LING 490 - SPECIAL TOPICS IN LINGUISTICS

Fundamentals of Digital Signal Processing

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Week 11

Last week…

- Time and frequency resolution
  - Frequency resolution is proportional to the inverse of time resolution, and vice versa
- Spectrogram: short-term spectra over time
- Windowing: analyse signal in small segments
  - The size/length of window
  - Overlap between two adjacent windows
  - Window function
- Implementation of spectrogram

More about LTI systems

- A surprising property of a LTI system
  - Knowing its response to ***sinusoidal*** signals allows predictions on its response to ***any*** input

- The property above arises from two factors:
  - The response of LTI systems to sinusoids
  - The possibility of expressing all signals as a sum of sinusoids of the appropriate amplitude, frequency and phase (Fourier's theorem)

More about LTI systems

Three important concepts of LTI systems

- Sinusoids are only changed in amplitude & phase as they are processed through LTI systems - **homogeneity and time invariance**
- If the responses of an LTI system to sinusoids of all frequencies are known, the system output to any input signal can be expressed as a sum of sinusoids with different frequencies, amplitudes and phases – ***additivity and Fourier theorem***
- Any input signals can be expressed so

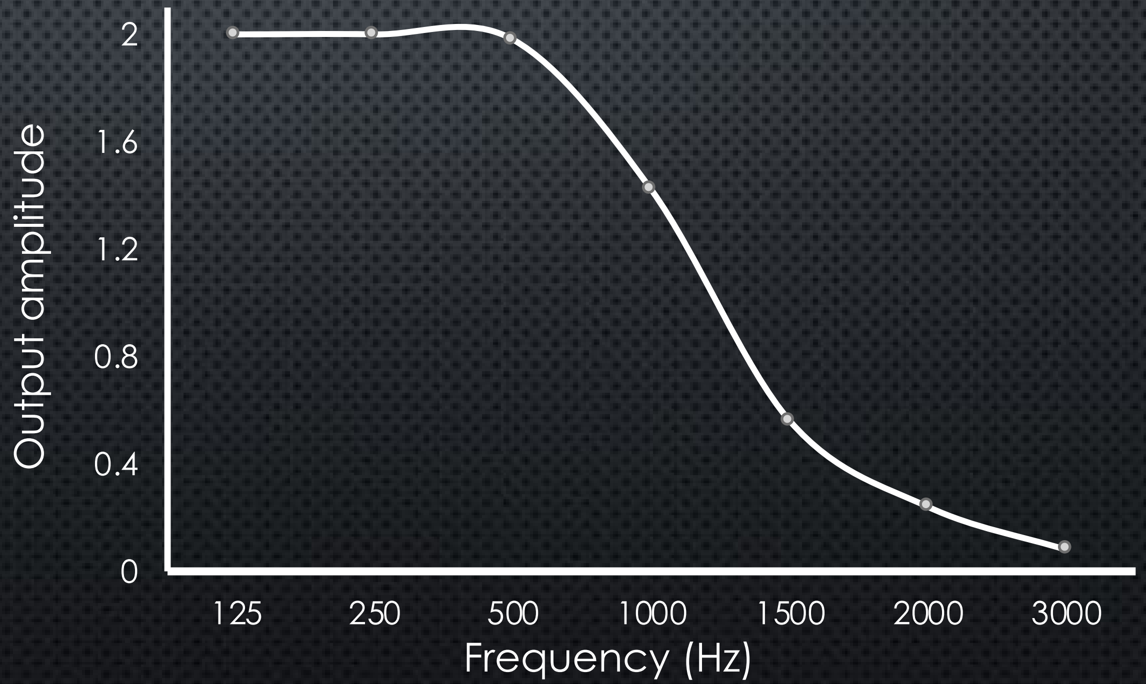
The frequency response of systems

- Characterisation of the way an LTI responds to sinusoid signals
  - Transfer function
- A transfer function may be considered to have two parts:
  - The amplitude response and phase response
- Mostly amplitude response is referred to as frequency response

The amplitude response of systems

Consider passing sinusoids with same amplitude (2) but different frequencies though an LTI system:

Frequency of input signal	Amplitude of output signal
125 Hz	2
250 Hz	2
500 Hz	1.98
1000 Hz	1.42
1500 Hz	0.56
2000 Hz	0.24
3000 Hz	0.08



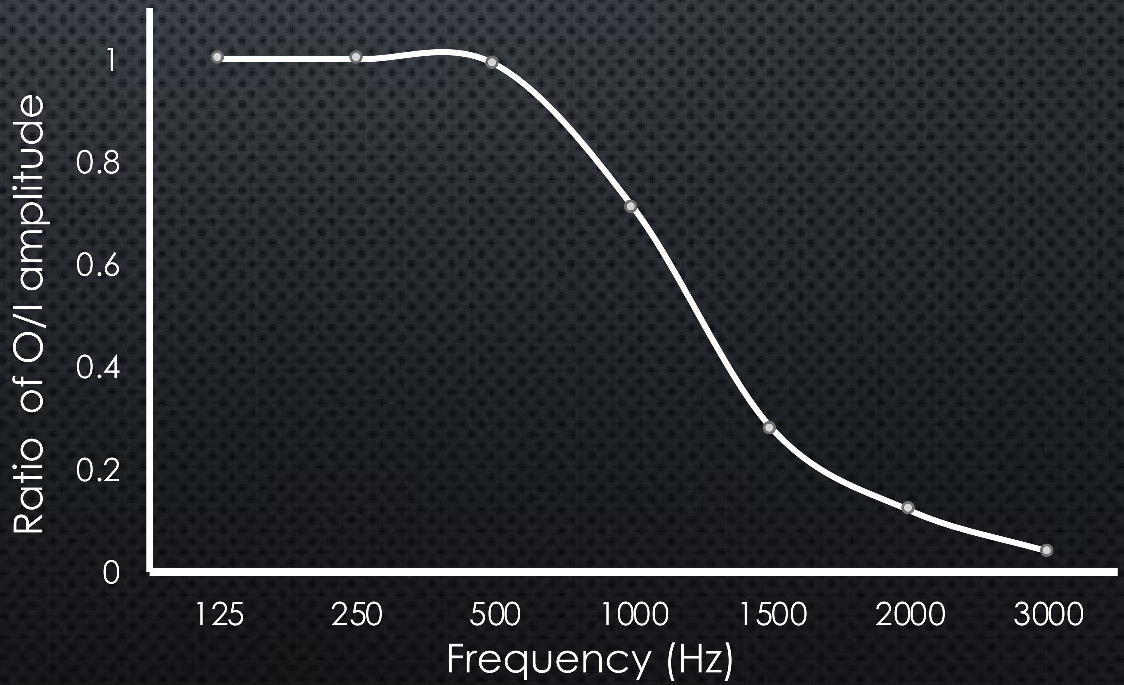
Amplitude response as ratio

Amplitude response can be normalised as such that it is dependent of input and output amplitude measured:

