LING 490 - SPECIAL TOPICS IN LINGUISTICS

Fundamentals of Digital Signal Processing

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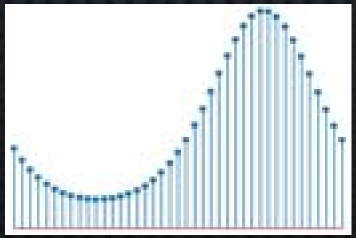
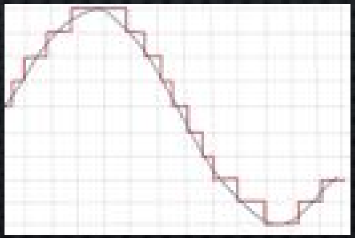
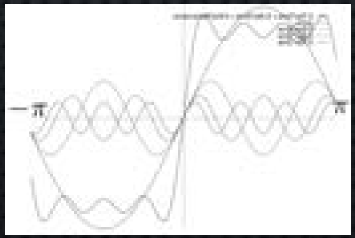
Week 2

Last week…

- Applications of DSP
- Motivations of learning DSP
- Definition of signal
- Analog vs digital signal
    - Advantages of performing DSP
- General workflow of DSP systems
    - Three fundamental ideas of DSP

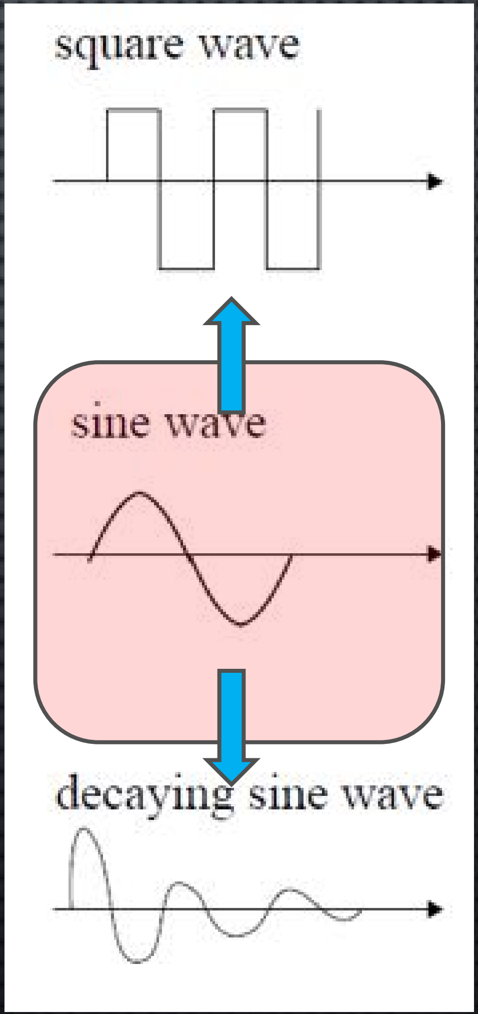




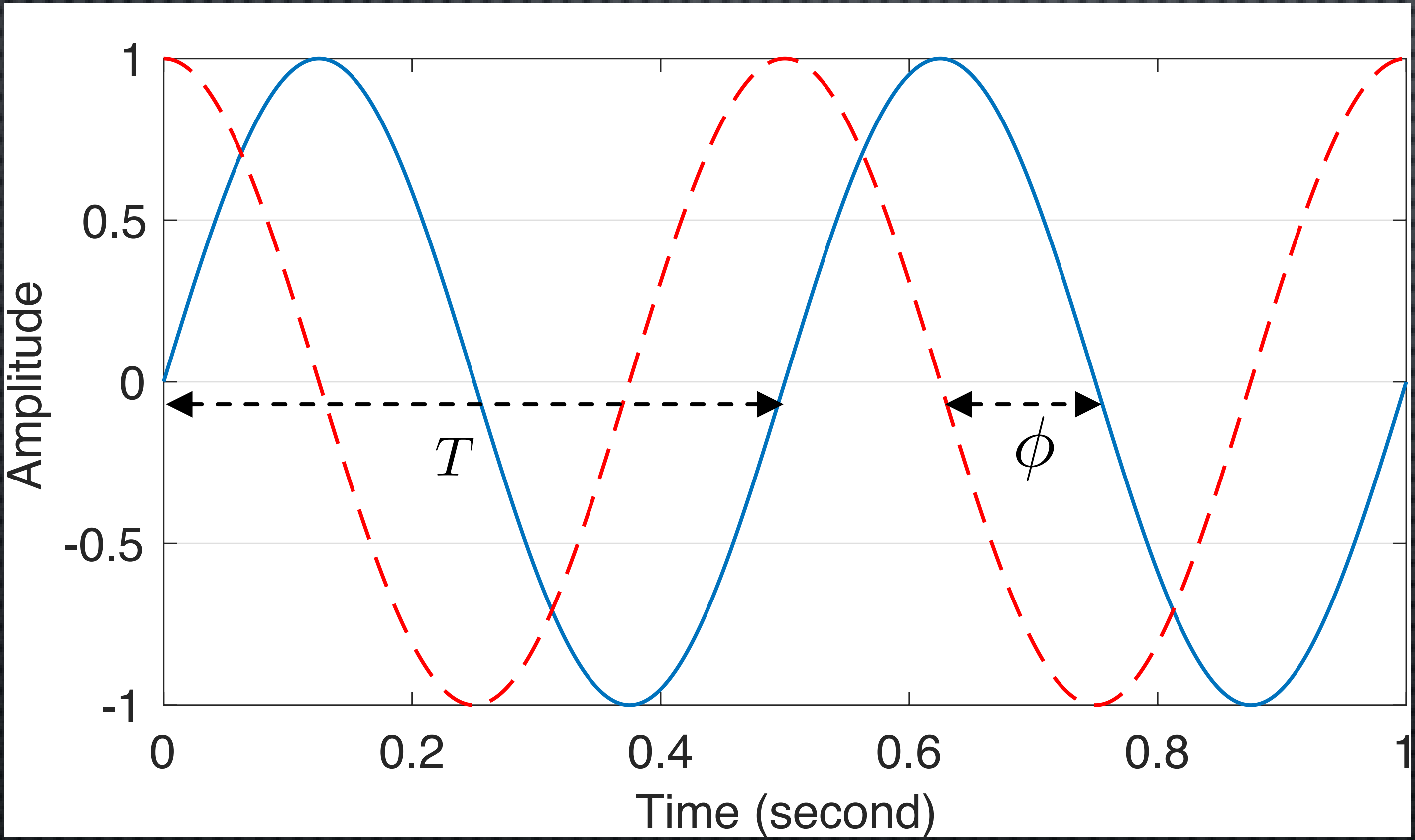


Waveform of signals

- Key idea of DSP
  - Describe complex signal in terms of the sum of basis functions which are simpler to interpret
- Some examples
  - Sinusoid: useful for describing many types of continuous signal
  - Square wave: useful for describing binary signals in communication
  - Decaying sine wave: useful for pitch-periods of speech



Properties of continuous signals



Peak amplitude: ***A***

frequency: ***f, ω***

phase: ***ϕ***

period: ***T***

Periodical signal

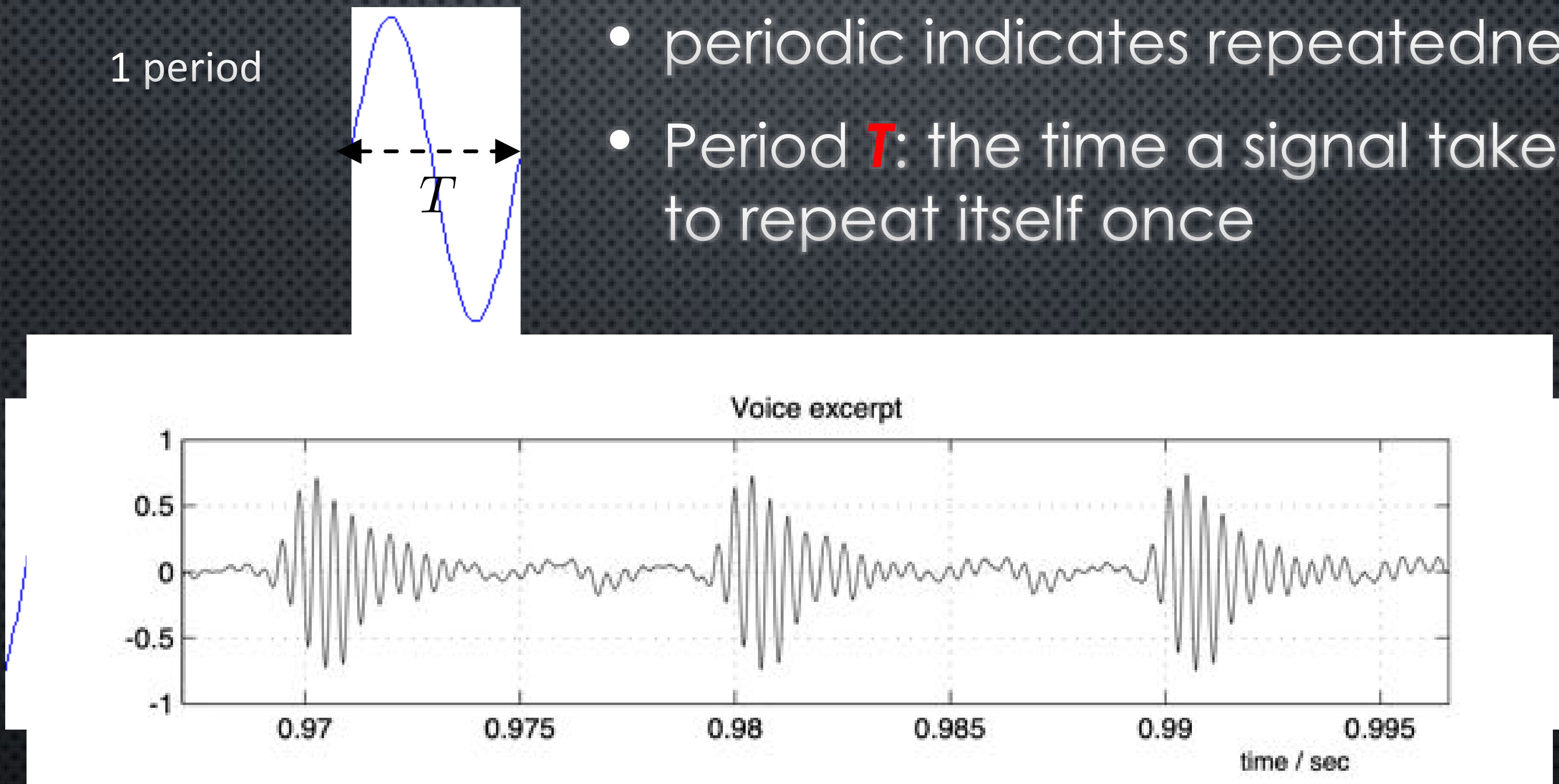
$$x(t) = A \cdot \sin(2\pi \cdot f \cdot t - \phi)$$

$$x(t) = A \cdot \sin(\omega \cdot t - \phi)$$

What is periodicity?

