Online Approximate Query Processing with Pilot Sampling and Block Sampling (Technical Report)

AUTHOR

Symbol	Meaning
G	number of groups in a ground-truth query result
$\hat{G}$	number of groups in an approximate query result
C	number of aggregate columns
R	table
P_i	i-th page in the table
Q_i	number of rows in P_i
S_i	sum of values in P_i
$r_{i,j}$	j-th row in the i-th page
e	desired relative error of each aggregate
p	desired failure probability
$a_{i,j}$	ground-truth aggregate at group i and aggregate column j
$\hat{a}_{i,j}$	estimated aggregate at group i and aggregate column j
$p^{(g)}$	probability of $\hat{G} < G$
$p^{(a)}_{i,j}$	probability of the error of $\hat{a}_{i,j}$ being larger than e

Table 1. Notations

1 SUPPORTED QUERY AND ERRORS

In this section, we first introduce the scope of queries we support. Then, we introduce the error semantics we accept for approximate queries. Finally, we set up the analysis framework for the error.

Query Scope. We consider aggregation queries with optional `WHERE`, `GROUP BY`, `HAVING`, `ORDER BY`, and `LIMIT` clauses (Fig. 1a). We support `COUNT`, `SUM`, and `AVG` aggregates, and their arithmetic combinations, including additions, multiplications, and divisions. We do not support `COUNT DISTINCT`, `MAX`, and `MIN`. In the `FROM` clause, users may specify a single table or multiple tables merged with `JOIN` and/or `UNION ALL` operations. In Figure 1b, we show an example query from the TPC-H benchmark that we supported.

Error Semantics. Users may specify required error rates as the last three lines of our query syntax (Fig. 1a). We support two types of errors: *Group Error* and *Aggregate Error*. The group error happens if the approximate query with a group-by clause misses a group whose size is larger than a user-specified value g. We include the group error since random sampling inevitably misses groups [12] and stratified sampling is prohibitively expensive for online query processing. Following prior work, we support the aggregate error, which happens if there exists an estimated aggregate from a group of size larger than g whose relative error to the ground-truth aggregate is larger than a user-specified rate e. We define the *Failure Probability* as the probability of either type of error happening. Finally, we provide a probabilistic guarantee that the failure probability is fewer than a user-specified rate p.

```
SELECT AGG(ac, [, ac1, ...]), ...
FROM T
    [JOIN T1, ...]
    [UNION ALL T1, ...]
[WHERE WP]
[GROUP BY gc [, gc1, ...]]
[ORDER BY gc [, gc1, ...] [LIMIT k]]
ERROR < e
GROUPSIZE > g {ROWS | PAGES}
FAILURE < p
```

(a) Abstract query syntax.

```
SELECT 100.00 *
       SUM(CASE WHEN p_type like 'PROMO%'
           THEN l_extendedprice * (1 - l_discount)
           ELSE 0 END) /
       SUM(l_extendedprice * (1 - l_discount))
FROM lineitem JOIN part
WHERE
    l_partkey = p_partkey AND ...
ERROR WITHIN e
FAILURE WITHIN p
```

(b) An example query with TPC-H query 14.

Fig. 1. Query syntax with an example query.

Error Analysis Setup. Next, we formally describe the group error and the aggregate error. Let G, G' be the groups from the input table used in the exact query and the approximate query, respectively, C be the number of aggregate columns, $A_{G_i,j}, \hat{A}_{G_i,j}$ be the exact aggregate and approximate aggregate at group G_i and column j. The group error happens if

$$\{G_i | G_i \in G, |G_i| > g\} \neq \{G_i | G_i \in G', |G_i| > g\}$$

The aggregate error happens if

$$\exists_{G_i \in G', |G_i| > g}, \quad \exists_{j \in [1,C]}, \quad \left| \frac{A_{G_i,j} - \hat{A}_{G_i,j}}{A_{G_i,j}} \right| > e$$

We guarantee that

$$\mathbb{P}\left[\left(\{G_i | G_i \in G, |G_i| > g\} \neq \{G_i | G_i \in G', |G_i| > g\} \right) \bigcup \left(\exists_{G_i \in G', |G_i| > g}, \ \exists_{j \in [1,C]}, \ \left| \frac{A_{G_i,j} - \hat{A}_{G_i,j}}{A_{G_i,j}} \right| > e \right) \right] \leq p$$

Due to the possible correlation between the aggregate error and group error, it is difficult to analyze the total failure probability directly. Therefore, we introduce Lemma 1.1 to decompose the failure probability into the probabilities of the group error and individual aggregate error. Lemma 1.1 shows that, to ensure the total failure probability, it is sufficient to guarantee the sum of probabilities of the group error and individual aggregate error. Therefore, it is sufficient to analyze the group error and individual aggregate error separately and ensure the summation of probability is less than p. In the rest of this report, we will introduce the theories to achieve the guarantee and the sampling design to efficiently process queries.

Lemma 1.1. *Let $p^{(g)}$ be the probability of group error and $p_{i,j}^{(a)}$ be the probability of the estimated aggregate $\hat{A}_{G_i,j}$ having a relative error larger than e.*

$$p^{(g)} := \mathbb{P}\big[\{G_i | G_i \in G, |G_i| > g\} \neq \{G_i | G_i \in G', |G_i| > g\} \big] \tag{1}$$

$$p_{i,j}^{(a)} := \mathbb{P}\left[\left| \frac{A_{G_i,j} - \hat{A}_{G_i,j}}{A_{G_i,j}} \right| > e \right] \tag{2}$$

The overall probability of the group error or aggregate error occurring is upper bounded by the sum of $p^{(g)}$ and all $p^{(a)}_{i,j}$. Namely,

$$\mathbb{P}\left[\text{Group Error} \bigvee \text{Aggregate Error}\right] \leq \sum_{\substack{G_i \in G' \\ |G_i|>g}} \sum_{j=1}^{C} p^{(a)}_{i.j} + p^{(g)}$$

Proof. We first rewrite the probability of Aggregate Error or Group Error with a union of events.

$$\begin{aligned}
&\mathbb{P}\left[\text{Group Error} \bigvee \text{Aggregate Error}\right] \qquad (3)\\
&= \mathbb{P}\left[\left(\{G_i | G_i \in G, |G_i| > g\} \neq \{G_i | G_i \in G', |G_i| > g\}\right) \bigcup \left(\exists_{G_i \in G', |G_i|>g},\ \exists_{j \in [1,C]},\ \left|\frac{A_{G_i,j} - \hat{A}_{G_i,j}}{A_{G_i,j}}\right| > e\right)\right]\\
&= \mathbb{P}\left[\left(\{G_i | G_i \in G, |G_i| > g\} \neq \{G_i | G_i \in G', |G_i| > g\}\right) \bigcup \left(\bigcup_{\substack{G_i \in G', |G_i|>g \\ j \in [1,C]}} \left(\left|\frac{A_{G_i,j} - \hat{A}_{G_i,j}}{A_{G_i,j}}\right| > e\right)\right)\right]
\end{aligned}$$

Next, we apply Boole's inequality to rewrite the probability of unions of events with the sum of probabilities of events.

$$\begin{aligned}
&\mathbb{P}\left[\text{Group Error} \bigvee \text{Aggregate Error}\right] \qquad (4)\\
&\leq \mathbb{P}\left[\left(\{G_i | G_i \in G, |G_i| > g\} \neq \{G_i | G_i \in G', |G_i| > g\}\right)\right] + \sum_{\substack{G_i \in G' \\ |G_i|>g}} \sum_{j=1}^{C} \mathbb{P}\left[\left|\frac{a_{i,j} - \hat{a}_{i,j}}{a_{i,j}}\right| > e\right]\\
&= p^{(g)} + \sum_{\substack{G_i \in G' \\ |G_i|>g}} \sum_{j=1}^{C} p^{(a)}_{i.j}
\end{aligned}$$

□

2 PRELIMINARIES OF SIMPLE RANDOM SAMPLING

In this section, we introduce the preliminaries of simple random sampling that results in independently and identically distributed (i.i.d.) samples. We present two lemmas that derive the concentration inequalities for the sample mean and standard deviation of an i.i.d. sample and the number of successes in a sequence of independent Bernoulli experiments.

Lemma 2.1. *Let $X_1, \ldots, X_n$ be an i.i.d. random sample from a distribution with the expected value given by μ and finite variance given by σ^2. Suppose $\bar{X}$ is the sample mean (Eq. 40) and $\hat{\sigma}^2$ is the unbiased sample variance (Eq. 41).*

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \qquad (5)$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} \left(X_i - \bar{X}\right)}{n-1} \qquad (6)$$

We have the following probabilistic bounds for μ and σ.

$$\lim_{n\to\infty} \mathbb{P}\left[L_\mu^{(t)}(\bar{X}, \hat{\sigma}, n, \delta/2) \le \mu \le U_\mu^{(t)}(\bar{X}, \hat{\sigma}, n, \delta/2)\right] = 1 - \delta$$
$$\lim_{n\to\infty} \mathbb{P}\left[L_\sigma(\hat{\sigma}, n, \delta/2) \le \sigma \le U_\sigma(\hat{\sigma}, n, \delta/2)\right] = 1 - \delta$$

where

$$L_\mu^{(t)}(\bar{X}, \hat{\sigma}, n, \delta) = \bar{X} - t_{n-1,1-\delta}\frac{\hat{\sigma}}{\sqrt{n}}, \quad U_\mu^{(t)}(\bar{X}, \hat{\sigma}, n, \delta) = \bar{X} + t_{n-1,1-\delta}\frac{\hat{\sigma}}{\sqrt{n}} \tag{7}$$

$$L_\sigma(\hat{\sigma}, n, \delta) = \sqrt{\frac{n-1}{\chi^2_{n-1,1-\delta}}}\hat{\sigma}, \quad U_\sigma(\hat{\sigma}, n, \delta) = \sqrt{\frac{n-1}{\chi^2_{n-1,\delta}}}\hat{\sigma} \tag{8}$$

$t_{n-1,1-\delta/2}$ is the upper $\delta/2$ critical point of student's t distribution with $n-1$ degrees of freedom. Namely, $\delta/2 = \mathbb{P}\left[T > t_{n-1,1-\delta/2}\right]$, where T follows student's t distribution with $n-1$ degrees of freedom. $\chi^2_{n-1,\delta/2}$ is the lower $\delta/2$ critical point of chi-squared distribution with $n-1$ degrees of.