$$R(f) = \frac{A_o(f)}{A_I(f)}$$

Input amplitude = 2

Frequency of input signal	Ratio of O/I amplitude
125 Hz	1
250 Hz	1
500 Hz	0.99
1000 Hz	0.71
1500 Hz	0.28
2000 Hz	0.12
3000 Hz	0.04



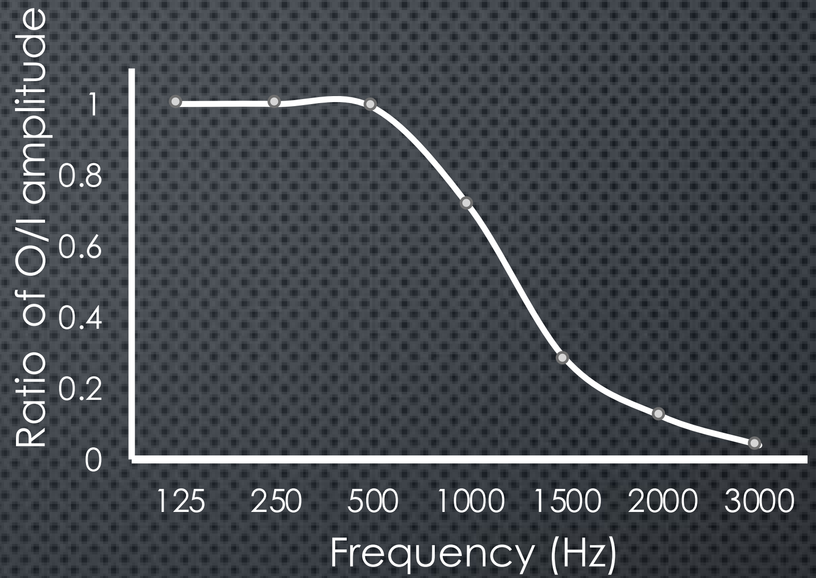
Amplitude response as ratio

- Amplitude response is then defined based on an input with an amplitude value of unity
- The output amplitude of a system can be calculated as

