LING 490 - SPECIAL TOPICS IN LINGUISTICS

Fundamentals of Digital Signal Processing

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Week 12

Last week…

- Another property of LTI system: Sinusoids are only changed in amplitude & phase as they are passed through LTI systems
- In the frequency domain, LTI can be characterised by its *transfer function*
  - The amplitude response and phase response
- The amplitude response can be expressed as the ratio between the output and the input

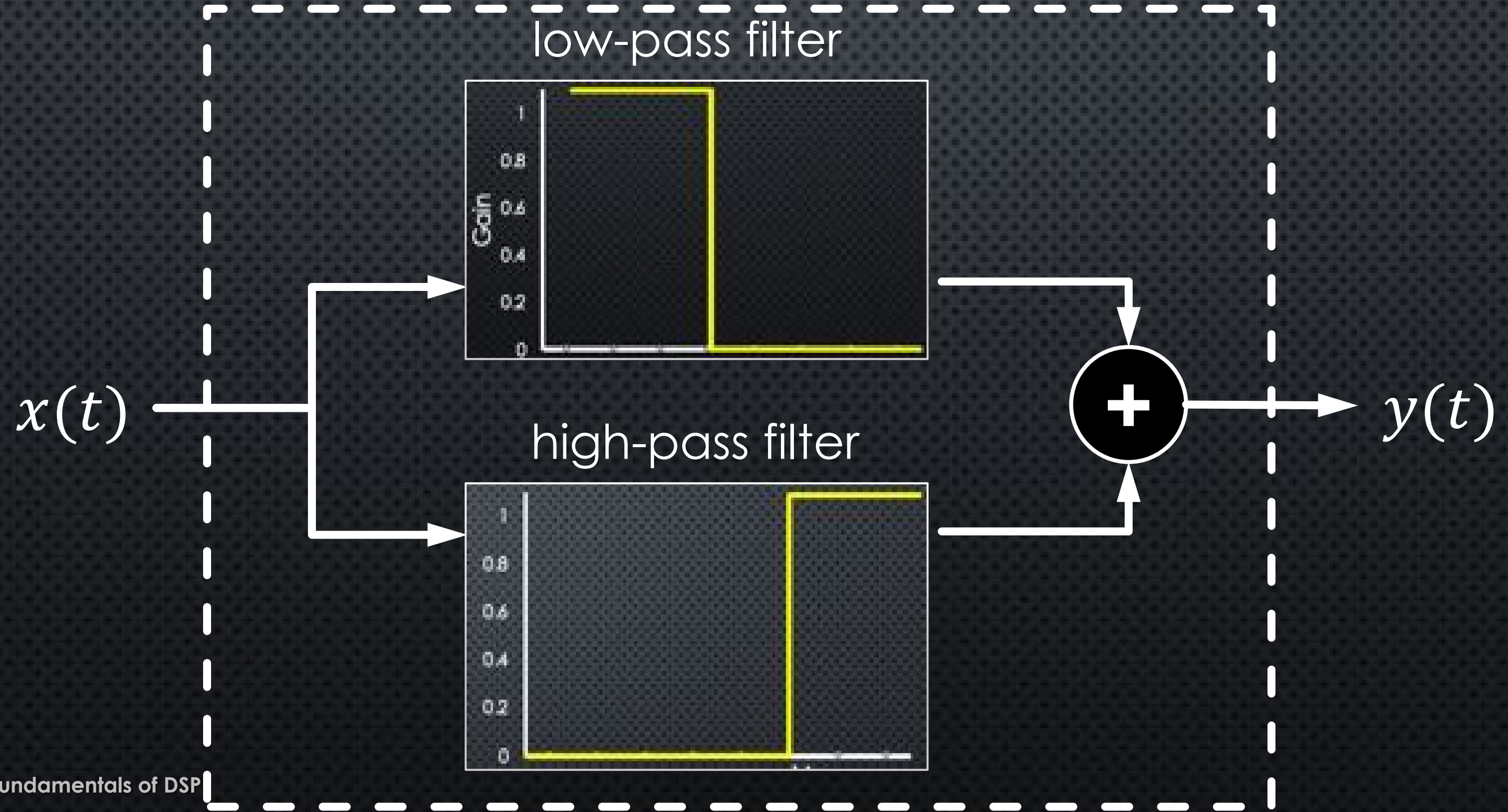
$$R(f) = \frac{A_o(f)}{A_I(f)}$$

Last week…

- Filter: systems which let some band of frequencies pass better than others
- Two basic filters:
  - Low-pass filter
  - High-pass filter
- Filter specifications:
  - Cut-off frequency
  - Roll-off
- Filters in combination:
  - Band-stop filter
  - Band-pass filter : band width