If we know the variance σ^2, we can have the following probabilistic bound for μ.

$$\lim_{n\to\infty} \mathbb{P}\left[L_\mu^{(z)}(\bar{X}, \sigma, n, \delta/2) \le \mu \le U_\mu^{(z)}(\bar{X}, \sigma, n, \delta/2)\right] = 1 - \delta$$

where

$$L_\mu^{(z)}(\bar{X}, \sigma, n, \delta) = \bar{X} - z_{1-\delta}\frac{\sigma}{\sqrt{n}}, \quad U_\mu^{(z)}(\bar{X}, \sigma, n, \delta) = \bar{X} + z_{1-\delta}\frac{\sigma}{\sqrt{n}}, \tag{9}$$

and $z_{1-\delta/2}$ is the upper $\delta/2$ critical point of standard normal distribution.

Lemma 2.2. *Suppose we have a sequence of N independent experiments. Each experiment is either successful or not successful with a fixed success probability θ. Suppose $\hat{n}$ is the number of successes. We have the following probabilistic bounds for $\hat{n}$.*

$$\lim_{N\to\infty} \mathbb{P}\left[N\theta - z_{1-\delta/2}\sqrt{N\theta(1-\theta)} \le \hat{n} \le N\theta + z_{1-\delta/2}\sqrt{N\theta(1-\theta)}\right] = 1 - \delta \tag{10}$$

or equivalently

$$\lim_{N\to\infty} \mathbb{P}\left[N - z_{1-\delta/2}\sqrt{\frac{N(1-\theta)}{\theta}} \le \frac{\hat{n}}{\theta} \le N + z_{1-\delta/2}\sqrt{\frac{N(1-\theta)}{\theta}}\right] = 1 - \delta \tag{11}$$

where $z_{1-\delta/2}$ is the upper $\delta/2$ critical point of standard normal distribution.

Lemma 2.1 defines the confidence interval of the expected value and variance with an i.i.d. sample. It follows from the Central Limit Theorem [9]. Lemma 5.3 defines the confidence interval of the number of successes in a binomial distribution. It follows the Central Limit Theorem with a standard normal approximation [9].

Based on probabilistic bounds derived in Lemma 2.1 and Lemma 5.3, we observe that the error of using an i.i.d. sample to estimate mean (Eq. 42 and 44) or total number of Bernoulli experiments (Eq. 46) monotonically decreases at the order of $-1/2$ with respect to the sample size n or sample rate θ. Therefore, the key to guaranteeing errors is to estimate a sufficient sample size or sample rate to ensure the error is lower than the user's specification, which will be introduced in the next section.

3 PREDICTING AGGREGATE ERRORS WITH PILOT SAMPLING

In this section, we first introduce the idea of using pilot sampling to estimate the sample rate for a given error requirement. Next, we introduce the theorems that define the conditions for the sample rate to ensure a given relative error for mean and cardinality estimations with a given failure probability.

Principle of Tail Bounds. We use the example of the relative error of mean estimation to demonstrate the idea of using the bounds derived from a pilot sample to estimate the unknown parameters in the error analysis. Based on the probabilistic bounds derived in Lemma 2.1 (Eq. 44), we have the following relative error of using the sample mean $\bar{Y}$ to estimate the population mean μ.

$$\left|\frac{\bar{Y}-\mu}{\mu}\right| \leq \frac{z_{1-\delta/2}}{\sqrt{n}} \cdot \underbrace{\frac{\sigma}{|\mu|}}_{unkown}$$

We observe that the relative error is inpredictable since the population parameters σ and μ are unknown. However, Lemma 2.1 shows that we can estimate those population parameters with a probabilistically bounded error that decreases as the sample size increases. Therefore, we draw a large enough sample to calculate the probabilistic bounds of desired parameters, which is *Pilot Sampling*. Then, we can calculate the relative error with the upper or lower bounds of population parameters. Since the bounds are probabilistic, we need to allocate failure probabilities at this step correspondingly (Eq. 12).

$$\left|\frac{\bar{Y}-\mu}{\mu}\right| \leq \frac{z_{1-\delta/2}}{\sqrt{n}} \cdot \underbrace{\frac{U_\sigma(\hat{\sigma}, n_1, \delta_1)}{L_\mu^{(t)}(\bar{X}, \hat{\sigma}, n_1, \delta_2)}}_{estimate\ with\ bounds} \quad \Rightarrow \quad \mathbb{P}\left[\left|\frac{\bar{Y}-\mu}{\mu}\right| \leq \frac{z_{1-\delta/2}}{\sqrt{n}} \cdot \frac{\sigma}{\mu}\right] \geq 1 - \underbrace{(\delta_1+\delta_2)}_{bounds'failure} \tag{12}$$

Via pilot sampling, we can conveniently analyze the relative errors to estimate the sample rate. In the rest of this section, we introduce two concrete applications of pilot sampling that analyze the relative error of mean and cardinality, which are necessary and sufficient to analyze `COUNT`, `SUM`, and `AVG`.

3.1 Sample Rate Estimation for the Mean Estimation

The estimation of the mean of a column is necessary to estimate `SUM` and `AVG`. Given a sampling design that outputs i.i.d. samples of a population with an unknown distribution, we can estimate the population mean with the sample mean. Theorem 5.4 introduces the estimation of sample size for a given relative error using the statistics calculated with the pilot sample.

Theorem 3.1. *Let $X_1, \ldots, X_{n_1}$ be an i.i.d. random sample (i.e., pilot sample) from a distribution with the positive expected value given by μ and finite variance given by σ^2, $\bar{X}$ be the sample mean, $\hat{\sigma}^2$ be the unbiased sample variance, and $z_{1-\delta/2}$ be the upper $\delta/2$ critical point of standard normal distribution. Suppose $Y_1, \ldots, Y_n$ is another i.i.d. sample. To ensure the relative error between the sample mean $\bar{Y}$ and μ being less than e with a maximum failure probability of p, i.e.,*

$$\mathbb{P}\left[\left|\frac{\bar{Y}-\mu}{\mu}\right| \geq e\right] \leq p,$$

it is sufficient to ensure

$$n \geq \left(\frac{z_{1-\delta_3/2}}{e} \frac{U_\sigma(\hat{\sigma}, n_1, \delta_1)}{L_\mu^{(t)}(\bar{X}, \hat{\sigma}, n_1, \delta_2)}\right)^2, \\ \delta_1+\delta_2+\delta_3 \leq p$$

where δ_1 is the failure probability of $U_\sigma(\hat{\sigma}, n_1, \delta_1)$ being the upper bound of σ, δ_2 is the failure probability of $L_\mu^{(t)}$ being the lower bound of μ, and δ_3 is the failure probability of the confidence interval of mean derived in Lemma 2.1.

Proof. We apply the one-sided version of the probabilistic bounds derived in Lemma 2.1 on the pilot sample result, resulting in

$$\mathbb{P}\left[\sigma \le U_\sigma(\hat{\sigma}, n_1, \delta_1)\right] = 1 - \delta_1 \tag{13}$$

$$\mathbb{P}\left[\mu \ge L_\mu^{(t)}(\bar{X}, \hat{\sigma}, n_1, \delta_2)\right] = 1 - \delta_2 \tag{14}$$

Next, we apply the probabilistic bounds derived in Lemma 2.1 on the final sample result, resulting in

$$\begin{aligned}
&\mathbb{P}\left[\bar{Y} - z_{1-\delta_3/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{Y} + z_{1-\delta_3/2}\frac{\sigma}{\sqrt{n}}\right] = 1 - \delta_3 \\
\Leftrightarrow\ &\mathbb{P}\left[-z_{1-\delta_3/2}\frac{\sigma}{\sqrt{n}} \le \bar{Y} - \mu \le z_{1-\delta_3/2}\frac{\sigma}{\sqrt{n}}\right] = 1 - \delta_3 \\
\Leftrightarrow\ &\mathbb{P}\left[\left|\bar{Y} - \mu\right| \le z_{1-\delta_3/2}\frac{\sigma}{\sqrt{n}}\right] = 1 - \delta_3
\end{aligned} \tag{15}$$

Based on Equation 15, we then derive the probabilistic bounds for the relative error between the sample mean and population mean.

$$\mathbb{P}\left[\left|\frac{\bar{Y} - \mu}{\mu}\right| \le \frac{z_{1-\delta_3/2}}{\sqrt{n}}\frac{\sigma}{\mu}\right] \ge 1 - \delta_3 \tag{16}$$

Suppose the sample size of the final sample satisfies

$$n \ge \left(\frac{z_{1-\delta_3/2}}{e}\frac{U_\sigma(\hat{\sigma}, n_1, \delta_1)}{L_\mu^{(t)}(\bar{X}, \hat{\sigma}, n_1, \delta_2)}\right)^2 \tag{17}$$

Then, we have

$$\sigma \le U_\sigma(\hat{\sigma}, n_1, \delta_1) \quad \text{AND} \quad \mu \ge L_\mu^{(t)}(\bar{X}, \hat{\sigma}, n_1, \delta_2) \quad \text{AND} \quad \left|\frac{\bar{Y} - \mu}{\mu}\right| \le \frac{z_{1-\delta_3/2}}{\sqrt{n}}\frac{\sigma}{\mu} \tag{18}$$

$$\Rightarrow \left|\frac{\bar{Y} - \mu}{\mu}\right| \le z_{1-\delta_3/2}\left(\frac{z_{1-\delta_3/2}}{e}\frac{U_\sigma(\hat{\sigma}, n_1, \delta_1)}{L_\mu^{(t)}(\bar{X}, \hat{\sigma}, n_1, \delta_2)}\right)^{-1}\frac{U_\sigma(\hat{\sigma}, n_1, \delta_1)}{L_\mu^{(t)}(\bar{X}, \hat{\sigma}, n_1, \delta_2)} = e$$

