LING 490 - SPECIAL TOPICS IN LINGUISTICS

Fundamentals of Digital Signal Processing

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Week 7

Last week…

- What is system?
- Properties of ideal system
  - Memory and memoryless
  - Causality
- Types of systems:
  - Chain, parallel, feedback loop, etc
- Linear time-invariant (LTI) system
  - Linearity = homogeneity + additivity
  - Time invariance

Linear systems: behind the scene

- What all linear systems have in common:
  - e.g. $y(t) = 3x(t) + 7x(t-1) + 2y(t-1) - 9y(t-2)$
  - The output is a sum of scaled inputs and scaled previous outputs

- Conventions:
  - Input coefficients: $b_0, b_1, b_2 \ldots b_n$
  - Output coefficients: $a_0, a_1, a_2 \ldots a_n$

- Linear combination equation:

$$y(t) = \sum_{j=0}^{M} b_j x(t-j) - \sum_{i}^{N} a_i y(t-i)$$

Linear systems: Linear combination equation

$$y(t) = \sum_{j=0}^{M} b_j x(t-j) - \sum_{i}^{N} a_i y(t-i)$$

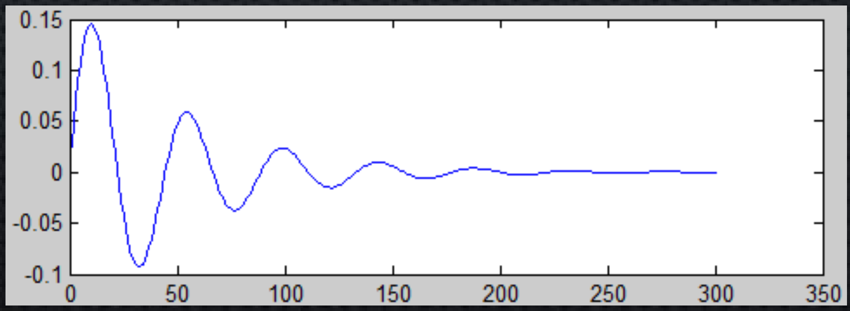
- **Advantage**
  - Easy to see how it is implemented (e.g. using the filter function)
- **Disadvantage**
  - Unclear what kind of frequency-domain behaviour we get from a given choice of a's and b's
    - Preemphasis: $y(t) = x(t) - 0.97 * x(t-1)$
    - Smoothing: $y(t) = x(t) + 0.992 * y(t-1)$

Another view - impulse response

Suppose the input to the system is a **unit impulse** (a one followed by lots of zeros). The output is called the **impulse response** of the system.


```
# create an impulse followed by 300 zeros
imp = np.vstack((1, np.zeros((300, 1))))

# plot response of crude telephone
plot(np.lfilter(1/40,[1 -1.94 0.96],imp))
```



Impulse response (IR)

- IR tells how a linear system reacts to a very simple stimulus
- IR tells how a system reacts to **any** stimulus!!!
  - Only knowing the IR allows to **fully** characterise the system
- In some sense, the IR (usually denoted as $\boldsymbol{h}$) is related to the a's and b's
- The second perspective on linear systems

Justification

- In discrete time, a signal x can be viewed as a **sum of delayed, scaled impulses**
  - e.g. The signal $x(t) = [1, -2, 7]$
- Notation: let $d(t)$ represent a signal which is 1 when t = 0 and zero elsewhere, i.e. [1, 0 , 0]
  - ***Delta function***
- $x(t)$ can be rewritten as:

$$x(t) = 1{\times}d(t-1) - 2{\times}d(t-2) + 7{\times}d(t-3)$$

since $t\text{-}1 = 0$ when $t=1$ (definition of delta function)
$t\text{-}2 = 0$ when $t=2$, etc

Justification - continued

- Any waveform $x(t)$ can be written as a sum of delayed, scaled impulses

$$x(t) = \sum_i x(i)d(t-i)$$

- Recall definition of IR, $h(t)$, and additivity of LTI system, the output of a system $y(t)$ to an arbitrary input $x(t)$ can be calculated as,

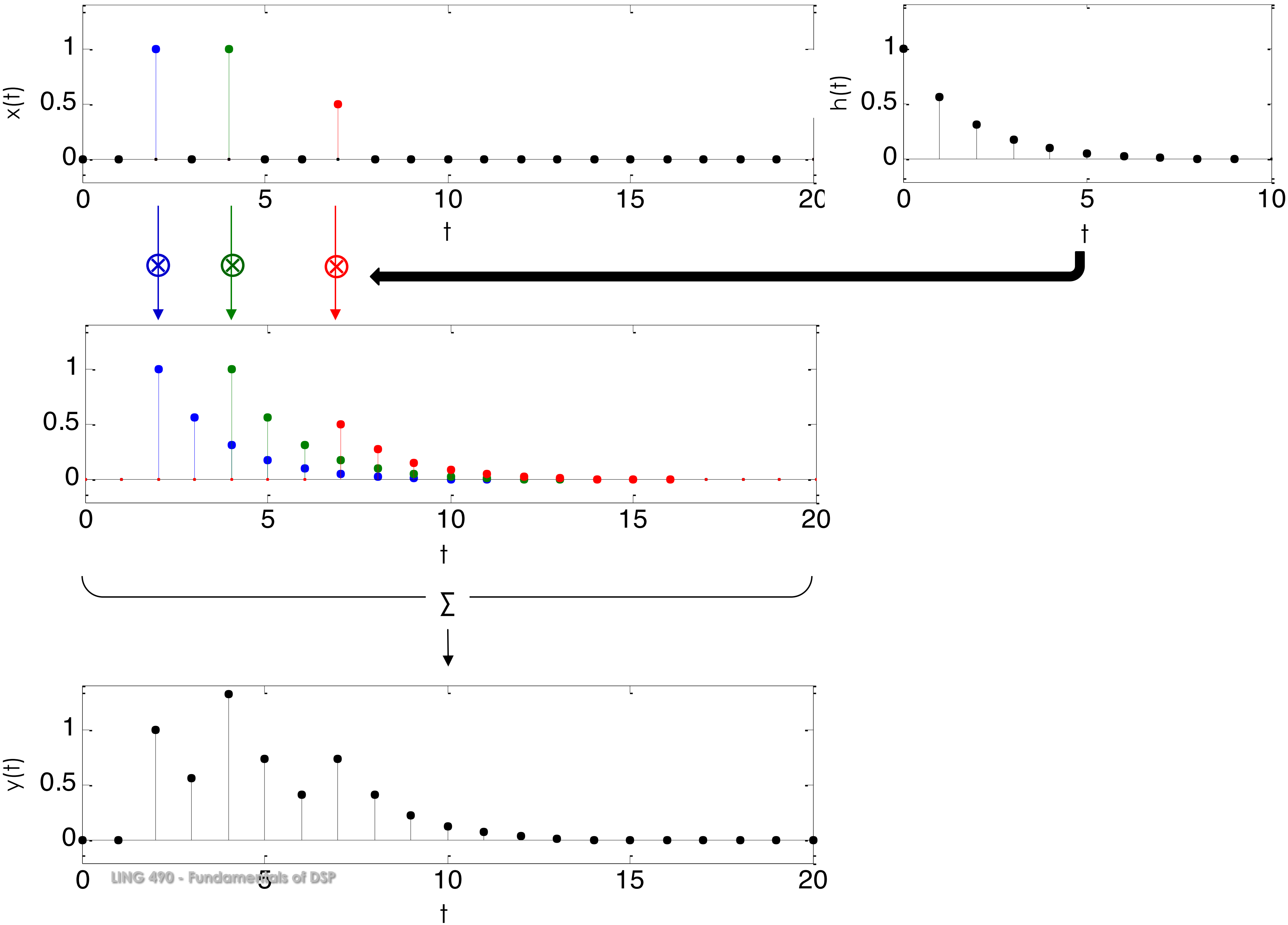
$$y(t) = \sum_i x(i)h(t-i) = \sum_i h(i)x(t-i)$$

