A.5 Below are the 2015 *GDP* ($US) figures provided by the World Bank for a few countries.

a. Express each in scientific notation.

i. Maldives *GDP* $3,142,812,004

ii. Nicaragua *GDP* $12,692,562,187

iii. Ecuador *GDP* $100,871,770,000

iv. New Zealand *GDP* $173,754,075,210

v. India *GDP* $2,073,542,978,208

vi. United States *GDP* $17,946,996,000,000

b. Using scientific notation divide the U.S. *GDP* by the *GDP* in (i) Maldives (ii) Ecuador.

c. The population of New Zealand in 2015 was 4.595 million. Use calculations with scientific notation to compute the per capita income in New Zealand. Express the result in scientific notation.

d. The 2015 population of St. Lucia was 184,999 and its *GDP* was $1,436,390,325. Use calculations with scientific notation to compute the per capita income in St. Lucia. Express the result in scientific notation.

e. Using scientific notation, express the sum of the U.S. and New Zealand *GDP* values. [*Hint*: Write each number as $a10^x$ where x is a convenient number for both and a is a numerical value, then simplify.]

A.6 Technology affects agricultural production by increasing yield over time. Let $WHEAT_t$ = average wheat production (tonnes per hectare) for the period 1950–2000 ($t = 1, \ldots, 51$) in Western Australia's Mullewa Shire.

a. Suppose production is defined by $WHEAT_t = 0.58 + 0.14\ln(t)$. Plot this curve. Find the slope and elasticity at the point $t = 49$ (1998).

b. Suppose production is defined by $WHEAT_t = 0.78 + 0.0003t^2$. Plot this curve. Find the slope and elasticity at the point $t = 49$ (1998).

A.7 Consider the function $WAGE = f(AGE) = 10 + 200AGE - 2AGE^2$.

a. Sketch the curve for values of AGE between $AGE = 20$ and $AGE = 70$.

b. Find the derivative $dWAGE/dAGE$ and evaluate it at $AGE = 30$, $AGE = 50$, and $AGE = 60$. On the curve in part (a), sketch the tangent to the curve at $AGE = 30$.

c. Find the AGE at which $WAGE$ is maximized.

d. Compute $WAGE_1 = f(29.99)$ and $WAGE_2 = f(30.01)$. Locate these values (approximately) on your sketch from part (a).

e. Evaluate $m = [f(30.01) - f(29.99)]/0.02$. Compare this value to the value of the derivative computed in (b). Explain, geometrically, why the values should be close. The value m is a "numerical derivative," which is useful for approximating derivatives.

A.8 Sketch each of the demand curves below. (i) Indicate the area under the curve between prices $P = 1$ and $P = 2$ on the sketch. (ii) Using integration, calculate the area under the curve between prices $P = 1$ and $P = 2$.

a. $Q = 15 - 5P$

b. $Q = 10P^{-1/2}$

c. $Q = 10/P$

A.9 Consider the function $f(y) = 1/100$ over the interval $0 < y < 100$ and $f(y) = 0$ otherwise.

a. Calculate the area under the curve $f(y)$ for the interval $30 < y < 50$ using a geometric argument.

b. Calculate the area under the curve $f(y)$ for the interval $30 < y < 50$ as an integral.

c. What is a general expression for the area under $f(y)$ over the interval $[a, b]$, where $0 < a < b < 100$?

d. Calculate the integral from $y = 0$ to $y = 100$ of the function $yf(y) = y/100$.

A.10 Consider the function $f(y) = 2e^{-2y}$ for $0 < y < \infty$.

a. Draw a sketch of the function.

b. Compute the integral of $f(y)$ from $y = 1$ to $y = 2$ and illustrate the value on the part (a) sketch.

A.11 Let $y_0 = 1$. For each o[illegible] y_1 = 1.01, 1.05, 1.10, 1.15, 1.20, and 1.25 compute [illegible] 2).

a. The [illegible] percentage change in y using equation [illegible] using equation (A.3).

b. The approximate perce[illegible] equation (A.3).

c. Comment on how well the approximation in equation (A.3) works as the value of y_1 increases.

A.19 Suppose your wage rate is determined by

$$WAGE = -19.68 + 2.52EDUC + 0.55EXPER - 0.007EXPER^2$$

where *EDUC* is years of schooling and *EXPER* is years of work experience. Using calculus, what value of *EXPER* maximizes *WAGE* for a person with 16 years of education? Show your work.