Namely, given a sample size satisfying Equation 17, if the relative error between $\bar{Y}$ and μ is larger than e, then one of the inequalities in 18 does not hold. Given the failure probability of inequalities in Equation 13, 14, and 16, we can derive the upper bound of the failure probability of the relative error

$$\begin{aligned}
&\mathbb{P}\left[\left|\frac{\bar{Y} - \mu}{\mu}\right| \ge e\right] \le \mathbb{P}\left[\left(\sigma \ge U_\sigma(\hat{\sigma}, n_1, \delta_1)\right) \vee \left(\mu \le L_\mu^{(t)}(\bar{X}, \hat{\sigma}, n_1, \delta_2)\right) \vee \left(\left|\frac{\bar{Y} - \mu}{\mu}\right| \ge \frac{z_{1-\delta_3/2}}{\sqrt{n}}\frac{\sigma}{\mu}\right)\right] \\
&\le \mathbb{P}\left[\sigma \ge U_\sigma(\hat{\sigma}, n_1, \delta_1)\right] + \mathbb{P}\left[\mu \le L_\mu^{(t)}(\bar{X}, \hat{\sigma}, n_1, \delta_2)\right] + \mathbb{P}\left[\left|\frac{\bar{Y} - \mu}{\mu}\right| \ge \frac{z_{1-\delta_3/2}}{\sqrt{n}}\frac{\sigma}{\mu}\right] \\
&= \delta_1 + \delta_2 + \delta_3
\end{aligned}$$

If we configure $\delta_1 + \delta_2 + \delta_3 \le p$, we have

$$\mathbb{P}\left[\left|\frac{\bar{Y} - \mu}{\mu}\right| \ge e\right] < p$$

□

Theorem 5.4 calculates the size of an i.i.d. sample for a required relative error and failure probability. To obtain an i.i.d. sample, we use Bernoulli sampling, which can be easily expressed in SQL [12] and implemented by many database management systems (DBMSs) [5–7, 11, 13]. In Bernoulli sampling, we conduct a Bernoulli trial on each sampling unit (i.e., a row or a page) independently, given a success probability for each trial (i.e., sample rate). Therefore, Bernoulli sampling does not guarantee the sample size ahead of time. We introduce Theorem 5.5 to estimate the sample rate for a required sample size in Bernoulli sampling.

Theorem 3.2. *Let n_1 be the number of successes in a sequence of N Bernoulli trials (i.e., pilot sample) with a success probability θ_1, where N is unknown. Suppose we conduct another sequence of N trials with a different success probability θ and result in $\hat{n}$ successes. To ensure that $\hat{n}$ is at least n with probability larger than $1-p$, i.e.,*

$$\mathbb{P}\left[\hat{n} \le n\right] \le p,$$

it is sufficient to ensure

$$\theta \ge \left(\frac{z_{1-\delta_2} + \sqrt{z_{1-\delta_2}^2 + 4n}}{2\sqrt{L_N(n_1, \theta_1, \delta_1)}}\right)^2$$
$$\delta_1 + \delta_2 \le p$$

where

$$L_N(n, \theta, \delta) = \left(\sqrt{\frac{n}{\theta} + z_{1-\delta}^2 \frac{1-\theta}{2\theta}} - \sqrt{z_{1-\delta}^2 \frac{1-\theta}{2\theta}}\right)^2$$

δ_1 and δ_2 are the failure probabilities of the confidence interval of the number of successes for the pilot sample and the final sample, respectively.

Proof. We apply the one-sided version of Lemma 5.3 on the first sequence of Bernoulli trials, resulting in

$$\begin{aligned}&\mathbb{P}\left[n_1 \le N\theta_1 + z_{1-\delta_1}\sqrt{N\theta_1(1-\theta_1)}\right] = 1-\delta_1\\ \Leftrightarrow &\mathbb{P}\left[\sqrt{N} \ge \sqrt{\frac{n_1}{\theta_1} + z_{1-\delta_1}^2 \frac{1-\theta_1}{2\theta_1}} - \sqrt{z_{1-\delta_1}^2 \frac{1-\theta_1}{2\theta_1}}\right] = 1-\delta_1\end{aligned} \tag{19}$$

Namely, we obtain a lower bound for the total number of Bernoulli trials based on a pilot sample. We denote the lower bound as

$$L_N(n, \theta, \delta) = \left(\sqrt{\frac{n}{\theta} + z_{1-\delta}^2 \frac{1-\theta}{2\theta}} - \sqrt{z_{1-\delta}^2 \frac{1-\theta}{2\theta}}\right)^2$$

Next, we apply the one-sided version of Lemma 5.3 on the second sequence of Bernoulli trials, resulting in

$$\mathbb{P}\left[\hat{n} \ge N\theta - z_{1-\delta_2}\sqrt{N\theta(1-\theta)}\right] = 1-\delta_2 \tag{20}$$

Furthermore, We observe that

$$\begin{aligned}\sqrt{\theta} \ge \frac{z_{1-\delta_2} + \sqrt{z_{1-\delta_2}^2 + 4n}}{2\sqrt{N}} \quad &\Rightarrow \quad N\theta - z_{1-\delta_2}\sqrt{N\theta} \ge n\\ &\Rightarrow \quad N\theta - z_{1-\delta_2}\sqrt{N\theta(1-\theta)} \ge n\end{aligned} \tag{21}$$

Based on inequalities 19, 20, and 21, we can have the following conclusion. Suppose the success probability θ satisfies

$$\theta \geq \left(\frac{z_{1-\delta_2} + \sqrt{z_{1-\delta_2}^2 + 4n}}{2\sqrt{L_N(n_1, \theta_1, \delta_1)}} \right)^2 \tag{22}$$

Then we have

$$\begin{aligned} &\sqrt{N} \geq L_N(n_1, \theta_1, \delta_1) \quad \texttt{AND} \quad \hat{n} \geq N\theta - z_{1-\delta_2}\sqrt{N\theta(1-\theta)} \qquad (23) \\ \Rightarrow \quad &\hat{n} \geq N\theta - z_{1-\delta_2}\sqrt{N\theta(1-\theta)} \geq N\theta - z_{1-\delta_2}\sqrt{N\theta} \geq n \end{aligned}$$

Namely, given a success probability satisfying 22, if the number of successes is less than n, then one of the inequalities in 23 does not hold. Given the failure probability of those inequalities in 19 and 20, we can derive the upper bound of the failure probability of the sample size

$$\begin{aligned} \mathbb{P}\left[\hat{n} \leq n\right] &\leq \mathbb{P}\left[\left(\sqrt{N} \leq L_N(n_1, \theta_1, \delta_1)\right) \vee \left(\hat{n} \leq N\theta - z_{1-\delta_2}\sqrt{N\theta(1-\theta)}\right)\right] \qquad (24) \\ &\leq \mathbb{P}\left[\left(\sqrt{N} \leq L_N(n_1, \theta_1, \delta_1)\right)\right] + \mathbb{P}\left[\left(\hat{n} \leq N\theta - z_{1-\delta_2}\sqrt{N\theta(1-\theta)}\right)\right] \qquad (25) \\ &= \delta_1 + \delta_2 \end{aligned}$$

If we configure $\delta_1 + \delta_2 \leq p$, we have

$$\mathbb{P}\left[\hat{n} \leq n\right] \leq p$$

□

Combining the results of Theorem 5.4 and 5.5, we can use the statistics calculated with a pilot sample to estimate the sample rate such that a given relative error of mean is guaranteed with a given failure probability. We observe that the final failure probability is a summation of the failure probabilities of multiple inequalities. In this work, we evenly allocate the failure probability budget for simplicity.

3.2 Sample Rate Estimation for the Cardinality Estimation

The estimation of the cardinality of a derived table is necessary to estimate `SUM` and `COUNT`. In Bernoulli sampling, we can estimate the total number of Bernoulli trials by the ratio of the number of successes and the success probability of the Bernoulli trial (i.e., sample rate). Theorem 5.6 uses the statistics calculated with a pilot sample to estimate the sample rate for a given relative error of estimating the cardinality with a given failure probability.

Theorem 3.3. *Let n_1 be the number of successes in a sequence of N Bernoulli trials (i.e., pilot sample) with a success probability θ_1, where N is unknown. Suppose we conduct another sequence of N trials with a different success probability θ and result in $\hat{n}$ successes. To ensure the probability of the relative error of estimating N being greater than e is less than p, i.e.,*

$$\mathbb{P}\left[\left|\frac{n/\theta - N}{N}\right| \geq e\right] \leq p,$$

it is sufficient to ensure

$$\theta \geq \left(1 + \frac{e^2 \cdot L_N(n_1, \theta_1, \delta_1)}{z_{1-\delta_2/2}}\right)^{-1}$$
$$\delta_1 + \delta_2 \leq p$$

where

$$L_N(n,\theta,\delta) = \left(\sqrt{\frac{n}{\theta} + z_{1-\delta}^2\frac{1-\theta}{2\theta}} - \sqrt{z_{1-\delta}^2\frac{1-\theta}{2\theta}}\right)^2 \tag{26}$$

δ_1 and δ_2 are the failure probabilities of the confidence interval of the number of successes for the pilot sample and the final sample, respectively.