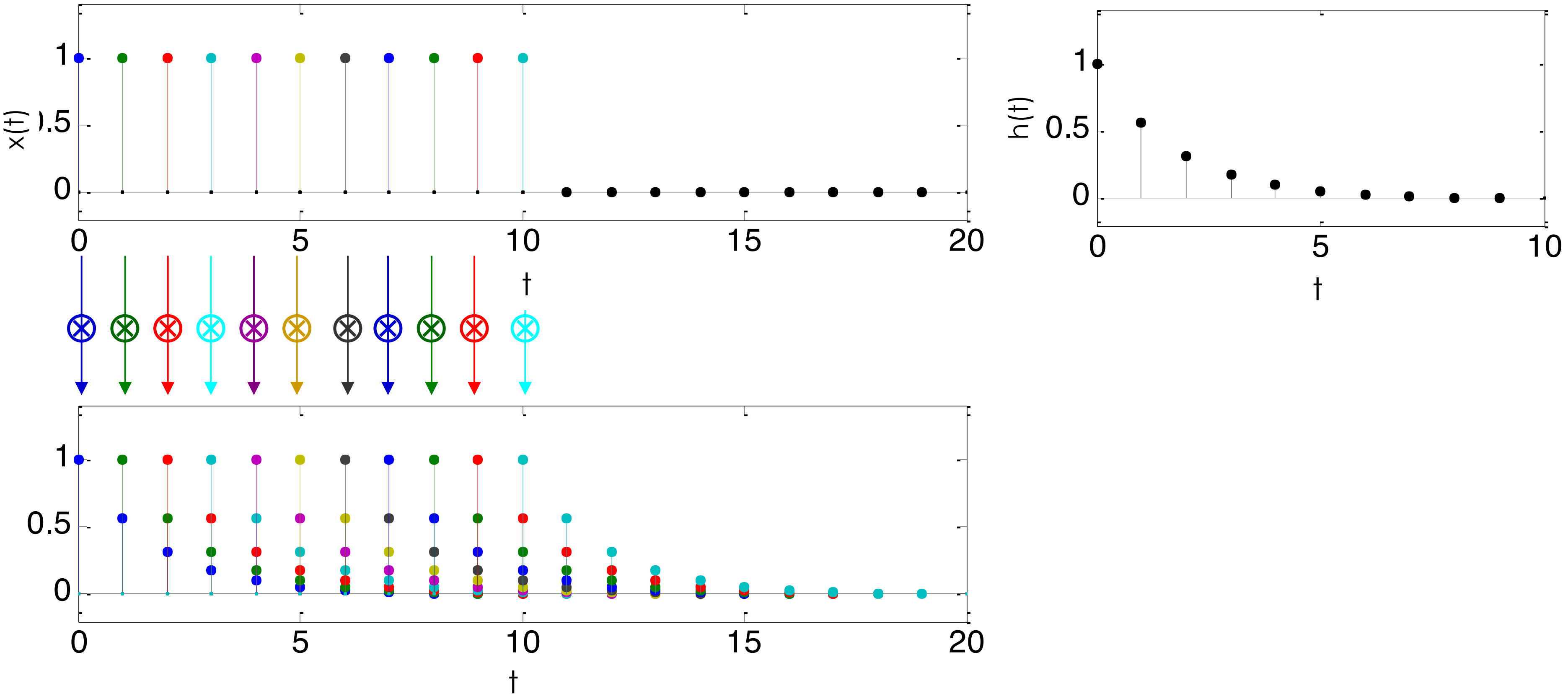
Convolution

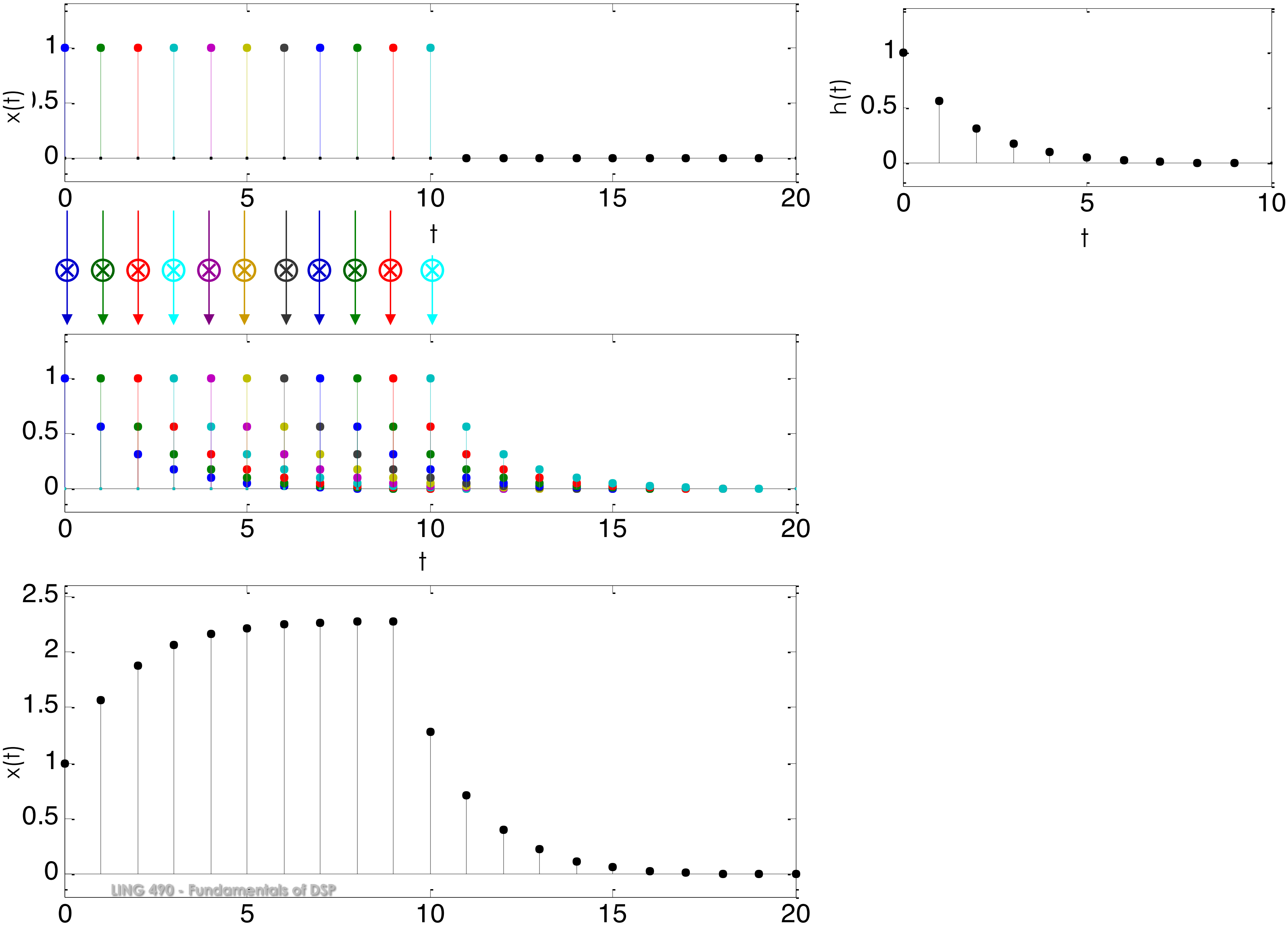
- A sum of the form is called the **convolution** of x and h, written