$$A_o(f) = R(f) \cdot A_I(f)$$

- This is no need of making measurements at the same input level

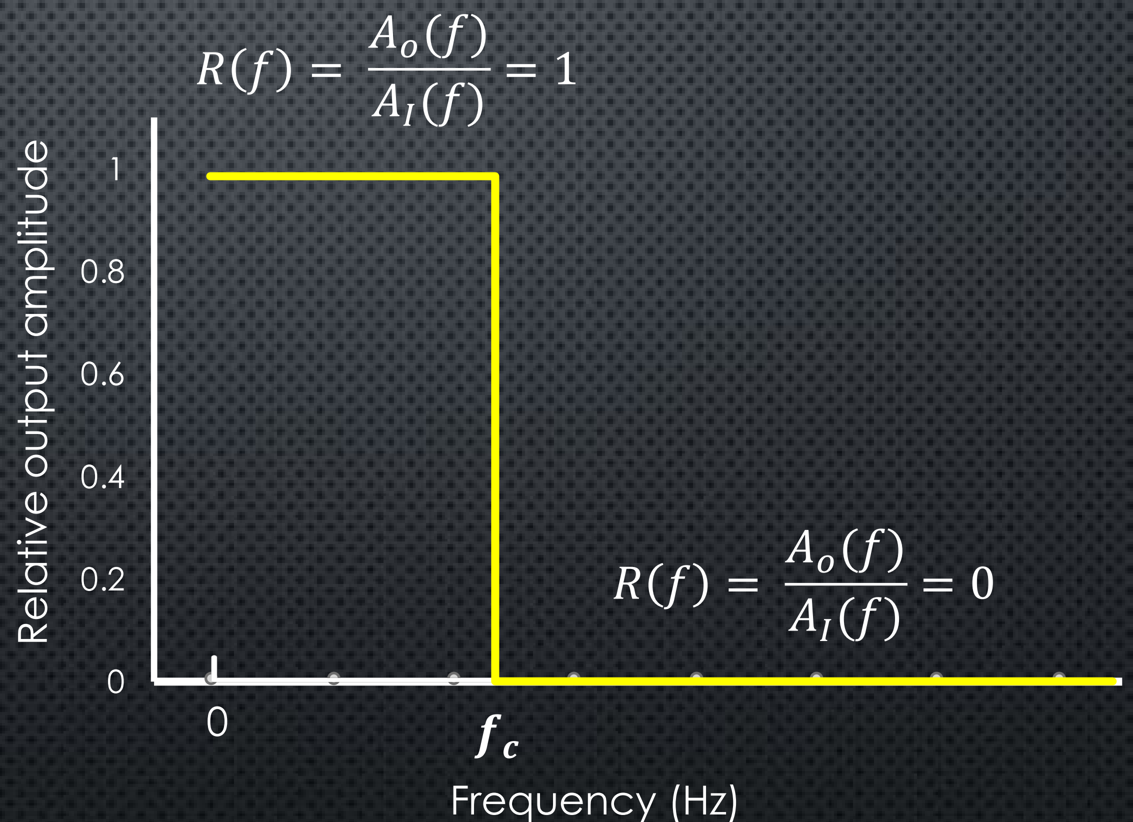
Filters



Filters: systems which let some band of frequencies pass better than others

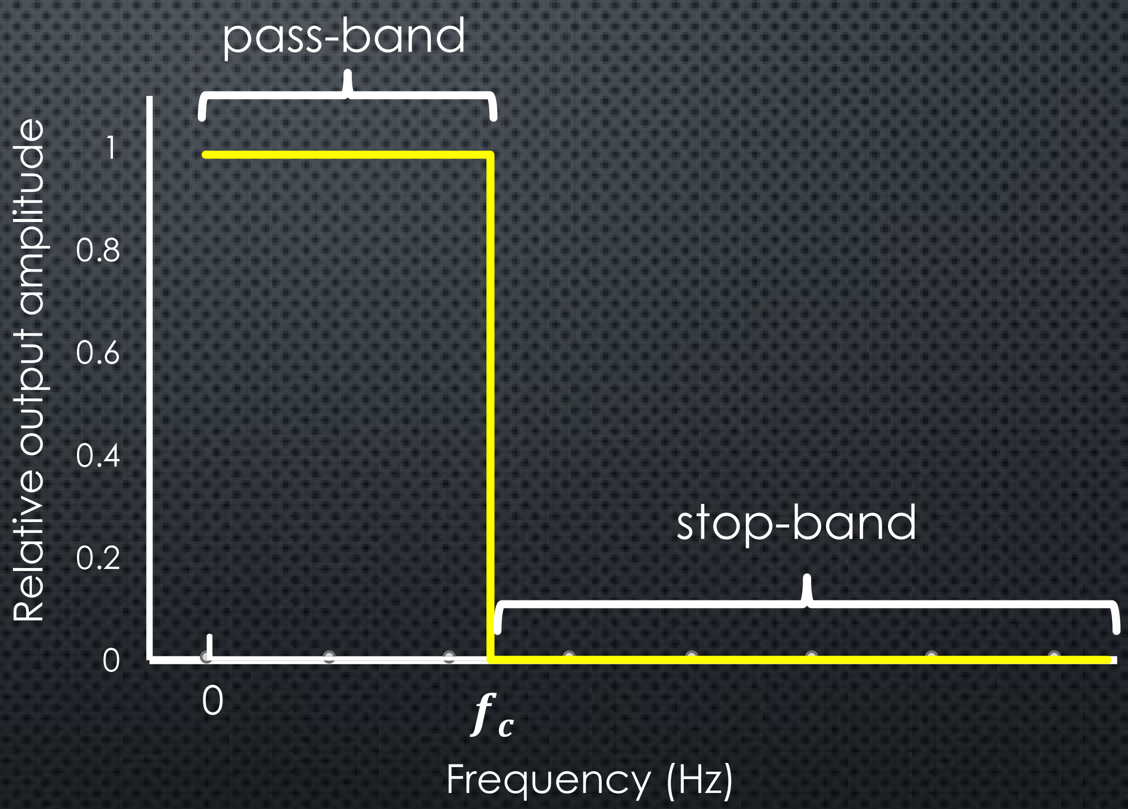
Low-pass filters

- Low-pass filters attenuate high-frequency sinusoids more than low-frequency sinusoids
- Cutoff frequency
  - The frequency location below/above which the filter passes all sinusoids



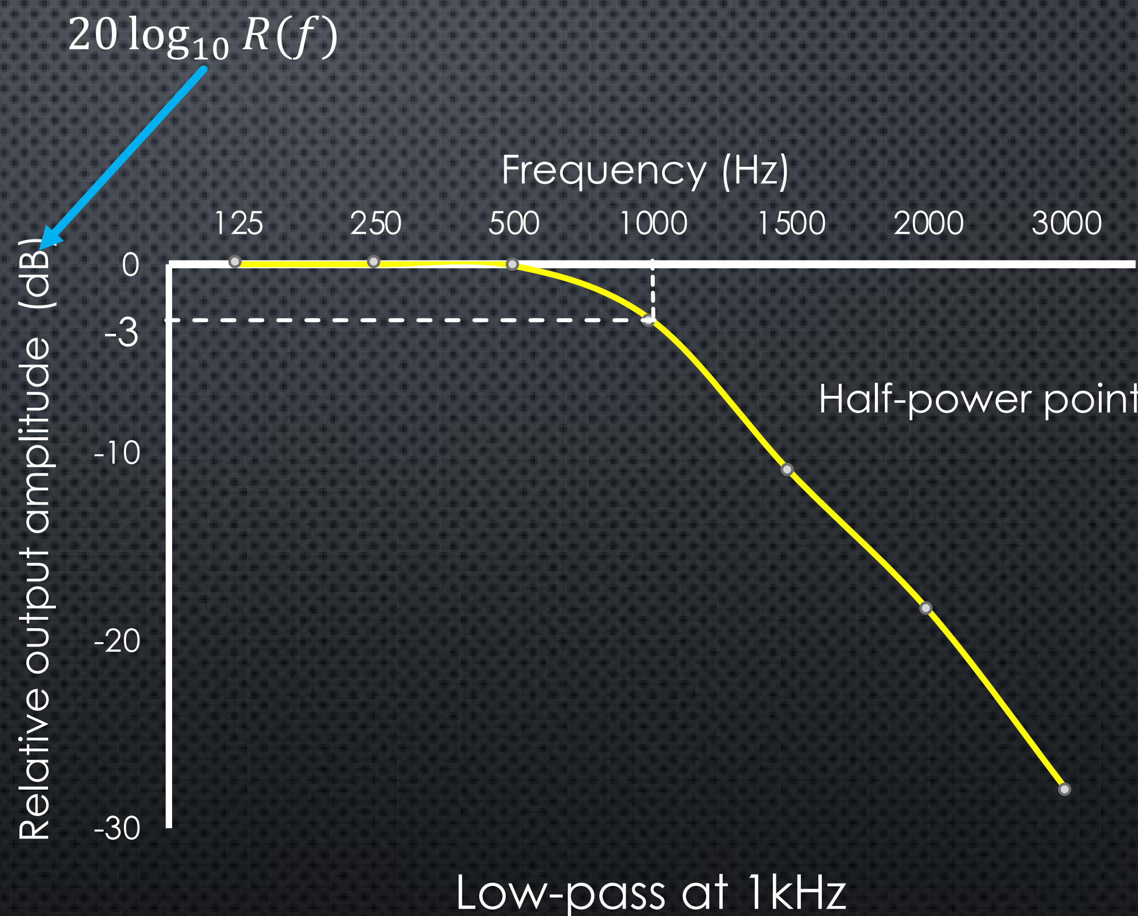
Low-pass filters: more terminologies

- Pass-band:
  - The frequency region over which the sinusoids can *pass* the system
- Stop-band:
  - The frequency region over which the sinusoids are *attenuated*