The frequency response of a parallel LTI system

- A system formed by any number of LTI systems in ***parallel*** is an LTI system, e.g. band-stop filter



The frequency response of a parallel LTI system

$\because R^{LP}(f) = \dfrac{A_O^{LP}(f)}{A_I^{LP}(f)}$

$\therefore A_O^{LP}(f) = R^{LP}(f) \cdot A_I^{LP}(f)$

$\because R^{HP}(f) = \dfrac{A_O^{HP}(f)}{A_I^{HP}(f)}$

$\therefore A_O^{HP}(f) = R^{HP}(f) \cdot A_I^{HP}(f)$

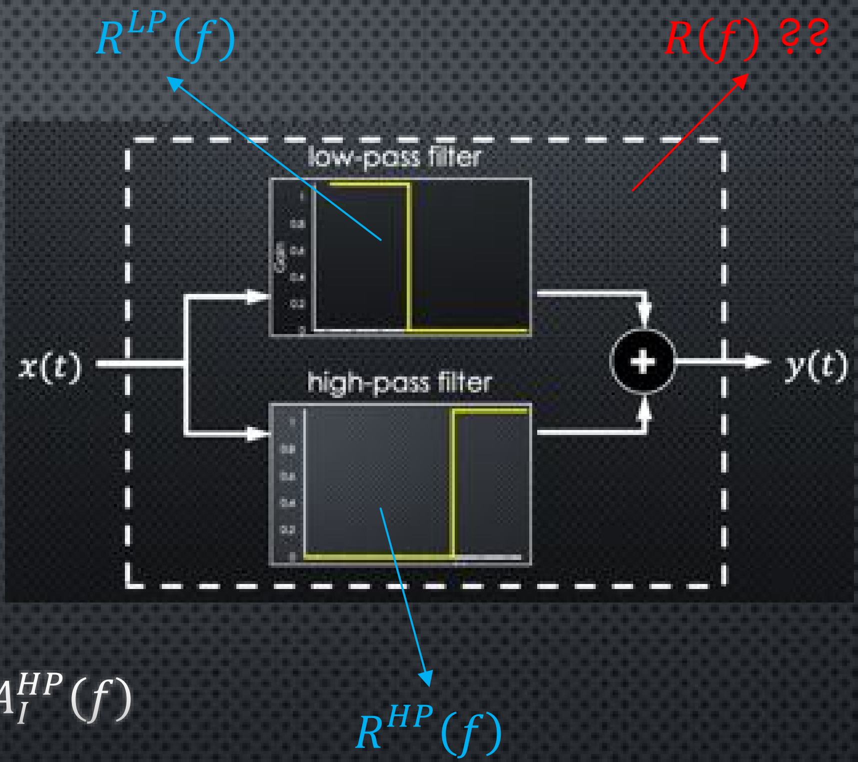
$\because A_O(f) = A_O^{LP}(f) + A_O^{HP}(f)$