Proof. Similar to the proof of Theorem 5.5, we obtain a lower bound of the N by applying Lemma 5.3 on the first sequence of Bernoulli trials.

$$\begin{aligned}&\mathbb{P}\left[n_1 \le N\theta_1 + z_{1-\delta_1}\sqrt{N\theta_1(1-\theta_1)}\right] = 1-\delta_1\\ \Leftrightarrow&\mathbb{P}\left[N \ge L_N(n_1,\theta_1,\delta_1)\right] = 1-\delta_1\end{aligned} \tag{27}$$

where

$$L_N(n,\theta,\delta) = \left(\sqrt{\frac{n}{\theta} + z_{1-\delta}^2\frac{1-\theta}{2\theta}} - \sqrt{z_{1-\delta}^2\frac{1-\theta}{2\theta}}\right)^2$$

Nest, we apply the probabilistic bounds derived in Lemma 5.3 on the second sequence of Bernoulli trails, resulting in the relative error bounds of estimating N.

$$\begin{aligned}&\mathbb{P}\left[N\theta - z_{1-\delta_2/2}\sqrt{N\theta(1-\theta)} \le n \le N\theta + z_{1-\delta_2/2}\sqrt{N\theta(1-\theta)}\right] = 1-\delta_2\\ \Leftrightarrow\ &\mathbb{P}\left[\left|\frac{n}{\theta} - N\right| \le z_{1-\delta_2/2}\sqrt{\frac{N(1-\theta)}{\theta}}\right] = 1-\delta_2\\ \Leftrightarrow\ &\mathbb{P}\left[\left|\frac{n/\theta - N}{N}\right| \le \frac{z_{1-\delta_2/2}}{\sqrt{N}}\sqrt{\frac{1}{\theta}-1}\right] = 1-\delta_2\end{aligned} \tag{28}$$

Suppose the success probability of the second sequence of Bernoulli trials satisfies

$$\theta \ge \left(1 + \frac{e^2 \cdot L_N(n_1,\theta_1,\delta_1)}{z_{1-\delta_2/2}}\right)^{-1} \tag{29}$$

We have

$$N \ge L_N(n_1,\theta_1,\delta_1) \quad \text{AND} \quad \left|\frac{n/\theta - N}{N}\right| \le \frac{z_{1-\delta_2/2}}{\sqrt{N}}\sqrt{\frac{1}{\theta}-1} \tag{30}$$

$$\Rightarrow \left|\frac{n/\theta - N}{N}\right| \le \frac{z_{1-\delta_2/2}}{\sqrt{L_N(n_1,\theta_1,\delta_1)}}\sqrt{1 + \frac{e^2 \cdot L_N(n_1,\theta_1,\delta_1)}{z_{1-\delta_2/2}} - 1} = e$$

Namely, given a sample rate satisfying 29, if the estimation of the number of Bernoulli trials has a relative error larger than e, then one of the inequalities in 30 does not hold. Given the failure probability of those inequalities in 27 and 28, we can derive the upper bound of the overall failure probability.

$$\mathbb{P}\left[\left|\frac{n/\theta - N}{N}\right| > e\right] \le \mathbb{P}\left[\left(N \le L_N(n_1,\theta_1,\delta_1)\right) \vee \left(\left|\frac{n/\theta - N}{N}\right| \ge \frac{z_{1-\delta_2/2}}{\sqrt{N}}\sqrt{\frac{1}{\theta}-1}\right)\right] \tag{31}$$

$$\le \mathbb{P}\left[N \le L_N(n_1,\theta_1,\delta_1)\right] + \mathbb{P}\left[\left|\frac{n/\theta - N}{N}\right| \ge \frac{z_{1-\delta_2/2}}{\sqrt{N}}\sqrt{\frac{1}{\theta}-1}\right] \tag{32}$$

$$= \delta_1 + \delta_2$$

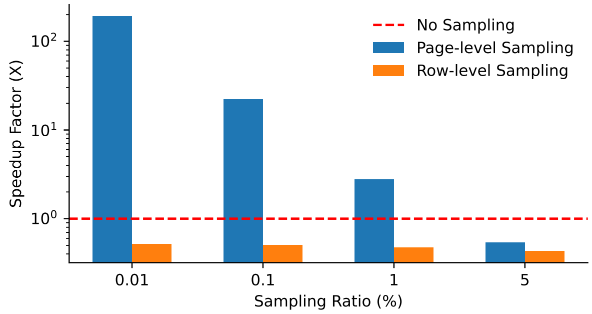

Fig. 2. Page-level sampling is up to 370 × faster than row-level sampling.

If we configure $\delta_1 + \delta_2 \leq p$, we have

$$\mathbb{P}\left[\left|\frac{n/\theta - N}{N}\right| > e\right] \leq p$$

□

4 EFFICIENT ONLINE SAMPLING WITH BLOCK SAMPLING

In this section, we introduce the concrete sampling design that leverages the sample rate estimation introduced in Section 3 and achieves the fast execution that is necessary for online query processing. We first introduce block sampling for approximate query processing. Next, we discuss the aggregate error analysis when the table executing block sampling is the same as the table calculating aggregates. Furthermore, we introduce the theory of commutativity of block sampling and common relation operations to conveniently analyze the sampling in complex aggregation queries. Finally, we derive the estimation of group error in approximate queries with block sampling.

4.1 Block Sampling

Row-level random sampling is widely used in prior work to approximately process queries since it leads to i.i.d. row samples, making the error estimation convenient [12]. However, as shown in Figure 2, row-level random sampling can be more expensive than a full scan of the entire table (i.e., no sampling) since row-level random access is notoriously expensive in modern database management systems (DBMSs) [1, 4]. Consequently, it is infeasible to process online queries with row-level sampling.

To address this challenge, we use a page-level sampling method: block sampling[1] [2, 3, 10]. Block sampling produces i.i.d. samples of blocks while each block contains a certain number of data. Since modern DBMSs store and read a page or a partition of data at a time [1, 14], using block sampling in DBMSs avoids the full scans of tables, which significantly reduces disk I/O costs. Empirically, block sampling can be up to 370 × faster than row-level sampling and faster than exact queries[1] (Fig. 2). Many DBMSs implement the block sample as a page-level Bernoulli sampling, which can be specified as a `TABLESAMPLE` or `TABLESAMPLE SYSTEM` clause [6, 7, 13].

However, because of the statistical property of block sampling and the complexity of queries, it is challenging to apply block sampling in approximate query processing. First, due to the possible correlations within a block, block sampling results in identical but dependent samples of data. Consequently, the Central Limit Theorem does not apply, which in turn invalidates the aforementioned Theorems for error analysis. Therefore, it is

[1]Blocking sampling is also called cluster sampling in statistics literature [8, 15]

[1]Speedups measured in Postgres with TPC-H query 6.

Aggregate	Exact value		Estimated value	
	Row-level	Page-level	Row-level	Page-level
COUNT	$\sum_{i=1}^{N} Q_i$	$N \cdot \mu_Q$	$\left(\sum_{i=1}^{n} Q_i\right)/\theta$	$(n/\theta) \cdot \bar{Q}$
SUM	$\sum_{i=1}^{N} \sum_{j=1}^{Q_i} X_{i,j}$	$N \cdot \mu_S$	$\left(\sum_{i=1}^{n} \sum_{j=1}^{Q_i} X_{i,j}\right)/\theta$	$(n/\theta) \cdot \bar{S}$
AVG	μ_X	μ_S/μ_Q	$\left(\sum_{i=1}^{n} \sum_{j=1}^{Q_i} X_{i,j}\right) / \left(\sum_{i=1}^{N} Q_i\right)$	$\bar{S}/\bar{Q}$

Table 2. Calculations of common aggregates with row-level statistics and page-level statistics equivalently.

challenging to analyze the aggregate errors given a sample of data with dependence. Furthermore, block sampling is executed on input tables to save costs, while aggregates are calculated in derived tables, such as the result table of filter operations. Therefore, we need to analyze the interaction between block sampling and complex relational operations. To sum up, we will address the follow challenges in the rest of this section.

(1) How to analyze the errors of aggregates computed over the sampled data by block sampling.
(2) How to analyze the errors of aggregates computed over the table derived from complex relational operations after block sampling.

4.2 Aggregate Error Estimation

Intuition. Although block sampling does not result in i.i.d. samples of data, it produces i.i.d. samples of blocks. Therefore, the values associated with a block of data also constitute an i.i.d. sample. We define *Page Statistic* as the value associate with a data block. Since page statistics are i.i.d., we can analyze aggregate errors with the theorems in Section 3 if aggregates can be estimated with page statistics.

We find that COUNT, SUM, and AVG can be estimated with page statistics in an equivalent way they are estimated with an i.i.d. sample of data. We will introduce the estimation and their corresponding error analysis in the rest of this subsection.

Notation. Suppose we apply the block sampling with sample rate θ over a table R with N pages, resulting in a sample of n pages $P_1, \ldots, P_n$. We note that N is usually unknown if R is a derived table in the query. Suppose page P_i has Q_i rows: $X_{i,1}, \ldots, X_{i,Q_i}$, where each row represents a numeric value. We call the number of rows of each page as the page size. Furthermore, we compute the sum of values on page P_i as S_i (i.e., page sum). We denote the sample mean and the expected value of page sizes as $\bar{Q}$ and μ_Q and the sample mean and the expected value of page sums as $\bar{S}$ and μ_S. Q_i and S_i are the key page statistics we will leverage to estimate aggregates.

Table 2 summarizes the estimations of common aggregates using page statistics which are equivalent to the corresponding estimations in the row level. Specifically, to estimate COUNT, we use the estimator that divides the COUNT of the sample by the sample rate. We observe that this estimator can be rewritten as the product of page statistics: n/θ and $\bar{Q}$. Similarly, to estimate SUM, we use the estimator that divides the SUM of the sample by the sample rate. We observe that this estimator can be rewritten as the product of page statistics: n/θ and $\bar{S}$. Finally, to estimate AVG, we use the ratio estimator that divides the sample sum and the sample size. We observe that this estimator can be rewritten as the ratio of page statistics: $\bar{S}$ and $\bar{Q}$.

Error Propagation. We observe that COUNT, SUM, and AVG can be calculated as a product or ratio of multiple page statistics whose error can be analyzed with the theorems we derived in Section 3. Specifically, we can leverage Theorem 5.4 and Theorem 5.5 to estimate the sample rate for a given relative error between $\bar{Q}$ and μ_Q or between $\bar{S}$ and μ_S. Similarly, we can apply Theorem 5.6 to estimate the sample rate for a given relative error between n/θ and N. Therefore, to estimate the sample rate for a given relative error of aggregates, it is sufficient to

(1) first calculate the relative errors of page statistics that achieve the required relative errors of aggregates,
(2) then estimate the sample rate for the calculated relative error of page statistics.