- periodic indicates repeatedness
- Period ***T***: the time a signal takes to repeat itself once

1 period



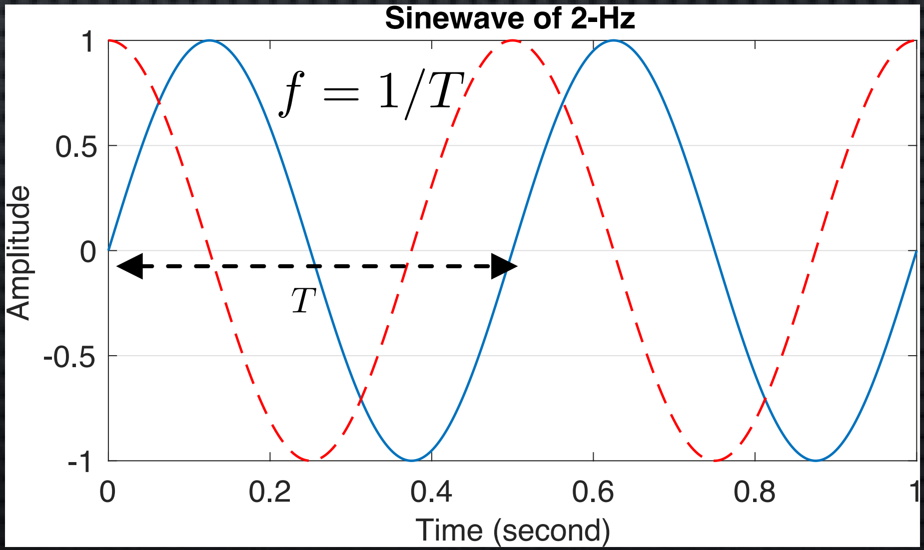
In fact, speech waveforms are not periodic (later we will see how to handle this)

What is frequency?

- Frequency indicates how frequently a signal repeats itself in a unit time

- An inverse function of ***T***

$$f = \frac{1}{T}$$

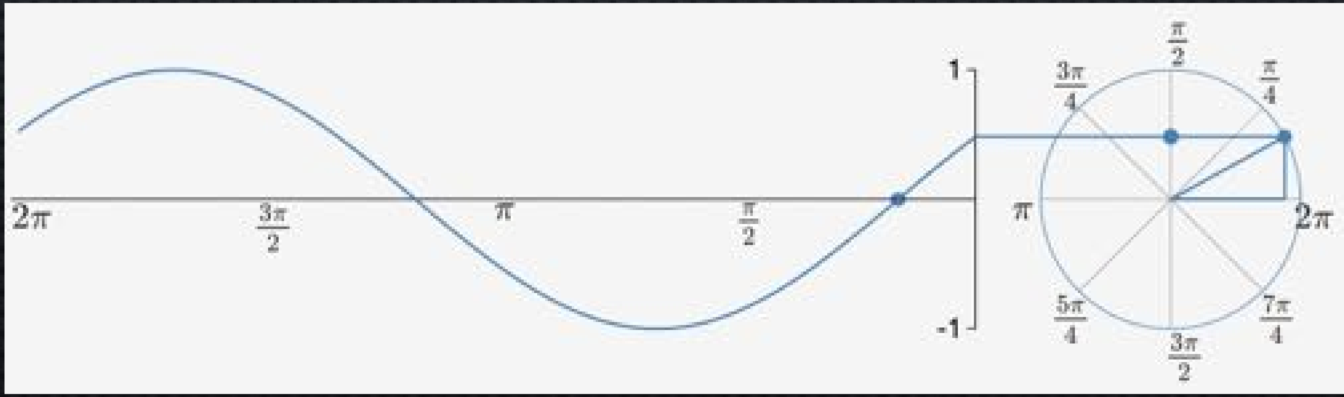


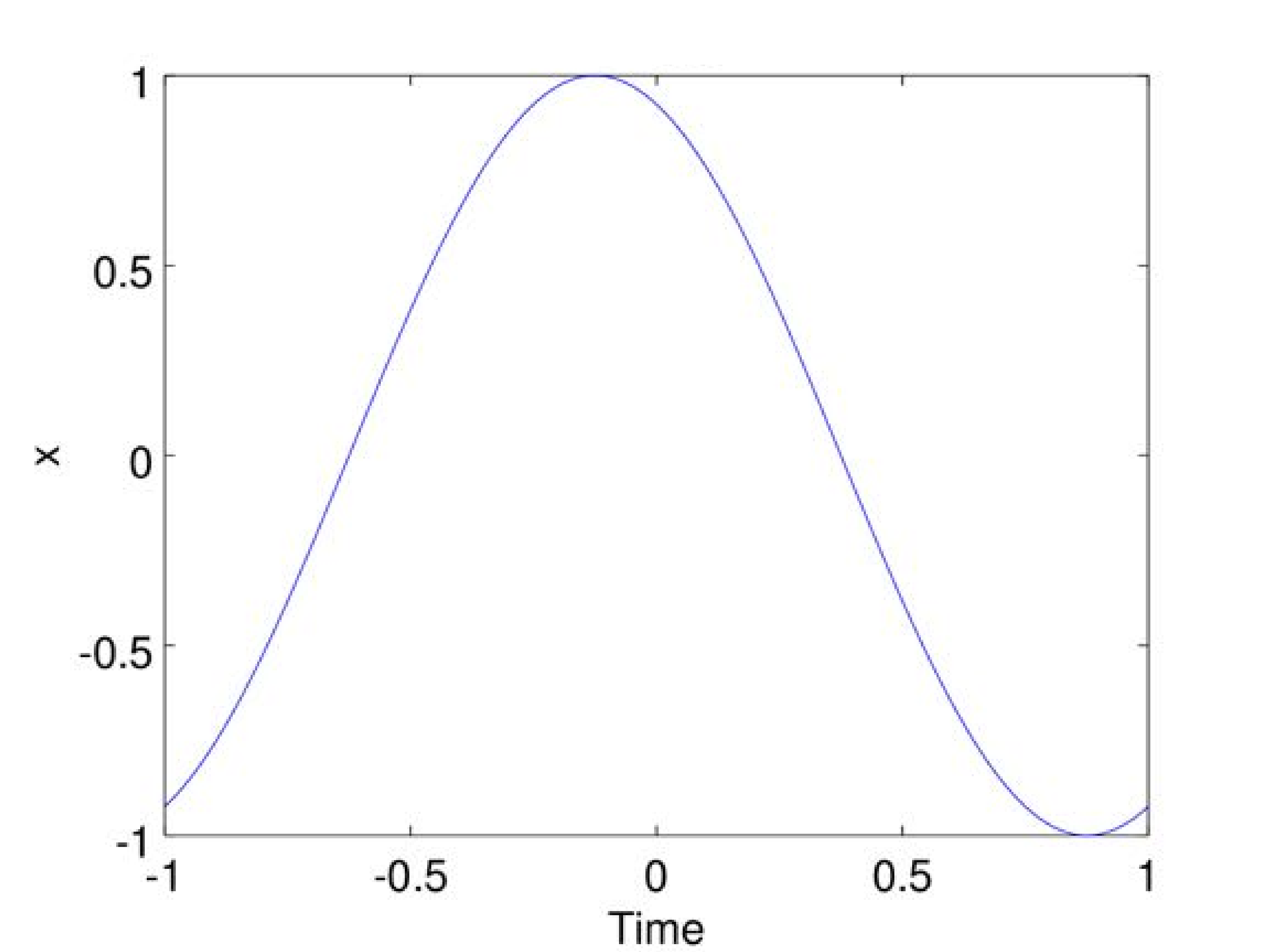
Relationship between *f* and ω

- ***f*** : regular/linear frequency (circle/s, or Hz)
  - The number of times the signal repeats itself per second
- **ω:** angular/radial/circular frequency (radians/s)
  - Angular displacement per/second

