$\mu_X = \mu_Y = 5$, $\sigma_X = \sigma_Y = 3$, and $\rho = 0.7$. The covariance between X and Y is $\sigma_{XY} = \rho\sigma_X\sigma_Y = 0.7 \times 3 \times 3 = 6.3$ so that $\beta = \sigma_{XY}/\sigma_X^2 = 6.3/9 = 0.7$ and $\alpha = \mu_Y - \beta\mu_X = 5 - 0.7 \times 5 = 1.5$. The conditional mean of Y given $X = 10$ is $E(Y|X = 10) = \alpha + \beta X = 1.5 + 0.7X = 1.5 + 0.7 \times 10 = 8.5$. The conditional variance is $\text{var}(Y|X = 10) = \sigma_Y^2(1 - \rho^2) = 3^2(1 - 0.7^2) = 9(0.51) = 4.59$. That is, the conditional distribution is $(Y|X = 10) \sim N(8.5, 4.59)$.

P.8 Exercises

Answers to odd-numbered exercises are on the book website www.principlesofeconometrics.com/poe5.

P.1 Let $x_1 = 17, x_2 = 1, x_3 = 0; y_1 = 5, y_2 = 2, y_3 = 8$. Calculate the following:

a. $\sum_{i=1}^{2} x_i$
b. $\sum_{t=1}^{3} x_t y_t$
c. $\bar{x} = \left(\sum_{i=1}^{3} x_i\right)/3$ [Note: $\bar{x}$ is called the arithmetic average or arithmetic mean.]
d. $\sum_{i=1}^{3}(x_i - \bar{x})$
e. $\sum_{i=1}^{3}(x_i - \bar{x})^2$
f. $\left(\sum_{i=1}^{3} x_i^2\right) - 3\bar{x}^2$
g. $\sum_{i=1}^{3}(x_i - \bar{x})(y_i - \bar{y})$ where $\bar{y} = \left(\sum_{i=1}^{3} y_i\right)/3$
h. $\left(\sum_{j=1}^{3} x_j y_j\right) - 3\bar{x}\bar{y}$

P.2 Express each of the following sums in summation notation.

a. $(x_1/y_1) + (x_2/y_2) + (x_3/y_3) + (x_4/y_4)$
b. $y_2 + y_3 + y_4$
c. $x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4$
d. $x_3y_5 + x_4y_6 + x_5y_7$
e. $(x_3/y_3^2) + (x_4/y_4^2)$
f. $(x_1 - y_1) + (x_2 - y_2) + (x_3 - y_3) + (x_4 - y_4)$

P.3 Write out each of the following sums and compute where possible.

a. $\sum_{i=1}^{3}(a - bx_i)$
b. $\sum_{t=1}^{4} t^2$
c. $\sum_{x=0}^{2}(2x^2 + 3x + 1)$
d. $\sum_{x=2}^{4} f(x + 3)$
e. $\sum_{x=1}^{3} f(x, y)$
f. $\sum_{x=3}^{4}\sum_{y=1}^{2}(x + 2y)$

P.4 Show algebraically that

a. $\sum_{i=1}^{n}(x_i - \bar{x})^2 = \left(\sum_{i=1}^{n} x_i^2\right) - n\bar{x}^2$
b. $\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \left(\sum_{i=1}^{n} x_i y_i\right) - n\bar{x}\bar{y}$
c. $\sum_{j=1}^{n}(x_j - \bar{x}) = 0$

P.5 Let *SALES* denote the monthly sales at a bookstore. Assume *SALES* are normally distributed with a mean of \$50,000 and a standard deviation of \$6000.

a. Compute the probability that the firm has a month with *SALES* greater than \$60,000. Show a sketch.
b. Compute the probability that the firm has a month with *SALES* between \$40,000 and \$55,000. Show a sketch.
c. Find the value of *SALES* that represents the 97th percentile of the distribution. That is, find the value $SALES_{0.97}$ such that $P(SALES > SALES_{0.97}) = 0.03$.
d. The bookstore knows their *PROFITS* are 30% of *SALES* minus fixed costs of \$12,000. Find the probability of having a month in which *PROFITS* were zero or negative. Show a sketch. [*Hint:* What is the distribution of *PROFITS*?]

P.6 A venture capital company feels that the rate of return (X) on a proposed investment is approximately normally distributed with a mean of 40% and a standard deviation of 10%.

a. Find the probability that the return X will exceed 55%.

b. The banking firm who will fund the venture sees the rate of return differently, claiming that venture capitalists are always too optimistic. They perceive that the distribution of returns is $V = 0.8X - 5\%$, where X is the rate of return expected by the venture capital company. If this is correct, find the probability that the return V will exceed 55%.

P.7 At supermarkets sales of "Chicken of the Sea" canned tuna vary from week to week. Marketing researchers have determined that there is a relationship between sales of canned tuna and the price of canned tuna. Specifically, $SALES = 50000 - 100\ PRICE$. $SALES$ is measured as the number of cans per week and $PRICE$ is measured in cents per can. Suppose $PRICE$ over the year can be considered (approximately) a normal random variable with mean $\mu = 248$ cents and standard deviation $\sigma = 10$ cents.

a. Find the expected value of $SALES$.

b. Find the variance of $SALES$.

c. Find the probability that more than 24,000 cans are sold in a week. Draw a sketch illustrating the calculation.

d. Find the $PRICE$ such that $SALES$ is at its 95th percentile value. That is, let $SALES_{0.95}$ be the 95th percentile of $SALES$. Find the value $PRICE_{0.95}$ such that $P\,(SALES > SALES_{0.95}) = 0.05$.