We call the first step as *Error Propagation*. We show the error propagation through multiplication, division, and addition, respectively.

Lemma 4.1. *(Multiplication) Let z be a quantity calculated as the product of two positive quantities x and y (i.e., $z = xy$). We estimate x with $\hat{x}$ and y with $\hat{y}$. Let $e_x(< 1)$ be the relative error between $\hat{x}$ and x, and $e_y(< 1)$ be the relative error between $\hat{y}$ and y. The relative error between z and $\hat{x}\hat{y}$ has an upper bound of $e_x + e_y + e_x \cdot e_y$.*

Proof. By definition of the relative error and the positiveness of x, we have

$$\left|\frac{\hat{x} - x}{x}\right| \le e_x \quad \Leftrightarrow \quad (1 - e_x)x \le \hat{x} \le (1 + e_x)x$$

Similarly, we have

$$\left|\frac{\hat{y} - y}{y}\right| \le e_y \quad \Leftrightarrow \quad (1 - e_y)y \le \hat{y} \le (1 + e_y)y$$

Then, we have

$$\begin{aligned} &(1 - e_x)(1 - e_y)xy \le \hat{x}\hat{y} \le (1 + e_x)(1 + e_y)xy \\ \Leftrightarrow\ &(e_xe_y - e_x - e_y)xy \le \hat{x}\hat{y} - xy \le (e_xe_y + e_x + e_y)xy \\ \Leftrightarrow\ &\left|\frac{\hat{x}\hat{y} - xy}{xy}\right| \le e_x + e_y + e_x \cdot e_y \end{aligned}$$

□

Lemma 4.2. *(Division) Let z be a quantity calculated as the ratio of two positive quantities x and y (i.e., $z = x/y$). We estimate the x with $\hat{x}$ and y with $\hat{y}$. Let $e_x(< 1)$ be the relative error between $\hat{x}$ and x, and $e_y(< 1)$ be the relative error between $\hat{y}$ and y. The relative error between z and $\hat{x}/\hat{y}$ has an upper bound of $\frac{e_x+e_y}{1+\min(e_x,e_y)}$.*

Proof. By the definition of the relative error and the positiveness of x and y, we have

$$\begin{aligned} &\frac{1 - e_x}{1 + e_y}\frac{x}{y} \le \frac{\hat{x}}{\hat{y}} \le \frac{1 + e_x}{1 - e_y}\frac{x}{y} \\ \Leftrightarrow\ &-\frac{e_x + e_y}{1 + e_x}\frac{x}{y} \le \frac{\hat{x}}{\hat{y}} - \frac{x}{y} \le \frac{e_x + e_y}{1 + e_y}\frac{x}{y} \\ \Rightarrow\ &\left|\frac{\hat{x}/\hat{y} - x/y}{x/y}\right| \le \frac{e_x + e_y}{1 + \min(e_x, e_y)} \end{aligned}$$

□

Lemma 4.3. *(Addition) Let z be a quantity calculated as the linear combination of two positive quantities x and y. Namely, $z = \lambda_1 x + \lambda_2 y$, where λ_1 and λ_2 are positive. We estimate x with $\hat{x}$ and y with $\hat{y}$. Let $e_x(< 1)$ be the relative error between $\hat{x}$ and x, and $e_y(< 1)$ be the relative error between $\hat{y}$ and y. The relative error between z and $\lambda_1\hat{x} + \lambda_2\hat{y}$ has an upper bound of $\max(e_x, e_y)$.*

Proof. By the definition of the relative error and the positiveness of x and y, we have

$$\begin{aligned} &(1 - e_x)\lambda_1 x + (1 - e_y)\lambda_2 y \le \lambda_1\hat{x} + \lambda_2\hat{y} \le (1 + e_x)\lambda_1 x + (1 + e_y)\lambda_2 y \\ \Leftrightarrow\ &-\lambda_1 e_x x - \lambda_2 e_y y \le \lambda_1\hat{x} + \lambda_2\hat{y} - (\lambda_1 x + \lambda_2 y) \le \lambda_1 e_x x + \lambda_2 e_y y \\ \Rightarrow\ &\left|\frac{\lambda_1\hat{x} + \lambda_2\hat{y} - (\lambda_1 x + \lambda_2 y)}{\lambda_1 x + \lambda_2 y}\right| \le \max(e_x, e_y) \end{aligned}$$

```sql
SELECT SUM(l_extendedprice * l_discount)
FROM lineitem TABLESAMPLE SYSTEM (0.5%)
WHERE
    l_shipdate >= DATE '1994-01-01'
    AND ...
```

(a) Executing block sampling on the input table saves the cost of disk read.

```sql
SELECT SUM(l_extendedprice * l_discount)
FROM (
    SELECT * FROM lineitem
    WHERE
        l_shipdate >= DATE '1994-01-01'
        AND ...
    ) cte TABLESAMPLE SYSTEM (0.5%)
```

(b) Executing block sampling on the intermediate table simplifies the error analysis of aggregates.

Fig. 3. Trade-off and comparison of executing block sampling at different stages of processing TPC-H query 6.

□

With error propagation, we can translate the error requirements of aggregates into the error requirements of page statistics. Additionally, we support the multiplication, division, and addition of multiple aggregates naturally.

4.3 Commutativity of Block Sampling and Relational Operations

We have shown that aggregate errors can be estimated with the help of page statistics when block sampling is applied to the table of aggregation. However, realistic queries usually compute aggregates over an intermediate table derived by complex relational operations after block sampling. The possible interactions of block sampling and relational operations make the analysis challenging.

To illustrate the challenge, we take the TPC-H query 6 as an example (Fig. 3). To save the cost of disk access, we would like to execute block sampling on the table-scan stage (Query 3a), while the error analysis becomes challenging due to the filtering operation between the sampling and aggregation. Query 3b simplifies the error analysis by executing sampling after the filter operation, while the query cost is not significantly reduced due to the full scan of the table `lineitem`. In the rest of this section, we introduce the theories that allow us to execute Query 3a and analyze the errors as it is executed as Query 3b.

We first introduce the notion of the commutativity between a relational operation and block sampling in Definition 5.10. Then, we show that the commutativity leads to an important property that the statistical distribution of the aggregate is independent of the order of the relational operation and block sampling when the relational operation and block sampling commute.

Definition 4.4. A relational operation (RO) commute with block sampling (BS) if

$$\mathbb{P}\left[RO\big(BS(R_1,\dots,R_n)\big) = R^*\right] = \mathbb{P}\left[BS\big(RO(R_1,\dots,R_n)\big) = R^*\right]$$

for any input tables $R_1,\dots,R_n (n \geq 1)$ and any output table R^*

Proposition 4.5. *If a relational operation (RO) and block sampling (BS) commutes, for any input tables $R_1,\dots,R_n (n \geq 1)$ and aggregate function f that maps a table to a real value, we have*

$$\forall x \in \mathbb{R}, \quad \mathbb{P}\left[f\Big(RO\big(BS(R_1,\dots,R_n)\big)\Big) = x\right] = \mathbb{P}\left[f\Big(BS\big(RO(R_1,\dots,R_n)\big)\Big) = x\right] \tag{33}$$

Namely, the probability distribution of the aggregate is independent of the order of executing RO and BS.

Proof. For any value $x \in \mathbb{R}$, let $T = \{R_1^*, \ldots, R_k^*\}$ be the set of all tables on which the aggregate function f results in x. We can rewrite

$$\mathbb{P}\left[f\Big(RO\big(BS(R_1,\ldots,R_n)\big)\Big) = x\right] = \mathbb{P}\left[\bigcup_{i=1}^{k}\left(RO\big(BS(R_1,\ldots,R_n)\big) = R_i^*\right)\right] = \sum_{i=1}^{k}\mathbb{P}\left[RO\big(BS(R_1,\ldots,R_n)\big) = R_i^*\right]$$
$$\mathbb{P}\left[f\Big(BS\big(RO(R_1,\ldots,R_n)\big)\Big) = x\right] = \mathbb{P}\left[\bigcup_{i=1}^{k}\left(BS\big(RO(R_1,\ldots,R_n)\big) = R_i^*\right)\right] = \sum_{i=1}^{k}\mathbb{P}\left[BS\big(RO(R_1,\ldots,R_n)\big) = R_i^*\right]$$

We know that RO and BS commutes. Namely, the following equation holds for any $R_i^* \in T$.

$$\mathbb{P}\left[RO\big(BS(R_1,\ldots,R_n)\big) = R_i^*\right] = \mathbb{P}\left[BS\big(RO(R_1,\ldots,R_n)\big) = R_i^*\right]$$

Therefore, for any $x \in \mathbb{R}$, we have

$$\mathbb{P}\left[f\Big(RO\big(BS(R_1,\ldots,R_n)\big)\Big) = x\right] = \mathbb{P}\left[f\Big(BS\big(RO(R_1,\ldots,R_n)\big)\Big) = x\right] \tag{34}$$

□

Next, we prove that block sampling commutes to common relation operations, including Selection, Join, Union, and Projection.

Proposition 4.6. *(Selection-BS Commutativity) Suppose σ_ϕ is a selection operation that selects a subset of the table based on the predicate ϕ. On any table R, BS_θ commutes with σ_ϕ for any sampling rate θ.*

Proof. First, we describe two events whose probability is zero and independent of the order of block sampling and selection. Suppose the input table R has N pages: $P_1, \ldots, P_N$. Let R^* be the result table of applying block sampling and selection on R.

$$\begin{aligned} E_1 &:= \quad \exists_{r \in R^*},\ \phi(r) = 0 \\ E_2 &:= \quad \forall_{i \in [1,N]}\ \exists_{r_1, r_2 \in P_i},\ \phi(r_1) = \phi(r_2) = 1 \texttt{ AND } r_1 \in R^* \texttt{ AND } r_2 \notin R^* \end{aligned} \tag{35}$$

As long as the selection operation is applied, every row in the result table must evaluate the predicate ϕ to 1. Therefore, the probability of E_1 is 0. Furthermore, block sampling ensures that if one row is sampled, the whole page is sampled. Therefore, it is not likely to have two rows from the same page satisfying the predicate, but only one of them is in the result table. Namely, the probability of E_2 is 0.