$$y(t) = x(t) \otimes h(t) = h(t) \otimes x(t)$$

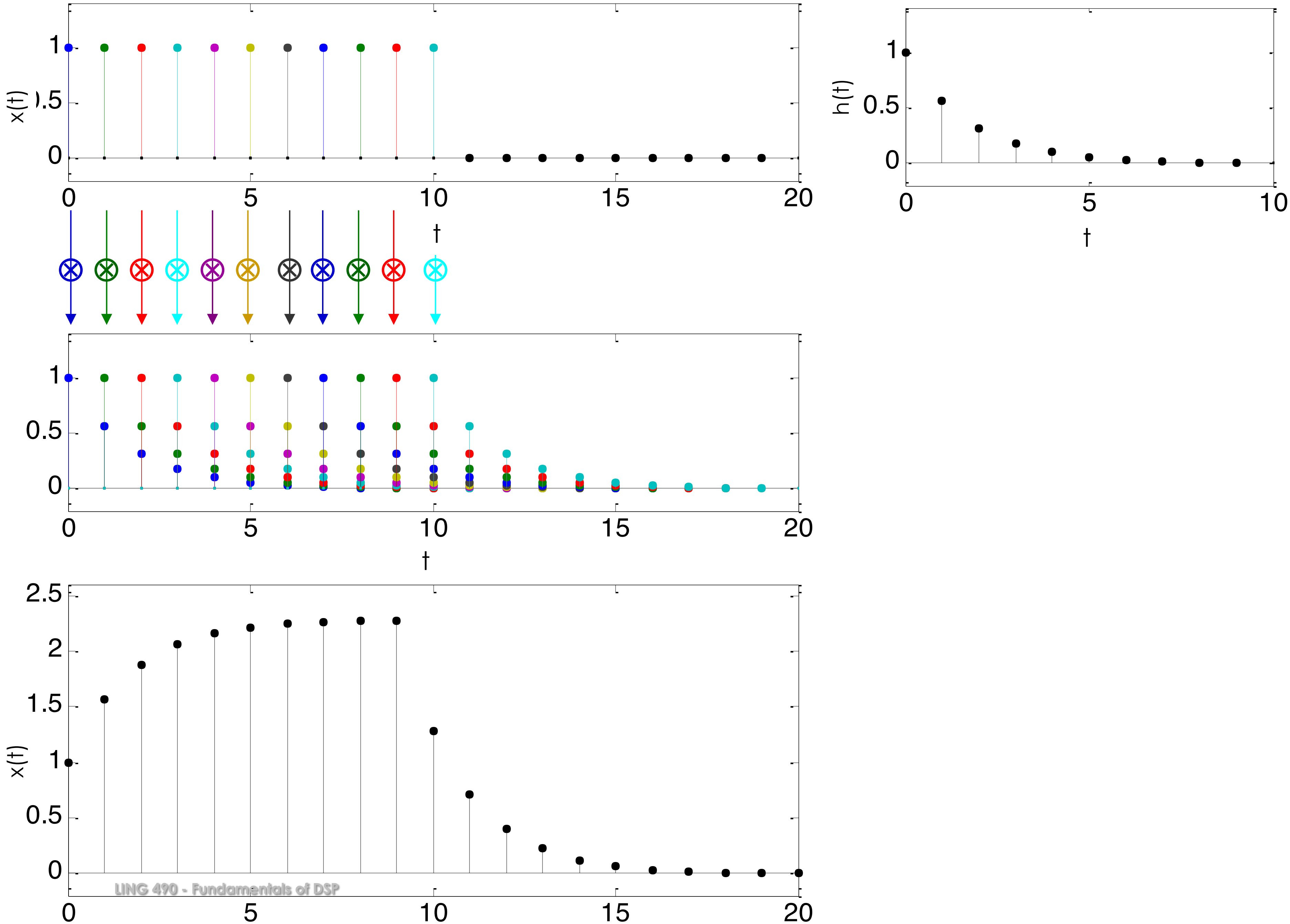
- The output of any linear system is given by convolving the input with the response of the system to an impulse


x(t)
h(t)
t
Σ
y(t)

