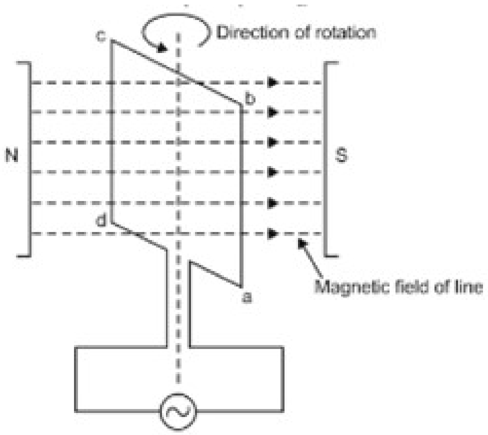
Unit III: Single Phase AC Circuit Analysis (10 hrs)

3.1 Generation of EMF by electromagnetic induction
3.2 Generation of alternating voltage, Sinusoidal Functions-terminology (phase, phase angle, amplitude, frequency, peak to peak value), average value and RMS or effective value of any type of alternating voltage or current waveform
3.3 Phase algebra, power triangle, impedance triangle, steady state response of circuits (RL, RC, RLC series and parallel) and concept about admittance, impedance reactance and its triangle), instantaneous power, average power, real-power, reactive power, power factor and significance of power factor, resonance in series and parallel RLC circuit. bandwidth, effect of Q-factor in resonance.

Generation of EMF by electromagnetic induction

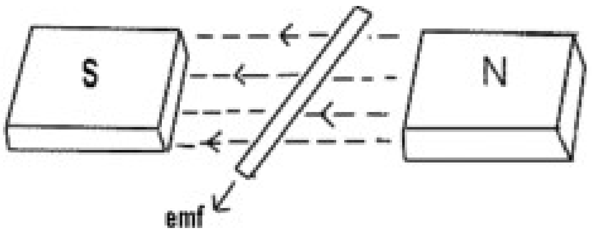
Electromagnetic Induction or Induction is a process in which a conductor is put in a particular position and magnetic field keeps varying or magnetic field is stationary and a conductor is moving. This produces a Voltage or EMF (Electromotive Force) across the electrical conductor. Michael Faraday discovered Law of Induction in 1830.



Basic principle of generation of alternating quantity.

- The induction of an electromotive force by the motion of a conductor across a magnetic field or by a change in magnetic flux in a magnetic field is called **'Electromagnetic Induction'.**
- This either happens when a conductor is set in a moving magnetic field (when utilizing AC power source) or when a conductor is always moving in a stationary magnetic field.
- This law of electromagnetic induction was found by **Michael Faraday.** He organized a leading wire according to the setup given bottom, connected to a gadget to gauge the voltage over the circuit. So when a bar magnet passes through the snaking, the voltage is measured in the circuit. The importance of this is a way of producing electrical energy in a circuit by using magnetic fields and not just batteries anymore. The machines like generators, transformers also the motors work on the principle of electromagnetic induction.

Faraday's law of Electromagnetic Induction



- First law: Whenever a conductor is placed in a varying magnetic field, EMF induces and this emf is called an induced emf and if the conductor is a closed circuit than the induced current flows through it.
- Second law: The magnitude of the induced EMF is equal to the rate of change of flux linkages.
- Based on his experiments we now have Faraday's law of electromagnetic induction according to which the amount of voltage induced in a coil is proportional to the number of turns and the changing magnetic field of the coil.

So now, the induced voltage is as follows:

$$\mathbf{e = N \times d\Phi/dt}$$

Where,

e is the induced voltage
N is the number of turns in the coil
Φ is the magnetic flux
t is the time

Applications of Electromagnetic Induction

1. Electromagnetic induction in AC generator
2. Electrical Transformers

Generation of alternating voltage: The voltage which changes polarity at regular interval of time is known as the alternating voltage. The one complete cycle of an alternating quantity consists two half cycles. And the direction of a half cycle changes after every particular interval of time. The machine which generates the alternating voltage is known as the alternator.

The alternating voltage is generated in two ways.

- By rotating the coil inside the uniform magnetic field at constant speed
- By rotating the magnetic field around the stationary coil at the constant speed.

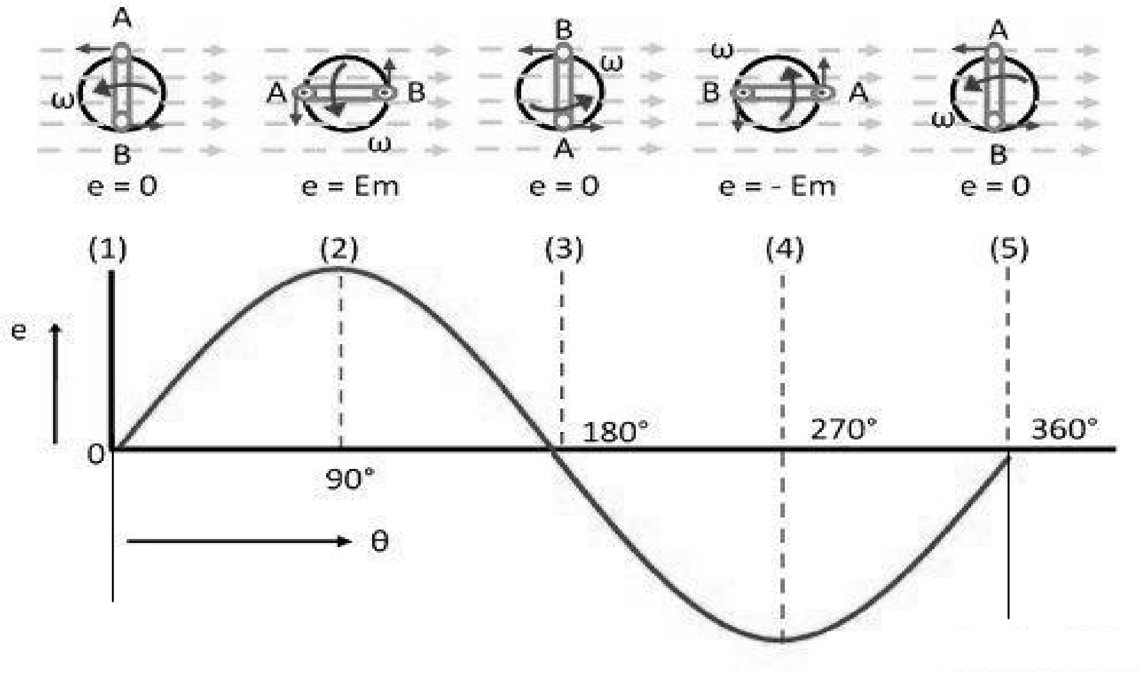
In small AC generators, the coil rotates between the magnetic field whereas in large ac generator the magnetic field rotates around the coil.

Note: *The major difference between the alternator and the generator is that in alternator the armature is stationary and the field system rotates whereas in the generator armature rotates and field is stationary. The armature of the alternator is mounted on the stationary element called stator and field winding on a rotating element, while the connection of a generator is just the reverse of it.*

Process of Generating Alternating Voltage

Consider the stationary coil places inside the uniform magnetic field. The load is connected across the coil with the help of brushes and the slip rings. When the coil rotates in the anticlockwise direction at constant angular velocity ω the electromotive force induces in the coil. The cross-sectional view of the coil at the different position is shown in the figure below.

The magnitude of the emf induced in the coil depends on the rate of the flux cut by the conductor. The figure below shows that the **no current induces in the coil when they are parallel to the magnetic line of forces. i.e., at the position (1), (2) and (3)**. And the total flux cut by the conductor becomes zero.



The magnitude of the induced emf becomes maximum when the conductor becomes perpendicular to the magnetic line of force. The conductor cuts the maximum flux at this position.

The generated alternating voltage (emf) can be expressed as

e = E_m Sin $\theta = E_m Sinwt$

Where e= Instantaneous value of induced emf(voltage)

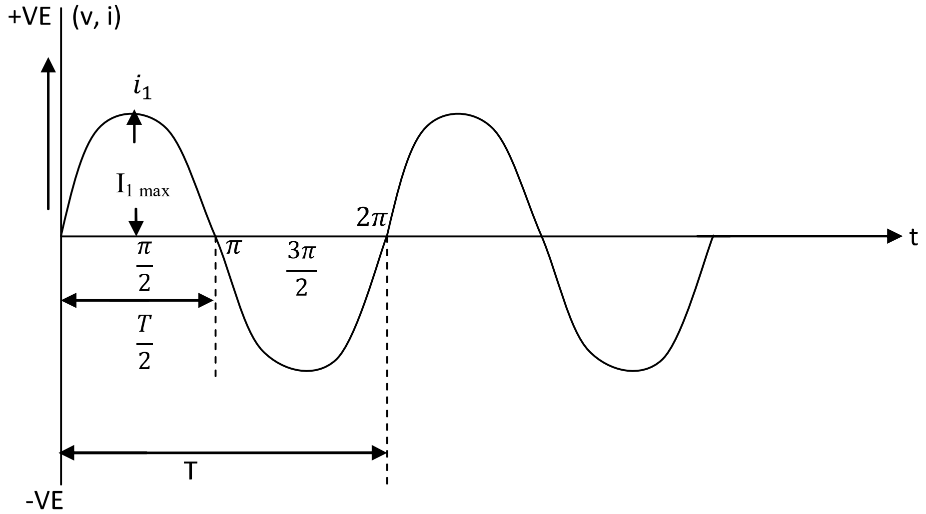
E_m = Maximum value of induced emf(voltage)

Since $w = 2\pi f$, where f is the frequency of rotation of the coil, the equation of voltage can be written as $e = E_m \sin 2\pi f t = E_m \sin\left(\frac{2\pi}{T}\right) t$

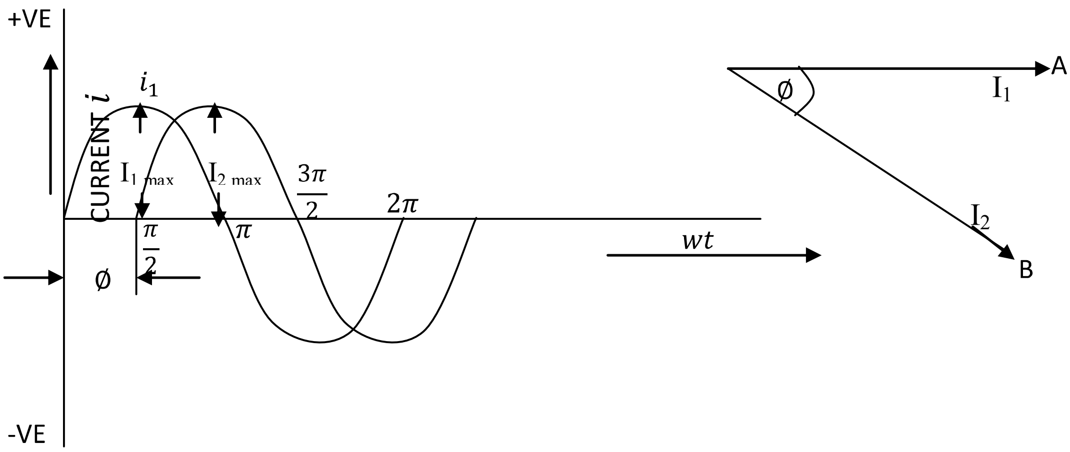
Sinusoidal Functions-terminology (phase, phase angle, amplitude, frequency, peak to peak value)

Alternating quantity (voltage or current)

An alternating quantity (voltage or current) is one that changes which changes its magnitude and polarity at regular intervals of time is called an alternating current. The major advantage of using the alternating current instead of direct current is that the alternating current is easily transformed from higher voltage level to lower voltage level.