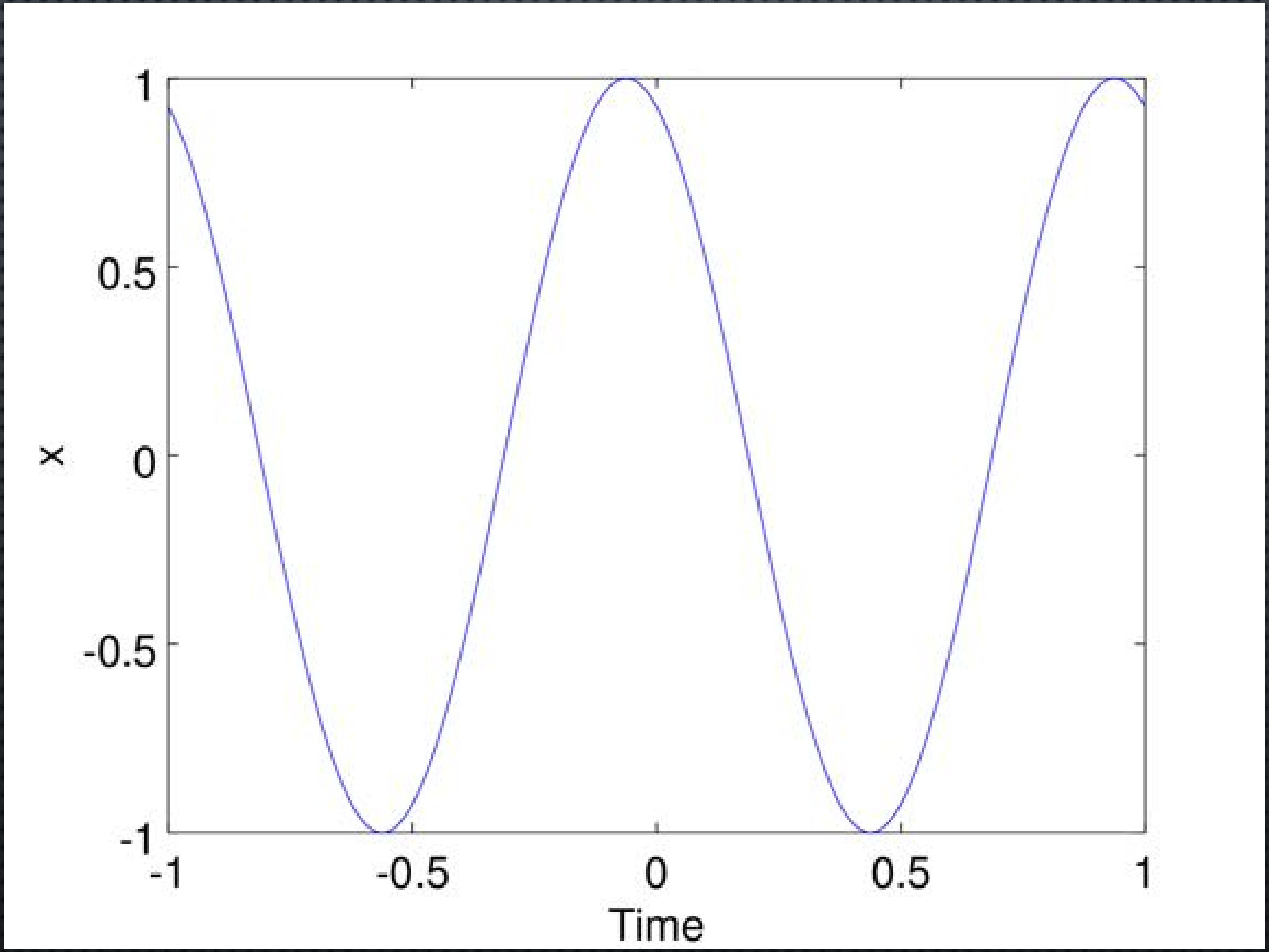
$$\omega = 2\pi \cdot f$$

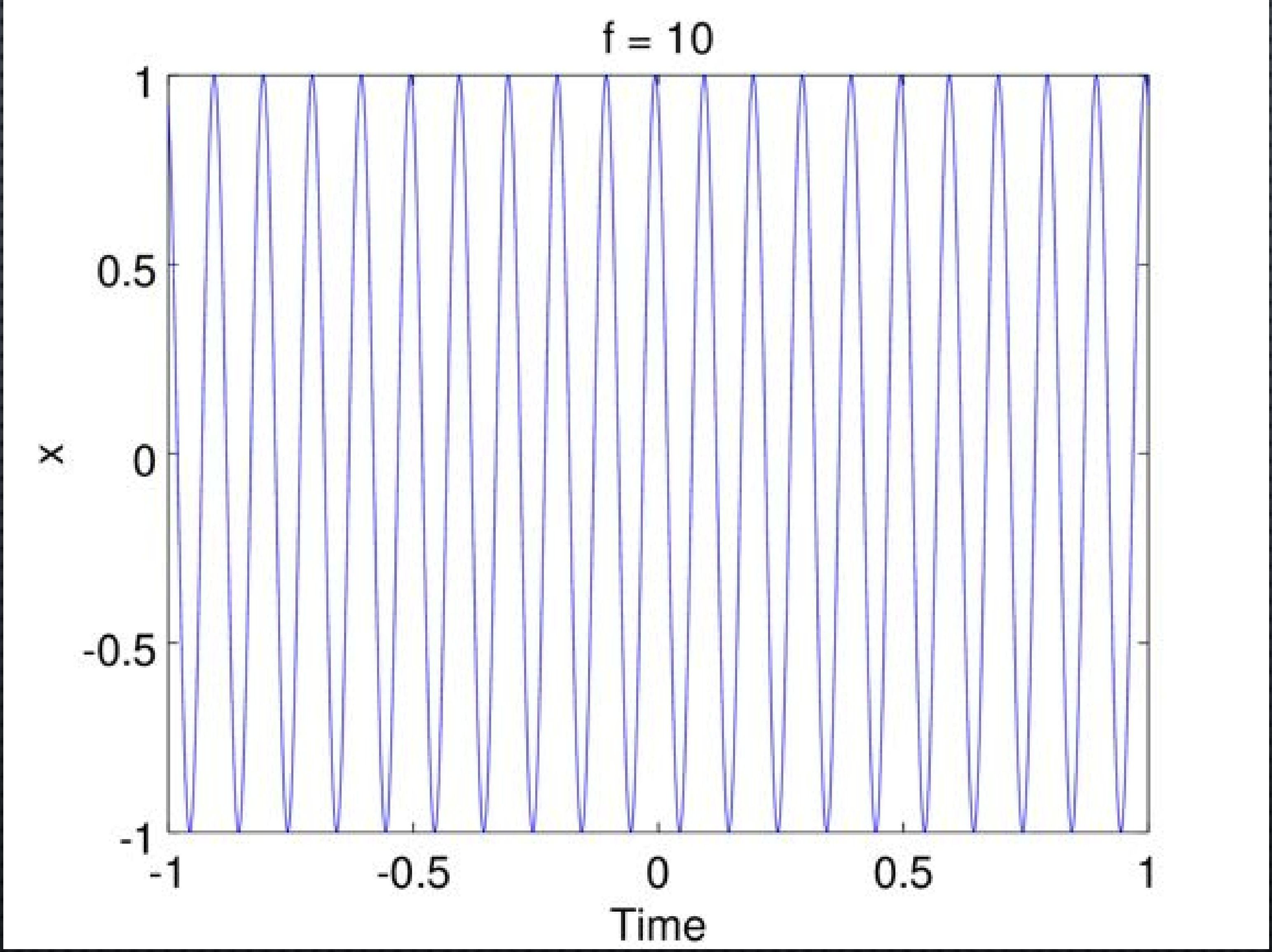
1 Hz ≈ 6.28 rad/s; 1 radian ≈ 57.3°

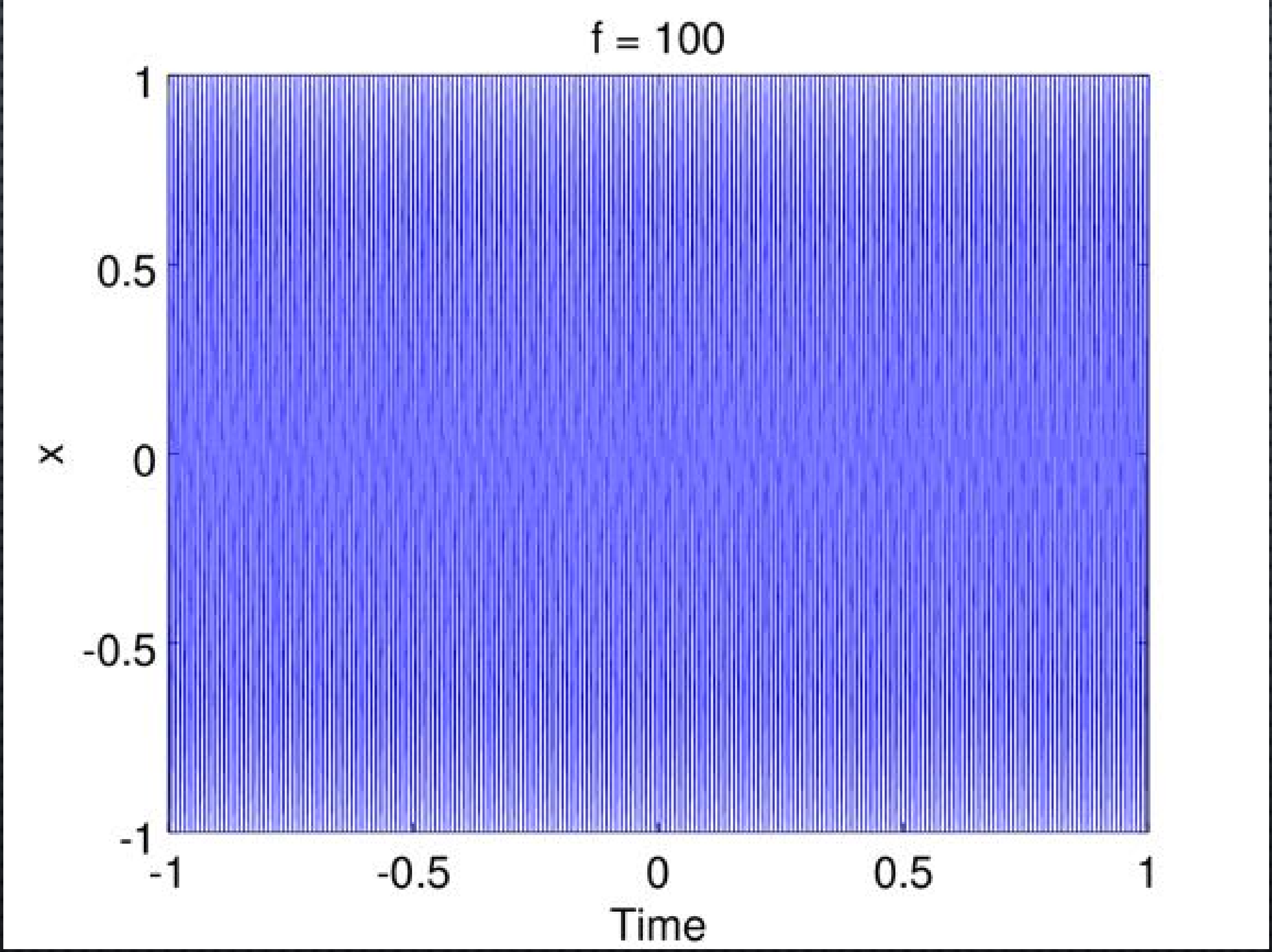




1
0.5
x
0
-0.5
-1
-1
-0.5
0
0.5
1
Time


1
0.5
0
-0.5
-1
x
-1
-0.5
0
0.5
1
Time

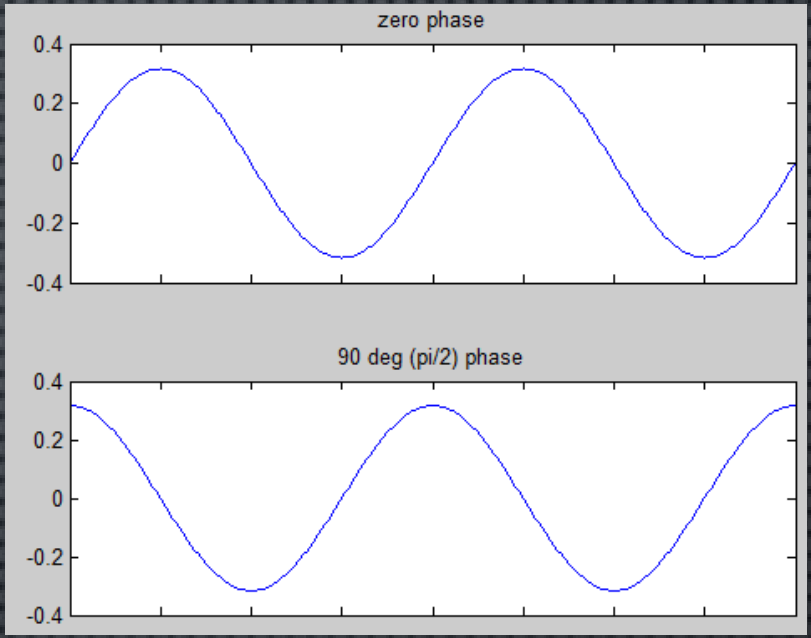

f = 10
1
