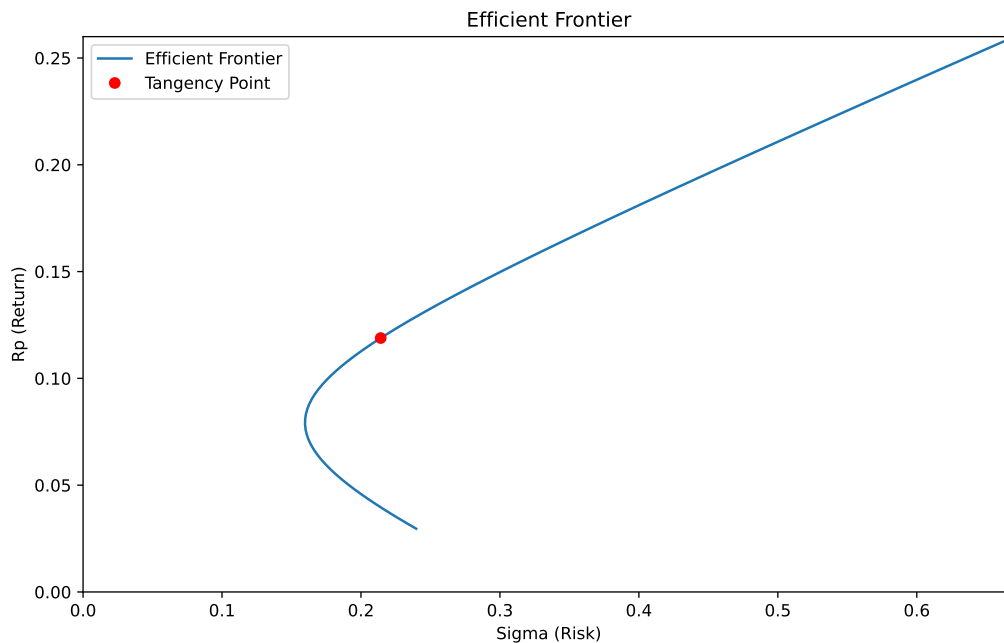



Legger til punkte for den optimale porteføljen:

```
# Calculating the tangency point of the normalized
# optimal portfolio
tangency_sigma = portfolio_front(C/B, A, B, C)

#plotting it, after annualizing the weekly data
ax.plot(plot_scale**0.5*tangency_sigma,
        plot_scale*(C/B + rf),
        'ro',label='Tangency Point')

ax.legend()
fig
```



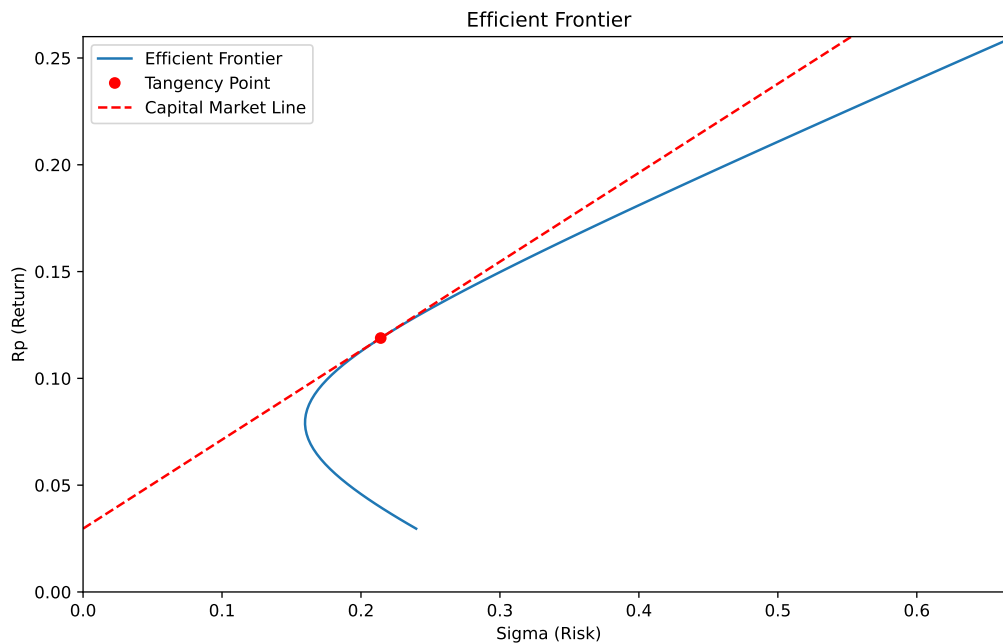
3.3 Porteføljefronten med optimal portefølje og tangeringslinje

```
sigma_range = np.linspace(0, np.max(sigma_values), 100)

# Plotting the portfolio front, after annualizing the
# weekly data
ax.plot(plot_scale**0.5*sigma_range, plot_scale*(rf
        + sigma_range*(C/B)/tangency_sigma),
        color='r', linestyle='--',
        label='Capital Market Line')

ax.legend()

fig
```