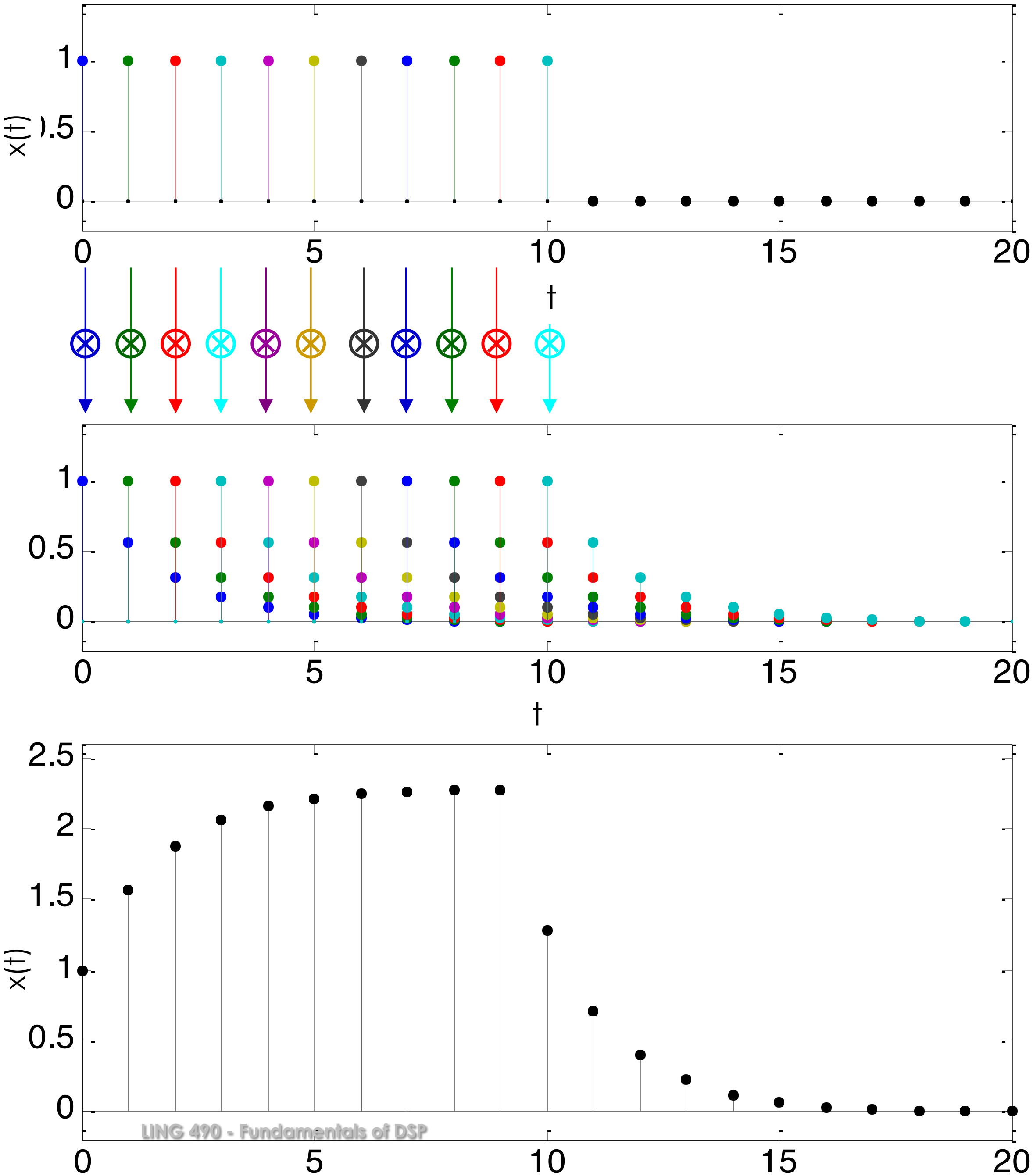

x(t)
1
0.5
0
0
5
10
15
20
t
h(t)
1
0.5
0
0
5
10
t
1
0.5
0
0
5
10
15
20
t

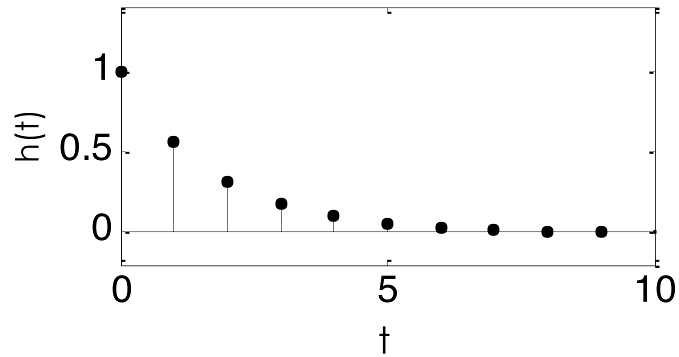


x(t)
h(t)
t
x(t)