0.5
0
-0.5
-1
x
-1
-0.5
0
0.5
1
Time

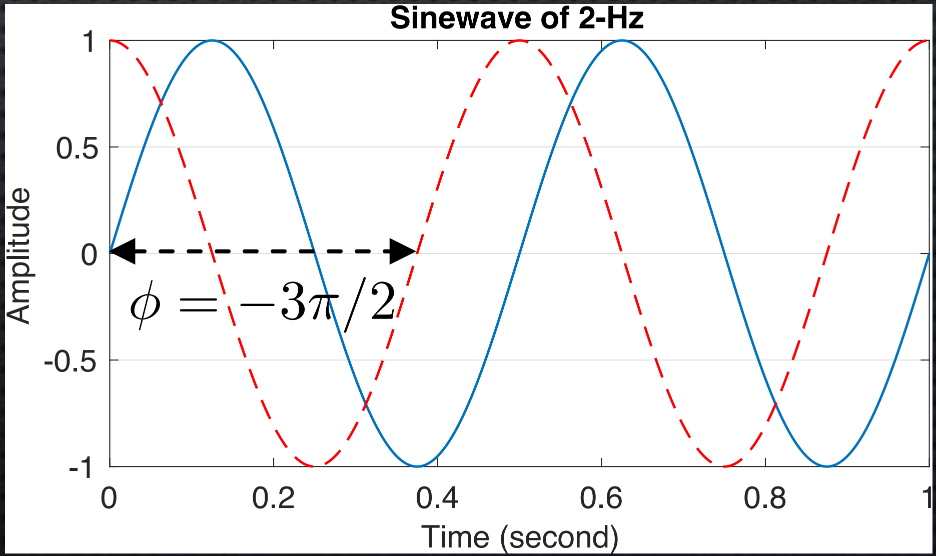

f = 100
1
0.5
0
-0.5
-1
x
-1
-0.5
0
0.5
1
Time

What is phase?

- For a periodic waveform, phase defines the point within the period at which the waveform starts (relative to 0)

- Measured in degrees or radians (360 degrees = 2*pi radians)