4 Forelesning 4: Faktorer

Finner volatilitet og avkastning til faktorene:

```
import pandas as pd

df = pd.read_pickle('data/factors.df')
df['Date'] = pd.to_datetime(df['Date'])
df = df.set_index('Date')
df_weekly = df.resample('W').sum()
df_weekly = df_weekly[['SMB', 'HML', 'LIQ', 'MOM']].dropna()
df = df[['SMB', 'HML', 'LIQ', 'MOM']].dropna()
means = df_weekly.mean()*52
std = df_weekly.std()*52**0.5
print(std)
print(means)
df_weekly
```

```
SMB    0.778593
HML    0.595021
LIQ    0.949455
```

```

MOM      1.208006
dtype: float64
SMB      1.076634
HML     -0.878219
LIQ      0.165220
MOM      1.891799
dtype: float64

```

	SMB	HML	LIQ	MOM
Date				
2016-01-10	0.040139	-0.038205	0.065538	0.039485
2016-01-17	-0.004794	-0.053537	0.044127	0.013392
2016-01-24	0.016701	0.025072	-0.006152	-0.014409
2016-01-31	0.002747	-0.001928	-0.017278	-0.024070
2016-02-07	-0.008014	-0.029920	0.001621	-0.008874
...
2025-07-06	-0.008904	0.000000	-0.003560	0.025957
2025-07-13	-0.036752	0.000000	0.003986	-0.033976
2025-07-20	-0.025081	0.000000	-0.036009	0.013855
2025-07-27	0.013498	0.000000	0.013135	0.006886
2025-08-03	0.000000	0.000000	0.000000	0.000000

Plotter punktene i grafen fra forrige kapittel:

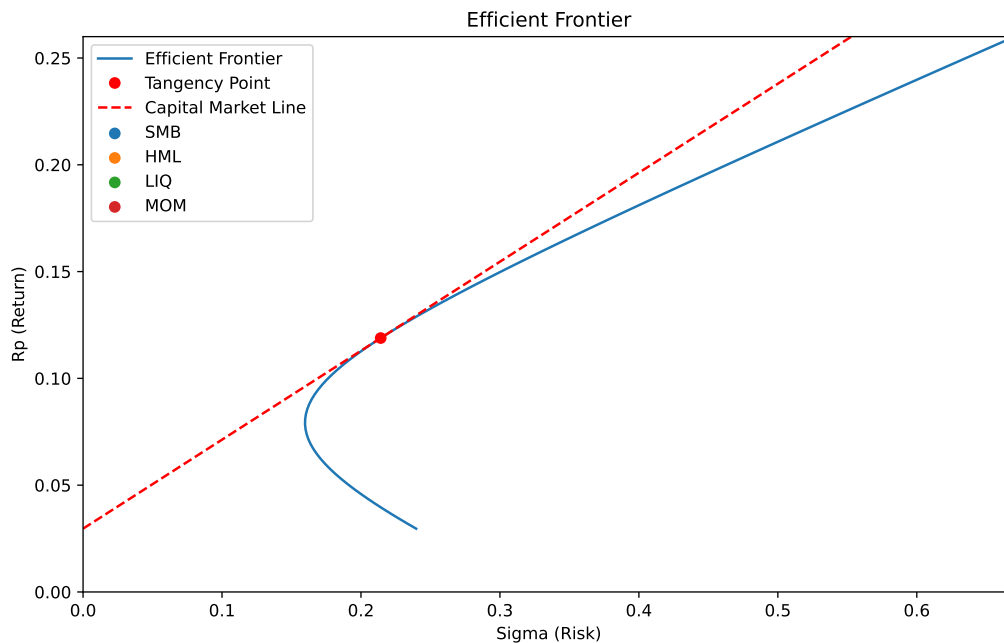
```

for series in means.index:
    ax.scatter(std[series], means[series], label=series)

ax.legend()

fig

```



5 Forelesning 5: VaR

5.0.0.1 Utregninger

```
import numpy as np
def generate_backtest(f, df, name, estimation_win_size):
    # Initialize lists to store calculated values
    datelist = []
    sigmalist = []
    d95list = []
    d99list = []
    ret = []

    # Iterate over returns to calculate and store VaR and volatility
    for t in range(estimation_win_size, len(df)):

        # Record date and current return
        datelist.append(df.index[t].date())
        ret.append(df[name].iloc[t])

        # Extract data from the estimation window (t-estimation_win_
```

```

        x = df[name].iloc[t-estimation_win_size:t-1]

        # Apply the provided VaR estimation function using the histo
        d95, d99, sigma = f(x, sigmalist)

        # Append the estimates to their respective lists
        sigmalist.append(sigma)
        d95list.append(d95)
        d99list.append(d99)

    # Return the results as numpy arrays for ease of analysis
    return (np.array(d95list),
            np.array(d99list),
            np.array(sigmalist),
            np.array(datelist),
            np.array(ret))

import matplotlib.pyplot as plt
def evaluate(plt, d95, d99, ret, dates, heading):
    # Clear the plot area to avoid overlapping plots
    plt.cla()

    # Plot the 95% VaR, 99% VaR, and actual returns
    plt.plot(dates, d95, label='95% Confidence Level')
    plt.plot(dates, d99, label='99% Confidence Level')
    plt.plot(dates, ret, label='Actual Return')

    # Highlight instances where returns breach the 95% VaR
    maxret = max(ret)
    breaches_95 = [maxret if d > r else 0 for d, r in zip(d95, ret)]
    plt.bar(dates, breaches_95, color='gray', alpha=0.5, width=0.5,

    # Set labels and title
    plt.ylabel('VaR')
    plt.xlabel('Date')
    plt.title(heading)
    plt.xticks(rotation=90)
    plt.legend(loc="lower right")
    plt.subplots_adjust(bottom=0.15)