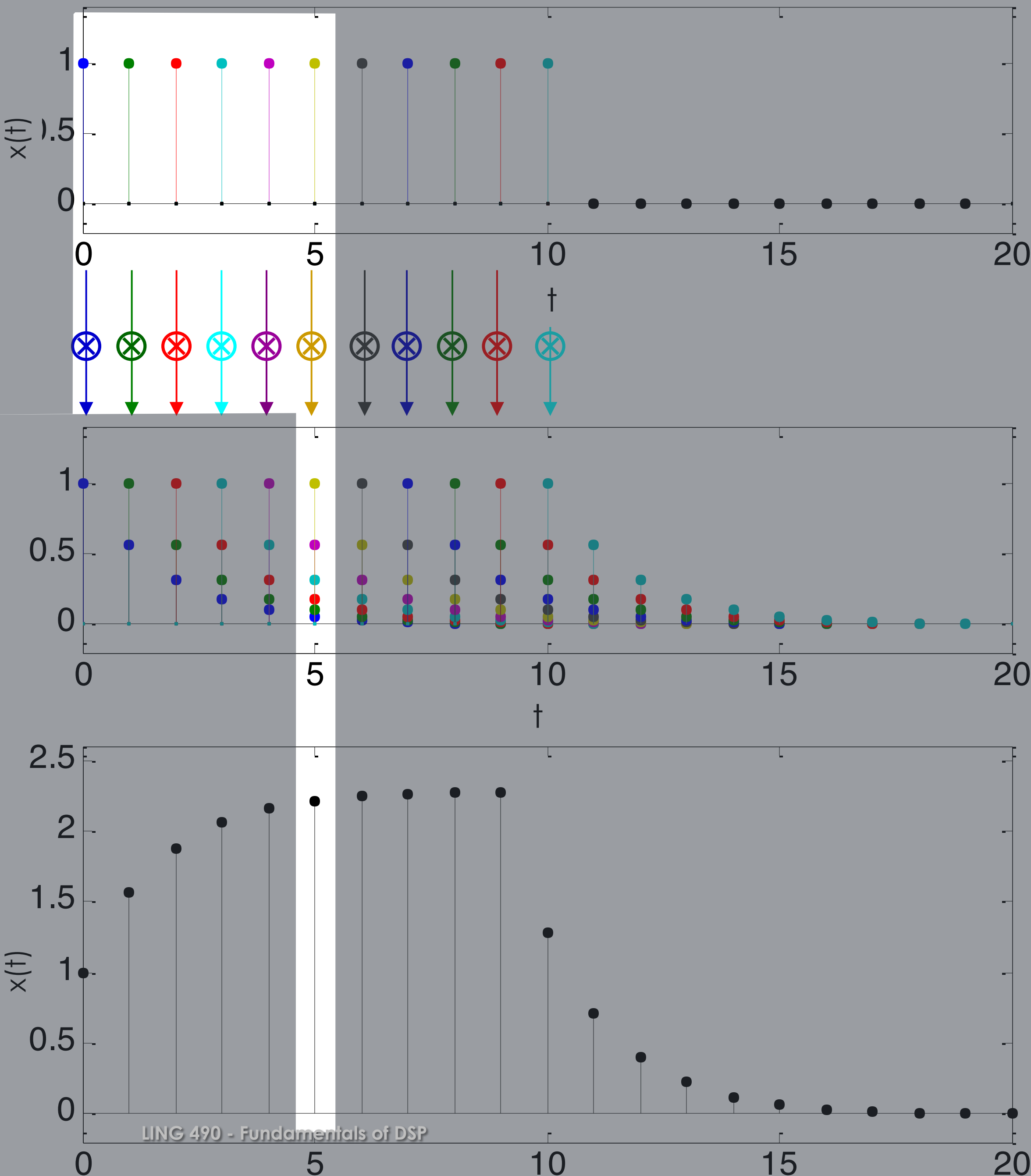


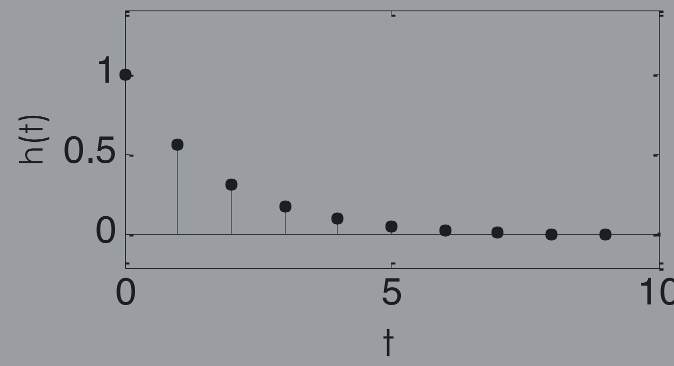
x(t)
h(t)
t
0
0.5
1
1.5
2
2.5
5
10
15
20