$\therefore A_O(f) = R^{LP}(f) \cdot A_I^{LP}(f) + R^{HP}(f) \cdot A_I^{HP}(f)$

$\because A_I^{LP}(f) = A_I^{HP}(f) = A_I(f)$

$\therefore A_O(f) = (R^{LP}(f) + R^{HP}(f)) \cdot A_I(f)$

$$R^{LP}(f) + R^{HP}(f) = \frac{A_O(f)}{A_I(f)} = R(f)$$

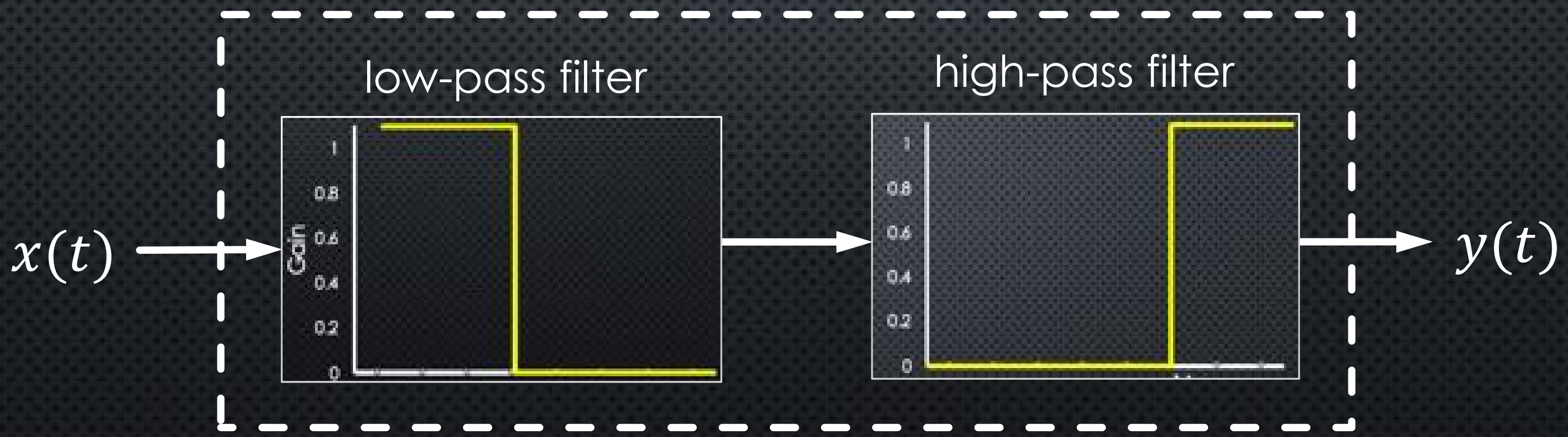


$$R(f) = R_1(f) + R_2(f) + \cdots + R_n(f)$$

$$R(f) = R^{HP}(f) + R^{LP}(f)$$

The frequency response of a chain LTI system

- A system formed by any number of LTI systems in cascade is also an LTI system, e.g. band-pass filter



The frequency response of a chain LTI system

$R(f)$??

$R^{LP}(f)$

$R^{HP}(f)$

$$\because R^{LP}(f) = \frac{A_O^{LP}(f)}{A_I^{LP}(f)}$$

$$\therefore A_O^{LP}(f) = R^{LP}(f) \cdot A_I^{LP}(f)$$

$$\because R^{HP}(f) = \frac{A_O^{HP}(f)}{\boxed{A_I^{HP}(f)}}$$

$$\because A_I^{HP}(f) = A_O^{LP}(f) = \boxed{R^{LP}(f) \cdot A_I^{LP}(f)}$$

$$\therefore R^{HP}(f) = \frac{A_O^{HP}(f)}{R^{LP}(f) \cdot A_I^{LP}(f)}$$

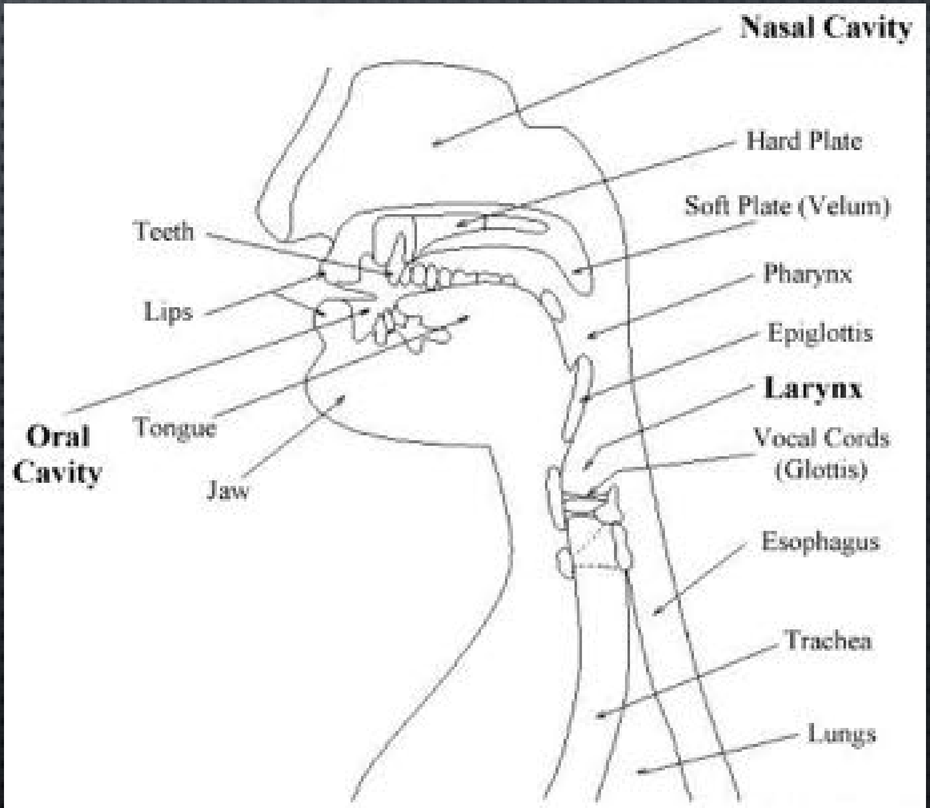
$$R^{HP}(f) \cdot R^{LP}(f) = \frac{A_O^{HP}(f)}{A_I^{LP}(f)} = R(f)$$