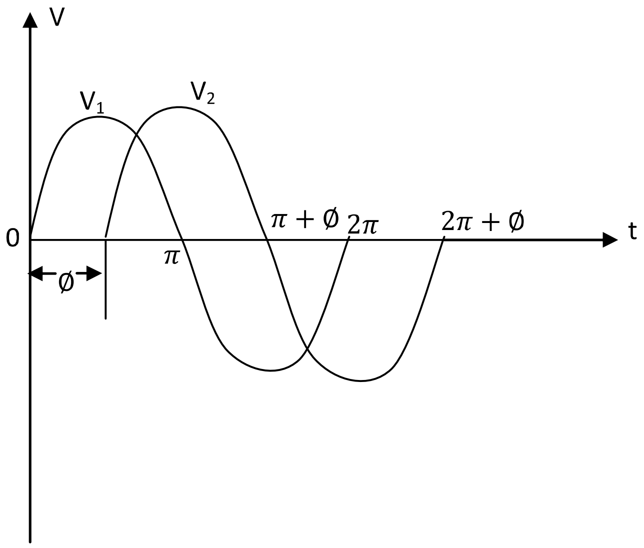


Phase and Phase angle: By phase of an alternating current is meant the fraction of time period of that has elapsed (beyond) since the current last passed through the zero position of reference. The phase angle of any quantity means the angle the phasor representing the quantity makes with the reference line (which is taken to be at zero degrees or radians)



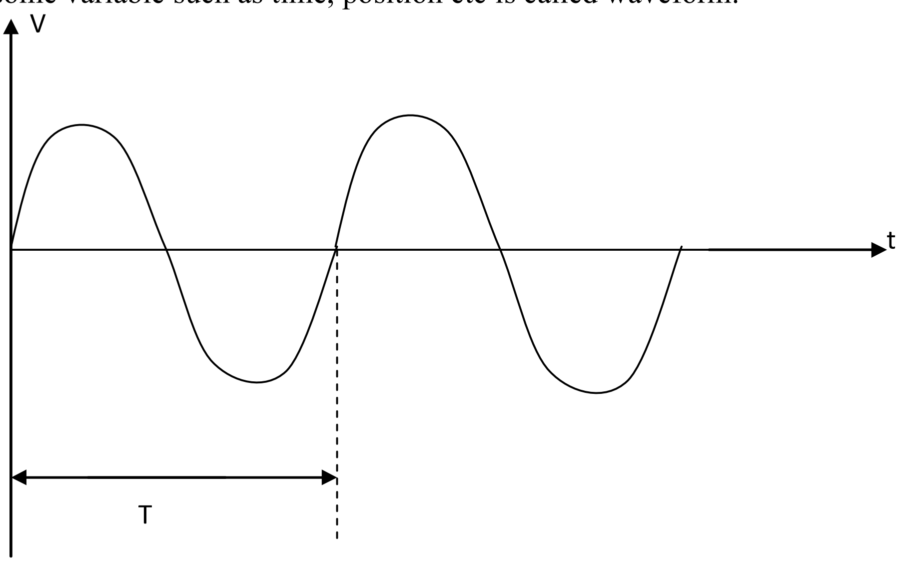
In the above figure phase angle of current I_2 is ($-\emptyset$)

Phase difference: Two or more alternating quantity are to be in phase difference if their zero values and maximum values don't occur at the same instant of time.



In the above figure V_1 and V_2 are said to be in phase difference because their zero values and maximum values are not occurring at the same instant. The max/zero value of V_1 are occurring earlier than the max/zero value of V_2. In this case V_1 is called leading quantity and V_2 is called lagging quantity. In other words it is written that V_1 leads V_2 by an angle $\emptyset$ or V_2 lags V_1 by an angle $\emptyset$.

Waveform: The path traced by a quantity such as AC voltage as shown in figure plotted as a function of some variable such as time, position etc is called waveform.



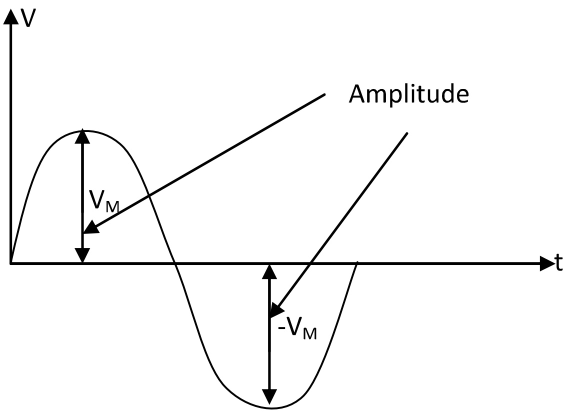
Time period: It is a time taken to complete one cycle is called time period. In the figure above T is the time period of alternating voltage V.

Frequency: The total number of cycle formed in one second is called frequency. It is given by

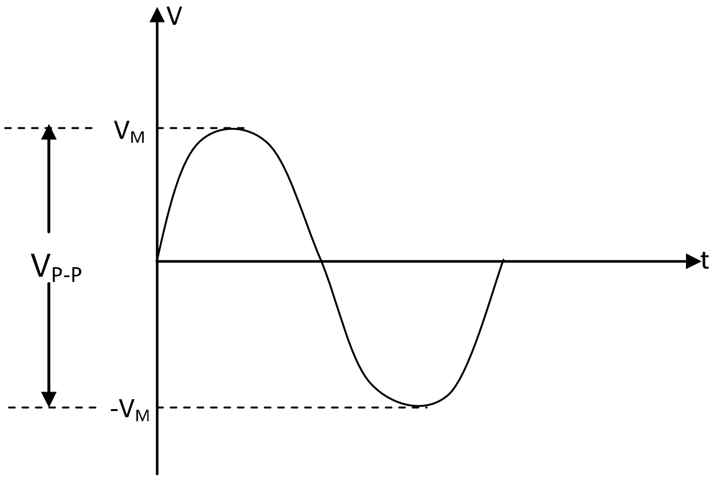
f = $\frac{1}{T}$ Hertz (Hz)

Cycle: One complete set of positive and negative values of alternating quantity is known as cycle. A cycle may also be sometimes specified in terms of angular measure. In that case, one complete cycle is said to be spread over 360^0 or 2π radians.

Amplitude: The maximum value, positive or negative, of an alternating quantity is known as its amplitude.



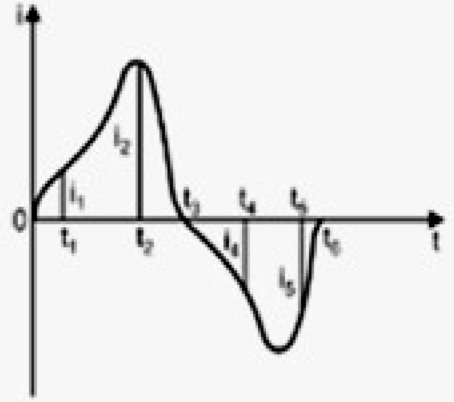
Peak to Peak value: The full value of alternating quantity from negative peak to positive peak of the waveform is called peak to peak value. Peak to peak value is the numerical sum of magnitudes of +ve peak and –ve peak. It is generally denoted by $V_{p\text{-}p}$ for voltage and $I_{p\text{-}p}$ for current. From figure given below $V_{p\text{-}p}$ is the peak to peak voltage for given alternating voltage and given by $V_{p\text{-}p} = 2V_m$



Average value and RMS or effective value of any type of alternating voltage or current waveform

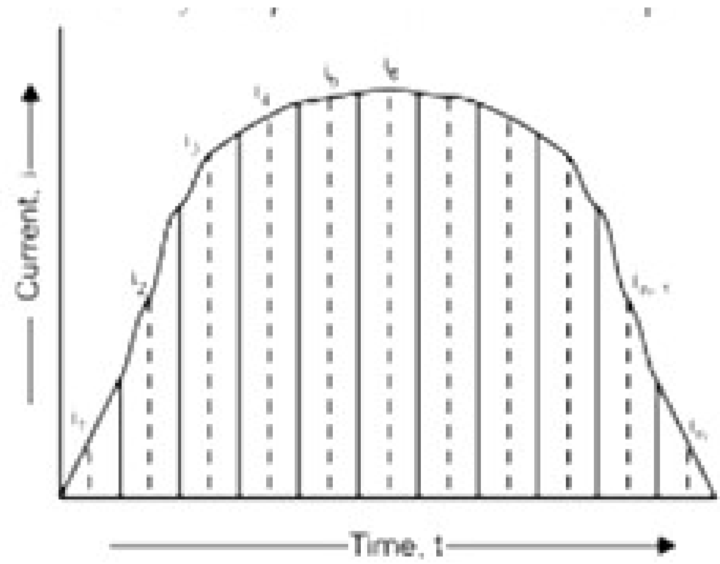
Instantaneous Value: The value of the alternating quantity at a given instant (time) is called the instantaneous value. It varies from instant to instant. Instantaneous values are denoted by small (lower case) letters. Thus, i denote the instantaneous current v denotes instantaneous voltage.