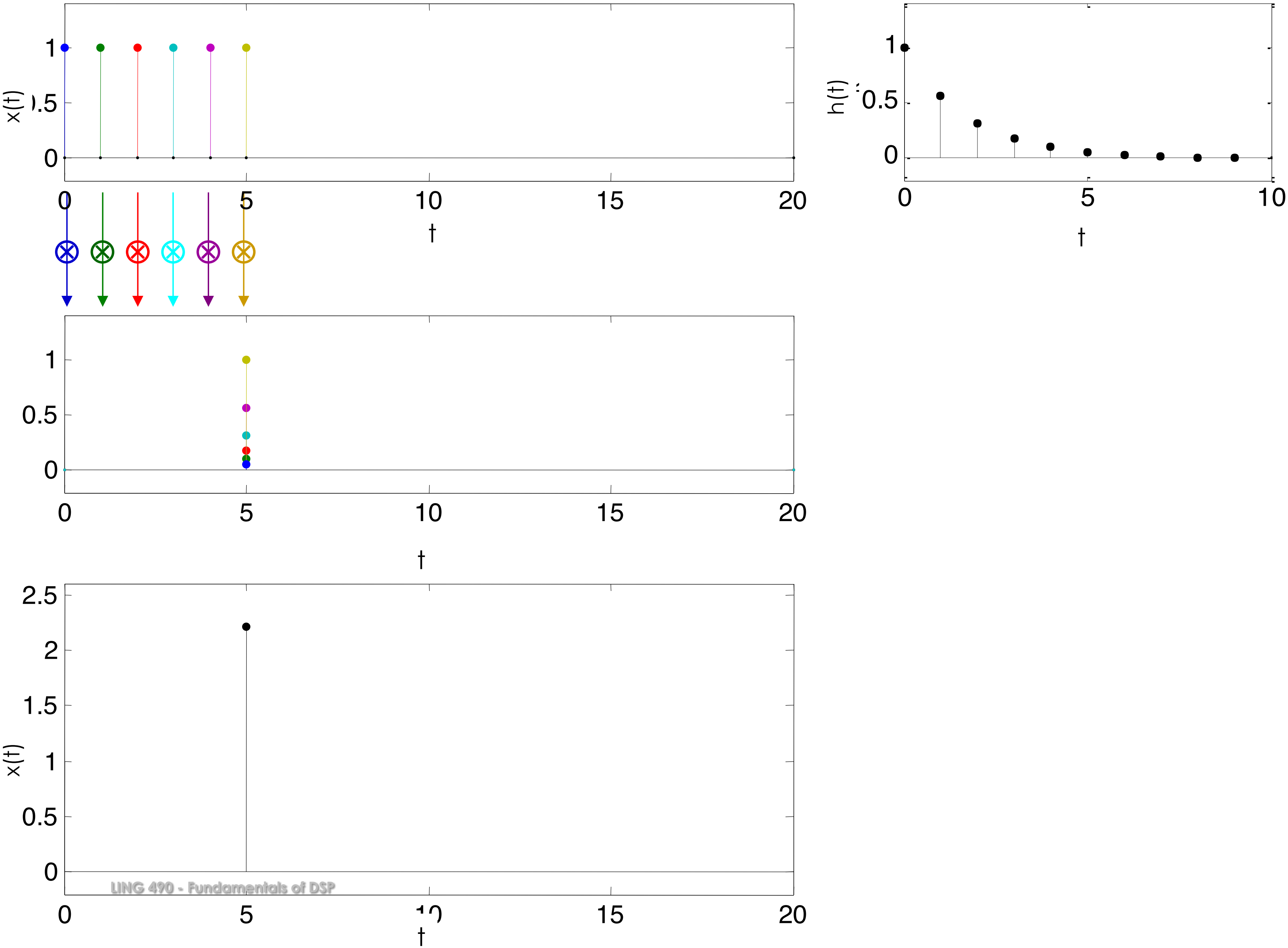


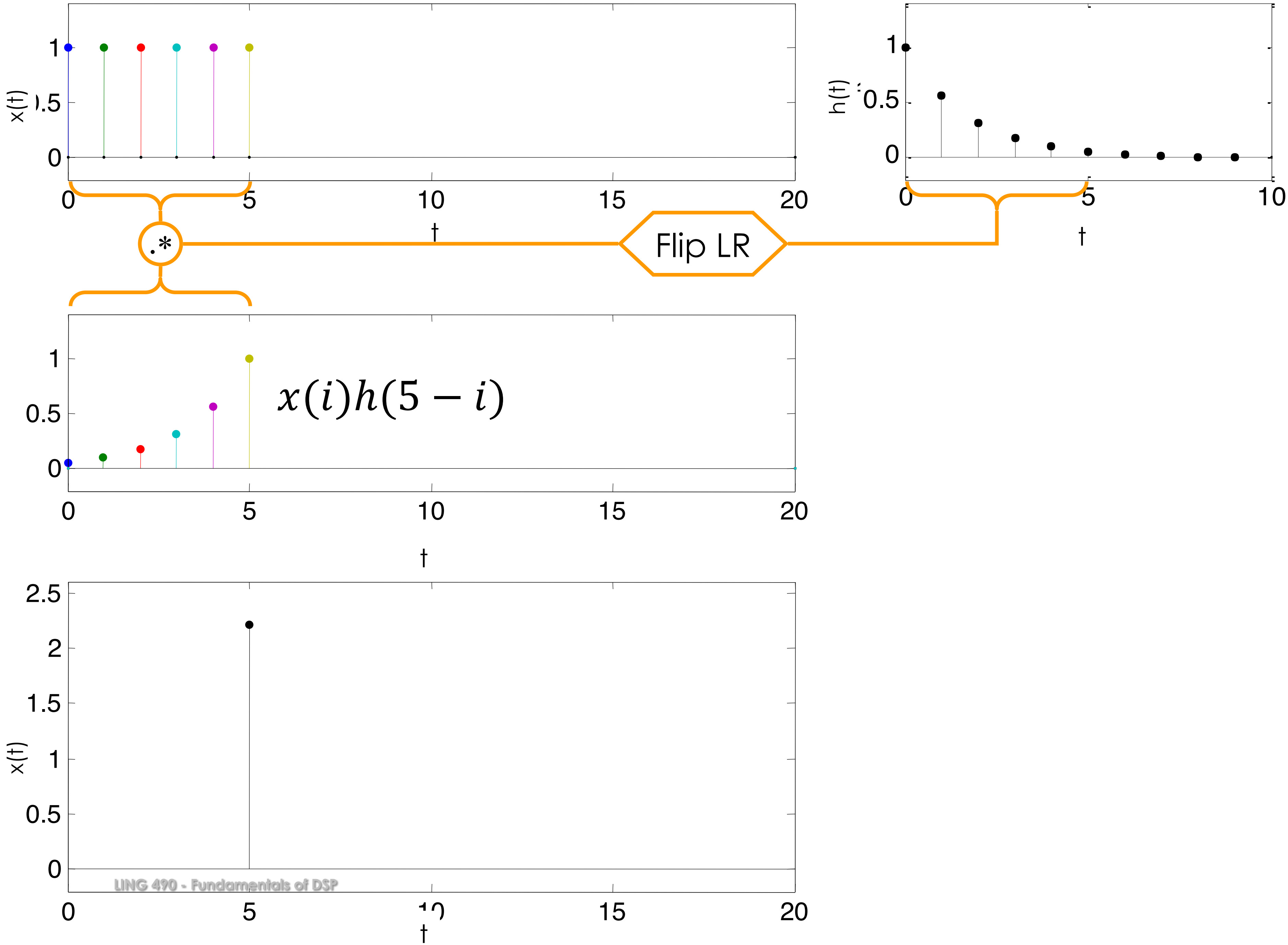
$$y(t) = \sum_{i} x(i)h(t-i)$$

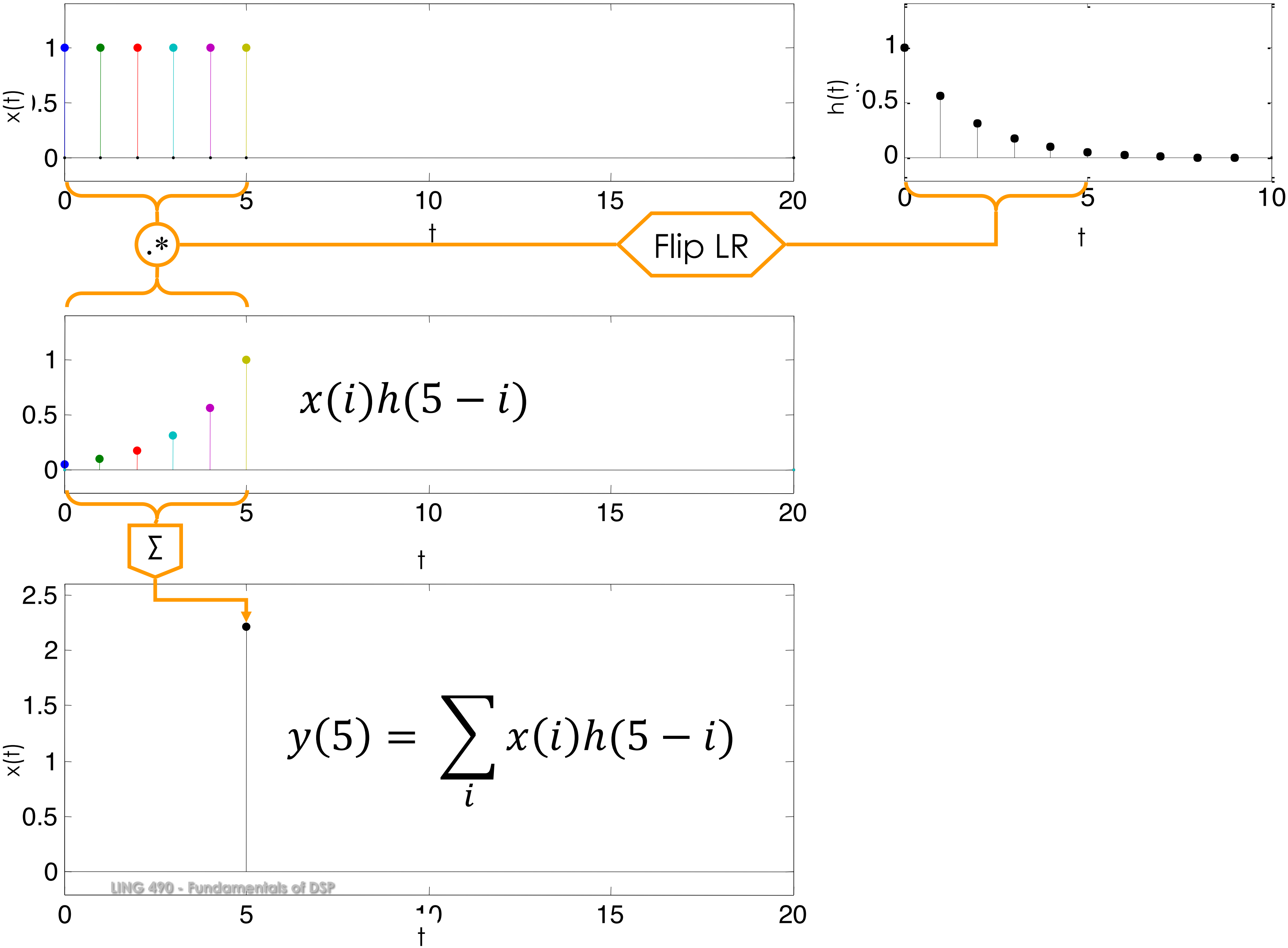




$$y(t) = \sum_i x(i)h(t-i)$$


x(t)
h(t)
t
x(t)
t
t
t


x(t)
h(t)
.*
Flip LR
$x(i)h(5-i)$
t
0
5
10
15
20
0.5
1
1.5
2
2.5


x(t)
h(t)
t
Flip LR
.*
Σ
$x(i)h(5-i)$
$y(5) = \sum_i x(i)h(5-i)$

Transforms

- The convolution of two signals is hard to visualise
- Any way to make convolution disappear?
  - Turn convolution into multiplication!
- Analogy: a product of two terms is more difficult to visualise than a sum; how to turn it to summation?
  - $\log(x\,y) = \log(x) + \log(y)$

Example

- Males tend to have longer vocal tracts than females, one consequence of which is a **scaling** of all formant frequencies.