$$R(f) = R_1(f) \cdot R_2(f) \cdot \cdots \cdot R_n(f)$$

$$R(f) = R^{HP}(f) \cdot R^{LP}(f)$$

A real-life LTI system: the vocal tract as a chain LTI

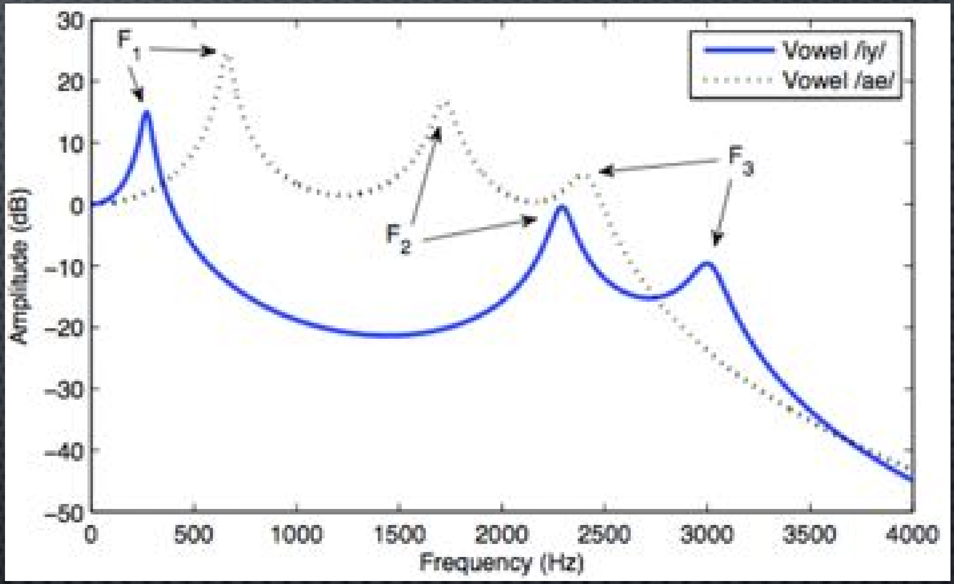
- The vocal tract is an acoustic system comprising a tube of varying width
- Frequency response for the vocal tract depends on its shape
- The shape of vocal tract can change
  - Different frequency responses
  - Different sounds!



A real-life LTI system: the vocal tract as a chain LTI

- While changing the shape, some frequency regions are enhanced while elsewhere is suppressed

- Frequency response of the vocal tract can be characterised by a set of resonances – formants



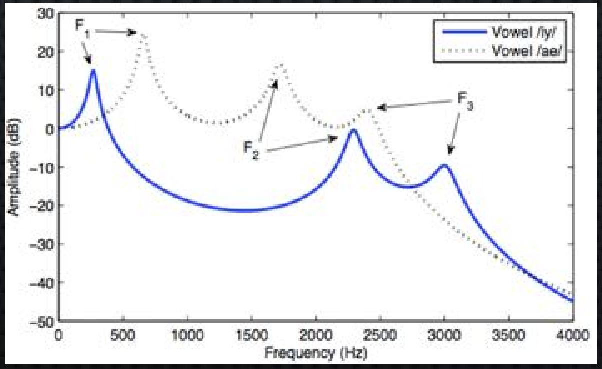
Two specifications of formants:

- Formant frequency
- Half-power bandwidth

A real-life LTI system: the vocal tract as a chain LTI

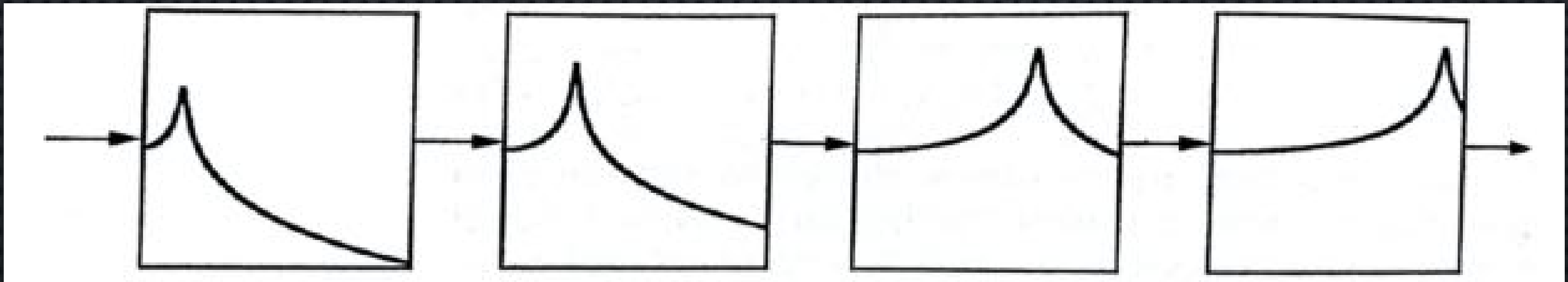
- Roll-off is hardly used to describe formants:
  - The effect of formants are mixed – no clear roll-off
  - The roll-offs of a clearly-distinguishable formants can be predicted from model