Phase in degrees and radians

- Phase ϕ in radians
- **d**: phase in angle

$$x(t) = A \cdot \sin(\omega \cdot t - \phi)$$

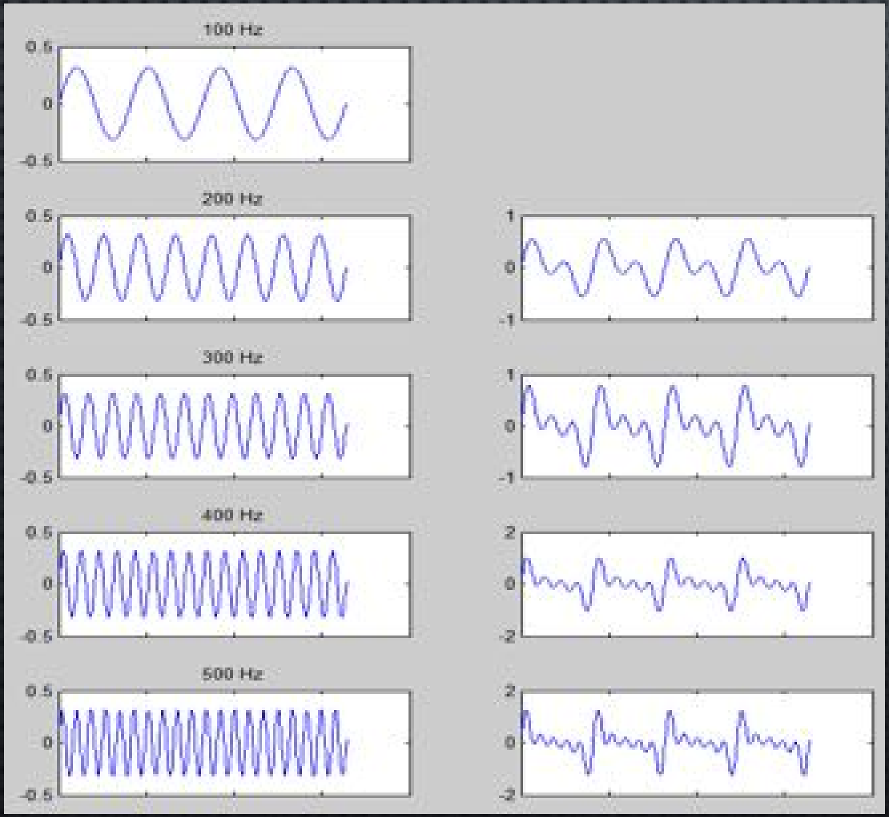
$$d = \frac{360 \cdot \phi}{2\pi}$$

$$\phi = \frac{2\pi \cdot d}{360}$$

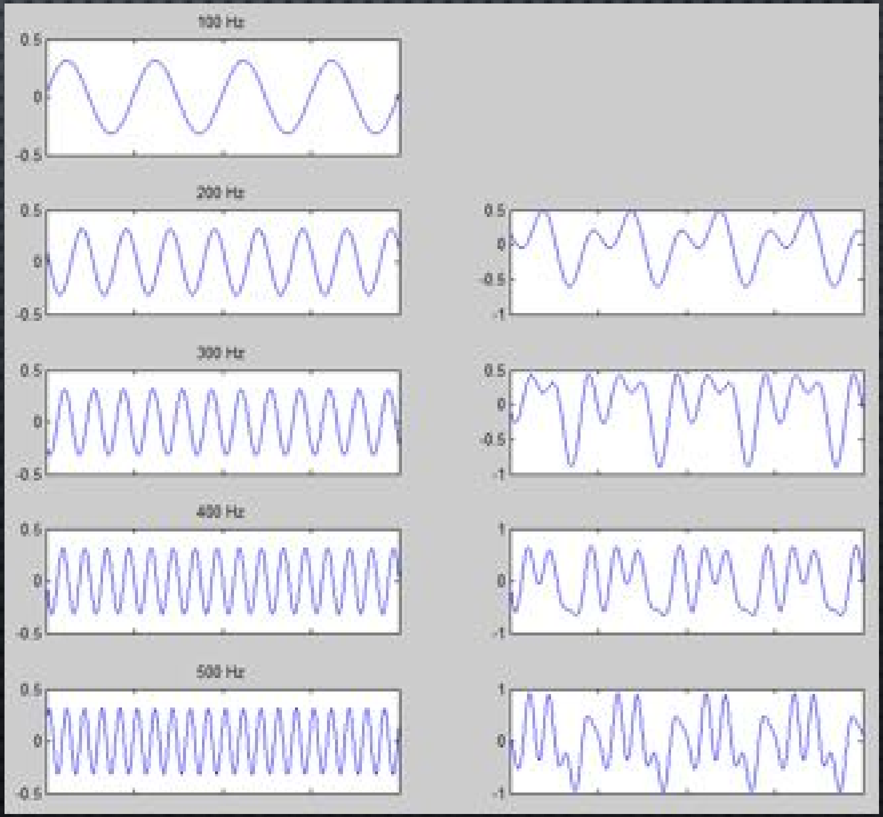
Phase in complex signals

Phase is required to be able to decompose waveforms exactly

phase = 0



phase = random



Properties of continuous signals

$$x(t) = A \cdot \sin(2\pi \cdot f \cdot t - \phi)$$

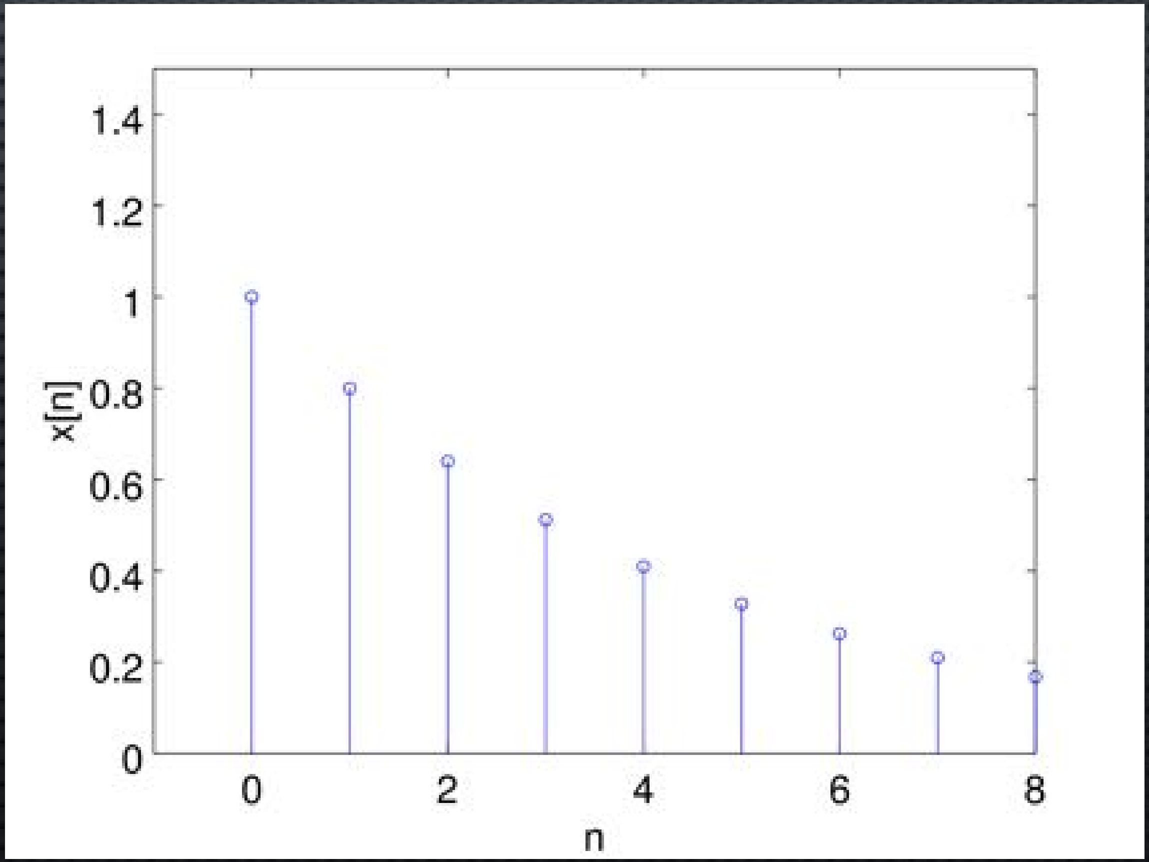
- For every fixed value of *f*, x(t) is periodic with period T = 1/*f*
- Continuous signals with distinct frequencies are themselves distinct
- Increasing *f* results in an increase in the rate of oscillation. As *t* is continuous, we can increase *f* without limit

Continuous vs Discrete time signals

- What does a digital signal look like?
- For a continuous time signal x(t), you can find a value for x at any value of t
- Discrete time signals are defined only at specific values of time

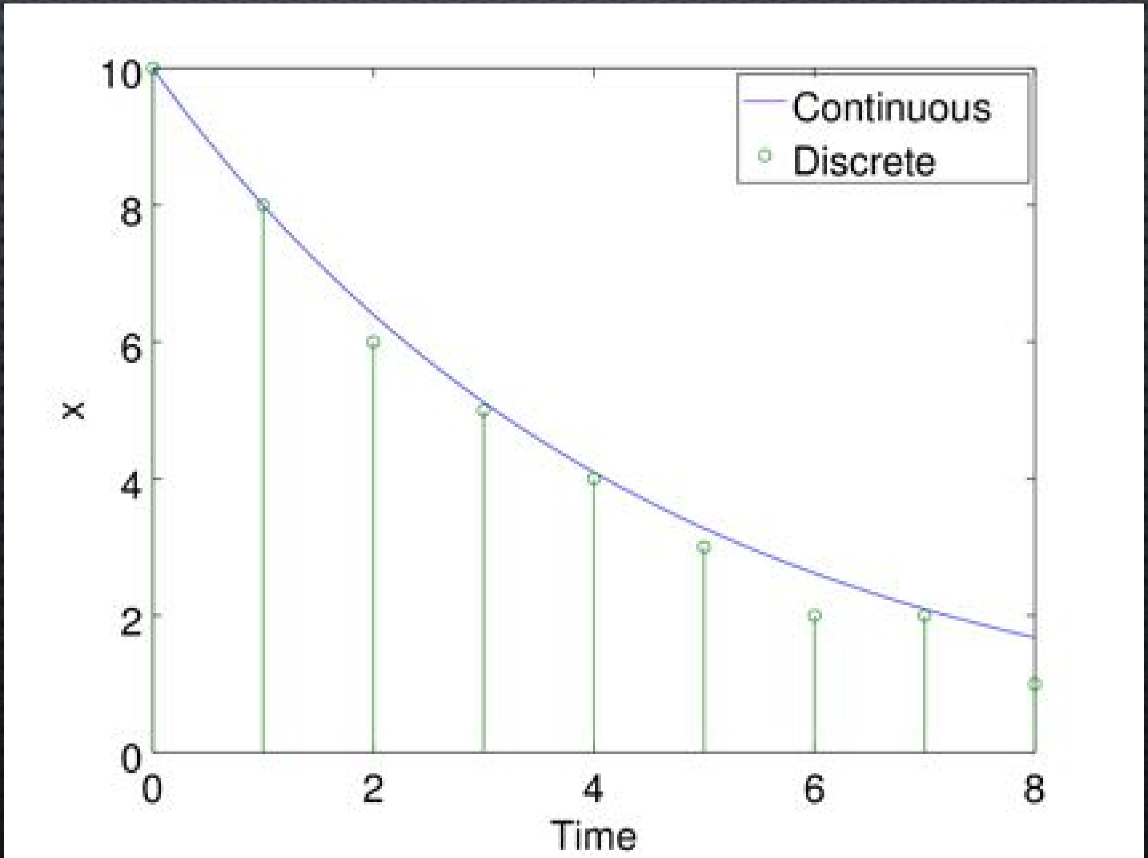
Continuous vs Discrete time signals

- Digital signals are often drawn using stem plots:



Continuous vs Discrete time signals

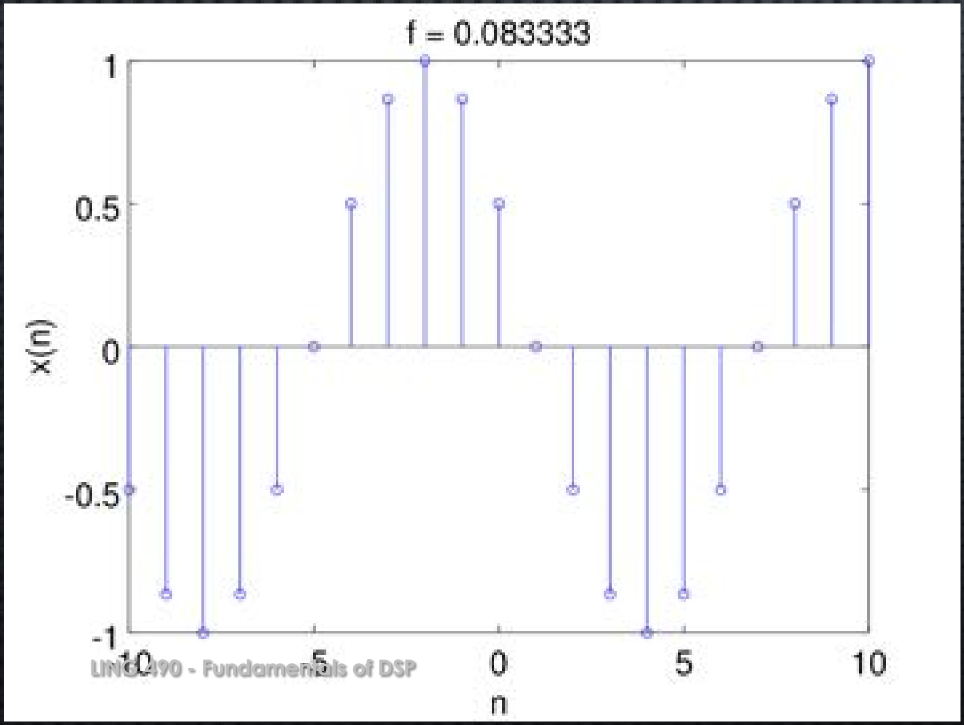
- Due to finite resolution in time, resolution in level is also finite