Time t	*Instantaneous Current*
0	0
t_1	i_1
t_2	i_2
t_3	0
t_4	$-i_4$
t_5	$-i_5$
t_6	0



The table gives some instantaneous values of the current wave form as shown in figure.

Average value: The average value of an alternating quantity is defined as the value which is obtained by averaging all the instantaneous value over a period of half cycle. In other words, the average or mean value of an alternating current is equal to the value of direct current which transfers across any circuit the same charge as it is transferred by alternating current during a given time.



The average value in a complete cycle is given by,

$V_{ave} = \int_0^{2\pi} \frac{V}{2\pi - 0}\, d(wt)$

$= \frac{1}{2\pi}\int_0^{2\pi} V_m \sin wt\, d(wt)$

$= \frac{V_m}{2\pi} |-coswt|_0^{2\pi}$

$= - \frac{V_m}{2\pi}[cos2\pi - cos0]$

$= -\frac{V_m}{2\pi}[cos2\pi - cos0]$

$= - \frac{V_m}{2\pi}[1 - 1]$

$= 0$

The average value in a half cycle is given by

$V_{ave} = \int_0^{\pi} \frac{V}{\pi - 0}\, d(wt)$

$= = \frac{1}{\pi}\int_0^{\pi} V_m \sin wt\, d(wt)$

$= \frac{V_m}{\pi} |-coswt|_0^{\pi}$

$= - \frac{V_m}{\pi}[cos\pi - cos0]$

$= - \frac{V_m}{\pi}[-1 - 1]$

$= \frac{2V_m}{\pi} = 0.63V_m$

Similarly,

$I_{ave} = \frac{2I_m}{\pi} = 0.63I_m$

So $\boxed{V_{ave} = \frac{2V_m}{\pi} = 0.63V_m \quad \text{and } I_{ave} = \frac{2I_m}{\pi} = 0.63I_m}$

Since in the case of a symmetrical alternating current (i.e. one whose two half cycles are exactly similar, whether sinusoidal or non sinusoidal) the average or mean value over a complete cycle is zero hence for such alternating quantities average or mean value means the value determined by taking the average of instantaneous value during half cycle or one alternation only. However, for unsymmetrical alternating current, as half-wave rectified current, the average value means the value determined by taking the mean of instantaneous values over the whole cycle.

Any function whose cycle is repeated continuously, irrespective of its wave shape, is termed as periodic function, such as sinusoidal function and its average value is given by

$F_{avg} = \frac{1}{T}\int_0^T f(t)dt \quad where\ T\ is\ time\ period\ of\ periodic\ function.$

In case of a symmetrical alternating current, whether sinusoidal or non sinusoidal, the average value is determined by taking average of one half cycle or one alteration only.

i.e. for symmetrical waveforms , $F_{avg} = \frac{2}{T}\int_0^{\frac{T}{2}} f(t)dt$

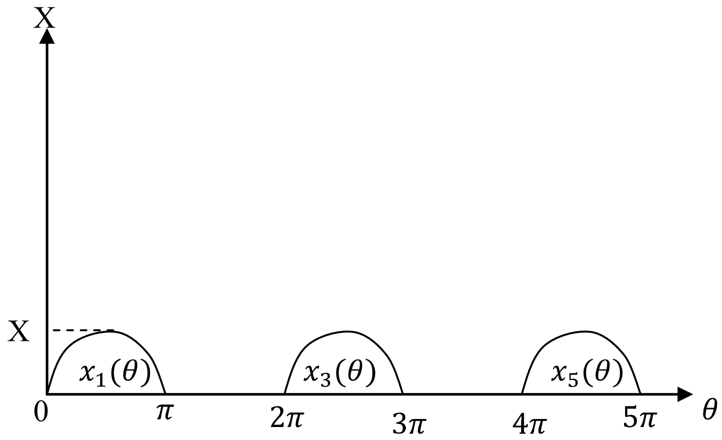
To find the average values follow the following steps:

- Find the time period and choose proper span for this time period.
- Area can be found by using integration i.e. $X_{avg} = \frac{1}{T}\int_0^T x(wt)dwt$
- Decompose $x(wt)$ if required. i.e. $\frac{1}{T}\left[\int_0^{T_1} x_1(wt)dwt + \int_{T_1}^{T_2} x_2(wt)dwt + \cdots\right]$
- Write the expressions for $x_1(t)$, $x_2(t)$, ... and substitute these values in above equation.
- Simplify the equation.

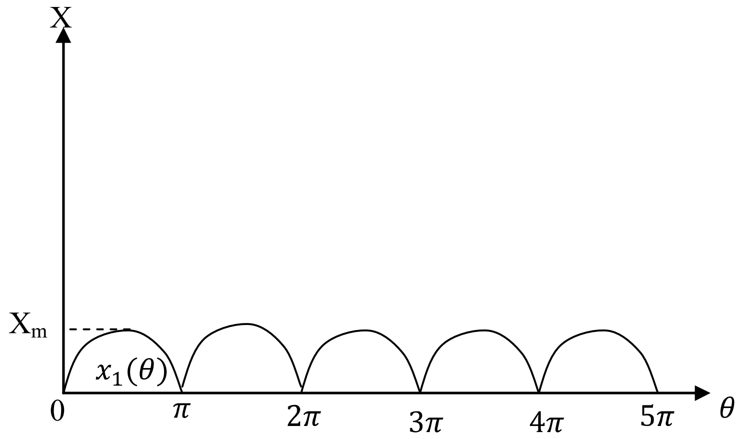
  Example: Find the average value of the wave shown in figure.

From figure:

- Time period T= 2π (0 to 2π)
- Note: We have to take the time period up to the point where it gets repeated.
- $X_{avg} = \frac{1}{2\pi}\int_0^{2\pi} x(\theta)d\theta$
- $X_{avg} = \frac{1}{2\pi}\left[\int_0^{\pi} x_1(\theta)d\theta + \int_{\pi}^{2\pi} x_2(\theta)d\theta\right]$
- $x_1(\theta) = X_m Sin\theta$ and $x_2(\theta) = 0$
- $X_{avg} = \frac{1}{2\pi}\left[\int_0^{\pi} X_m\ Sin\theta d\theta + \int_{\pi}^{2\pi} 0\ d\theta\right]$
- $X_{avg} = \frac{X_m}{\pi}$



Similarly for wave given below the average value is calculated as



From figure:

- Time period T= π (0 to π)
- Note: We have to take the time period up to the point where it gets repeated.
- $X_{avg} = \frac{1}{\pi}\int_0^{\pi} x(\theta)d\theta$
- $X_{avg} = \frac{1}{\pi}[\int_0^{\pi} x_1(\theta)d\theta]$
- $x_1(\theta) = X_m Sin\theta$
- $X_{avg} = \frac{1}{\pi}[\int_0^{\pi} X_m Sin\theta d\theta]$
- $X_{avg} = \frac{2X_m}{\pi}$

Practical importance of Average value:

- The average value is used for application like battery charging and rectifying circuits etc.
- The d.c. ammeters and voltmeters indicate the average value.
- The charge transferred in capacitor circuits is measured using average values.
- The average value of purely sinusoidal waveform is always zero.

RMS or effective value: The RMS value, also called effective value of an alternating quantity voltage (or current) is equal to that steady or DC voltage (or current) which when flows or applied to a given resistance for a given time produces the same amount of heat as when the alternating current or voltage is flowing or applied to the same resistance for the same time.

The effective value of alternating current or voltage is equal to the square root of the mean of the squares of successive ordinates and that is why it is known as root-mean square(rms) value. Using the integral calculus the root mean square (rms) or effective value of an alternating quantity over a time period is given by

$$F_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)dt}$$

A sinusoidal alternating current is represented by $i = I_{max}$ sinwt

$$I^2_{rms} = \frac{Area\ of\ first\ half\ cycle\ of\ i^2}{\pi} = \frac{1}{\pi}\int_0^{\pi} i^2 d(wt)$$

$$= \frac{1}{\pi}\int_0^{\pi} I^2_{max}\ Sin^2 wt\ d(wt)$$

$$= \frac{I^2_{max}}{2\pi}\int_0^{\pi}(1 - \cos 2wt\ )d(wt)$$

$$= \frac{I^2_{max}}{2\pi}\left[wt - \frac{1}{2}\sin 2wt\ \right]_0^{\pi}$$

$$= \frac{I^2_{max}}{2\pi} \times \pi = \frac{I^2_{max}}{2}$$

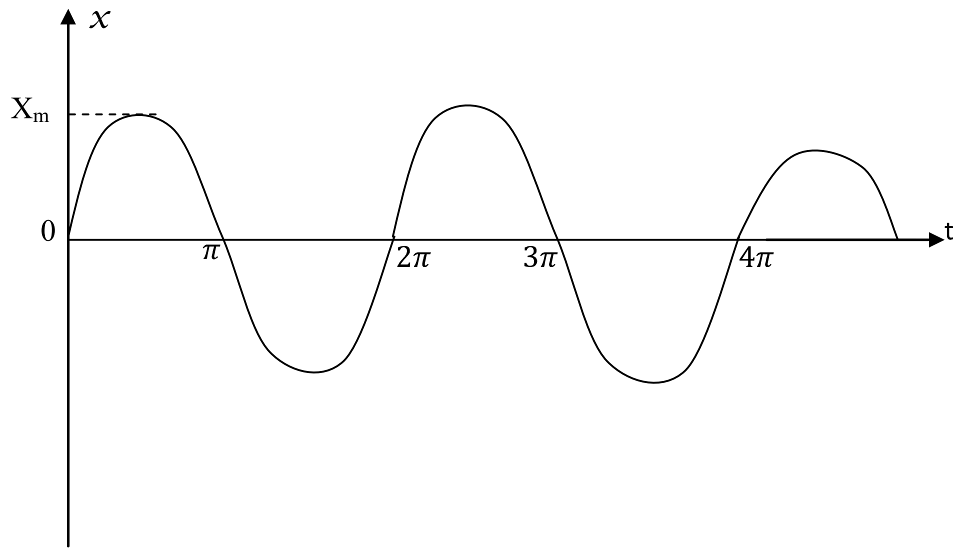
Or, $I_{rms} = \sqrt{\frac{I^2_{max}}{2}} = \frac{I_{max}}{\sqrt{2}}$

Similarly, $E_{rms} = \frac{E_{max}}{\sqrt{2}}$

To find the rms values follow the following steps:

- Find the time period and choose proper span for this time period.
- Area can be found by using integration i.e. $X^2_{rms} = \frac{1}{T}\int_0^T x^2(wt)dwt$
- Decompose $x(wt)$ if required. i.e. $X^2_{rms} = \frac{1}{T}[\int_0^{T_1} x_1^2(wt)dwt + \int_{T_1}^{T_2} x_2^2(wt)dwt + \cdots]$
- Write the expressions for $x_1(t)$, $x_2(t)$, ..and substitute these values in above equation.
- Simplify the equation.

