Own price elasticity from a sales model

Estimate

$$s = \beta_1 + \beta_2 \cdot p$$

where p is price, q is quantity, and $s = p \cdot q$.

So

$$p \cdot q = \beta_1 + \beta_2 \cdot p$$

Total differentiate

$$\begin{split} d(p\cdot q) &= \beta_2 \cdot dp \\ dp\cdot q + dq\cdot p &= \beta_2 \cdot dp \\ dq\cdot p &= \beta_2 \cdot dp - dp\cdot q \\ dq\cdot p &= dp \left(\beta_2 - q\right) \\ \frac{dq\cdot p}{dq} &= \left(\beta_2 - q\right) \end{split}$$

So

$$\begin{split} \frac{dq \cdot p}{dp \cdot q} &= \frac{(\beta_2 - q)}{q} \\ E &= \frac{\beta_2}{q} - 1 \end{split}$$

This derives the own price elasticity E from a sales model.