Using integration, the area under the curve $f(x) = 2x$ and above the x-axis between the limits $x = a$ and $x = b$ is obtained by finding the **definite integral** of $f(x) = 2x$. To use the fundamental theorem of calculus, we need the indefinite integral. Using the power rule, Integral Rule 4, we obtain

$$\int 2xdx = 2\int xdx = 2\left[\frac{1}{2}x^2 + C\right] = x^2 + 2C$$
$$= x^2 + C_1 = F(x) + C_1$$

where $F(x) = x^2$ and the constant of integration is C_1. The area we seek is given by

$$\int_a^b 2xdx = F(b) - F(a) = b^2 - a^2 \qquad \text{(A.14)}$$

This is the same answer we obtained in (A.13) using geometry.

Many times the algebra is abbreviated, because the constant of integration does not affect the definite integral. You will see for definite integrals

$$\int_a^b 2xdx = x^2\Big|_a^b = b^2 - a^2$$

The vertical bar notation means: evaluate the expression first at b and subtract from it the value of the expression at a.

A.5 Exercises

A.1 Each of the following formulas, (1), (2), and (3), represents a supply or demand relation.

(1) $Q = -3 + 2P$ where $P = 10$
(2) $Q = 100 - 20P$ where $P = 4$
(3) $Q = 50P^{-2}$ where $P = 2$

a. Calculate the slope of each function at the given point.
b. Interpret the slope found in (a). Do the slopes change for different values of P and Q? Is it a supply curve (positive relationship) or a demand curve (inverse relationship)?
c. Calculate the elasticity of each function at the given point.
d. Interpret the elasticity found in (c). Do the elasticities change for different values of P and Q?

A.2 The infant mortality rate (*MORTALITY*) for a country is related to the annual per capita income (*INCOME*, U.S. \$1000) in that country. Three relationships that may describe this relationship are

(1) $\ln(MORTALITY) = 7.5 - 0.5\ln(INCOME)$
(2) $MORTALITY = 1400 - 100INCOME + 1.67INCOME^2$
(3) $MORTALITY = 1500 - 50INCOME$

a. Sketch each of these relationships between *MORTALITY* and *INCOME* between $INCOME = 0$ and $INCOME = 30$.
b. For each of these relationships, calculate the elasticity of infant mortality with respect to income if (i) $INCOME = 1$, (ii) $INCOME = 3$, and (iii) $INCOME = 25$.

A.3 Suppose the rate of inflation *INF*, the annual percentage increase in the general price level, is related to the annual unemployment rate *UNEMP* by the equation $INF = -3 + 7 \times (1/UNEMP)$.

a. Sketch the curve for values of *UNEMP* between 1 and 10.
b. Where is the impact of a change in the unemployment rate the largest?
c. If the unemployment rate is 5%, what is the marginal effect of an increase in the unemployment rate on the inflation rate?

A.4 Simplify the following expressions:

a. $x^{2/3}x^{2/7}$
b. $x^{2/3} \div x^{2/7}$
c. $(x^6y^4)^{-1/2}$

A.5 Below are the 2015 *GDP* ($US) figures provided by the World Bank for a few countries.

a. Express each in scientific notation.
   i. Maldives *GDP* $3,142,812,004
   ii. Nicaragua *GDP* $12,692,562,187
   iii. Ecuador *GDP* $100,871,770,000
   iv. New Zealand *GDP* $173,754,075,210
   v. India *GDP* $2,073,542,978,208
   vi. United States *GDP* $17,946,996,000,000

b. Using scientific notation divide the U.S. *GDP* by the *GDP* in (i) Maldives (ii) Ecuador.

c. The population of New Zealand in 2015 was 4.595 million. Use calculations with scientific notation to compute the per capita income in New Zealand. Express the result in scientific notation.

d. The 2015 population of St. Lucia was 184,999 and its *GDP* was $1,436,390,325. Use calculations with scientific notation to compute the per capita income in St. Lucia. Express the result in scientific notation.

e. Using scientific notation, express the sum of the U.S. and New Zealand *GDP* values. [*Hint*: Write each number as $a10^x$ where x is a convenient number for both and a is a numerical value, then simplify.]