Example: Find the rms value of the wave shown in figure.



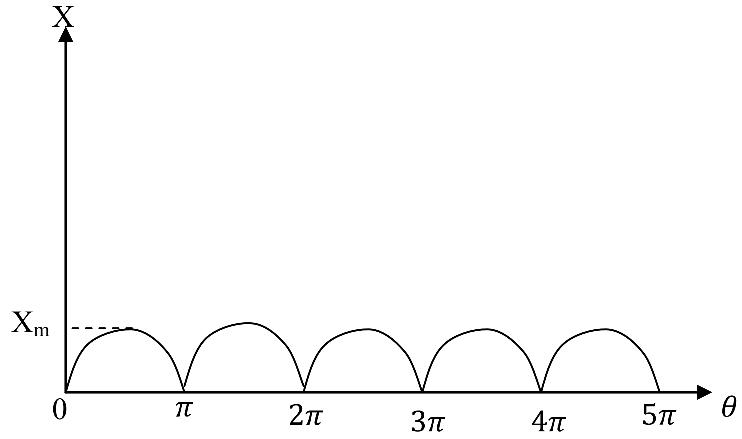
From figure:

- Time period T= 2π (Time from which signal gets repeated)
- Note: We have to take the time period up to the point where it gets repeated.
- $X_{rms}^2 = \frac{1}{2\pi}\int_0^{2\pi} x^2(\theta)d\theta$

No need to decompose the signal as it is repeated in same manner.

- $x(\theta) = X_m Sin\theta$
- $X_{rms}^2 = \frac{1}{2\pi}\left[\int_0^{2\pi} X_m^2 \, sin^2\theta d\theta \right]$
- $X_{rms}^2 = \frac{X_m^2}{2}$
- $X_{rms} = \frac{X_m}{\sqrt{2}}$

Similarly for the signal given below, we can calculate the rms value as:



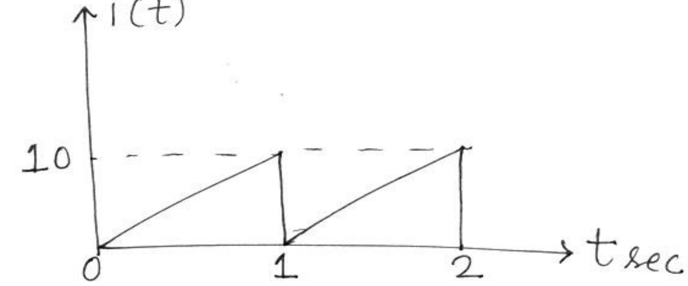
- Time period T= π (Time from which signal gets repeated)

- $X_{rms}^2 = \frac{1}{\pi}\int_0^{\pi} x^2(\theta)d\theta$

No need to decompose the signal as it is repeated in same manner.

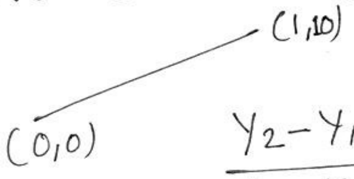
- $x(\theta) = X_m Sin\theta$
- $X_{rms}^2 = \frac{1}{\pi}\ [\int_0^{\pi} X_m^2\ sin^2\theta d\theta\ ]$
- $X_{rms}^2 = \frac{X_m^2}{\pi}\ [\int_0^{\pi}\ (\frac{1-cos2\theta}{2})d\theta\ ]$
- $X_{rms}^2 = \frac{X_m^2}{2\pi}\ [\int_0^{\pi}\ d\theta\ -\ \int_0^{\pi}\ cos2\theta d\theta\ ]$
- $X_{rms}^2 = \frac{X_m^2}{2\pi}\ [\int_0^{\pi}\ d\theta\ -\ \int_0^{\pi}\ cos2\theta d\theta\ ]$
- $X_{rms}^2 = \frac{X_m^2}{2\pi}\ [\ [\theta]_0^{\pi} - \left[\frac{sin2\theta}{2}\right]_0^{\pi}]$
- $X_{rms}^2 = \frac{X_m^2}{2\pi}\ [\ \pi - 0 - \frac{sin2\pi}{2} - \frac{sin0}{2}]$
- $X_{rms}^2 = \frac{X_m^2}{2\pi}\ [\ \pi - 0 - 0 - 0]$
- $X_{rms}^2 = \frac{\pi X_m^2}{2\pi}$
- $X_{rms} = \frac{X_m}{\sqrt{2}}$

Problem: Find RMS and average value of current for the signal ~~shown~~ (waveform shown in figure.



Soln Time period (T) = 1 sec

We can use equation of straight line



$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y}{x_1 - x}$$

$$\frac{10-0}{1-0} = \frac{0 - i(t)}{0 - t}$$

or $\frac{10}{1} = \frac{-i(t)}{-t}$

or $\boxed{i(t) = 10t}$

So, $i(t) = 10t$ .... for $0 < t < 1$ which is equation of waveform.

$$I_{avg} = \frac{1}{T}\int_0^T i\,dt$$

$$= \frac{1}{1}\int_0^1 10t\,dt$$

$$I_{avg} = 10\int_0^1 t\,dt$$
$$= 10\left[\frac{t^2}{2}\right]_0^1$$
$$= 10 \times \frac{1^2}{2} = 5\ A$$
$$\therefore \boxed{I_{avg} = 5\ A}$$

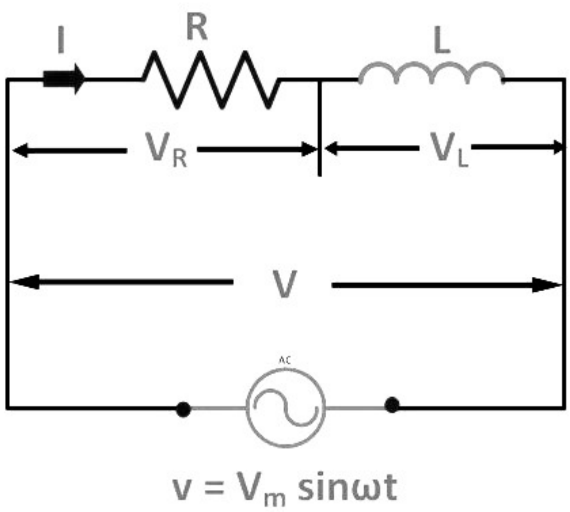
$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{1}\int_0^1 (10t)^2\,dt}$$
$$= \sqrt{\int_0^1 100t^2\,dt}$$
$$= 10\sqrt{\int_0^1 t^2\,dt} = 10\sqrt{\left(\frac{t^3}{3}\right)_0^1}$$
$$\therefore I_{rms} = \frac{10}{\sqrt{3}}\sqrt{\left[t^3\right]_0^1}$$
$$= \frac{10}{\sqrt{3}} \times \sqrt{1^3 - 0^3}$$
$$= \frac{10}{\sqrt{3}} = 5.7735\ A$$

3.4 Phase algebra, power triangle, impedance triangle, steady state response of circuits (RL, RC, RLC series and parallel) and concept about admittance, impedance reactance and its triangle), instantaneous power, average real-power, reactive power, power factor and significance of power factor, resonance in series and parallel RLC circuit. bandwidth, effect of Q-factor in resonance.

RL Series Circuit

A circuit that contains pure resistance R ohms connected in series with a coil having a pure inductance of L (Henry) is known as **RL Series Circuit**. When an AC supply voltage V is applied, the current, I flow in the circuit.

So, I_R and I_L will be the current flowing in the resistor and inductor respectively, but the amount of current flowing through both the elements will be same as they are connected in series with each other. The circuit diagram of RL Series Circuit is shown below:

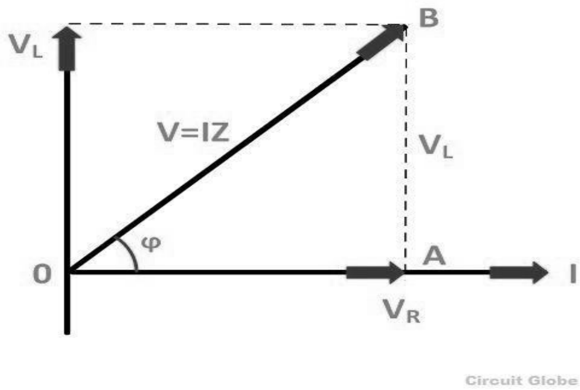


Where,

- V_R – voltage across the resistor R
- V_L – voltage across the inductor L
- V – Total voltage of the circuit

Phasor Diagram of the RL Series Circuit

The phasor diagram of the RL Series circuit is shown below:



VV V

Steps to draw the Phasor Diagram of RL Series Circuit

The following steps are given below w by step:

- Current I is taken as a reference.
- The Voltage drop across the resistance $V_R = I_R$ is drawn in phase with the current I.
- The voltage drop across the inductive reactance $V_L = IX_L$ is drawn ahead of the current I. As the current lags voltage by an angle of 90 degrees in the pure Inductive circuit.
- The vector sum of the two voltages drops V_R and V_L is equal to the applied voltage V.

Now,

In right-angle triangle OAB

$V_R = IR$ and $V_L = IX_L$ where $X_L = 2\pi fL$

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2} \quad \text{or}$$

$$I = L = \frac{V}{Z}$$

Where,

$$Z = \sqrt{R^2 + X_L^2}$$

Z is the total opposition offered to the flow of alternating current by an RL Series circuit and is called impedance of the circuit. It is measured in ohms (Ω).