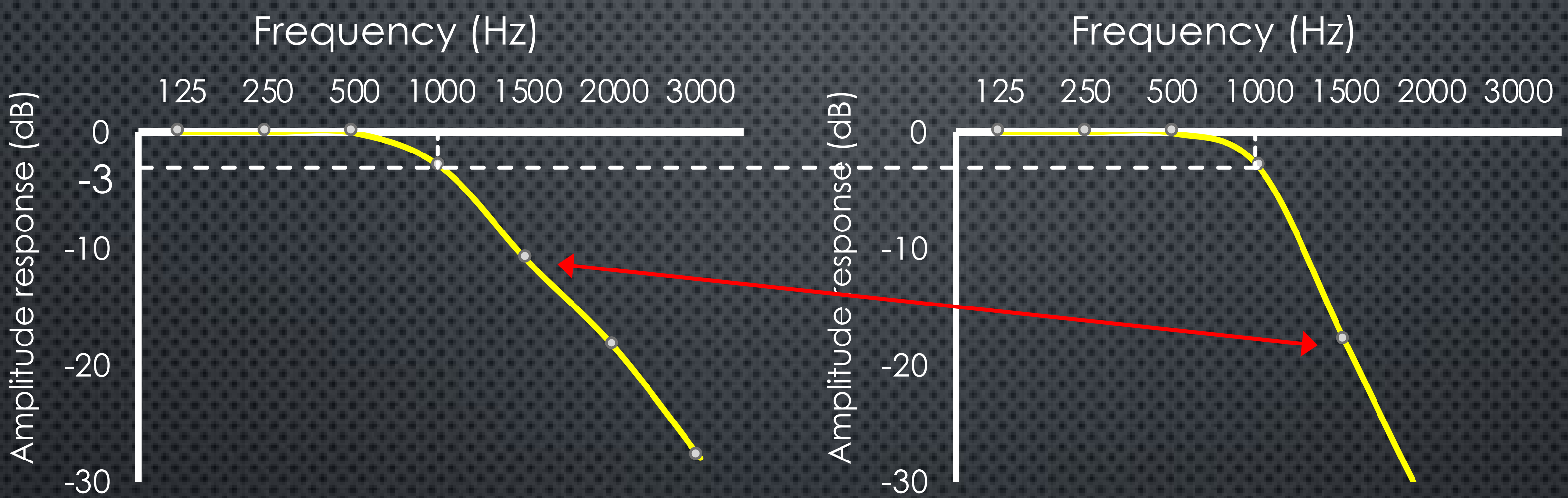


A real-life filter: half-power point

- Amplitude response in decibels
  - Signal is amplified, amplitude response **>** 0 dB
  - Signal is attenuated, amplitude response **<** 0 dB
- A realistic cut-off frequency – ***half-power point***
  - The frequency location at which the amplitude response is ***3 dB*** below the peak

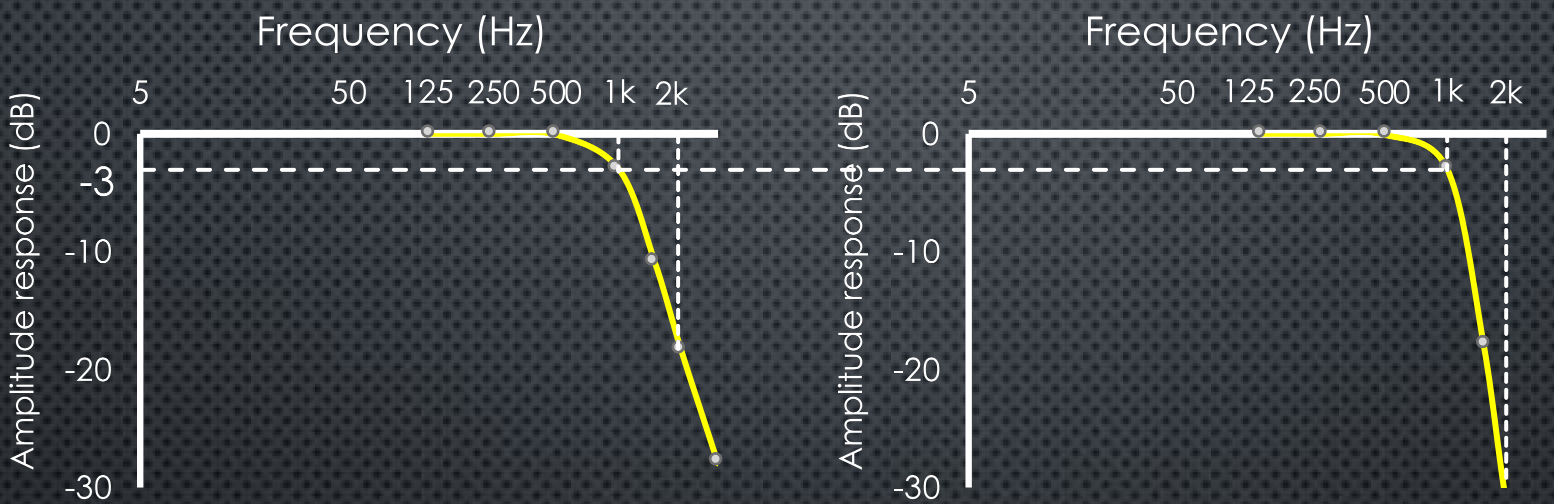


A real-life filter: roll-off



- ***Roll-off***: the sharpness with which a filter declines the stop-band
  - The less sharp the roll-off, he more the sinusoids with frequencies not in the pass-band of the filter will be represented in the output.