A.6 Technology affects agricultural production by increasing yield over time. Let $WHEAT_t$ = average wheat production (tonnes per hectare) for the period 1950–2000 ($t = 1, \ldots, 51$) in Western Australia's Mullewa Shire.

a. Suppose production is defined by $WHEAT_t = 0.58 + 0.14\ln(t)$. Plot this curve. Find the slope and elasticity at the point $t = 49$ (1998).

b. Suppose production is defined by $WHEAT_t = 0.78 + 0.0003t^2$. Plot this curve. Find the slope and elasticity at the point $t = 49$ (1998).

A.7 Consider the function $WAGE = f(AGE) = 10 + 200AGE - 2AGE^2$.

a. Sketch the curve for values of AGE between $AGE = 20$ and $AGE = 70$.

b. Find the derivative $dWAGE/dAGE$ and evaluate it at $AGE = 30$, $AGE = 50$, and $AGE = 60$. On the curve in part (a), sketch the tangent to the curve at $AGE = 30$.

c. Find the AGE at which $WAGE$ is maximized.

d. Compute $WAGE_1 = f(29.99)$ and $WAGE_2 = f(30.01)$. Locate these values (approximately) on your sketch from part (a).

e. Evaluate $m = [f(30.01) - f(29.99)]/0.02$. Compare this value to the value of the derivative computed in (b). Explain, geometrically, why the values should be close. The value m is a "numerical derivative," which is useful for approximating derivatives.

A.8 Sketch each of the demand curves below. (i) Indicate the area under the curve between prices $P = 1$ and $P = 2$ on the sketch. (ii) Using integration, calculate the area under the curve between prices $P = 1$ and $P = 2$.

a. $Q = 15 - 5P$

b. $Q = 10P^{-1/2}$

c. $Q = 10/P$

A.9 Consider the function $f(y) = 1/100$ over the interval $0 < y < 100$ and $f(y) = 0$ otherwise.

a. Calculate the area under the curve $f(y)$ for the interval $30 < y < 50$ using a geometric argument.

b. Calculate the area under the curve $f(y)$ for the interval $30 < y < 50$ as an integral.

c. What is a general expression for the area under $f(y)$ over the interval $[a, b]$, where $0 < a < b < 100$?

d. Calculate the integral from $y = 0$ to $y = 100$ of the function $yf(y) = y/100$.

A.10 Consider the function $f(y) = 2e^{-2y}$ for $0 < y < \infty$.

a. Draw a sketch of the function.

b. Compute the integral of $f(y)$ from $y = 1$ to $y = 2$ and illustrate the value on the part (a) sketch.

A.11 Let $y_0 = 1$. For each of the values $y_1 = 1.01, 1.05, 1.10, 1.15, 1.20$, and 1.25 compute

a. The actual percentage change in y using equation (A.2).

b. The approximate percentage change in y using equation (A.3).

c. Comment on how well the approximation in equation (A.3) works as the value of y_1 increases.

A.19 Suppose your wage rate is determined by

$$WAGE = -19.68 + 2.52EDUC + 0.55EXPER - 0.007EXPER^2$$

where $EDUC$ is years of schooling and $EXPER$ is years of work experience. Using calculus, what value of $EXPER$ maximizes $WAGE$ for a person with 16 years of education? Show your work.

A.20 Suppose wages are determined by the following equation. $EDUC$ = years of education, $EXPER$ = years of work experience, and $FEMALE = 1$ if person is female, 0 otherwise.

$$WAGE = -23.06 + 2.85EDUC + 0.80EXPER - 0.008EXPER^2 - 9.21FEMALE$$
$$+ 0.34(FEMALE \times EDUC) - 0.015(EDUC \times EXPER)$$

Find $\partial WAGE/\partial EDUC$ for a female with 16 years of schooling and 10 years of experience. Show your work.