$$F1_{female} = a * F1_{male}$$
$$F2_{female} = a * F2_{male}$$

- Taking logs, the male/female difference is simply a **constant shift of log(a) along the frequency axis**

Convolution to multiplication

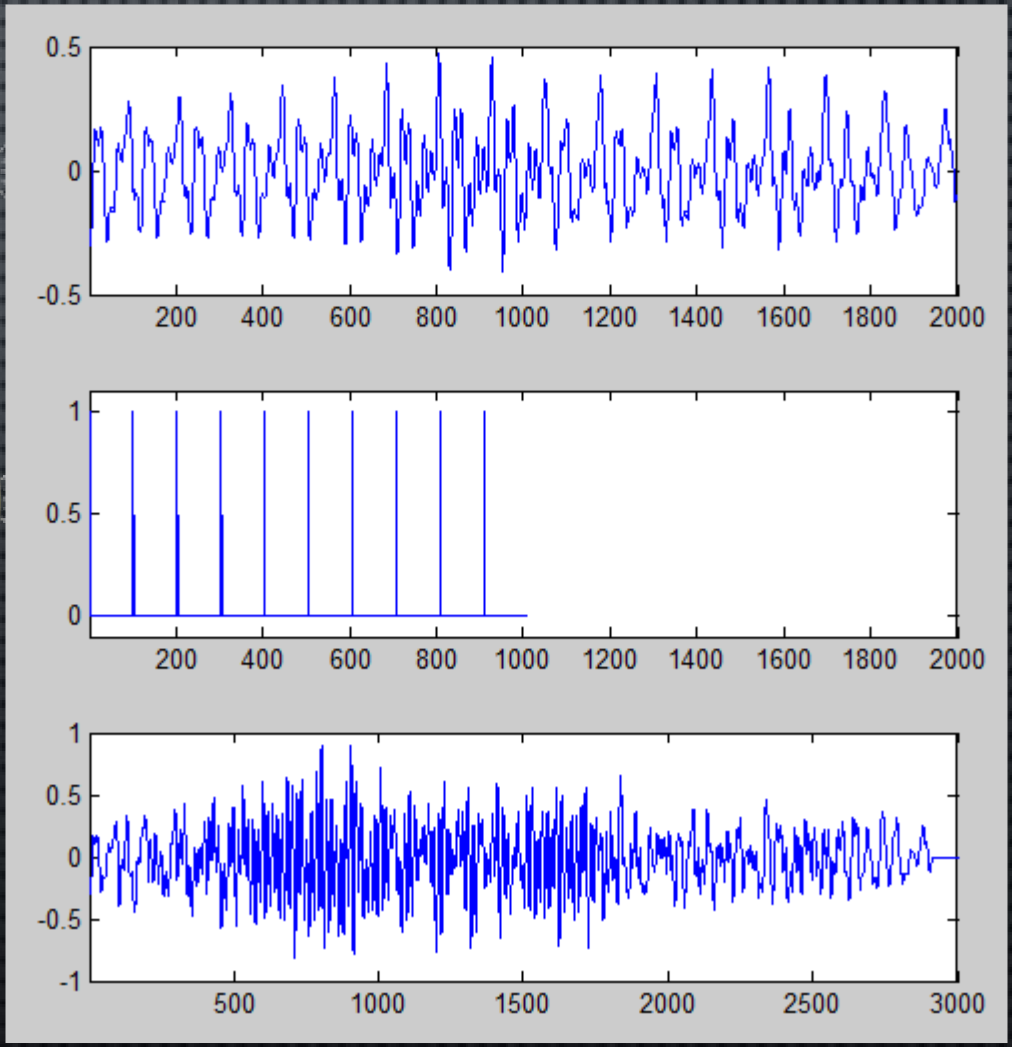
- The Fourier transform turns convolution into multiplication!
- The convolution of a signal and impulse response can be visualised in the frequency domain with the Fourier transform of the signal and the Fourier transform of the impulse response
- If the IR of a system is known or can be measured, then we can work out what **spectral filtering effect** it will have on *any* signal

Convolution of speech and impulses

speech

impulse series (~200 Hz)