- The entire frequency response is predictable from the formant frequencies and their bandwidth



A real-life LTI system: the vocal tract as a chain LTI

- Speech synthesisers generate vowel sounds by creating different frequency responses for their vocal tract models
  - One solution: cascade a series of band-pass filters, each of which has the frequency response of a resonance

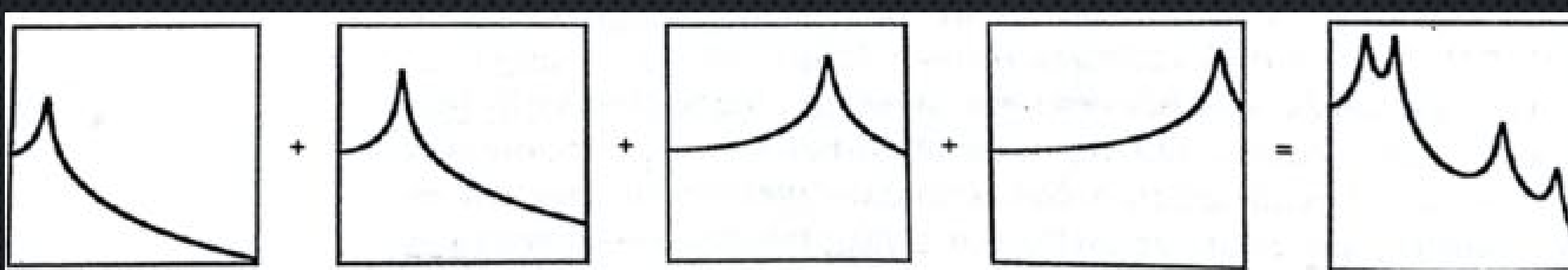


A real-life LTI system: the vocal tract as a chain LTI

- Example: synthesis of the vowel /ɔ/ as in "lot"
  - The formant frequencies: 600, 1000. 2700 and 3600
  - The bandwidths: 80, 60, 90 and 100 Hz

$$R(f) = R_1\ (f) \cdot R_2\ \ (f) \cdot \cdots \cdot R_n\ \ (f)$$

$\log(R^{600})$ $\log(R^{1000})$ $\log(R^{2700})$ $\log(R^{3600})$ $\log(R)$



Phase response of LTI systems

- Recall that an LTI system can only change sinusoid in *amplitude* and *phase*
- Changes in amplitude across frequency are more perceptually salient than in phase
- Changes in phase may cause large changes on waveform
- The phase change due to a system is frequency-specific
- **Phase response** of an LTI: the curve describing the phase change as a function of frequency

Phase response of LTI systems

- Given the phases of the input and output sinusoids, the phase response of the system $D(f)$ is,

