Own price elasticity from a sales model

Estimate

$$s = \beta_1 + \beta_2 \cdot p$$

where p is price, q is quantity, and sales s is: $s = p \cdot q$.

So

$$p \cdot q = \beta_1 + \beta_2 \cdot p$$

Total differentiate

$$d(p \cdot q) = \beta_2 \cdot dp$$
$$dp \cdot q + dq \cdot p = \beta_2 \cdot dp$$
$$dq \cdot p = \beta_2 \cdot dp - dp \cdot q$$
$$dq \cdot p = dp\,(\beta_2 - q)$$
$$\frac{dq \cdot p}{dq} = (\beta_2 - q)$$

So

$$\frac{dq \cdot p}{dp \cdot q} = \frac{(\beta_2 - q)}{q}$$
$$E = \frac{\beta_2}{q} - 1$$

This derives the own price elasticity E from a sales model.