A real-life filter: the octave scale



- A way to quantify the slop of the roll-off of a filter
  - The decrease of amplitude response becomes almost linear
- The ***octave*** scale
  - A logarithmic scale for frequency axis
  - An octave is a doubling in frequency, e.g. 125, 250, 500, 1k, etc
  - Equal ratios between values taking up equal distance

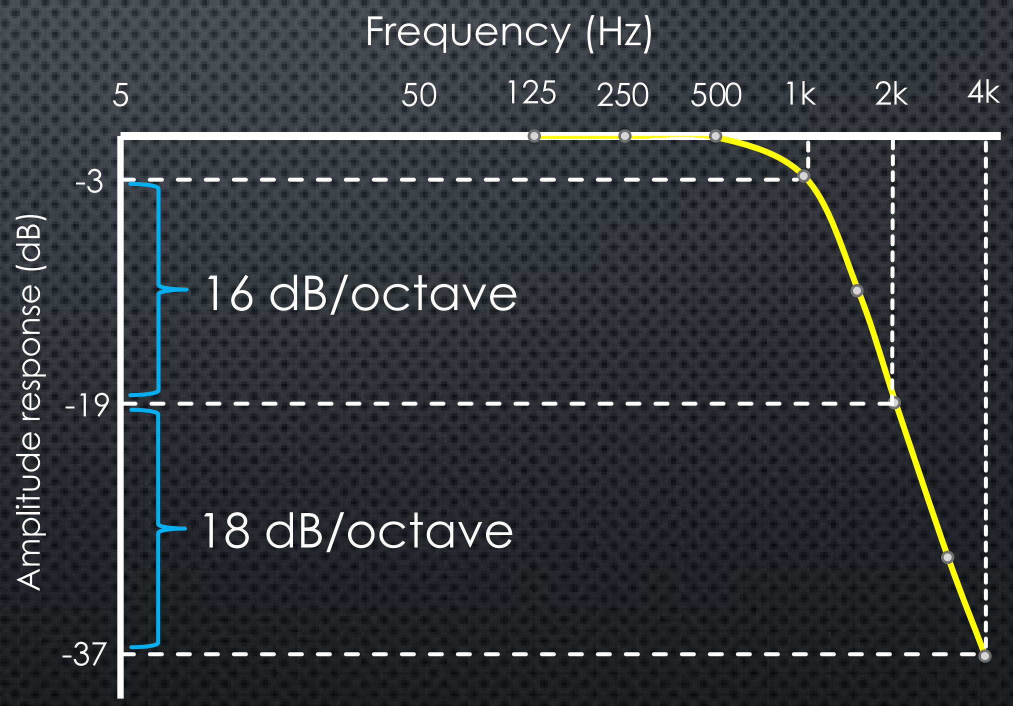
A real-life filter: the octave scale

- Difference between octave scales and the base-10 log scales
  - The ratio change that leads to a 1-unit logarithmic change
- Octave and base-10 log values can be mutually converted through multiplication by a constant:

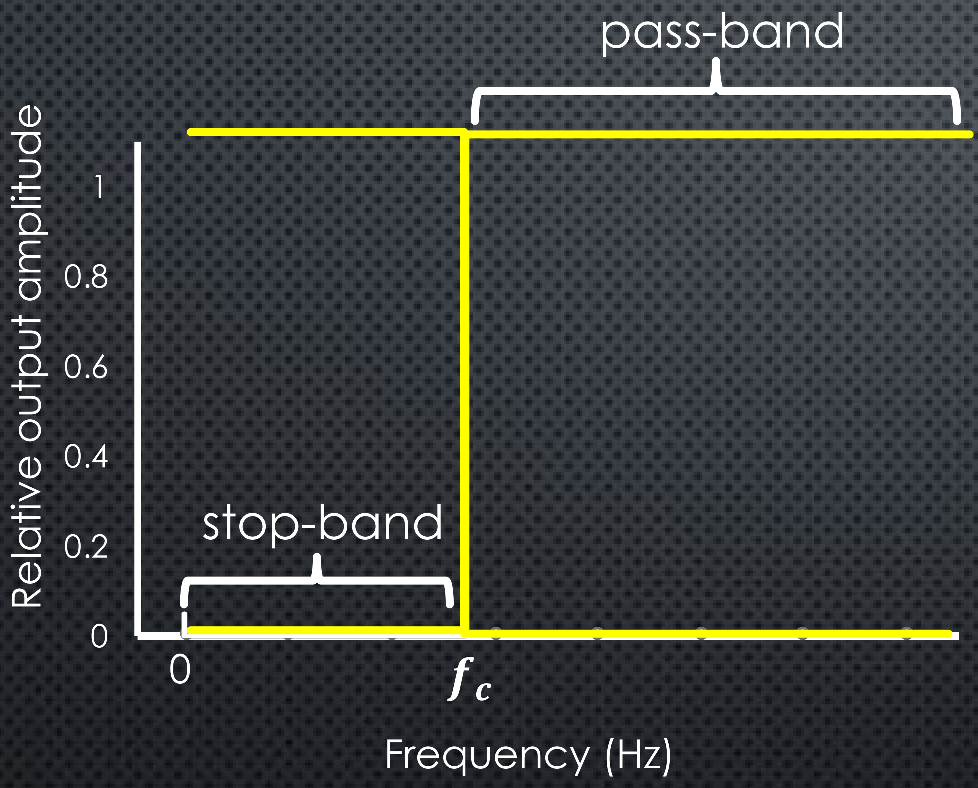
$$\log_2 x = \frac{\log_{10} x}{\log_{10} 2} \approx \frac{\log_{10} x}{0.301} \approx 3.32 \cdot \log_{10} x$$