Properties of discrete time signals

- A discrete time sinusoid can be expressed by:

$$x(n) = A \cdot \cos(\omega \cdot n - \phi)$$



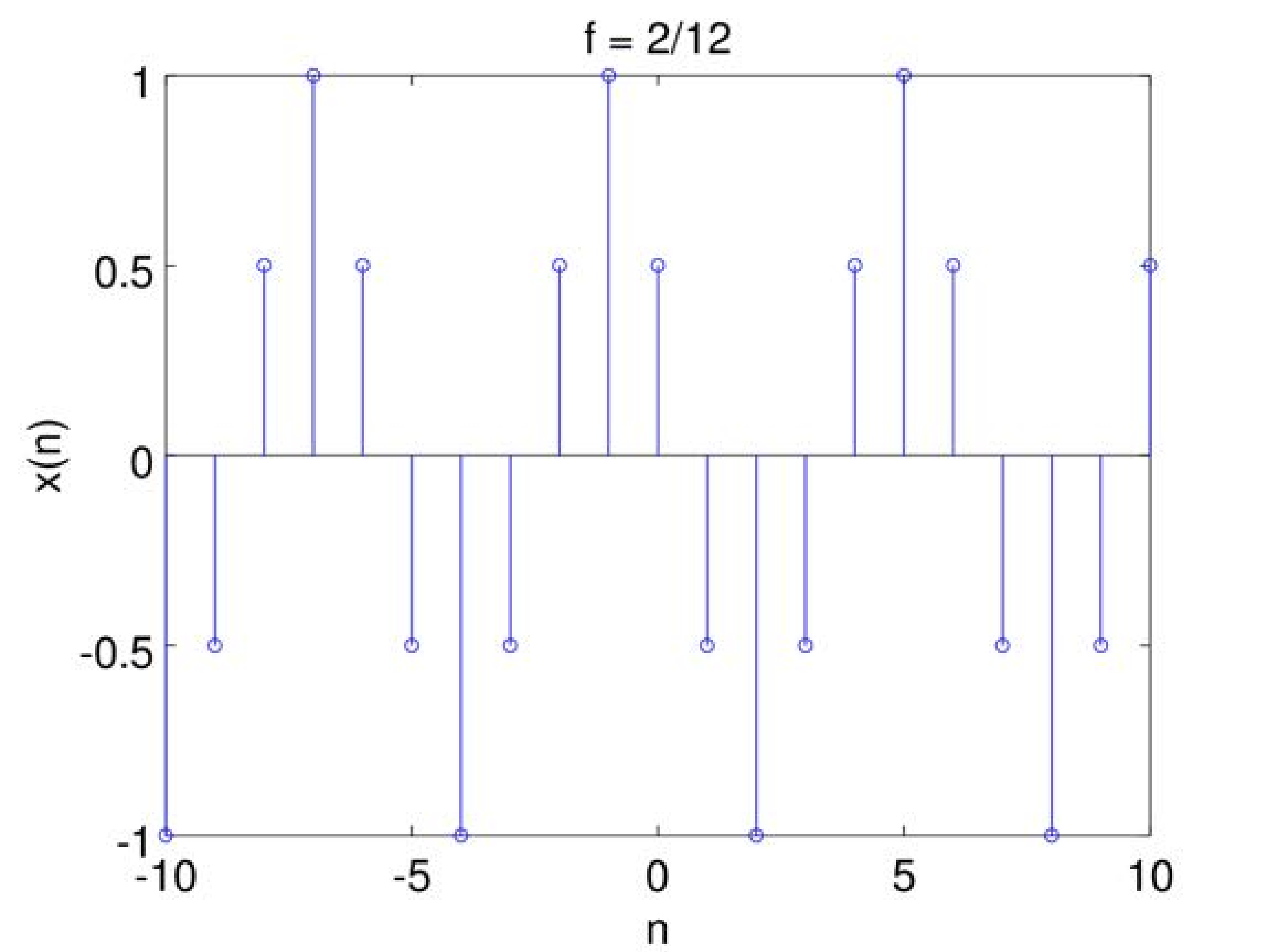
NOTE:

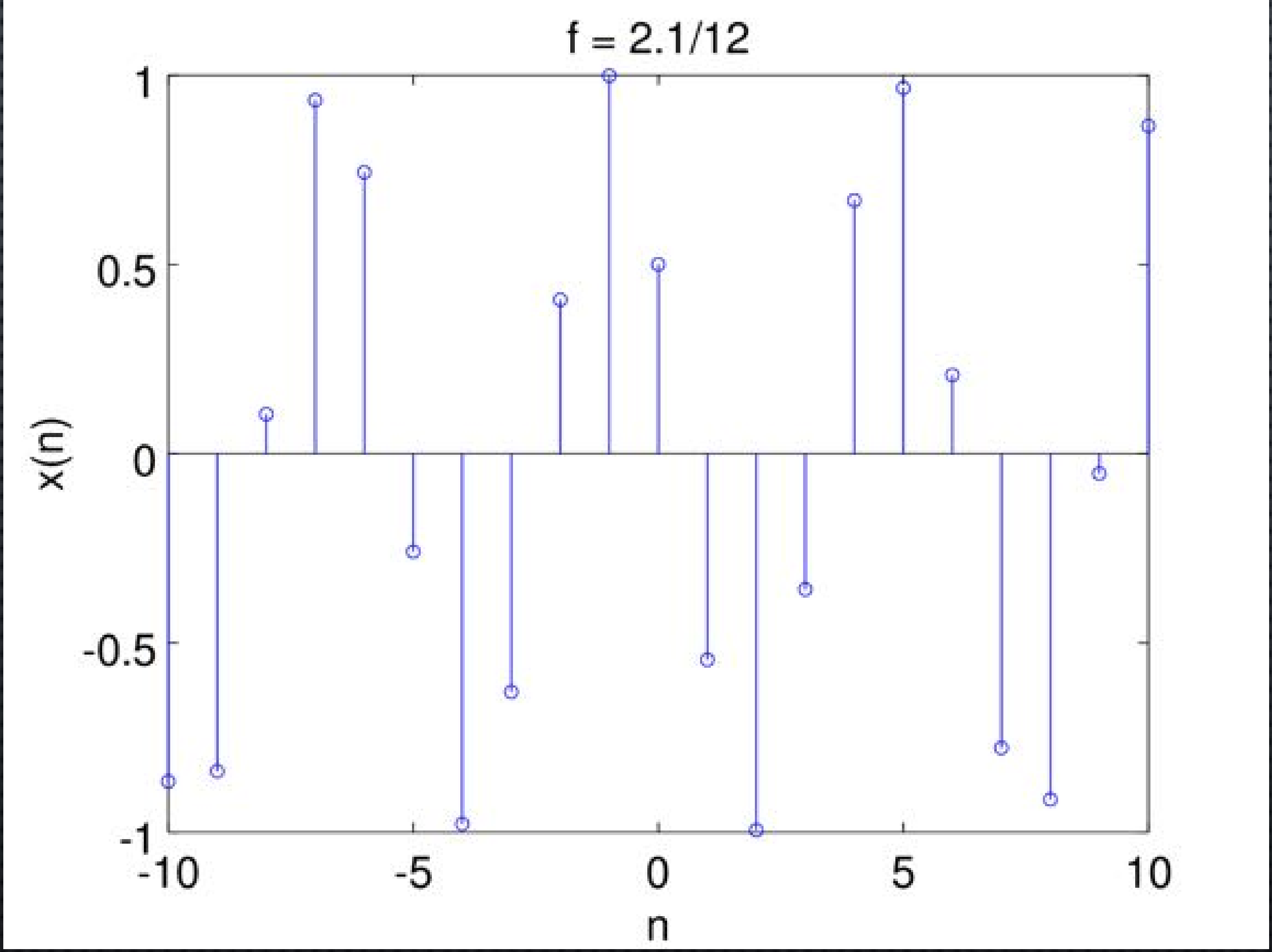
$\omega = 2\pi \cdot f$

f here is cycles/sample

Properties of discrete time signals

- A discrete time signal is periodic if f is a rational number
- Discrete time signals whose frequencies are separated by an integer multiple of 2π are identical
- The highest rate of oscillation is when $\omega = \pi$