$$D(f) = P_O(f) - P_I(f)$$

P_i: phase of input

P_O: phase of output

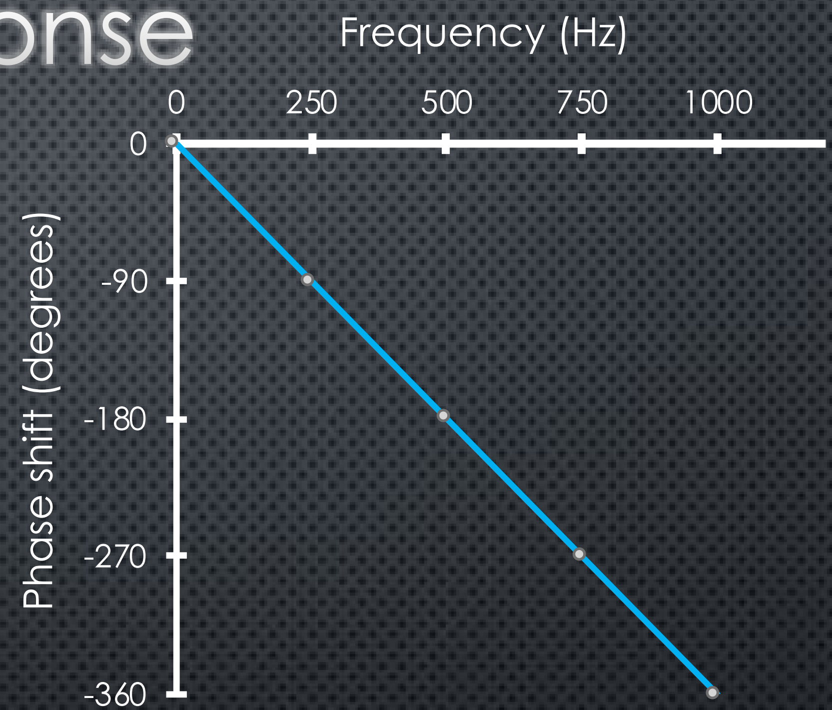
- The output phase of a system can be calculated as,

$$P_O(f) = P_I(f) + D(f)$$

- Regardless of the input phase, $D(f)$ holds constant
  - A shift in phase for sinusoids is equal to a shift in time
  - Time invariance of an LTI

Linear phase response

- The effect of an LTI with a linear phase response
  - All sinusoids going through the system are delayed by the same amount of time