Phase Angle

In RL Series circuit the current lags the voltage by 90 degrees angle known as phase angle. It is given by the equation:

$$\tan\varphi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L}{R}$$

Power in R L Series Circuit

If the alternating voltage applied across the circuit is given by the equation:

$$v = V_m \mathrm{Sin}\omega t \ldots\ldots\ldots\ldots\ldots (1)$$

The equation of current I is given as: $i = I_m \sin(\omega t - \varphi) \ldots \ldots \ldots . (2)$

Then the instantaneous power is given by the equation: $p = v\,i \ldots \ldots \ldots . (3)$

Putting the value of v and i from the equation (1) and (2) in the equation (3) we will get

$$P = (V_m \mathrm{Sin}\omega t) x\, I_m \sin(\omega t - \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t - \varphi) \sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} [\cos\varphi - \cos(2\omega t - \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t - \varphi)$$

The average power consumed in the circuit over one complete cycle is given by the equation shown below:

$$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{V_m}{\sqrt{2}} \cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}} \frac{V_m}{\sqrt{2}} \cos(2\omega t - \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s} \cos\varphi = V\, I \cos\varphi$$

$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \ldots\ldots\ldots\ldots (4)$$

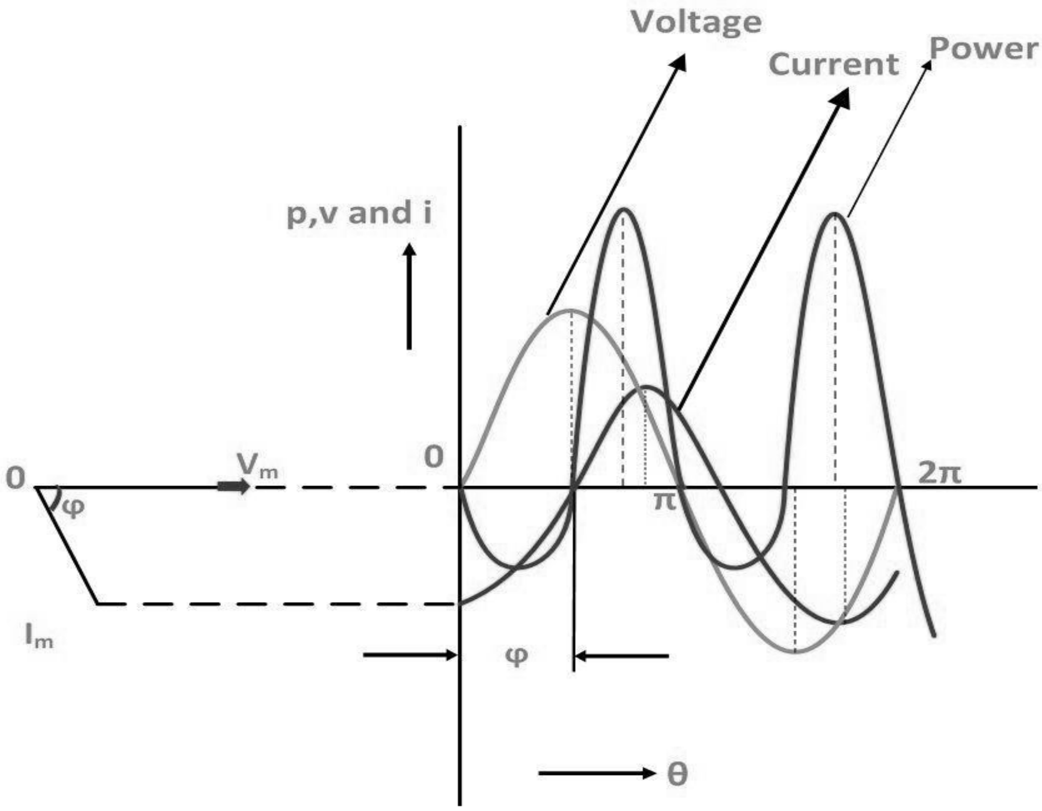
Where $\cos\phi$ is called the power factor of the circuit.
The power factor is defined as the ratio of resistance to the impedance of an AC Circuit.
Putting the value of V and $\cos\phi$ from the equation (4) the value of power will be:

$$P = (IZ)(I)(R/Z) = I^2 R \ldots\ldots\ldots (5)$$

From equation (5) it can be concluded that the inductor does not consume any power in the circuit.

Waveform and Power Curve of the RL Series Circuit

The **waveform** and **power curve** of the RL series circuit is shown below:

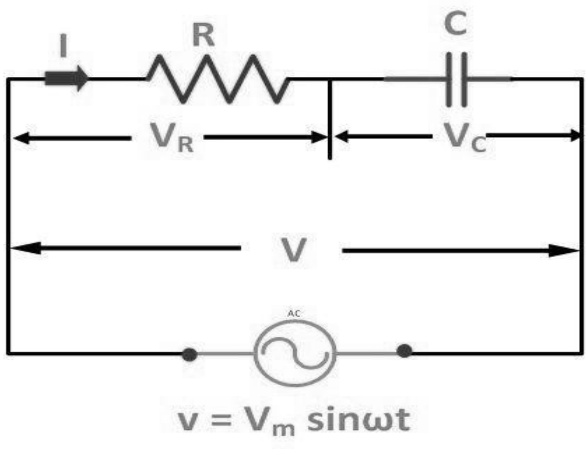


The various points on the power curve are obtained by the product of voltage and current.
If you analyze the curve carefully, it is seen that the power is negative between angle 0 and ϕ and between 180 degrees and (180 + ϕ) and during the rest of the cycle the power is positive. The current lags the voltage and thus they are not in phase with each other.

RC Series Circuit

A circuit that contains pure resistance R ohms connected in series with a pure capacitor of capacitance C farads is known as **RC Series Circuit.** A sinusoidal voltage is applied and current I flows through the resistance (R) and the capacitance (C) of the circuit.

The RC Series circuit is shown in the figure below:

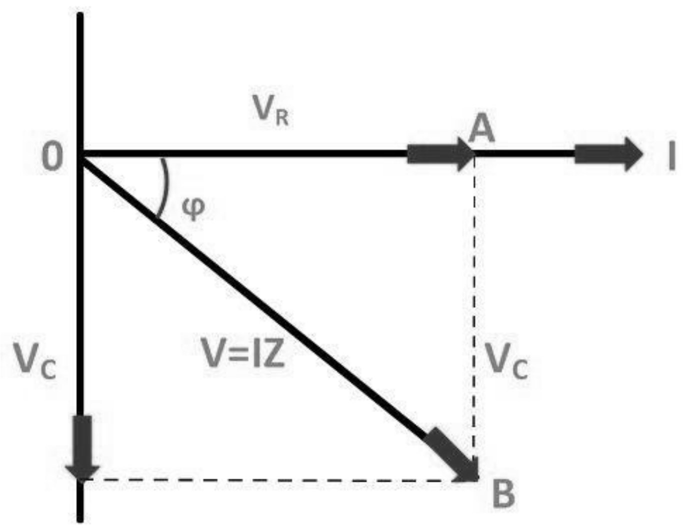


Where,

- V_R – voltage across the resistance R
- V_C – voltage across capacitor C
- V – total voltage across the RC Series circuit

Phasor Diagram of RC Series Circuit

The phasor diagram of the RC series circuit is shown below:



Steps to draw a Phasor Diagram

The following steps are used to draw the phasor diagram of RC Series circuit

- Take the current I (r.m.s value) as a reference vector
- Voltage drop in resistance VR = IR is taken in phase with the current vector
- Voltage drop in capacitive reactance VC = IXC is drawn 90 degrees behind the current vector, as current leads voltage by 90 degrees (in the pure capacitive circuit)
- The vector sum of the two voltage drops is equal to the applied voltage V (r.m.s value).

Now,

$\mathbf{V_R = I_R}$ and $\mathbf{V_C = IX_C}$

Where $X_C = I/2\pi fC$

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

In right triangle OAB,

Where,

$$Z = \sqrt{R^2 + X_C^2}$$

Z is the total opposition offered to the flow of alternating current by an RC series circuit and is called **impedance** of the circuit. It is measured in ohms (Ω).

Phase angle
From the phasor diagram shown above, it is clear that the current in the circuit leads the applied voltage by an angle ϕ and this angle is called the **phase angle**.

$$\tan\varphi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_C}{R}$$

Power in RC Series Circuit

If the alternating voltage applied across the circuit is given by the equation

$$v = V_m \mathrm{Sin}\omega t \,\ldots\ldots\ldots\ldots\ldots(1)$$

then,

$$i = I_m \sin(\omega t + \varphi) \ldots\ldots\ldots\ldots (2)$$

Therefore, the instantaneous power is given by p = vi

Putting the value of v and i from the equation (1) and (2) in p = vi

$$P = (V_m \mathrm{Sin}\omega t) x\, I_m \sin(\omega t + \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t + \varphi)\sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}[\cos\varphi - \cos(2\omega t + \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(2\omega t + \varphi)$$

The average power consumed in the circuit over a complete cycle is given by:

$$P = \text{ average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{ average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s} \cos\varphi = V\, I \cos\varphi$$

Where cosϕ is called the **power factor** of the circuit.