A real-life filter: the octave scale

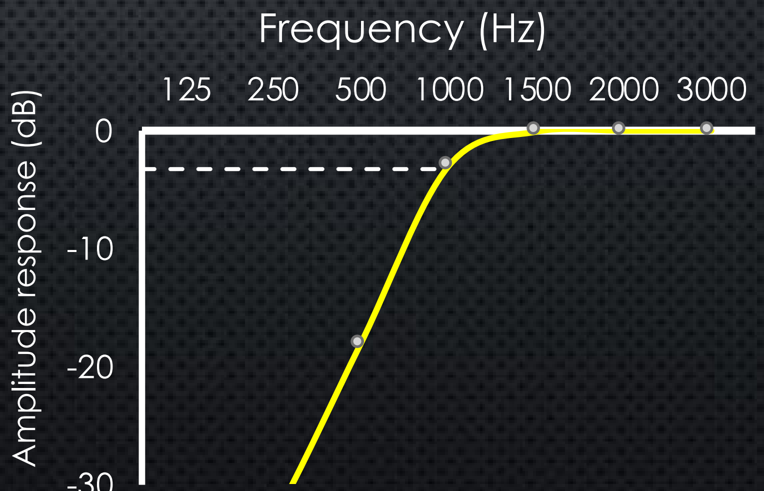
- Filter roll-offs are linear on dB vs octave frequency scales
- Filter roll-off is often expressed as dB/octave
- In practice,
  - slope < 18 dB/octave: shallow
  - 18 < slope 48: moderately seep
  - slop > 90: very steep



High-pass filters

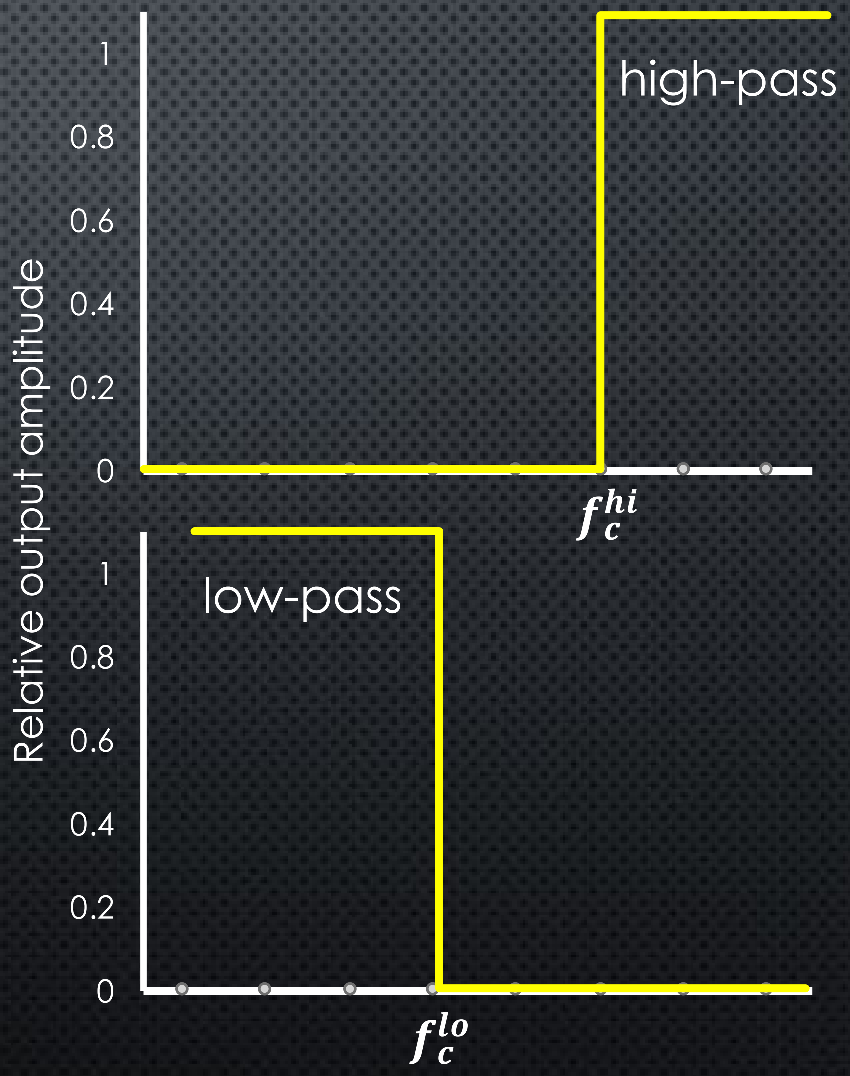


- High-pass filters only let sinusoids at or above certain frequency through

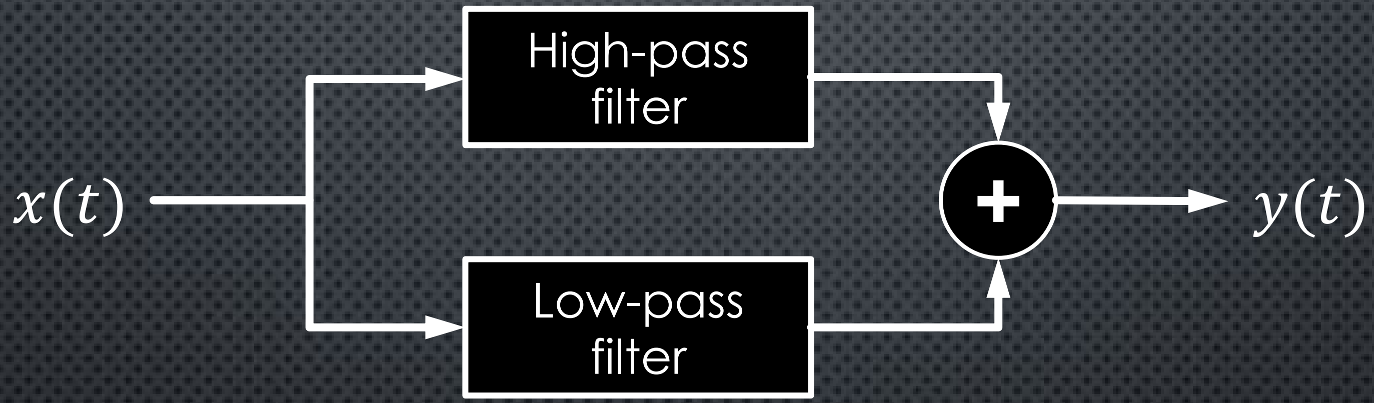


Band-stop filter: systems in parallel

- A **band-stop** filter can be thought of as a combination of one high-pass filter and one low-pass filter *in parallel* with *different* cutoff frequencies f_c^{hi} and f_c^{lo} respectively
  - $f_c^{hi} > f_c^{lo}$