- $\Delta t(f) = \frac{D(f)}{360} \cdot \frac{1}{f}$

f: linear frequency

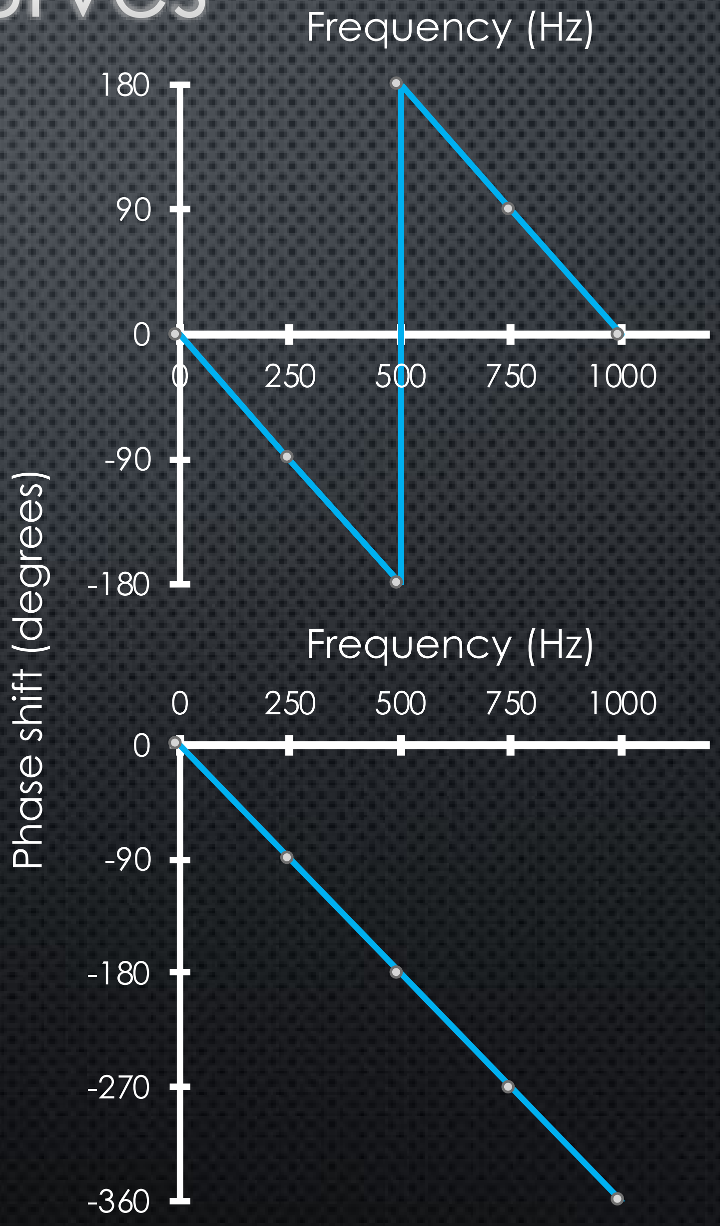
$$D(f) = k \cdot f$$

$$\Delta t(f) = \frac{k \cdot f}{360} \cdot \frac{1}{f} = \frac{k}{360}$$

$$\Delta t = \frac{k}{360}$$

Wrapping phase curves

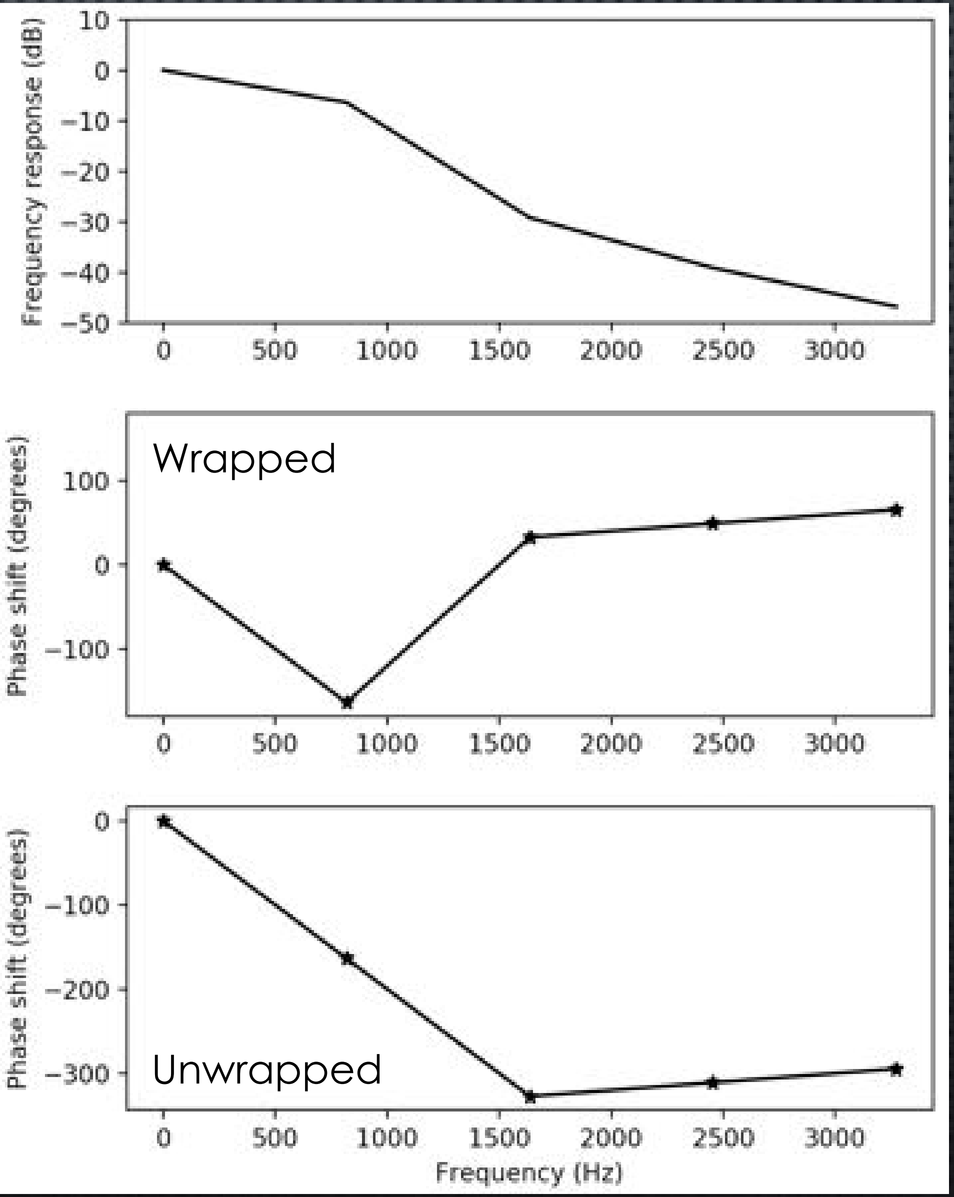
- Phase response curves often show range for 360° or 2π
  - -180 ° to 180 ° ($-\pi$ to π)
  - -360 ° to 0 ° (-2π to 0)
  - 0 ° to 360 ° (0 to 2π)
- Discontinuous phase response curve does not indicate any discontinuity of the phase response of a system
- Converting smooth phase curves to discontinuous is **phase wrapping**; the inverse operations is **phase unwrapping**



Other phase responses

- An example of a 10^{th}-order Finite Impulse Response low-pass filter
  - Cut-off: 40 Hz

- An idealised filter has linear phase response to preserve the shape of waveform
  - Most of practical systems do not!