$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \dots\dots\dots\dots (3)$$

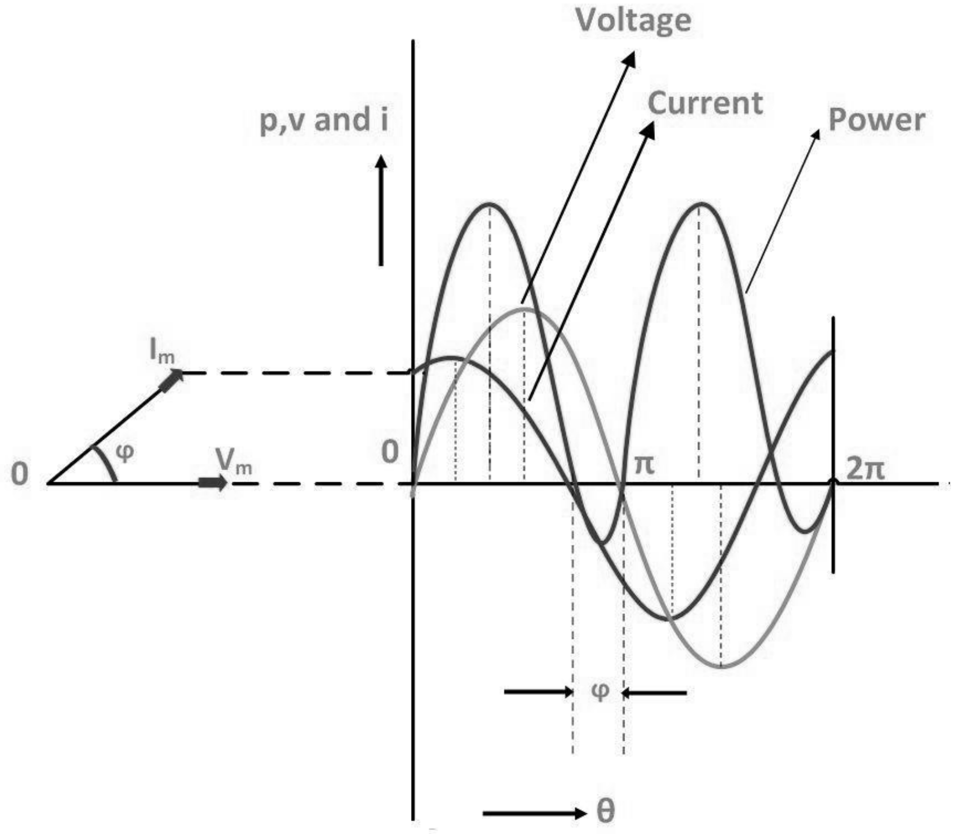
Putting the value of V and cosϕ from the equation (3) the value of power will be

$$P = (IZ)(I)(R/Z) = I^2 R \dots\dots\dots (4)$$

From the equation (4) it is clear that the power is actually consumed by the resistance only and the capacitor does not consume any power in the circuit.

Waveform and Power Curve of the RC Series Circuit

The waveform and power curve of the RC circuit is shown below:



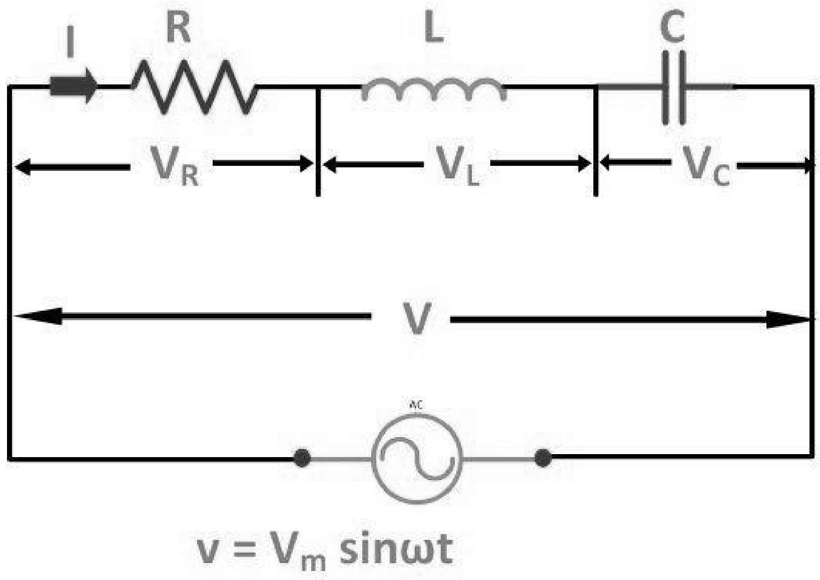
The various points on the power curve are obtained from the product of the instantaneous value of voltage and current.

The power is negative between the angle (180° – ϕ) and 180° and between (360° -ϕ) and 360° and in the rest of the cycle, the power is positive. Since the area under the positive loops is greater than that under the negative loops, therefore the net power over a complete cycle is **positive**.

RLC Series Circuit

When a pure resistance of R ohms, a pure inductance of L Henry and a pure capacitance of C farads are connected together in series combination with each other then **RLC Series Circuit** is formed. As all the three elements are connected in series so, the current flowing through each element of the circuit will be the same as the total current I flowing in the circuit.

The **RLC Circuit** is shown below:



In the RLC Series circuit

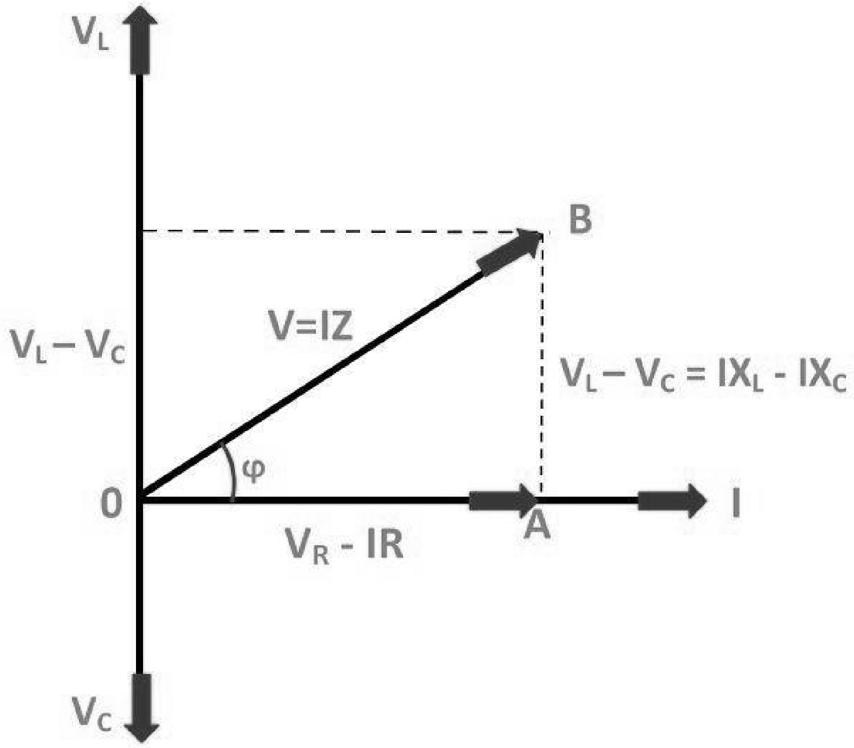
$X_L = 2\pi fL$ and $X_C = 1/2\pi fC$

When the AC voltage is applied through the RLC Series circuit the resulting current I flows through the circuit, and thus the voltage across each element will be:

- $V_R = IR$ that is the voltage across the resistance R and is in phase with the current I.
- $V_L = IX_L$ that is the voltage across the inductance L and it leads the current I by an angle of 90 degrees.
- $V_C = IX_C$ that is the voltage across capacitor C and it lags the current I by an angle of 90 degrees.

Phasor Diagram of RLC Series Circuit

The phasor diagram of the RLC series circuit when the circuit is acting as an inductive circuit that means ($V_L > V_C$) is shown below and if ($V_L < V_C$) the circuit will behave as a capacitive circuit.



Steps to draw the Phasor Diagram of the RLC Series Circuit

- Take current I as the reference as shown in the figure above
- The voltage across the inductor L that is V_L is drawn leads the current I by a 90-degree angle.
- The voltage across the capacitor c that is V_c is drawn lagging the current I by a 90-degree angle because in capacitive load the current leads the voltage by an angle of 90 degrees.
- The two vectors V_L and V_C are opposite to each other.

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad \text{or}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Where,

It is the total opposition offered to the flow of current by an RLC Circuit and is known as **Impedance** of the circuit.

Phase Angle

From the phasor diagram, the value of phase angle will be

$$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L - X_C}{R}$$

Power in RLC Series Circuit

The product of voltage and current is defined as power.

$$P = VI\cos\varphi = I^2R$$

Where $\cos\phi$ is the power factor of the circuit and is expressed as:

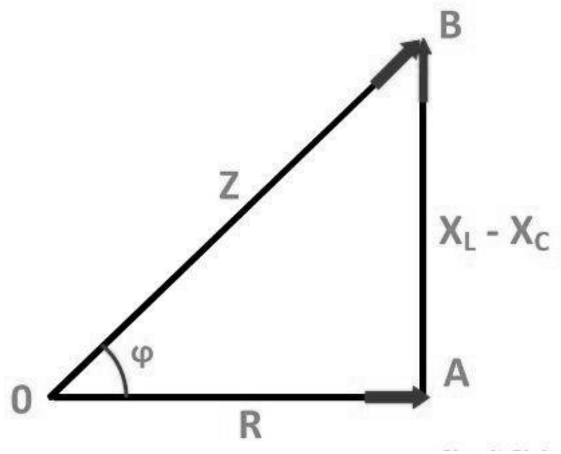
$$\cos\varphi = \frac{V_R}{V} = \frac{R}{Z}$$

The three cases of RLC Series Circuit

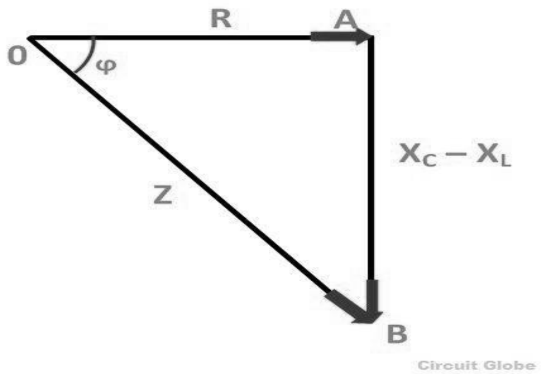
- When $X_L > X_C$, the phase angle ϕ is positive. The circuit behaves as RL series circuit in which the current lags behind the applied voltage and the power factor is lagging.
- When $X_L < X_C$, the phase angle ϕ is negative, and the circuit acts as a series RC circuit in which the current leads the voltage by 90 degrees.
- When $X_L = X_C$, the phase angle ϕ is zero, as a result, the circuit behaves like a purely resistive circuit. In this type of circuit, the current and voltage are in phase with each other. The value of the power factor is **unity**.