Next, we analyze the probability distributions excluding events E_1 and E_2. Let $R' = \{P_1', \ldots, P_{N'}'\}$ be the subset of R satisfying ϕ, where each page P_i' contains non-zero rows satisfying ϕ. If we exclude events E_1 and E_2, the result table must be a subset of pages in R', regardless of the order of operations. We can calculate the inclusion probability of P_i' in R^* for different orders of operations. It turns out that the inclusion probability is independent of the operation order.

$$\mathbb{P}\left[P_i' \in \sigma_\phi(BS_\theta(R))\right] = \mathbb{P}\left[P_i \in BS_\theta(R)\right] = \theta$$
$$\mathbb{P}\left[P_i' \in BS_\theta(\sigma_\phi(R))\right] = \mathbb{P}\left[P_i' \in BS_\theta(R')\right] = \theta$$

Since pages are independent of each other in the process of selection and block sampling, for the result table $R^* = \{P'_1, \dots, P'_n\}$, we have

$$\mathbb{P}\left[\sigma_\phi(BS_\theta(R)) = R^*\right] = \mathbb{P}\left[\left(\bigcap_{i=1}^{n} P'_i \in \sigma_\phi(BS_\theta(R))\right) \bigcap \left(\bigcap_{i=n+1}^{N'} P'_i \notin \sigma_\phi(BS_\theta(R))\right)\right] = \theta^n (1-\theta)^{N'-n}$$

$$\mathbb{P}\left[BS_\theta(\sigma_\phi(R)) = R^*\right] = \mathbb{P}\left[\left(\bigcap_{i=1}^{n} P'_i \in BS_\theta(\sigma_\phi(R))\right) \bigcap \left(\bigcap_{i=n+1}^{N'} P'_i \notin BS_\theta(\sigma_\phi(R))\right)\right] = \theta^n (1-\theta)^{N'-n}$$

Therefore, for any result table, the probability is independent of the order of selection and block sampling. Namely, selection and block sampling commutes. □

Proposition 4.7. *(Join-BS Commutativity) Suppose $\bowtie_\phi$ is a join operation that merges two tables based on the predicate ϕ. On any table R_1 and R_2, BS_θ commutes with $\bowtie_\phi$ for any sampling rate θ, in the case that BS_θ is executed on one of R_1 and R_2.*

Proof. Since block sampling commutes with selection, it is sufficient to prove that block sampling also commutes with cross-product. Without loss of generality, we assume block sampling is executed on R_1. We first describe an event whose probability is zero and is independent of the order of block sampling and cross-product. Suppose R_1 has N pages: $R_1 = \{P_1 \dots, P_N\}$. Let R^* be the result table after applying block sampling and cross-product over R_1 and R_2.

$$E := \quad \forall_{i \in [1,N]}\, \exists_{r_1, r_2 \in P_i},\ (r_1 \bowtie R_2) \in R^* \text{ AND } (r_2 \bowtie R_2) \notin R^*$$

Block sampling ensures that if one row is sampled, the whole page is sampled. Therefore, it is not likely to have one row in the result table while other rows from the same page are not. Namely, the probability of E is 0.

Next, we analyze the probability distributions excluding the event E. Let $R' = \{P'_1, \dots, P'_N\}$ be the full cross-product where $P'_i = P_i \bowtie R_2$ (i.e., the cross-product of page P_i and table R_2). If we exclude event E, the result table R must be a subset of pages in R'. We can calculate the inclusion probability of P'_i in R. It turns out that the inclusion probability is independent of the operation order.

$$\mathbb{P}\left[P'_i \in (BS_\theta(R_1) \bowtie R_2)\right] = \mathbb{P}\left[P_i \in BS_\theta(R_1)\right] = \theta$$
$$\mathbb{P}\left[P'_i \in BS_\theta(R_1 \bowtie R_2)\right] = \mathbb{P}\left[P'_i \in BS_\theta(R')\right] = \theta$$

Since pages are independent of each other in the process of cross-product and block sampling, for the arbitrary result table $R^* = \{P'_1, \dots, P'_n\}$, we have

$$\mathbb{P}\left[(BS_\theta(R_1) \bowtie R_2) = R^*\right] = \mathbb{P}\left[\left(\bigcap_{i=1}^{n} P'_i \in (BS_\theta(R_1) \bowtie R_2)\right) \bigcap \left(\bigcap_{i=n+1}^{N} P'_i \notin (BS_\theta(R_1) \bowtie R_2)\right)\right] = \theta^n (1-\theta)^{N-n}$$

$$\mathbb{P}\left[BS_\theta(R_1 \bowtie R_2) = R^*\right] = \mathbb{P}\left[\left(\bigcap_{i=1}^{n} P'_i \in BS_\theta(R_1 \bowtie R_2)\right) \bigcap \left(\bigcap_{i=n+1}^{N} P'_i \notin BS_\theta(R_1 \bowtie R_2)\right)\right] = \theta^n (1-\theta)^{N-n}$$

Therefore, for any result table, the probability is independent of the order of cross-product and block sampling. Namely, cross-product and block sampling commute. Since selection and block sampling commute, join and block sampling commute. □

Proposition 4.8. *(Union-BS Commutativity) Suppose $\cup$ is a union operation that merges two tables and does not drop duplicate values. On any two tables R_1 and R_2, BS_θ commutes with the union $\cup$ in the case that BS_θ is executed on both R_1 and R_2 with the same sampling probability θ.*

Proof. First, we describe an event whose probability is zero and independent of the order of union and block sampling. Suppose R_1 has N_1 pages: $R_1 = \{P_1^{(1)}, \ldots, P_{N_1}^{(1)}\}$, and R_2 has N_2 pages: $R_2 = \{P_1^{(2)}, \ldots, P_{N_2}^{(2)}\}$. Let R^* be the result table after applying block sampling and union over R_1 and R_2.

$$\begin{aligned} E_1 &:= \quad \forall_{i\in[1,N_1]}\ \exists_{r_1,r_2\in P_i^{(1)}},\ r_1 \in R^* \text{ AND } r_2 \notin R^* \\ E_2 &:= \quad \forall_{i\in[1,N_2]}\ \exists_{r_1,r_2\in P_i^{(2)}},\ r_1 \in R^* \text{ AND } r_2 \notin R^* \end{aligned}$$

Block sampling ensures that if one row is sampled, the whole page is sampled. Therefore, it is not likely to have one row in the result table while other rows from the same page are not. Namely, both the probability of E_1 and the probability of E_2 are zero.

Next, we analyze the probability distribution excluding E_1 and E_2. Let $R' = \{P_1^{(1)}, \ldots, P_{N_1}^{(1)}, P_1^{(2)}, \ldots, P_{N_2}^{(2)}\}$ be the union of R_1 and R_2. If we exclude E_1 and E_2, the result table R must be a subset of pages in R'. We can calculate the inclusion probability of $P_i^{(1)}$ and $P_i^{(2)}$ in R. It turns out that the inclusion probability is independent of the operation order. Without loss of generality, we only show the calculation of the inclusion probability of $P_i^{(1)}$.

$$\begin{aligned} &\mathbb{P}\left[P_i^{(1)} \in (BS_\theta(R_1) \cup BS_\theta(R_2))\right] = \mathbb{P}\left[P_i^{(1)} \in BS_\theta(R_1)\right] = \theta \\ &\mathbb{P}\left[P_i^{(1)} \in BS_\theta(R_1 \cup R_2)\right] = \theta \end{aligned}$$

Since pages are independent of each other in the process of union and block sampling, for the arbitrary result table $R^* = \{P_1^{(1)}, \ldots, P_{n_1}^{(1)}, P_1^{(2)}, \ldots, P_{n_2}^{(2)}\}$, we have

$$\begin{aligned} &\mathbb{P}\left[(BS_\theta(R_1) \cup BS_\theta(R_2)) = R^*\right] \\ &= \mathbb{P}\left[\left(\bigcup_{i=1}^{n_1} P_i^{(1)} \in BS_\theta(R_1)\right) \bigcup \left(\bigcup_{i=n_1+1}^{N_1} P_i^{(1)} \notin BS_\theta(R_1)\right) \bigcup \left(\bigcup_{i=1}^{n_2} P_i^{(2)} \in BS_\theta(R_2)\right) \bigcup \left(\bigcup_{i=n_2+1}^{N_2} P_i^{(2)} \notin BS_\theta(R_2)\right)\right] \\ &= \theta^{n_1} \cdot (1-\theta)^{N_1-n_1} \cdot \theta^{n_2} \cdot (1-\theta)^{N_2-n_2} = \theta^{n_1+n_2} \cdot (1-\theta)^{N_1+N_2-n_1-n_2} \\ &\mathbb{P}\left[BS_\theta(R_1 \cup R_2) = R^*\right] = \mathbb{P}\left[\left(\bigcup_{k=1}^{2}\bigcup_{i=1}^{n_k} P_i^{(k)} \in BS_\theta(R_1 \cup R_2)\right) \bigcup \left(\bigcup_{k=1}^{2}\bigcup_{i=n_k+1}^{N_k} P_i^{(k)} \notin BS_\theta(R_1 \cup R_2)\right)\right] \\ &= \theta^{n_1+n_2}(1-\theta)^{N_1-n_1+N_2-n_2} \end{aligned}$$

Therefore, for any result table, the probability is independent of the order of union and block sampling. Namely, union and block sampling commute. □

Proposition 4.9. *(Projection-BS Commutativity) Suppose π_A is a projection operation that returns certain columns of a table based on the attribute specification A. On any table R, BS commutes with π_A.*

Proof. Since the projection operation π_A modifies the columns of a table, which is orthogonal to the BS operation, BS naturally commutes with π_A. □

To conclude, Proposition 5.12, 5.13, 5.14, and 5.15 proves the commutativity of BS with common relation operations. In terms of SQL queries, BS commutes with `JOIN`, `GROUP BY`, `WHERE`, `HAVING`, `UNION ALL`, and `SELECT` clauses. Based on the commutativity property, we can efficiently process approximate queries by executing BS on the input table while conveniently analyzing aggregate errors with BS executed over any intermediate table.

