

Algorithms

Graph Reduction

We call the reduction of the graph, the operation of removing from the graphs all nodes and edges that cannot appear in the minimal path set. We have two different algorithms for graph reduction based on the type of graph. For directed graph it is REDUCE-DIRECTED-GRAPH and for undirected it is REDUCE-UNDIRECTED-GRAPH. Before we reduce the given graph, we connect an artificial source 's' to the source nodes and an artificial terminal 't' to the terminals nodes. In case of directed graph artificial source node 's' only have outgoing edges and artificial terminal node 't' have only incoming nodes.

Algorithm1: REDUCE-DIRECTED-GRAPH(G, s, t)

```
1: for all  $v$  in  $G.V$  do
2:   if  $\text{card}(\text{IN}_N(v)) == 0$  and  $n$  is not  $s$  then
3:      $\text{To\_delete\_in} \leftarrow \text{To\_delete\_in} \cup v$  // storing all those nodes which cannot be reached
4:   end if
5:   if  $\text{card}(\text{OUT}_N(v)) == 0$  and  $n$  is not  $s$  then
6:      $\text{To\_delete\_out} \leftarrow \text{To\_delete\_out} \cup v$  // storing all those nodes from where we cannot move further
7:   end if
8: end for
9: while  $\text{To\_delete\_in}$  is not empty do
10:   $x \leftarrow \text{element from to } \text{To\_delete\_in}$ 
11:   $\text{To\_delete\_in} \leftarrow \text{To\_delete\_in} - x$ 
12:  for all  $y$  in  $\text{OUT}_N(x)$  do
13:    if  $\text{card}(\text{IN}_N(y)) == 1$  then
14:       $\text{To\_delete\_in} \leftarrow \text{To\_delete\_in} \cup y$ 
15:    end if
16:  end for
17:   $G \leftarrow G/x$ 
18: end while
19: while  $\text{To\_delete\_out}$  is not empty do
20:   $x \leftarrow \text{element from to } \text{To\_delete\_out}$ 
21:   $\text{To\_delete\_out} \leftarrow \text{To\_delete\_out} - x$ 
22:  for all  $y$  in  $\text{IN}_N(x)$  do
23:    if  $\text{card}(\text{OUT}_N(y)) == 1$  then
24:       $\text{To\_delete\_out} \leftarrow \text{To\_delete\_out} \cup y$ 
25:    end if
26:  end for
27:   $G \leftarrow G/x$ 
28: end while
29: return  $G$ 
```

Algorithm2: REDUCE-UNDIRECTED-GRAPH(G, s, t)

```
1: for all  $v$  in  $G.V$  do
2:   if  $\text{card}(L[v]) == 1$  and  $n$  is not  $s$  then
3:      $\text{To\_delete} \leftarrow \text{To\_delete} \cup v$  // storing all those nodes from where we cannot move further
4:   end if
5: end for
6: while  $\text{To\_delete}$  is not empty do
7:   $x \leftarrow \text{element from to } \text{To\_delete}$ 
8:   $\text{To\_delete} \leftarrow \text{To\_delete} - x$ 
```

```

9:   for all y in L[x] do
10:       if card(L[y]) == 1 then
11:           To_delete <- To_delete U y
12:       end if
13:   end for
14:   G<-G/x
15: end while
16: return G

```

Finding paths

Once the graph is reduced, we call the finding path algorithm to compute the minimal path set. We have two different algorithms for finding the path sets, FIND-PATH-DIRECTED and FIND-PATH-UNDIRECTED. Finding paths is called after the graph is reduced. Before finding path in undirected graph we convert it into directed graph and then call FIND-PATH-UNDIRECTED. While finding path in undirected graph we maintain a stack where all the visited nodes are stored.

Algorithm 3: FIND-PATH-DIRECTED(G,S,T)

```

1: REDUCE-DIRECTED-GRAPH(G,s,t)
2: l<-[]
3: PATH(G,S,T)
4:   for all x in S do
5:       NEXT_NODE(G,S,T,x,R)
5:       l<-[]
7:   end for
8: NEXT-NODE(G,S,T,n,R)
9:   if n in T then
10:       ADD-NODE(n)
11:       l<- l - n
12:   else
13:       ADD-NODE(n)
14:       for x in OUT_N(n) do
15:           if x not in S then
16:               NEXT-NODE(G,S,T,x,R)
17:           end if
18:       end for
19:       l<- l - x
20:   end if
21: ADD-NODE(n)
22:   l<-l U n

```

Algorithm 4: FIND-PATH-UNDIRECTED(G,S,T)

```

1: REDUCE-UNDIRECTED-GRAPH(G,s,t)
2: CONVERT-UNDIRECTED(G,s,t)
3: l<-[]
4: visited_node<-[]
5: PATH(G,S,T)
6:   for all x in S do
7:       NEXT_NODE(G,S,T,x,R)
8:       l<-[]
9:       visited_node<-[]
10:   end for

```

```

11: NEXT-NODE(G,S,T,n,R)
12:   if n in T then
13:     ADD-NODE(n)
14:     l ← l – n
15:     visited_node ← visited_node – n
16:   else
17:     ADD-NODE(n)
18:     for x in OUT_N(n) do
19:       if x not in visited_node and x not in s then
20:         NEXT-NODE(G,S,T,x,R)
21:       end if
22:     end for
23:     l ← l – x
24:     visited_node ← visited_node – x
25:   end if
26: ADD-NODE(n)
27:   l ← l ∪ n
28:   visited_node ← visited_node ∪ n

```

Converting Graph

We call the converting graph, the operation of converting the undirected graph to directed graph. Converting Graph is called after the undirected graph is reduced.

- Source nodes only have outgoing edges.
- Terminal nodes only have incoming edges.

Algorithm 5: CONVERT-UNDIRECTED-TO-DIRECTED(G,S,T)

```

1: for x in G.V do
2:   if x in S then
3:     for y in L(x) do
4:       OUT_N(x) ← OUT_N(x) ∪ y
5:     end for
6:   else if x in T then
7:     for y in L(x) do
8:       IN_N(x) ← IN_N(x) ∪ y
9:     end for
10:  else
11:    for y in G.V do
12:      if y in T then
13:        OUT_N(x) ← OUT_N(x) ∪ y
14:      else if y in S then
15:        IN_N(x) ← IN_N(x) ∪ y
16:      else
17:        IN_N(x) ← IN_N(x) ∪ y
18:        OUT_N(x) ← OUT_N(x) ∪ y
19:      end if
20:    end if
21:  end for

```