Impedance Triangle of RLC Series Circuit

When the quantities of the phasor diagram are divided by the common factor I then the right angle triangle is obtained known as impedance triangle. The impedance triangle of the RL series circuit, when ($X_L > X_C$) is shown below:



If the inductive reactance is greater than the capacitive reactance than the circuit reactance is inductive giving a **lagging phase angle**.
Impedance triangle is shown below when the circuit acts as an RC series circuit ($X_L < X_C$)



When the capacitive reactance is greater than the inductive reactance the overall circuit reactance acts as capacitive and the phase an

Applications of RLC Series Circ

The following are the application of the RLC circuit:

- It acts as a variable tuned circuit
- It acts as a low pass, high pass, band pass, band stop filters depending upon the type of frequency.
- The circuit also works as an oscillator
- Voltage multiplier and pulse discharge circuit

RL, RC, RLC parallel circuits:

RL parallel circuit: Let us consider following R-L parallel circuit supplied by an ac source as shown in figure.

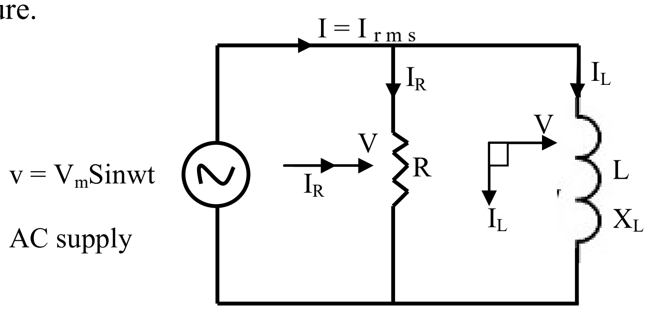
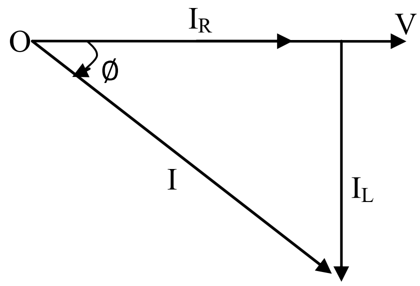


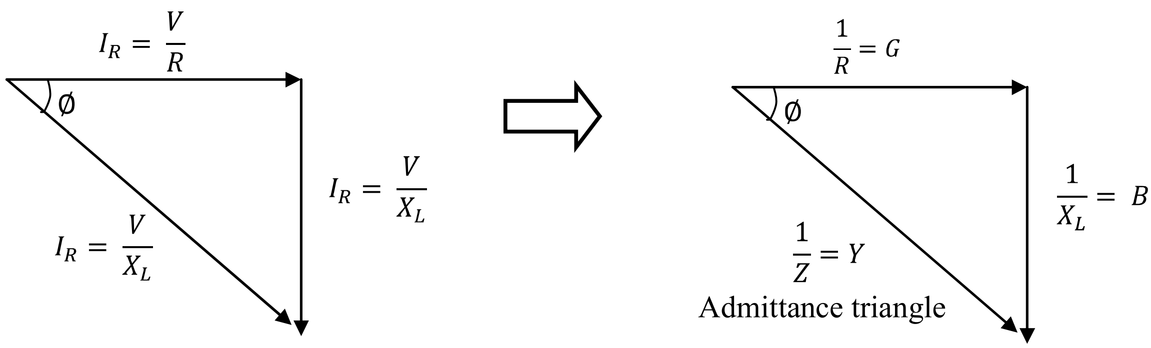
Figure: AC through parallel circuit

From figure R = resistance in Ω , X_L = inductive reactance in Ω which is equal to wL (X_L = wL).

- In parallel circuit voltage is same or equal through all parallel branches.
- Here $I_R = \left(\frac{V}{R}\right)$ Amp and $I_L = \left(\frac{V}{X_L}\right)$ Amp
- Now from phasor diagram as shown in figure
  Here we have,
  $I^2 = I_R^2 + I_L^2$
  $\Rightarrow I = \sqrt{I_R^2 + I_L^2}$
- Now phase angle $\text{Tan}\,\emptyset = \frac{I_L}{I_R}$
  Or, $\emptyset = Tan^{-1}\left(\frac{I_L}{I_R}\right)$
  $0 < \emptyset < 90$
- Here current lags behind voltage by angle $\emptyset$.
- Now power factor = $\text{Cos}\emptyset = \left(\frac{I_R}{I}\right)$ lagging.
- Now from phasor diagram we have



Phasor diagram of the circuit (Current triangle)



Where G = $\frac{1}{R}$ conductance (mho)

B = $\frac{1}{X_L}$ = inductive susceptance (mho)

And Y = $\frac{1}{Z}$ = Admittance (mho)

Now we have $Y^2 = G^2 + B^2 \Rightarrow Y = \sqrt{G^2 + B^2}$

From Ohm's law V = I.Z

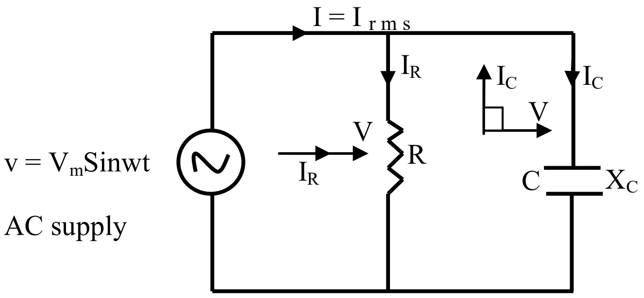
$\Rightarrow I = \frac{V}{Z} = V\left(\frac{1}{Z}\right) \Rightarrow I = V.Y$

& $P = \frac{V^2}{R}$ = VI Cos∅ Watts

AC through R-C parallel circuit

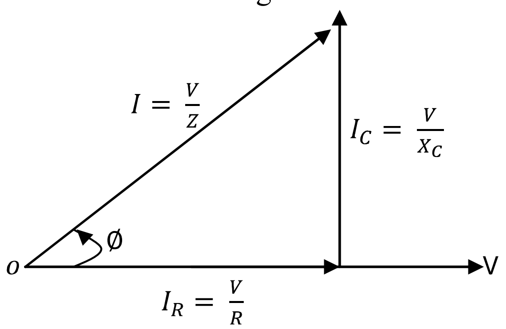
Let us consider following R-C parallel circuit supplied by ac voltage source.

Where R = resistance (Ω) and $\frac{1}{X_C} = \frac{1}{wC}$ = capacitive reactance (Ω)



AC through parallel R-C circuit

- Here voltage is common, so voltage is taken as reference.
- Where $I_R = \left(\frac{V}{R}\right)$ Amp and $I_C = \left(\frac{V}{X_C}\right)$ Amp
- Now Phasor diagram of the circuit is as shown as



Phasor diagram of the circuit

Where, $I^2 = I_R^2 + I_C^2$

Or, $I = \sqrt{I_R^2 + I_C^2}$

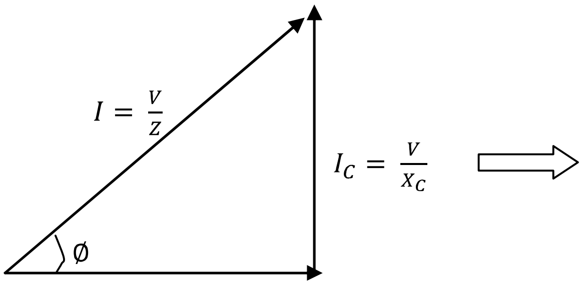
Current (I) leads the voltage V by an angle ∅

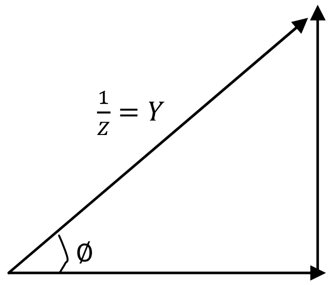
Phase angle ∅ = Tan^{-1} (I_C / I_R) ($0 < ∅ < 90^0$)

Power factor Cos ∅ = $\frac{I_R}{I}$ leading

Note: *Anticlockwise angle indicates lead behavior and clockwise angle indicates lag behavior.*

Now again from phasor diagram we have,





Admittance triangle

- From admittance triangle,

  Y = $\frac{1}{Z}$ = Admittance (mho)

  G = $\frac{1}{R}$ = conductance (mho)

  And $\frac{1}{X_C} = B =$ Capacitive susceptance (mho)

  And $Y^2 = G^2 + B^2 \Longrightarrow Y = \sqrt{G^{2+}B^2}$ mho

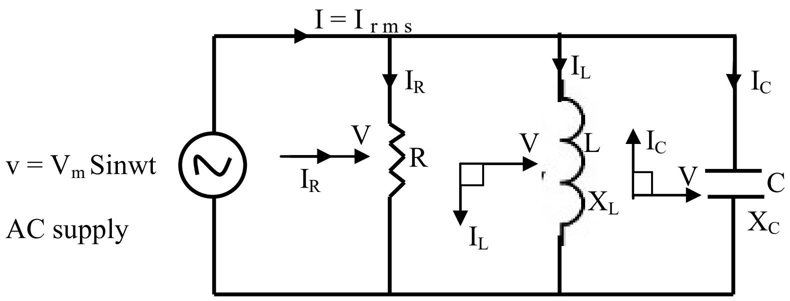
  From Ohm's law V = I.Z

  I= V.Y Amp

  And P = $\frac{V^2}{R} = VIcos\emptyset$

AC through R-L-C parallel circuit

Let us consider following R-L-C parallel circuit supplied by an ac voltage source.