convolution of speech
and impulses

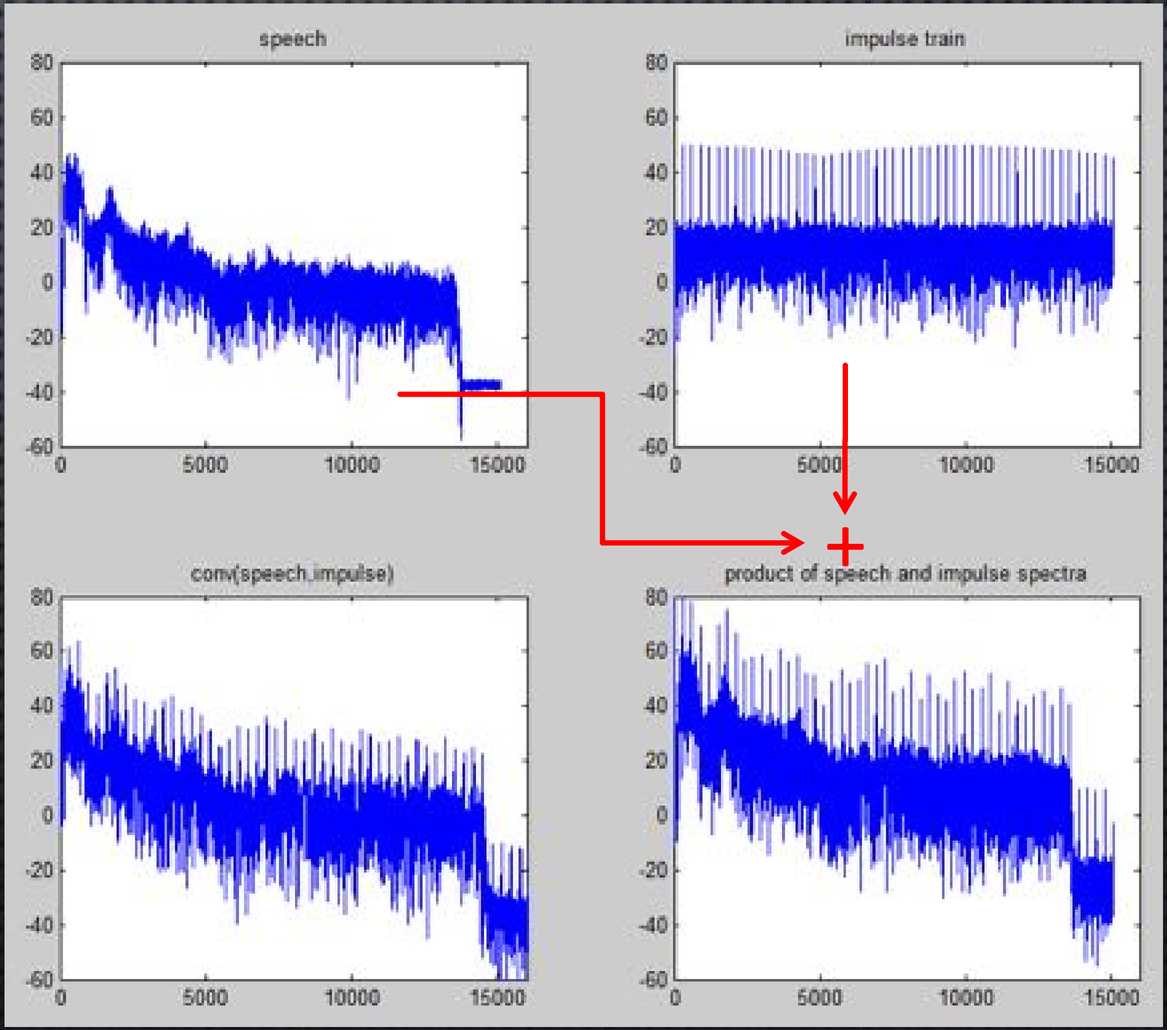


Difficult to interpret…

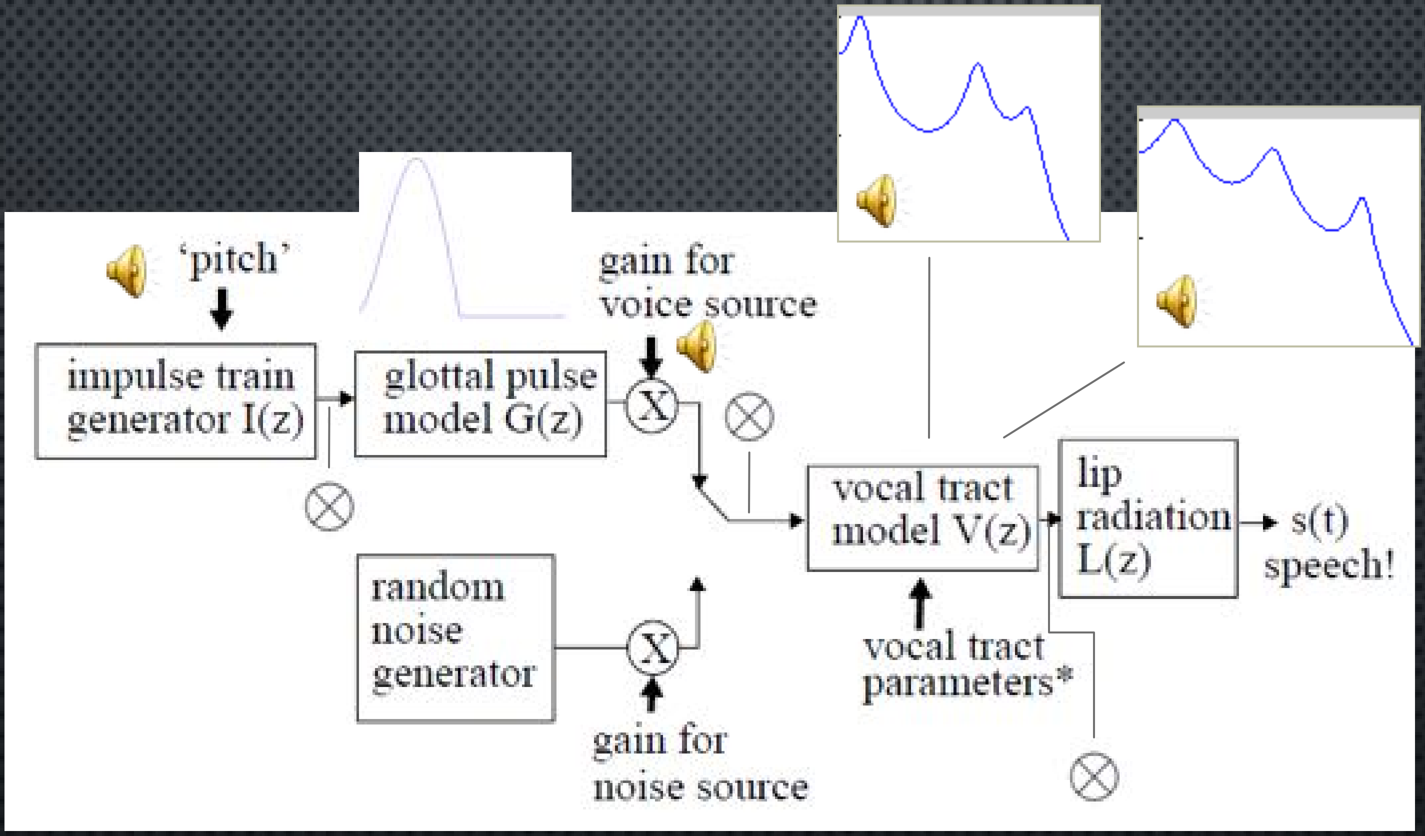
Convolution as multiplication or addition of spectra

- The effect of convolving an impulse series with speech is to weight the speech spectrum by that of the impulse train

- Because they are log spectra, the result is an addition of spectra



Convolutions in the standard model of speech production



Easier to visualise effects as multiplications of spectra!

Applications of convolution

- Convolution is the backbone of many common signal processing tasks:
  - Image processing
  - Signal filtering
  - Audio processing
  - Machine learning/AI

Convolution Reverb

Convolution Reverb

Reverb examples