A.20 Suppose wages are determined by the following equation. *EDUC* = years of education, *EXPER* = years of work experience, and *FEMALE* = 1 if person is female, 0 otherwise.

$$WAGE = -23.06 + 2.85EDUC + 0.80EXPER - 0.008EXPER^2 - 9.21FEMALE$$
$$+ 0.34(FEMALE \times EDUC) - 0.015(EDUC \times EXPER)$$

Find $\partial WAGE/\partial EDUC$ for a female with 16 years of schooling and 10 years of experience. Show your work.

C.16 Two independent food scientists are researching the shelf-life (Y) of "Bill's Big Red" spaghetti sauce. The first collects a random sample of 25 jars and finds their average shelf life to be $\overline{Y} = 48$ months. The second researcher collects a random sample of 100 jars and finds their average shelf life to be $\overline{Y}_2 = 40$ months.

a. Find the ratio of the standard error of $\overline{Y}_1$ relative to the standard error of $\overline{Y}_2$.

b. A combined estimate can be obtained by finding the weighted average $\bar{Y} = c\overline{Y}_1 + (1-c)\overline{Y}_2$. Is there any value of c that makes this estimator of μ unbiased?

c. What value of c yields the combined estimate with the smallest standard error? Explain the intuition behind your solution, and why weighting the two means equally, with $c = 0.5$, is not the best choice.

C.17 Suppose school children are subjected to a standardized math test each spring. In the population of comparable children, the test score Y is normally distributed with mean 500 and standard deviation 100, $Y \sim N(\mu = 500, \sigma^2 = 100^2)$. It is claimed that reducing class sample size will increase test scores.

a. How can we tell if reducing class size actually does increase test scores? Would you be convinced if a sample of $N = 25$ students from the smaller classes had an average test score of 510? Calculate the probability of obtaining a sample mean of $\overline{Y} = 510$, or more, even if smaller classes actually have no effect on test performance.

b. Show that a class average of 533 will be reached by chance only 5% of the time, if the smaller class sizes have no effect. Is the following statement correct or incorrect? "We can conclude that smaller classes raise average test scores if a class of 25 students has an average test score of 533 or better, with this result being due to sampling error with probability 5%."

c. Suppose that smaller classes actually do improve the average mean population test score to 550. What is the probability of observing a class of 25 with an average score of 533 or better? If our objective is to determine whether smaller classes increase test scores, is it better for this number to be larger or smaller?

d. If smaller classes increase average test score to 550, what is the probability of having a small class average of less than or equal to 533?

e. Draw a figure showing two normal distributions, one with mean 500 and standard deviation 100, and the other with mean 550 and standard deviation 100. On the figure locate the value 533. In part (b) we showed that if the change in class size has no effect on test scores, we would still obtain a class average of 533 or more by chance 5% of the time; we would incorrectly conclude that the smaller classes helped test scores, which is a Type I error. In part (d) we derived the probability that we would obtain a class average test score of less than 533, making us unable to conclude that smaller classes help, even though smaller classes did help. This is a Type II error. If we push the threshold to the right, say 540, what happens to Type I and Type II errors? If we push the threshold to the left, say 530, what happens to the probability of Type I and Type II errors?

C.11.2 Computer Exercises

C.18 Does being in a small class help primary school students learning, and performance on achievement tests? Use the sample data file *star5_small* to explore this question.

a. Consider students in regular-sized classes, with $REGULAR = 1$. Construct a histogram of $MATHSCORE$. Carry out the Jarque–Bera test for normality at the 5% level of significance. What do you conclude about the normality of the data?

b. Calculate the sample mean, standard deviation and standard error of the mean for $MATHSCORE$ in regular-sized classes. Use the t-statistic in equation (C.16) to test the null hypothesis that the population mean (the population of students who are enrolled in regular-sized classes) μ_R is 490 versus the alternative that it isn't. Use the 5% level of significance. What is your conclusion?

c. Given the result of the normality test in (a), do you think the test in part (b) is justifiable? Explain your reasoning.

d. Construct a 95% interval estimate for the mean μ_R.

e. Repeat the test in (b) for the population of students in small classes, $SMALL = 1$. Denote the population mean for these students as μ_S. Use the 5% level of significance. What is your conclusion?

f. Let μ_R and μ_S denote the population mean test scores on the math achievement test, $MATHSCORE$. Using the appropriate test, outlined in Section C.7.2, test the null hypothesis $H_0: \mu_S - \mu_R \le 0$ against the alternative $H_1: \mu_S - \mu_R > 0$. Use the 1% level of significance. Does it appear that being in a small class increases the expected math test score, or not?