f = 2/12
1
0.5
0
-0.5
-1
x(n)
-10
-5
0
5
10
n

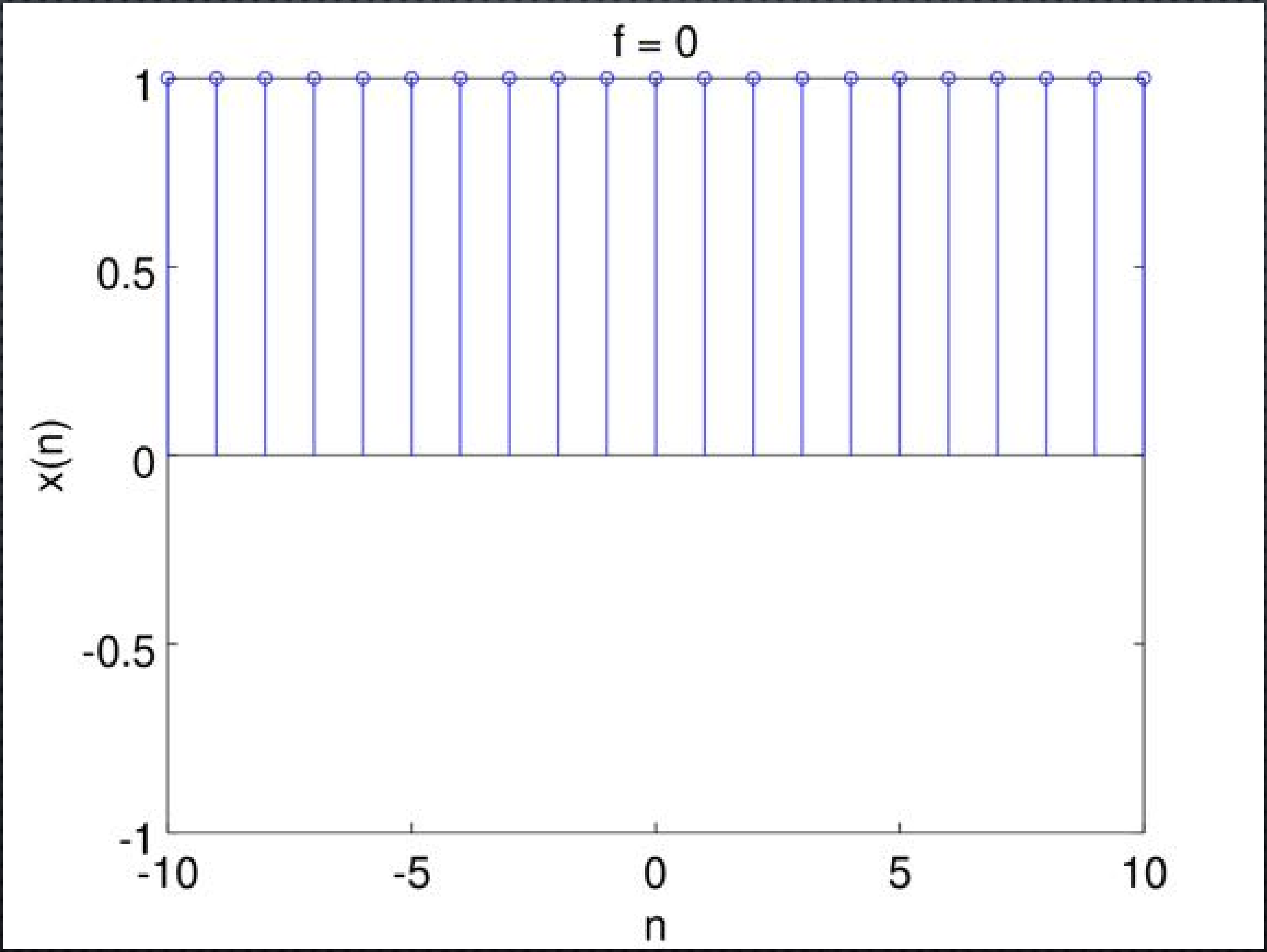

f = 2.1/12
x(n)
n
1
0.5
0
-0.5
-1
-10
-5
0
5
10

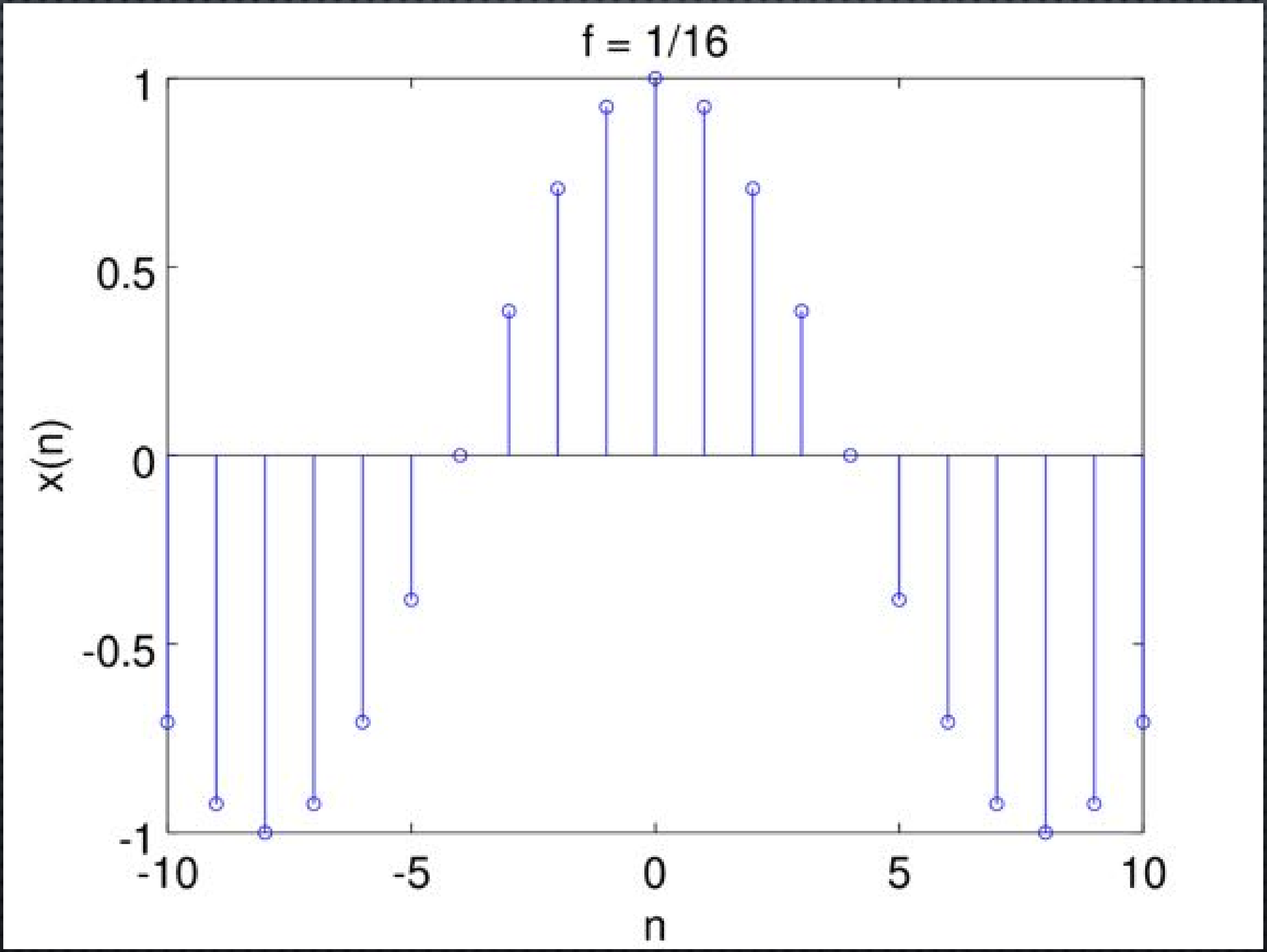
Properties of discrete time signals

- A discrete time signal is periodic if *f* is a rational number
- Discrete time signals whose frequencies are separated by an integer multiple of 2π are identical
- The highest rate of oscillation is when $\omega = \pi$.

