# Frisch-Waugh-Lowell (FWL) theorem

### Background

The FWL theorem states that if you want to estimate the effect of one variable on another, controlling for other variables, you can do so by:

1. Regressing the dependent variable on the control variables and saving the residuals.

2. Regressing the independent variable of interest on the control variables and saving the residuals.

3. Regressing the residuals from step 1 on the residuals from step 2.

The coefficient of the independent variable in this 3. regression will be the same as the coefficient of the independent variable in a multiple regression that includes all the variables.

Load the data, and estimate the multiple regression model:

$$\text{sales}_\text{i} = \beta_1 + \beta_2(\text{price}_\text{i}) + \beta_3(\text{advert}_\text{i}) + u_i$$

```r
load(url("http://www.principlesofeconometrics.com/poe5/data/rdata/andy.rdata"))
lm(sales ~ price + advert, data = andy)
```

```

Call:
lm(formula = sales ~ price + advert, data = andy)

Coefficients:
(Intercept)        price       advert  
    118.914       -7.908        1.863  
```

The parameters of the estimated regression model are:

$$\hat{\text{sales}} = 118.91 - 7.91(\text{price}) + 1.86(\text{advert})$$

The interpretation of the $\beta_3$ slope parameter of `advert` is "the change in monthly `sales` ($ 1000) when advertising expenditure `advert` is increased by one unit ($ 1000), and the price index (`price`) **is held constant**."

### R Code Explanation

1. Regressing sales on price and saving residuals (`u1`):

```
u1 <- resid(lm(sales~price, data=andy))
```

Here, you're regressing `sales` on `price` using the `andy` data. The residuals from this regression represent the variation in `sales` that cannot be explained by `price`. These residuals, or sales without price effect, are saved in `u1`.

2. Regressing advert on price and saving residuals (`u2`):

```
u2 <- resid(lm(advert~price, data=andy))
```

Similarly, you're regressing `advert` on `price`. The residuals represent the variation in `advert` that cannot be explained by `price`. These residuals, or advertising without price effects, are saved in `u2`.

3. Regressing the residuals:

```
lm(u1~u2) # with intercept
```

```
Call:
lm(formula = u1 ~ u2)

Coefficients:
(Intercept)           u2
  1.335e-16    1.863e+00
```

```
lm(u1~0+u2) # without intercept
```

```
Call:
lm(formula = u1 ~ 0 + u2)

Coefficients:
   u2
1.863
```

You're regressing the residuals `u1` on `u2`. So your dependent variable is sales without price effect, and your explanatory variable is advertising without price effect. The first regression includes an intercept, while the second one does not. For practical purposes they are identical. The coefficient of `u2` in these regressions is the same as the coefficient of `advert` in a regression of `sales` on both `price` and `advert`.

4. Repeating multiple regression for comparison:

```
lm(sales~price+advert, data=andy)
```

```
Call:
lm(formula = sales ~ price + advert, data = andy)

Coefficients:
(Intercept)        price       advert
    118.914       -7.908        1.863
```

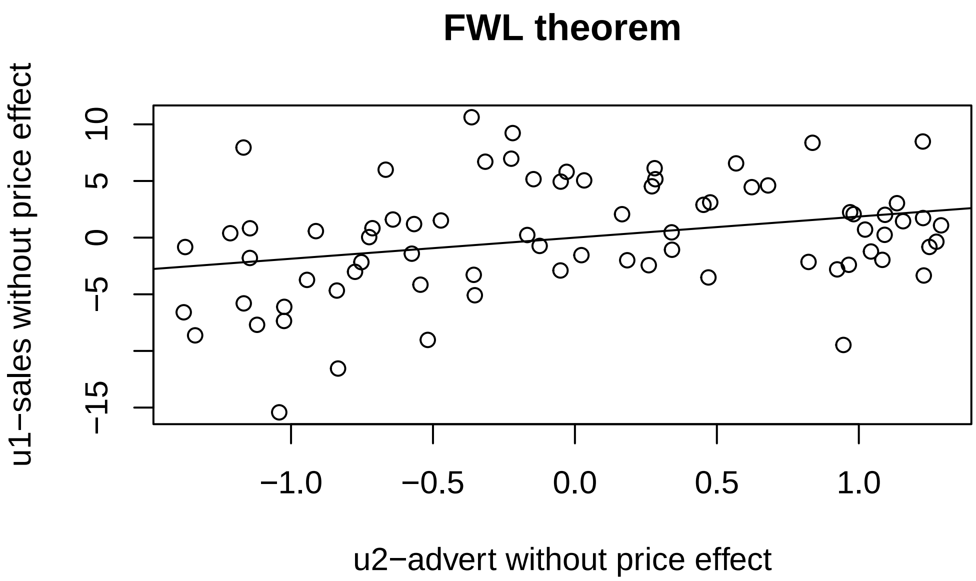
This is a multiple regression of `sales` on both `price` and `advert`. The coefficient of `advert` in this regression is the same as the coefficient of `u2` in the previous regressions, demonstrating the FWL theorem.

```
c(coef(lm(u1~u2))[2], coef(lm(u1~0+u2)), coef(lm(sales~price+advert, data=andy))[3])
```

```
      u2       u2   advert
1.862584 1.862584 1.862584
```

5. Plotting the residuals:

```
plot(u2,u1, main="FWL theorem",
     xlab="u2-advert without price effect",
     ylab="u1-sales without price effect")
abline(lm(u1~0+u2))
```




This code plots the residuals `u1` against `u2` and adds a regression line (without an intercept) to the plot. This visually demonstrates the relationship (slope) between the residuals as the slope ($\beta_3$) parameter of `advert` as "the change in monthly `sales` ($ 1000) when advertising expenditure `advert` is increased by one unit ($ 1000), and the price index (`price`) **is held constant**."