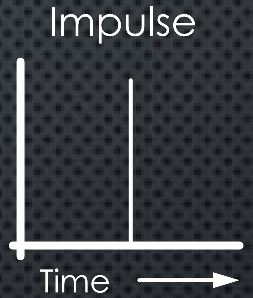
The relationship between impulse response and frequency response

- Two ways to characterise LTI system
  - Frequency domain: frequency response
  - Time domain: impulse response

- The *frequency response* and *impulse response* provide the same information about the system – two different ways of looking at the same thing

The relationship between impulse response and frequency response

- In the time domain, when an impulse is the input of an LTI, the output of the system is the impulse response of the system, H



- In the frequency domain:

$$A_O(f) = R(f) \cdot A_I(f)$$

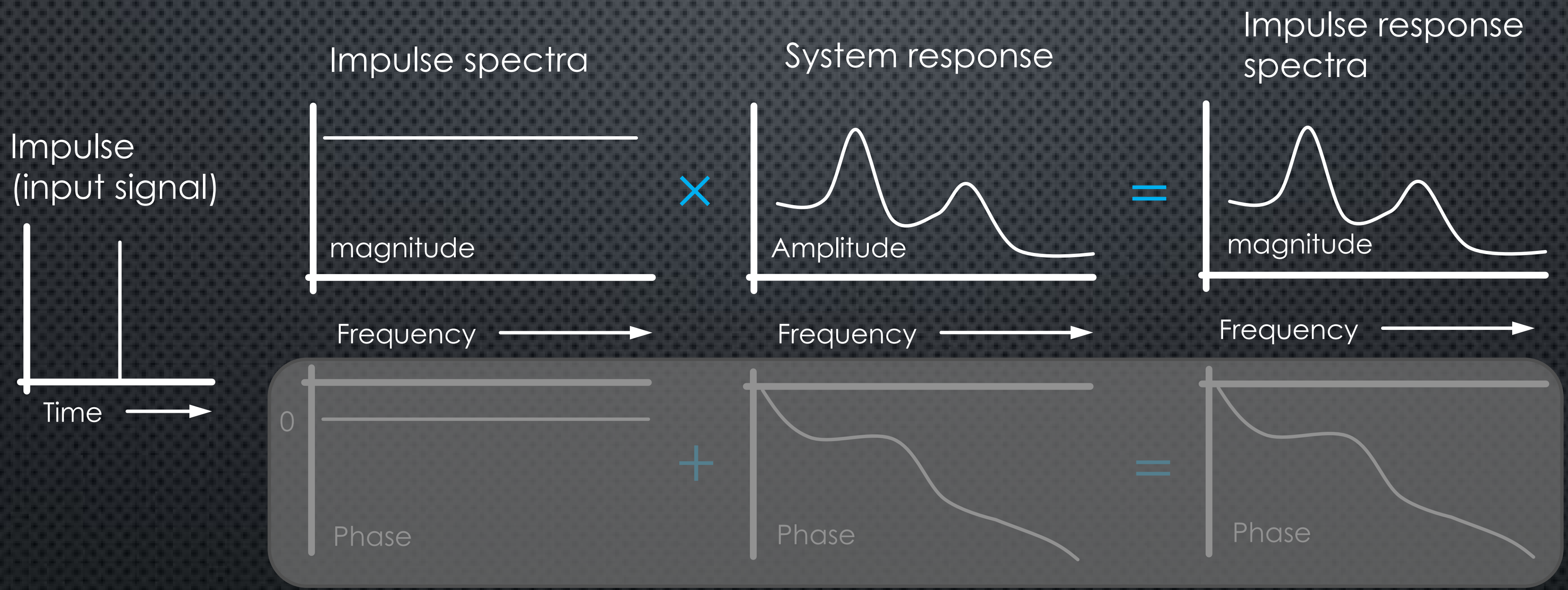
$$\because A_I(f) = k, \forall f$$

An impulse has the same amount of energy at all frequencies!!

$$\therefore A_O(f) = R(f) \cdot k$$

- The *amplitude spectrum of an impulse response* of a system is the *frequency response* of the system!!!

The relationship between impulse response and frequency response



From frequency response to impulse response

- Systems in parallel:
  - The frequency domain: $R(f) = R_1\ (f) + R_2\ (f) + \cdots + R_n\ (f)$
  - The time domain: $H(t) = H_1\ (t) + H_2\ (t) + \cdots + H_n\ (t)$

- Systems in cascade:
  - The frequency domain: $R(f) = R_1\ (f) \cdot R_2\ (f) \cdot \cdots \cdot R_n\ (f)$
  - The time domain: $H(t) = H_1\ (t) \otimes H_2\ (t) \otimes \ldots \otimes H_n\ (t)$