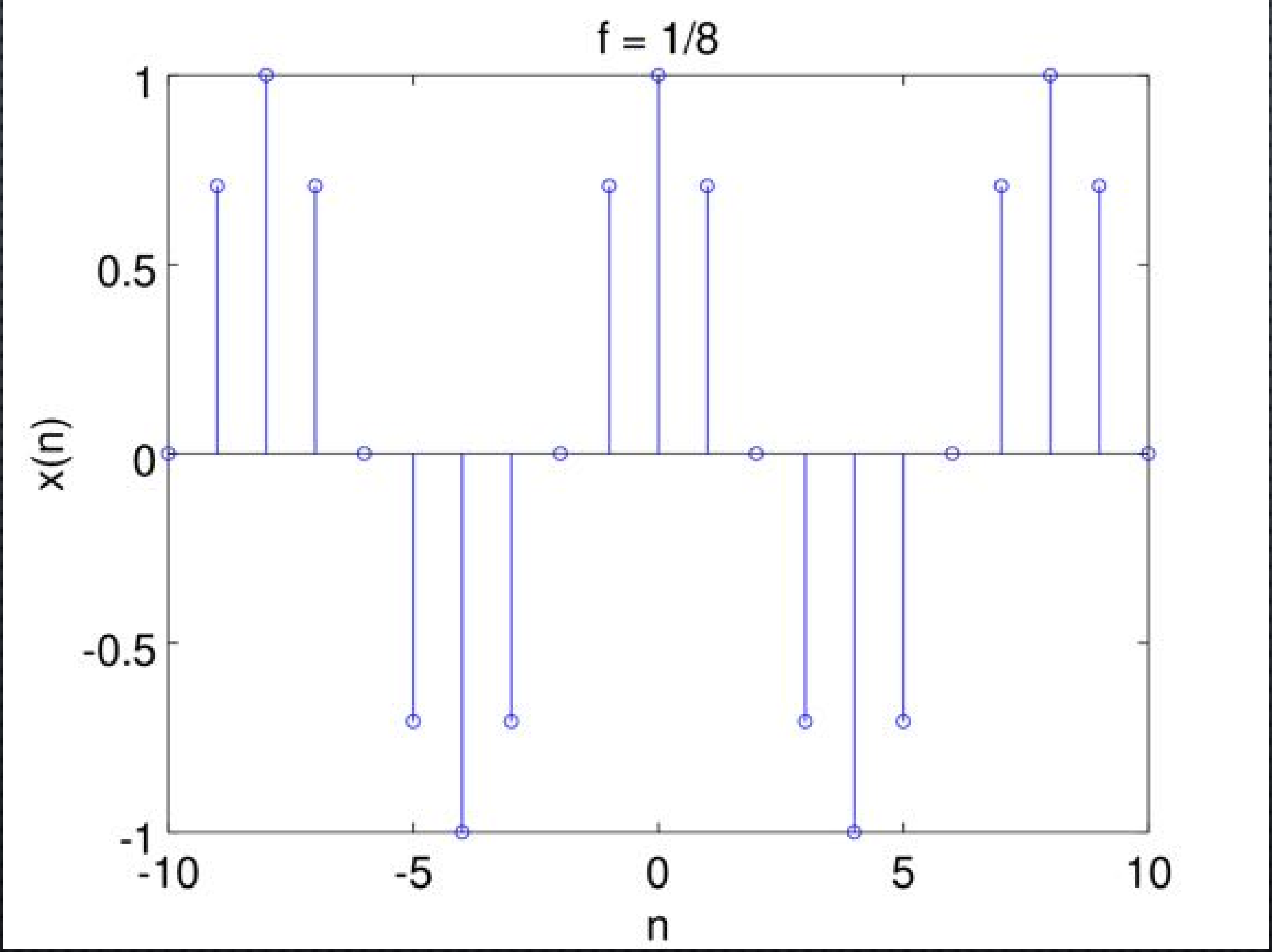
Properties of discrete time signals

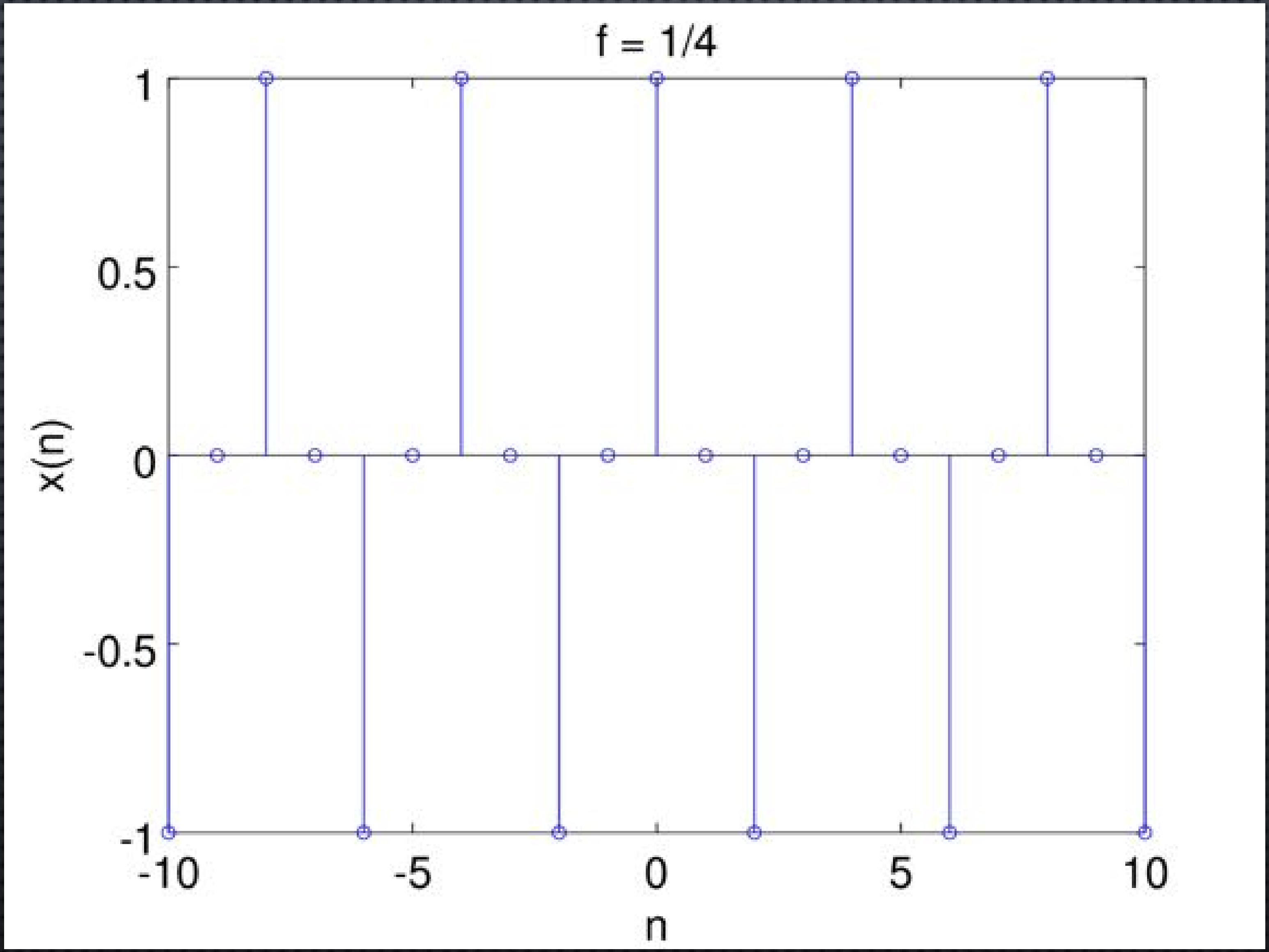
- A discrete time signal is periodic if f is a rational number.
- Discrete time signals whose frequencies are separated by an integer multiple of 2π are identical.
- The highest rate of oscillation is when $\omega = \pi$ or $f = \frac{1}{2}$

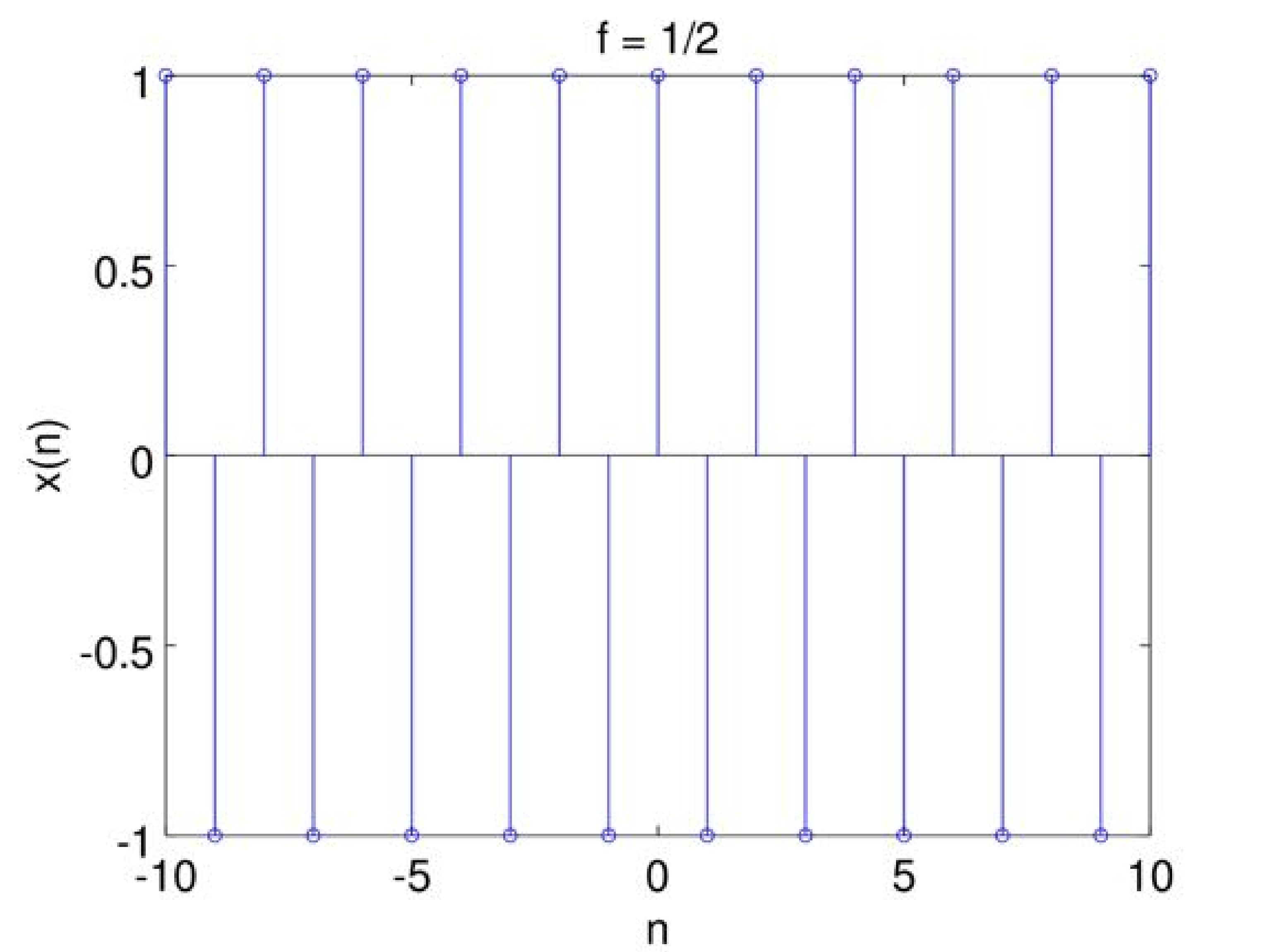

f = 0
1
0.5
0
-0.5
-1
x(n)
-10
-5
0
5
10
n



f = 1/16
x(n)
n
1
0.5
0
-0.5
-1
-10
-5
0
5
10


f = 1/8
1
0.5
0
-0.5
-1
x(n)
-10
-5
0
5
10
n


f = 1/4
1
0.5
0
-0.5
-1
x(n)
-10
-5
0
5
10
n


f = 1/2
x(n)
n
1
0.5
0
-0.5
-1
-10
-5
0
5
10

Properties of discrete time signals

- What happens between $\pi \leq \omega \leq 2\pi$?

- This is called aliasing…
  - If the angular frequency of a discrete time signal increased from π to 2π, its rate of oscillation will decrease!

Summary of key points

- For a continuous signal x(t), we can find the value of x at any point in time

- Discrete time signals are defined only at specific values of time

Summary of key points

Continuous signal	Discrete signal
For every fixed value of *f*, x(t) is periodic	A discrete time signal is periodic if *f* is a rational number
Continuous signals with distinct frequencies are themselves distinct	Discrete time signals whose frequencies are separated by an integer multiple of 2π are identical
Increasing *f* results in an increase in the rate of oscillation. As *t* is continuous, we can increase *f* without limit	The highest rate of oscillation is when $\omega = \pi$