```

```

plt.show()

# Calculate and print the breach percentage for each confidence level
backtest_results = [np.round(sum(d > ret) / len(ret) * 100, 1) for level in levels]

for i, level in enumerate([95, 99]):
    breaches = sum([d95, d99][i] > ret)
    print(f"{heading} with {level}% confidence interval:\n"
          f"Breaches: {breaches}\n"
          f"Backtesting (Realized VaR - % breaches): {backtest_results[i]}")

PVALS = [0.05, 0.01] # Confidence intervals (95% and 99%)
from scipy.stats import norm

def normal_est(x, sigmalist):
    z = norm.ppf(PVALS) # Z-scores for the specified confidence level
    sigma = np.std(x, ddof=1) # Sample standard deviation
    return z[0] * sigma, z[1] * sigma, sigma

def historical_est(x, sigmalist):
    q95 = abs(np.quantile(x, PVALS[0])) # 95th percentile of historical data
    q99 = abs(np.quantile(x, PVALS[1])) # 99th percentile of historical data
    return -q95, -q99, None # VaR values are negative to indicate potential loss

def last_volat(x, sigmalist):
    x = np.array(x)
    z = norm.ppf(PVALS)
    if not sigmalist: # If sigmalist is empty, use initial standard deviation
        sigma = np.std(x, ddof=1)
    else: # Update sigma based on past volatility and recent error
        sigma = (0.1 * (x[0] - np.mean(x))**2 + 0.9 * sigmalist[-1]**2)**0.5
    return z[0] * sigma, z[1] * sigma, sigma

```

5.0.0.2 Evaluating

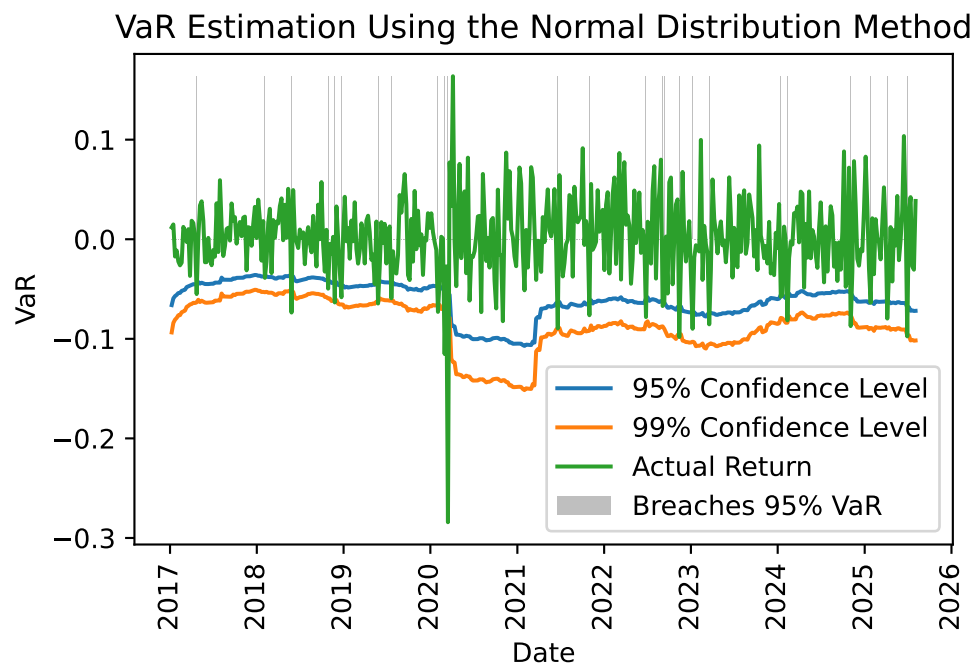
```

NAME = 'EQNR'
ESTIMATION_WINSIZE = 52

df = pd.read_pickle('data/X.df')

```

```
(normal95, normal99,
 sigma, dates, ret )= generate_backtest(normal_est,
 df, NAME, ESTIMATION_WINSIZE)
evaluate(plt, normal95, normal99, ret, dates,
 'VaR Estimation Using the Normal Distribution Method')
```



VaR Estimation Using the Normal Distribution Method with 95% confidence
Breaches: 25

Backtesting (Realized VaR - % breaches): 5.6%

VaR Estimation Using the Normal Distribution Method with 99% confidence
Breaches: 10

Backtesting (Realized VaR - % breaches): 2.2%