C.19 Does having a household member with an advanced degree increase household income relative to a household that includes a member having only a college degree? Use the sample data file *cex5_small* to explore this question.

a. Construct a histogram of incomes for households that include a member with an advanced degree. Construct another histogram of incomes for households that include a member with a college degree. What do you observe about the shape and location of these two histograms?

b. In the sample that includes a member with an advanced degree, what percentage of households have household incomes greater than \$10,000 per month? What is the percentage for households that include a member having a college degree?

c. Test the null hypothesis that the population mean income for households including a member with an advanced degree, μ_{ADV}, is less than, or equal to, \$9,000 per month against the alternative that it is greater than \$9,000 per month. Use the 5% level of significance.

d. Test the null hypothesis that the population mean income for households including a member with a college degree, μ_{COLL}, is less than, or equal to, \$9,000 per month against the alternative that it is greater than \$9,000 per month. Use the 5% level of significance.

e. Construct 95% interval estimates for μ_{ADV} and μ_{COLL}.

f. Test the null hypothesis $\mu_{ADV} \leq \mu_{COLL}$ against the alternative $\mu_{ADV} > \mu_{COLL}$. Use the 5% level of significance. What is your conclusion?

C.20 How much variation is there in household incomes in households including a member with an advanced degree? Use the sample data file *cex5_small* to explore this question. Let σ^2_{ADV} denote the population variance.

a. Test the null hypothesis $\sigma^2_{ADV} = 2500$ against the alternative $\sigma^2_{ADV} > 2500$. Use the 5% level of significance. Clearly state the test statistic and the rejection region. What is the **p-value** for this test?

b. Test the null hypothesis $\sigma^2_{ADV} = 2500$ against the alternative $\sigma^2_{ADV} < 2500$. Use the 5% level of significance. Clearly state the test statistic and the rejection region. What is the p-value for this test?

c. Test the null hypothesis $\sigma^2_{ADV} = 2500$ against the alternative $\sigma^2_{ADV} \neq 2500$. Use the 5% level of significance. Clearly state the test statistic and the rejection region.

C.21 School officials consider performance on a standardized math test acceptable if 40% of the population of students score at least 500 points. Use the sample data file *star5_small* to explore this topic.

a. Compute the sample proportion of students enrolled in regular-sized classes who score 500 points or more. Calculate a 95% interval estimate of the population proportion. Based on this interval can we reject the null hypothesis that the population proportion of students in regular-sized classes who score 500 points or better is $p = 0.4$?

b. Test the null hypothesis that the population proportion p of students in a regular-sized class who score 500 points or more is less than or equal to 0.4 against the alternative that the true proportion is greater than 0.4. Use the 5% level of significance.

c. Test the null hypothesis that the population proportion p of students in a regular-sized class who score 500 points or more is equal to 0.4 against the alternative that the true proportion is less than 0.4. Use the 5% level of significance.

d. Repeat parts (a)–(c) for students in small classes.

C.22 Consider two populations of Chinese chemical firms: those who export their products and those who do not. Let us consider the sales revenue for these two types of firms. Use the data file *chemical_small* for this exercise. It contains data on 1200 firms in 2006.

a. The variable *LSALES* is ln(*SALES*). Construct a histogram for this variable and test whether the data are normally distributed using the Jarque–Bera test with 10% level of significance.

b. [illegible] *SALES* = exp(*LSALES*). Construct a histogram for this variable and test whether the data are [illegible]ue–Bera test with 10% level of significance.

c. [illegible] of firms: those who export (*EXPORT* = 1) and those who do not (*EXPORT* = 0). Let μ_1 [illegible] mean of *LSALES* for firms that export, and let μ_0 be the population mean of *LSALES* for firms that do not export. Estimate the difference in means $\mu_1 - \mu_0$ and interpret this value. [*Hint:* Use the properties of differences in log-variables.]

d. Test the hypothesis that the means of these two populations are equal. Use the test that assumes