4.4 Group Error Estimation

In this subsection, we quantify the probability of missing groups when using block sampling to approximately process queries with `GROUP BY` clauses. We include the probability of missing groups as part of the failure probability budget, as decomposed in Lemma 1.1.

Scenario. We may miss groups only when the table executing block sampling contains group-by keys. If the sampled pages do not cover certain groups, we miss the groups. Due to the randomness of sampling, we cannot guarantee that no groups are missed. For example, a full table scan is necessary to guarantee groups with sizes = 1 are not missed. Therefore, we provide a conditioned probabilistic guarantee that the probability of missing groups with sizes larger than a user-specified value is small. We achieve the guarantee by configuring the sampling rate of the pilot sampling.

Notation. Let R be the table executing block sampling with a column being the group-by key, m be the size of each page, θ be the sampling rate, and g be the minimum group size. Page size can be retrieved from the meta-data of the DBMS. We introduce Lemma to estimate the required sampling rate of pilot sampling to ensure the block sampling does not miss groups of size larger than g.

Lemma 4.10. *On a table R with page size m, the probability of missing a group of size greater than g by block sampling is less than p if the sampling rate satisfies*

$$\theta \geq 1 - \left(1 - (1-p)^{\lceil g/m \rceil / |R|}\right)^{1/\lceil g/m \rceil}$$

Proof. Based on the meta-information, we calculate the number of pages in R as $N = |R|/m$. Because each group has at least g rows, each group takes at least $n_0 = \lceil g/m \rceil$ pages. Suppose there are t groups with size larger than g. Let n_i be the number of pages taken by the i-th group. We then have the following constraints

$$t \leq \frac{|R|}{n_0} \tag{36}$$

$$\forall_{1 \leq i \leq t},\ n_i \geq n_0 \tag{37}$$

Based on the process of block sampling, we can calculate the probability of including i-th group as following

$$\mathbb{P}\left[\text{include group i}\right] = 1 - \mathbb{P}\left[\text{miss group i}\right] = 1 - (1-\theta)^{n_i}$$

Next, we calculate the probability of including groups i and j ($i \neq j$). Suppose group i and group j share k pages ($k \geq 0$), then the probability of including two groups has the following lower bound.

$$\begin{aligned}
&\mathbb{P}\left[(\text{include group i}) \wedge (\text{include group j})\right] = \mathbb{P}\left[\text{include group i} \mid \text{include group j}\right] \cdot \mathbb{P}\left[\text{include group j}\right] \\
&= \big(\mathbb{P}\left[\text{include group i}\right] \cdot \mathbb{P}\left[\text{not include shared pages} \mid \text{include group j}\right] + \\
&\quad\ \mathbb{P}\left[\text{include shared pages} \mid \text{include group j}\right]\big) \cdot \mathbb{P}\left[\text{include group j}\right] \\
&= \left((1-(1-\theta)^{n_i}) \cdot \frac{n_j - k}{n_j} + \frac{k}{n_j}\right) \cdot (1-(1-\theta)^{n_j}) \\
&\geq (1-(1-\theta)^{n_i}) \cdot (1-(1-\theta)^{n_j})
\end{aligned}$$

We extend the results to all groups and calculate lower bound of the the probability of including all groups

$$\mathbb{P}\left[\text{include all groups}\right] \geq \prod_{i=1}^{t} (1-(1-\theta)^{n_i})$$

- Author

Applying the constraints in 36 and 37, we can have the following lower bound for the probability of including all groups

$$\mathbb{P}\left[\text{include all groups}\right] \geq \prod_{i=1}^{t} \left(1-(1-\theta)^{n_0}\right) \geq \prod_{i=1}^{|R|/n_0} \left(1-(1-\theta)^{n_0}\right) = \left(1-(1-\theta)^{\lceil g/m \rceil}\right)^{|R|/\lceil g/m \rceil}$$

Therefore, if the sampling rate θ satisfies

$$\theta \geq 1-\left(1-(1-p)^{\lceil g/m \rceil/|R|}\right)^{1/\lceil g/m \rceil}$$

then

$$\mathbb{P}\left[\text{miss a group}\right] = 1-\mathbb{P}\left[\text{include all groups}\right] \leq p$$

□

REFERENCES

[1] Azure Synapse Analytics. [n. d.]. *Designing tables.* https://learn.microsoft.com/en-us/azure/synapse-analytics/sql-data-warehouse/sql-data-warehouse-tables-overview Accessed: 2024-05-12.

[2] Surajit Chaudhuri, Gautam Das, and Utkarsh Srivastava. 2004. Effective use of block-level sampling in statistics estimation. In *SIGMOD*.

[3] Xingguang Chen, Fangyuan Zhang, and Sibo Wang. 2022. Efficient Approximate Algorithms for Empirical Variance with Hashed Block Sampling. In *KDD*.

[4] Douglas Comer. 1979. Ubiquitous B-tree. *ACM Computing Surveys (CSUR)* (1979).

[5] Databricks. [n. d.]. *TABLESAMPLE clause | Databricks on AWS.* https://docs.databricks.com/en/sql/language-manual/sql-ref-syntax-qry-select-sampling.html Accessed: 2024-05-12.

[6] Azure Databricks. 2024. *TABLESAMPLE clause.* https://learn.microsoft.com/en-us/azure/databricks/sql/language-manual/sql-ref-syntax-qry-select-sampling Accessed: 2024-05-12.

[7] DuckDB. [n. d.]. *Samples - DuckDB.* https://duckdb.org/docs/sql/samples.html Accessed: 2024-05-12.

[8] Ralph H Henderson and Thalanayar Sundaresan. 1982. Cluster sampling to assess immunization coverage: a review of experience with a simplified sampling method. *Bulletin of the World Health Organization* 60, 2 (1982), 253.

[9] Robert V Hogg, Joseph W McKean, and Allen T Craig. 2019. *Introduction to mathematical statistics.* Pearson.

[10] Wen-Chi Hou, Gultekin Ozsoyoglu, and Baldeo K Taneja. 1988. Statistical estimators for relational algebra expressions. In *SIGMOD*.

[11] Apache Impala. [n. d.]. *TABLESAMPLE Clause.* https://impala.apache.org/docs/build/html/topics/impala_tablesample.html Accessed: 2024-05-12.

[12] Yongjoo Park, Barzan Mozafari, Joseph Sorenson, and Junhao Wang. 2018. Verdictdb: Universalizing approximate query processing. In *SIGMOD*.

[13] PostgreSQL. [n. d.]. *TABLESAMPLE Implementation - PostgreSQL wiki.* https://wiki.postgresql.org/wiki/TABLESAMPLE_Implementation Accessed: 2024-05-12.

[14] Konstantin Shvachko, Hairong Kuang, Sanjay Radia, and Robert Chansler. 2010. The hadoop distributed file system. In *IEEE 26th symposium on mass storage systems and technologies (MSST)*.

[15] Steven K Thompson. 1990. Adaptive cluster sampling. *J. Amer. Statist. Assoc.* 85, 412 (1990), 1050–1059.

5 THEOREMS

Lemma 5.1. *(Error Decomposition) Let $p^{(g)}$ be the probability of missing groups with size larger than g (Eq. 38), and $p_{i,j}^{(a)}$ be the probability of the estimated aggregate $\hat{A}_{i,j}$ of group G_i and column j having a relative error larger than e.*

$$p^{(g)} := \mathbb{P}\left[\{G_i | G_i \in G, G_i \notin G', |G_i| > g\} \neq \emptyset\right] \tag{38}$$

$$p_{i,j}^{(a)} := \mathbb{P}\left[\left|\frac{A_{i,j} - \hat{A}_{i,j}}{A_{i,j}}\right| > e\right] \tag{39}$$

The overall probability of the group error or aggregate error occurring is upper bounded by the sum of $p^{(g)}$ and all $p_{i,j}^{(a)}$. Namely,

$$\mathbb{P}\left[\textit{Group Error} \bigvee \textit{Aggregate Error}\right] \leq \sum_{\substack{G_i \in G' \\ |G_i| > g}} \sum_{j=1}^{C} p_{i.j}^{(a)} + p^{(g)}$$

Lemma 5.2. *Let $X_1, \ldots, X_n$ be an i.i.d. random sample from a distribution with the expected value given by μ and finite variance given by σ^2. Suppose $\bar{X}$ is the sample mean (Eq. 40) and $\hat{\sigma}^2$ is the unbiased sample variance (Eq. 41).*

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \tag{40}$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} \left(X_i - \bar{X}\right)}{n-1} \tag{41}$$

We have the following probabilistic bounds for μ and σ.

$$\lim_{n \to \infty} \mathbb{P}\left[L_{\mu}^{(t)}(\bar{X}, \hat{\sigma}, n, \delta/2) \leq \mu \leq U_{\mu}^{(t)}(\bar{X}, \hat{\sigma}, n, \delta/2)\right] = 1 - \delta$$

$$\lim_{n \to \infty} \mathbb{P}\left[L_{\sigma}(\hat{\sigma}, n, \delta/2) \leq \sigma \leq U_{\sigma}(\hat{\sigma}, n, \delta/2)\right] = 1 - \delta$$

where

$$L_{\mu}^{(t)}(\bar{X}, \hat{\sigma}, n, \delta) = \bar{X} - t_{n-1,1-\delta} \frac{\hat{\sigma}}{\sqrt{n}}, \quad U_{\mu}^{(t)}(\bar{X}, \hat{\sigma}, n, \delta) = \bar{X} + t_{n-1,1-\delta} \frac{\hat{\sigma}}{\sqrt{n}} \tag{42}$$

$$L_{\sigma}(\hat{\sigma}, n, \delta) = \sqrt{\frac{n-1}{\chi^2_{n-1,1-\delta}}}\hat{\sigma}, \quad U_{\sigma}(\hat{\sigma}, n, \delta) = \sqrt{\frac{n-1}{\chi^2_{n-1,\delta}}}\hat{\sigma} \tag{43}$$

$t_{n-1,1-\delta/2}$ is the upper $\delta/2$ critical point of student's t distribution with $n-1$ degrees of freedom. Namely, $\delta/2 = \mathbb{P}\left[T > t_{n-1,1-\delta/2}\right]$, where T follows student's t distribution with $n-1$ degrees of freedom. $\chi^2_{n-1,\delta/2}$ is the lower $\delta/2$ critical point of chi-squared distribution with $n-1$ degrees of.