P.8 The Shoulder and Knee Clinic knows that their expected monthly revenue from patients depends on their level of advertising. They hire an econometric consultant who reports that their expected monthly revenue, measured in \$1000 units, is given by the following equation $E(REVENUE|ADVERT) = 100 + 20\,ADVERT$, where $ADVERT$ is advertising expenditure in \$1000 units. The econometric consultant also claims that $REVENUE$ is normally distributed with variance $\text{var}(REVENUE|ADVERT) = 900$.

a. Draw a sketch of the relationship between expected $REVENUE$ and $ADVERT$ as $ADVERT$ varies from 0 to 5.

b. Compute the probability that $REVENUE$ is greater than 110 if $ADVERT = 2$. Draw a sketch to illustrate your calculation.

c. Compute the probability that $REVENUE$ is greater than 110 if $ADVERT = 3$.

d. Find the 2.5 and 97.5 percentiles of the distribution of $REVENUE$ when $ADVERT = 2$. What is the probability that $REVENUE$ will fall in this range if $ADVERT = 2$?

e. Compute the level of $ADVERT$ required to ensure that the probability of $REVENUE$ being larger than 110 is 0.95.

P.9 Consider the U.S. population of registered voters, who may be Democrats, Republicans or independents. When surveyed about the war with ISIS, they were asked if they strongly supported war efforts, strongly opposed the war, or were neutral. Suppose that the proportion of voters in each category is given in Table P.8:

TABLE P.8 Table for Exercise P.9

		War Attitude		
		Against	Neutral	In Favor
	Republican	0.05	0.15	0.25
Political Party	Independent	0.05	0.05	0.05
	Democrat	0.35	0.05	0

a. Find the "marginal" probability distributions for war attitudes and political party affiliation.

b. What is the probability that a randomly selected person is a political independent given that they are in favor of the war?

c. Are the attitudes about war with ISIS and political party affiliation statistically independent or not? Why?

d. For the attitudes about the war assign the numerical values $AGAINST = 1$, $NEUTRAL = 2$, and $IN\ FAVOR = 3$. Call this variable WAR. Find the expected value and variance of WAR.

e. The Republican party has determined that monthly fundraising depends on the value of WAR from month to month. In particular the monthly contributions to the party are given by the relation (in millions of dollars) $CONTRIBUTIONS = 10 + 2 \times WAR$. Find the mean and standard deviation of $CONTRIBUTIONS$ using the rules of expectations and variance.

P.10 A firm wants to bid on a contract worth \$80,000. If it spends \$5000 on the proposal it has a 50–50 chance of getting the contract. If it spends \$10,000 on the proposal it has a 60% chance of winning the contract. Let X denote the net revenue from the contract when the \$5000 proposal is used and let Y denote the net revenue from the contract when the \$10,000 proposal is used.

X	$f(x)$
−5,000	0.5
75,000	0.5

y	$f(y)$
−10,000	0.4
70,000	0.6

a. If the firm bases its choice solely on expected value, how much should it spend on the proposal?
b. Compute the variance of X. [*Hint*: Using scientific notation simplifies calculations.]
c. Compute the variance of Y.
d. How might the variance of the net revenue affect which proposal the firm chooses?

P.11 Prior to presidential elections citizens of voting age are surveyed. In the population, two characteristics of voters are their registered party affiliation (republican, democrat, or independent) and for whom they voted in the previous presidential election (republican or democrat). Let us draw a citizen at random, defining these two variables.

$$PARTY = \begin{cases} -1 & \text{registered republican} \\ 0 & \text{independent or unregistered} \\ 1 & \text{registered democrat} \end{cases}$$

$$VOTE = \begin{cases} -1 & \text{voted republican in previous election} \\ 1 & \text{voted democratic in previous election} \end{cases}$$

a. Suppose that the probability of drawing a person who voted republication in the last election is 0.466, and the probability of drawing a person who is registered republican is 0.32, and the probability that a randomly selected person votes republican given that they are a registered republican is 0.97. Compute the joint probability Prob$[PARTY = -1, VOTE = -1]$. Show your work.
b. Are these random variables statistically independent? Explain.

P.12 Based on years of experience, an economics professor knows that on the first principles of economics exam of the semester 13% of students will receive an A, 22% will receive a B, 35% will receive a C, 20% will receive a D, and the remainder will earn an F. Assume a 4 point grading scale (A = 4, B = 3, C = 2, D = 1, and F = 0). Define the random variable $GRADE = 4, 3, 2, 1, 0$ to be the grade of a randomly chosen student.

a. What is the probability distribution $f(GRADE)$ for this random variable?
b. What is the expected value of $GRADE$? What is the variance of $GRADE$? Show your work.
c. The professor has 300 students in each class. Suppose that the grade of the ith student is $GRADE_i$ and that the probability distribution of grades $f(GRADE_i)$ is the same for all students. Define $CLASS\_AVG = \sum_{i=1}^{300} GRADE_i/300$. Find the expected value and variance of $CLASS\_AVG$.
d. The professor has estimated that the number of economics majors coming from the class is related to the grade on the first exam. He believes the relationship to be $MAJORS = 50 + 10CLASS\_AVG$. Find the expected value and variance of $MAJORS$. Show your work.

P.13 The LSU Tigers baseball team will play the Alabama baseball team in a weekend series of two games. Let $W = 0$, 1, or 2 equal the number of games LSU wins. Let the weekend's weather be designated as Cold or Not Cold. Let $C = 1$ if the weather is cold and $C = 0$ if the weather is not cold. The joint probability function of these two random variables is given in Table P.9, along with space for the marginal distributions.