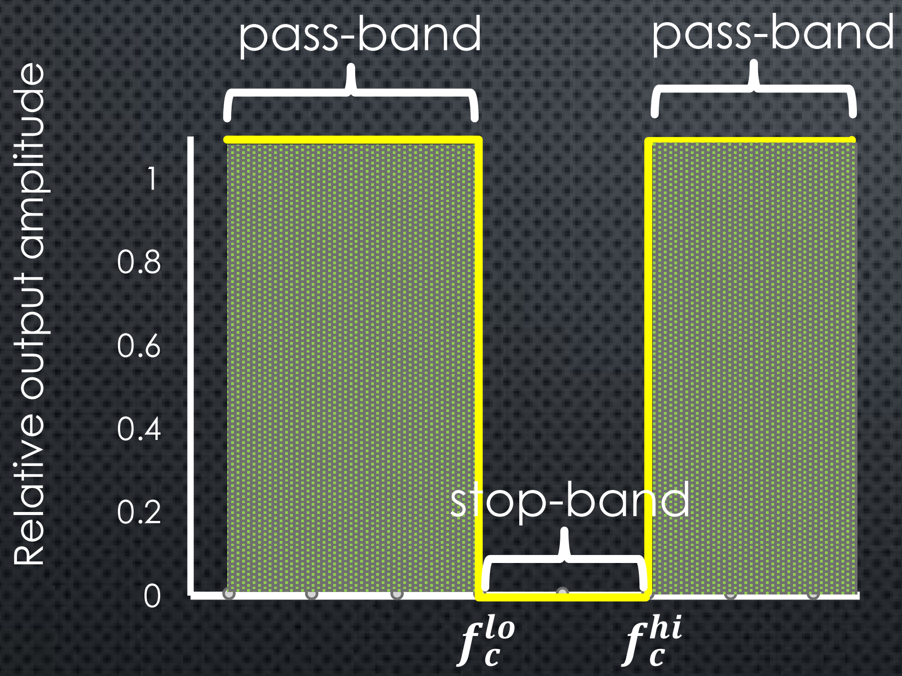
Band-stop filter: systems in parallel

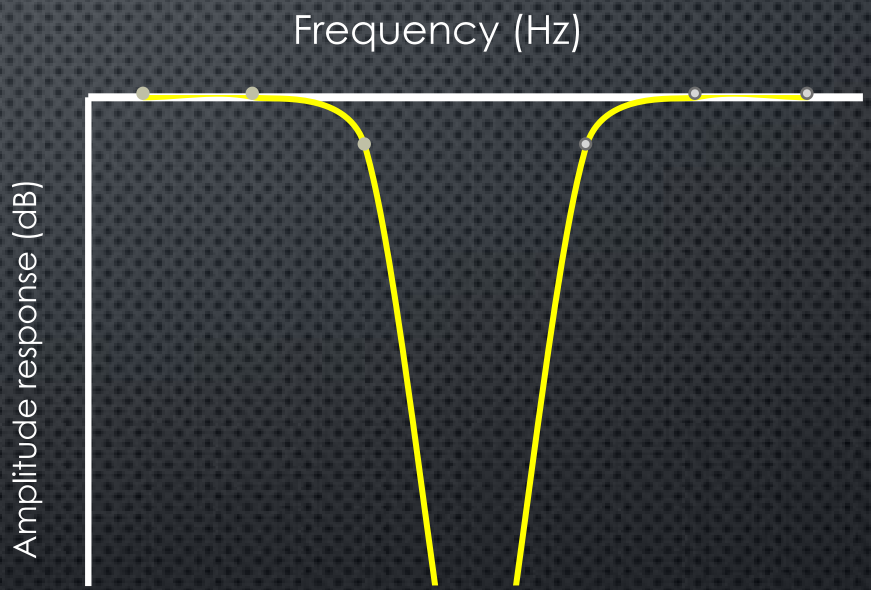


- The input signal is processed by two filters concurrently
- The output is the sum of the outputs by the two filters

- What does the frequency response of the new system look like?

