Rabin-Karp Algorithm CS 491 – Competitive Programming

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Objectives

- Explain how a rolling hash works
- ► Use a rolling hash to find a pattern in a string quickly



Näive String Matching

Introduction and Objectives

Consider this code

```
int find(string s, string desire) {
     int found = -1;
     for(int i=0; i<s.length() - desire.length(); ++i) {</pre>
       found = 0:
4
       for(int j=0; j<desire.length(); ++j)</pre>
          if (s[i] != s[j])
6
            break:
          else ++found:
        if (found == desire.length())
9
10
          return i:
     } // end for i
11
     return -1; // not fouund
12
13
```

What is the time complexity?

Hash Functions

- Remember hash functions!
 - \blacktriangleright h(s) should be fast to compute
 - $h(s_1) = h(s_2)$ probably means $s_1 = s_2$
 - $h(s_1) \neq h(s_2)$ definitely means $s_1 \neq s_2$
- Can this help us with string matching?

Hashing

Consider this code

```
int find(string s, string desire) {
14
     int found = -1;
15
     for(int i=0; i<s.length() - desire.length(); ++i) {</pre>
16
        if (h(desire) == h(s.substr(i,desire.length())) &&
17
            desire = s.substr(i,desire.length()))
18
          return i
19
     \} // end for i
20
     return -1; // not fouund
21
   }
22
```

▶ How about now?

Rolling Hashes

Consider this hash function:

$$h(c_0 \cdots c_{n-1}) = c_0 a^n + c_1 a^{n-1} + c_2 a^{n-2} + \cdots + c_{n-1} \text{ modulo } b$$

- a is a constant (256 is reasonable)
- b is a large prime number (let's use 100007)
- $ightharpoonup c_i$ is the *i*th character in a string.
- Try it yourself!
 - ► Compute the hash for abc
 - Compute the hash for bci
- ► Hint: ASCII for a is 95, i is 103

Rolling Hashes, ctd

Introduction and Objectives

Consider this hash function:

$$h(c_0 \cdots c_{n-1}) = c_0 a^n + c_1 a^{n-1} + c_2 a^{n-2} + \cdots + c_{n-1}$$
 modulo b

- $h("abc") = 95 \times 256^2 + 96 \times 256 + 97 \mod 100007 = 50159$
- $h("bci") = 96 \times 256^2 + 97 \times 256 + 103 \mod 100007 = 15950$
- Can you convert from one to the other quickly?
 - Add 100007 to prevent "going negative"
 - ightharpoonup "Subtract off" a by subtracting $(95 \times 256^2 \mod 100007)$
 - Multiply the remainder by 256 and modulo 100007.
 - Add 103
 - Take the modulus again.
- So: $h("bci") = ((h("abc") + 100007 + (95 \times 100007)))$ $256^2 \mod 100007$) $\times 256 + 103$) $\mod 100007$

Hashing

Setting up

Introduction and Objectives

```
int a = 256;
23
   int b = 100007;
24
25
   int pow = 1;
26
   for(i=0; i<desire.length(); ++i)</pre>
27
      pow = (pow * a) \% b;
28
29
   int hashs = 0;
30
   int hashd = 0;
31
32
   for(i=0; i<desire.length(); ++i) { % assume desire < s</pre>
33
        hashs = hashs * a + s[i];
34
        hashd = hashd * a + desire[i];
35
   }
36
```

Matching Part

Introduction and Objectives

```
while (i < s.length() - desire.length()) {</pre>
37
      if (hashs == hashsd &&
38
          desire = s.substr(i,desire.length()))
39
        return i:
40
     hashs = (b + hashs) - (s[i] * pow % b)
41
        + s[i+desire.length()];
42
     hashs = hashs * a % b;
43
   }
44
   return -1; // if failed.
45
```