If we know the variance σ^2, we can have the following probabilistic bound for μ.

$$\lim_{n \to \infty} \mathbb{P}\left[L_{\mu}^{(z)}(\bar{X}, \sigma, n, \delta/2) \leq \mu \leq U_{\mu}^{(z)}(\bar{X}, \sigma, n, \delta/2)\right] = 1 - \delta$$

where

$$L_{\mu}^{(z)}(\bar{X}, \sigma, n, \delta) = \bar{X} - z_{1-\delta} \frac{\sigma}{\sqrt{n}}, \quad U_{\mu}^{(z)}(\bar{X}, \sigma, n, \delta) = \bar{X} + z_{1-\delta} \frac{\sigma}{\sqrt{n}}, \tag{44}$$

and $z_{1-\delta/2}$ is the upper $\delta/2$ critical point of standard normal distribution.

Lemma 5.3. *Suppose we have a sequence of N independent experiments. Each experiment is either successful or not successful with a fixed success probability θ. Suppose $\hat{n}$ is the number of successes. We have the following probabilistic bounds for $\hat{n}$.*

$$\lim_{N\to\infty} \mathbb{P}\left[N\theta - z_{1-\delta/2}\sqrt{N\theta(1-\theta)} \le \hat{n} \le N\theta + z_{1-\delta/2}\sqrt{N\theta(1-\theta)}\right] = 1-\delta \tag{45}$$

or equivalently

$$\lim_{N\to\infty} \mathbb{P}\left[N - z_{1-\delta/2}\sqrt{\frac{N(1-\theta)}{\theta}} \le \frac{\hat{n}}{\theta} \le N + z_{1-\delta/2}\sqrt{\frac{N(1-\theta)}{\theta}}\right] = 1-\delta \tag{46}$$

where $z_{1-\delta/2}$ is the upper $\delta/2$ critical point of standard normal distribution.

Theorem 5.4. *Let $X_1, \ldots, X_{n_1}$ be an i.i.d. random sample (i.e., pilot sample) from a distribution with the positive expected value given by μ and finite variance given by σ^2, $\bar{X}$ be the sample mean, $\hat{\sigma}^2$ be the unbiased sample variance, and $z_{1-\delta/2}$ be the upper $\delta/2$ critical point of standard normal distribution. Suppose $Y_1, \ldots, Y_n$ is another i.i.d. sample. To ensure the relative error between the sample mean $\bar{Y}$ and μ being less than e with a maximum failure probability of p, i.e.,*

$$\mathbb{P}\left[\left|\frac{\bar{Y}-\mu}{\mu}\right| \ge e\right] \le p,$$

it is sufficient to ensure

$$n \ge \left(\frac{z_{1-\delta_3/2}}{e}\frac{U_\sigma(\hat{\sigma}, n_1, \delta_1)}{L_\mu^{(t)}(\bar{X}, \hat{\sigma}, n_1, \delta_2)}\right)^2,$$
$$\delta_1 + \delta_2 + \delta_3 \le p$$

where δ_1 is the failure probability of $U_\sigma(\hat{\sigma}, n_1, \delta_1)$ being the upper bound of σ, δ_2 is the failure probability of $L_\mu^{(t)}$ being the lower bound of μ, and δ_3 is the failure probability of the confidence interval of mean derived in Lemma 2.1.

Theorem 5.5. *Let n_1 be the number of successes in a sequence of N Bernoulli trials (i.e., pilot sample) with a success probability θ_1, where N is unknown. Suppose we conduct another sequence of N trials with a different success probability θ and result in $\hat{n}$ successes. To ensure that $\hat{n}$ is at least n with probability larger than $1-p$, i.e.,*

$$\mathbb{P}[\hat{n} \le n] \le p,$$

it is sufficient to ensure

$$\theta \ge \left(\frac{z_{1-\delta_2} + \sqrt{z_{1-\delta_2}^2 + 4n}}{2\sqrt{L_N(n_1, \theta_1, \delta_1)}}\right)^2$$
$$\delta_1 + \delta_2 \le p$$

where

$$L_N(n, \theta, \delta) = \left(\sqrt{\frac{n}{\theta} + z_{1-\delta}^2\frac{1-\theta}{2\theta}} - \sqrt{z_{1-\delta}^2\frac{1-\theta}{2\theta}}\right)^2$$

δ_1 and δ_2 are the failure probabilities of the confidence interval of the number of successes for the pilot sample and the final sample, respectively.

THEOREM 5.6. *Let n_1 be the number of successes in a sequence of N Bernoulli trials (i.e., pilot sample) with a success probability θ_1, where N is unknown. Suppose we conduct another sequence of N trials with a different success probability θ and result in $\hat{n}$ successes. To ensure the probability of the relative error of estimating N being greater than e is less than p, i.e.,*

$$\mathbb{P}\left[\left|\frac{n/\theta - N}{N}\right| \geq e\right] \leq p,$$

it is sufficient to ensure

$$\theta \geq \left(1 + \frac{e^2 \cdot L_N(n_1, \theta_1, \delta_1)}{z_{1-\delta_2/2}}\right)^{-1}$$
$$\delta_1 + \delta_2 \leq p$$

where

$$L_N(n, \theta, \delta) = \left(\sqrt{\frac{n}{\theta} + z_{1-\delta}^2 \frac{1-\theta}{2\theta}} - \sqrt{z_{1-\delta}^2 \frac{1-\theta}{2\theta}}\right)^2 \tag{47}$$

δ_1 and δ_2 are the failure probabilities of the confidence interval of the number of successes for the pilot sample and the final sample, respectively.

LEMMA 5.7. *(MULTIPLICATION) Let z be a quantity calculated as the product of two positive quantities x and y (i.e., $z = xy$). We estimate x with $\hat{x}$ and y with $\hat{y}$. Let $e_x(< 1)$ be the relative error between $\hat{x}$ and x, and $e_y(< 1)$ be the relative error between $\hat{y}$ and y. The relative error between z and $\hat{x}\hat{y}$ has an upper bound of $e_x + e_y + e_x \cdot e_y$.*

LEMMA 5.8. *(DIVISION) Let z be a quantity calculated as the ratio of two positive quantities x and y (i.e., $z = x/y$). We estimate the x with $\hat{x}$ and y with $\hat{y}$. Let $e_x(< 1)$ be the relative error between $\hat{x}$ and x, and $e_y(< 1)$ be the relative error between $\hat{y}$ and y. The relative error between z and $\hat{x}/\hat{y}$ has an upper bound of $\frac{e_x+e_y}{1+\min(e_x,e_y)}$.*

LEMMA 5.9. *(ADDITION) Let z be a quantity calculated as the linear combination of two positive quantities x and y. Namely, $z = \lambda_1 x + \lambda_2 y$, where λ_1 and λ_2 are positive. We estimate x with $\hat{x}$ and y with $\hat{y}$. Let $e_x(< 1)$ be the relative error between $\hat{x}$ and x, and $e_y(< 1)$ be the relative error between $\hat{y}$ and y. The relative error between z and $\lambda_1\hat{x} + \lambda_2\hat{y}$ has an upper bound of $\max(e_x, e_y)$.*

Definition 5.10. A relational operation (*RO*) commute with block sampling (*BS*) if

$$\mathbb{P}\left[RO\big(BS(R_1, \ldots, R_n)\big) = R^*\right] = \mathbb{P}\left[BS\big(RO(R_1, \ldots, R_n)\big) = R^*\right]$$

for any input tables $R_1, \ldots, R_n (n \geq 1)$ and any output table R^*

PROPOSITION 5.11. *If a relational operation (RO) and block sampling (BS) commutes, for any input tables $R_1, \ldots, R_n (n \geq 1)$ and aggregate function f that maps a table to a real value, we have*

$$\forall x \in \mathbb{R}, \quad \mathbb{P}\left[f\Big(RO\big(BS(R_1, \ldots, R_n)\big)\Big) = x\right] = \mathbb{P}\left[f\Big(BS\big(RO(R_1, \ldots, R_n)\big)\Big) = x\right] \tag{48}$$

Namely, the probability distribution of the aggregate is independent of the order of executing RO and BS.

PROPOSITION 5.12. *(SELECTION-BS COMMUTATIVITY) Suppose σ_ϕ is a selection operation that selects a subset of the table based on the predicate ϕ. On any table R, BS_θ commutes with σ_ϕ for any sampling rate θ.*

PROPOSITION 5.13. *(JOIN-BS COMMUTATIVITY) Suppose $\bowtie_\phi$ is a join operation that merges two tables based on the predicate ϕ. On any table R_1 and R_2, BS_θ commutes with $\bowtie_\phi$ for any sampling rate θ, in the case that BS_θ is executed on one of R_1 and R_2.*

Proposition 5.14. *(Union-BS Commutativity) Suppose $\cup$ is a union operation that merges two tables and does not drop duplicate values. On any two tables R_1 and R_2, BS_θ commutes with the union $\cup$ in the case that BS_θ is executed on both R_1 and R_2 with the same sampling probability θ.*

Proposition 5.15. *(Projection-BS Commutativity) Suppose π_A is a projection operation that returns certain columns of a table based on the attribute specification A. On any table R, BS commutes with π_A.*

Lemma 5.16. *On a table R with page size m, the probability of missing a group of size greater than g by block sampling is less than p if the sampling rate satisfies*

$$\theta \geq 1 - \left(1 - (1-p)^{\lceil g/m \rceil / |R|}\right)^{1/\lceil g/m \rceil}$$