Band-stop filter: amplitude response

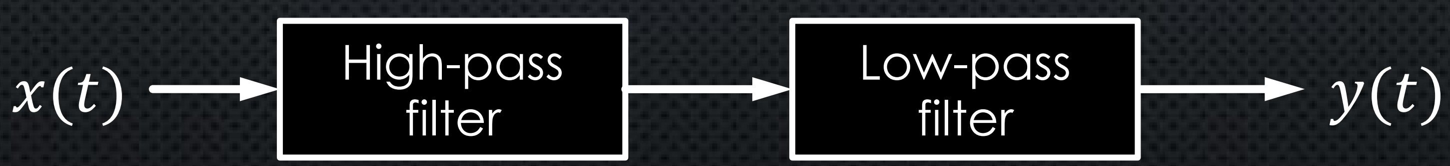




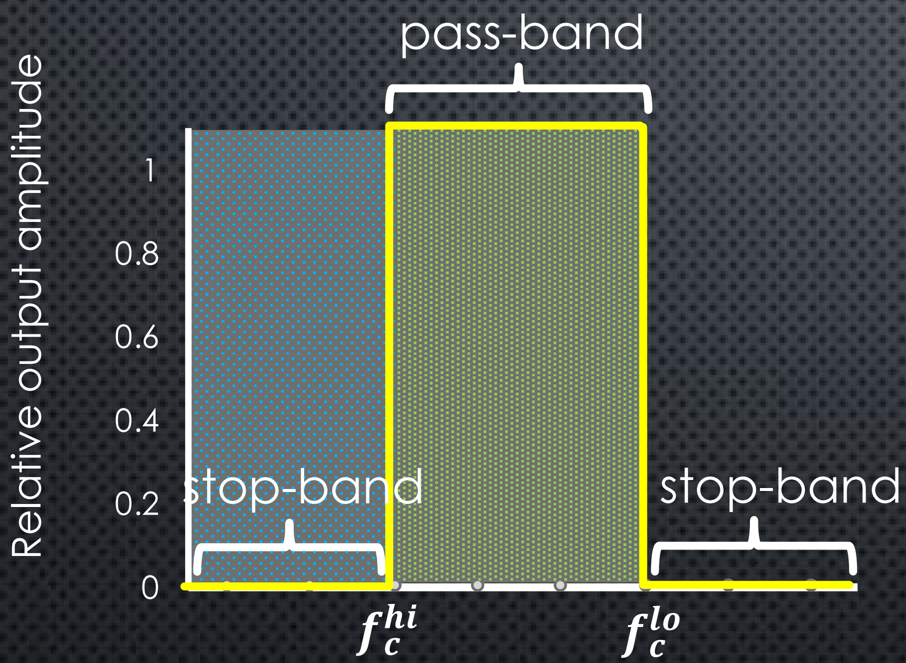
Band-pass filter: systems in cascade

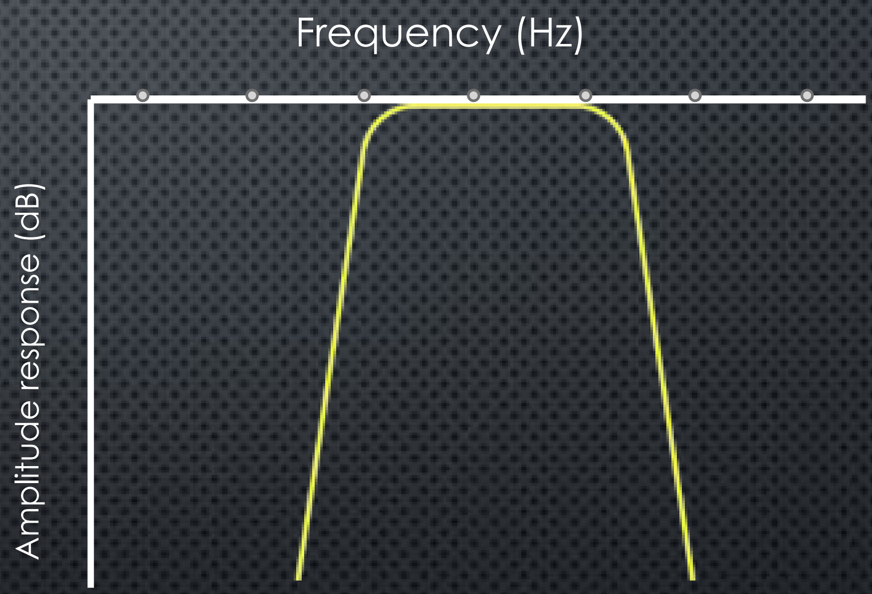
- A **band-pass** filter can be thought of as a combination of one high-pass filter and one low-pass filter *in sequence* with *different* cutoff frequencies $\boldsymbol{f_c^{hi}}$ and $\boldsymbol{f_c^{lo}}$ respectively
  - $\boldsymbol{f_c^{hi} < f_c^{lo}}$

- Construct a ***chain/cascading*** system using the two filters



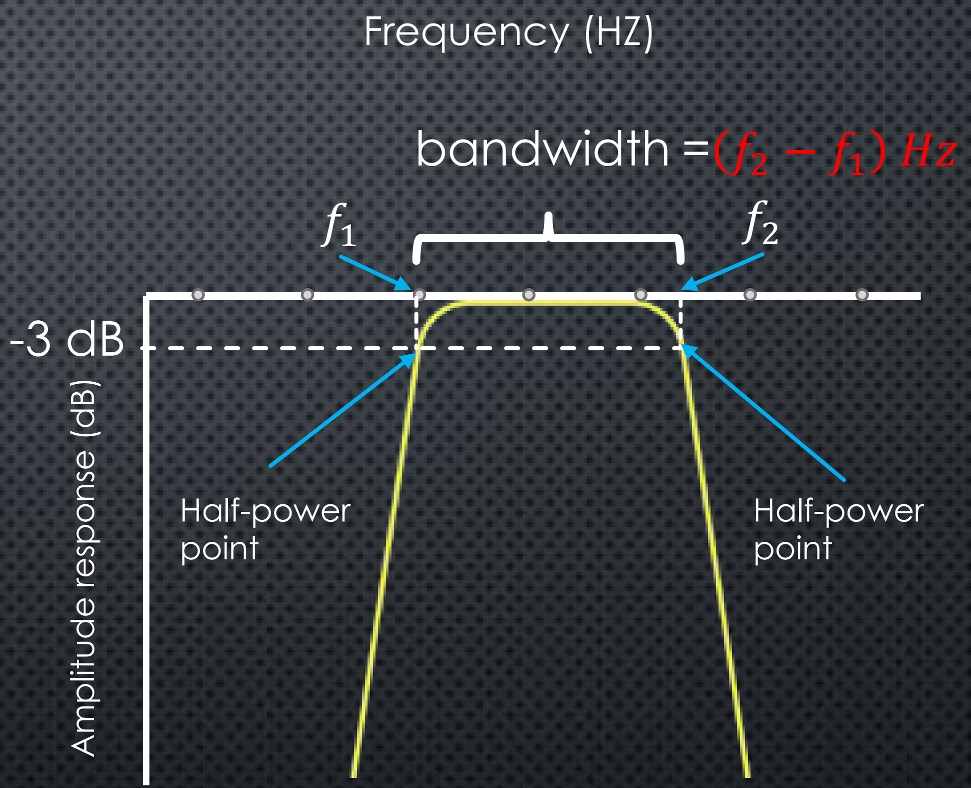
Band-pass filter: amplitude response





Band-pass filter: bandwidth

- Band-pass filter are specified by the width of their pass-band in Hz – **bandwidth**
  - The distance between its upper and lower cut-off frequencies
  - Usually the difference between the two half-power points



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Comparison on basic filters

	No. of cut-off	Comb. method	Resp. to freq. < cut-off	Resp. to freq. > cut-off	Resp. to freq. in between cut-offs	Resp. to freq. outside of cut-offs
Low-pass	1	-	Pass	Stop	-	-
High-pass	1	-	Stop	Pass	-	-
Band-stop	2	Parallel	-	-	Stop	Pass
Band-bass	2	Chain	-	-	Pass	Stop