R-L-C parallel circuit

Where $I_R = \frac{V}{R}$ Amp

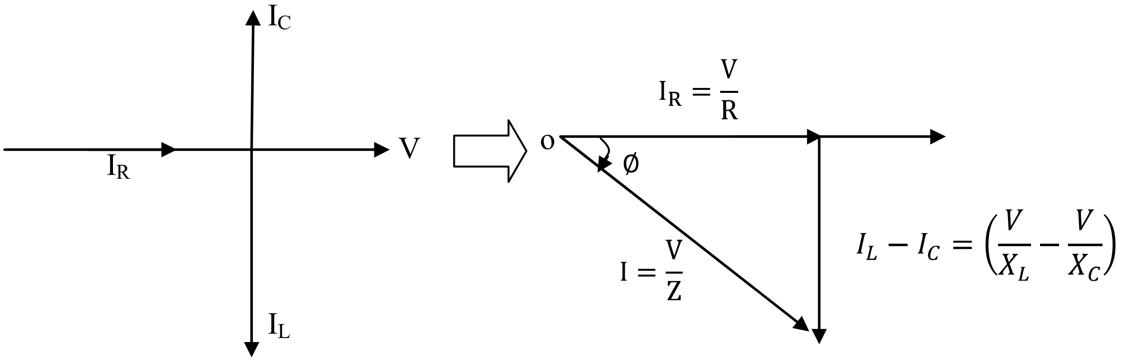
$$I_L = \frac{V}{X_L} = \frac{V}{wL} \text{ Amp}$$

$$I_L = \frac{V}{X_C} = \frac{V}{\left(\frac{1}{wC}\right)} \text{ Amp}$$

- AC through R-L-C parallel circuit
  - Here voltage is common, so voltage is taken reference.
  - Now phasor diagrams for different cases are as shown below-

- **Case –I**

  $I_L > I_C$ or $X_C > X_L$ (lower frequency case)



Phasor diagram

From phasor diagram,

$I^2 = I_R^2 + (I_L - I_C)^2$

$\therefore I = \sqrt{I_R^2 + (I_L - I_C)^2}$ ....(i)

Current lags behind the voltage by an angle $\emptyset$

$\therefore$ Power factor = Cos$\emptyset$ = $\left(\frac{I_R}{I}\right)$ lagging

Now from equation (i) we have

$$I = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2}$$

$$\text{Or, } I = V\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

$$\text{Or, } \frac{I}{V} = Y = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2} = \text{Admittance (mho)}$$

$\Longrightarrow Y = \sqrt{G^2 + (B_L - B_C)^2}$ mho

From Ohm's law V = I.Z and I = $\frac{V}{Z}$ = V.Y and power

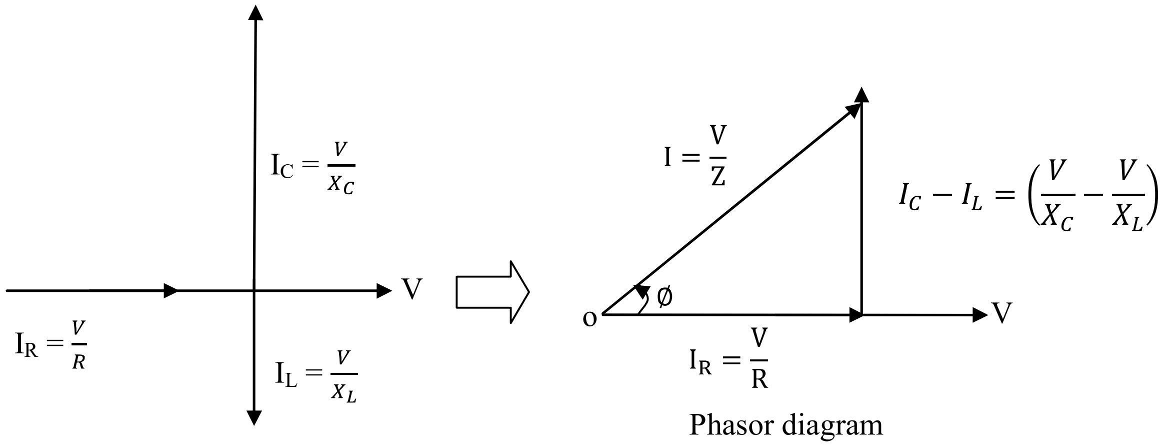
$P = \frac{V^2}{R} = VI\cos\emptyset$ watt

Where $G = \frac{1}{R}$ = conductance (mho)

$B_L = \frac{1}{X_L}$ = inductive susceptance (mho)

$B_C = \frac{1}{X_C}$ = capacitive susceptance (mho)

- **Case –II**
  $I_C > I_L$ or $X_L > X_C$ (higher frequency case)



Phasor diagram

From phasor diagram,

$I^2 = I_R^2 + (I_C - I_L)^2$

$\therefore I = \sqrt{I_R^2 + (I_C - I_L)^2}$ .... (i)

Here current leads the voltage by an angle ∅

$\therefore$ Power factor = Cos∅ = $\left(\frac{I_R}{I}\right)$ leading

Now from equation (i) we have

$$I = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_C} - \frac{V}{X_L}\right)^2}$$

$$\text{Or, } I = V\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$$

$$\text{Or, } \frac{I}{V} = Y = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2} \text{ mho}$$

$\Rightarrow Y = \sqrt{G^2 + (B_C - B_L)^2}$ (mho) = Admittance

Where $G = \frac{1}{R}$ = conductance (mho)

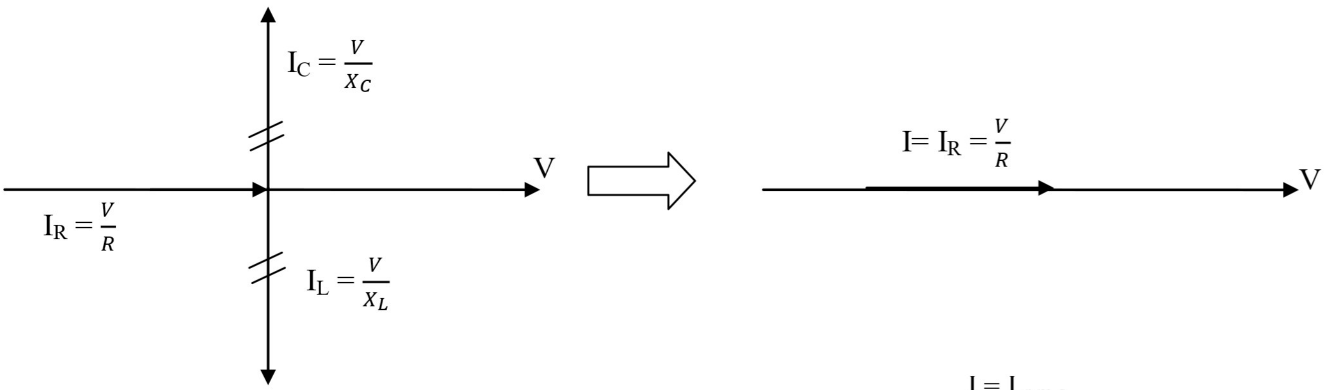
$B_L = \frac{1}{X_L}$ = inductive susceptance (mho)

$B_C = \frac{1}{X_C}$ = capacitive susceptance (mho)

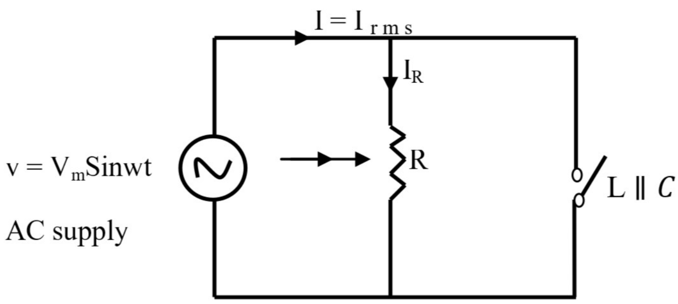
From Ohm's law V = I.Z and $I = \frac{V}{Z}$ = V.Y Amp and

Power loss in the circuit $P = \frac{V^2}{R}$ = VIcos∅ watt

- **Case –III**
  $I_L = I_C$ or $X_L = X_C$ (medium frequency case)



- Circuit will be purely resistive
- This is called parallel resonance case
- L || C behaves like open circuit



Since $Y = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$

A parallel resonance $X_L = X_C$

$Y = \frac{1}{Z} = \frac{1}{R} = G$ (mho)

And power factor $\cos\emptyset = \cos 0 = 1$(unity) as in case of purely resistive circuit

So the power (P) = $\frac{V^2}{R} = P_{min}$

Thus at resonance or parallel resonance with constant voltage source, circuit behaves like band stop filter.

Instantaneous Power

The instantaneous electric power in an AC circuit is given by P=VI where V and I are the instantaneous voltage and current.

Since,

$V = V_m Sinwt$

$I = I_m \sin(wt - \emptyset)$

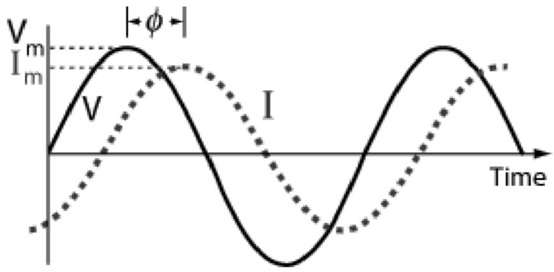
Then the instantaneous power at any time t can be expressed as

$P_{instantaneous} = V_m I_m \sin wt \sin(wt - \emptyset)$

And using the trigonometric identity

$\sin(wt - \emptyset) = \sin wt \cos\emptyset - \cos wt \sin\emptyset$

The power becomes:



$\mathbf{P_{instantaneous} = V_m I_m \cos\emptyset \sin^2 wt - V_m I_m \sin\emptyset \sin wtcoswt}$

Averaging this power over a complete cycle gives the average power.

Average Power

Normally the average power is the power of interest in AC circuits. Since the expression for the instantaneous power

$P_{instantaneous} = V_m I_m \cos\emptyset \sin^2 wt - V_m I_m \sin\emptyset \sin wtcoswt$

is a continuously varying one with time, the average must be obtained by integration. Averaging over one period T of the sinusoidal function will give the average power. The second term in the power expression above averages to zero since it is an odd function of t. The average of the first term is given by

$$P_{average} = V_m I_m \cos\emptyset \frac{\int_0^T \sin^2 wtdt}{T} = \frac{V_m I_m \cos\emptyset}{2}$$

Since the rms voltage and current are given by $V = \frac{V_m}{\sqrt{2}}$ and $I = \frac{I_m}{\sqrt{2}}$, the average power can be expressed as $\mathbf{P_{average} = VIcos\emptyset}$

Power in AC circuit: In AC circuits there are three types of power. They are

1. Apparent power (S)
2. Active or (real) power (P)
3. Reactive power (Q)

Let,

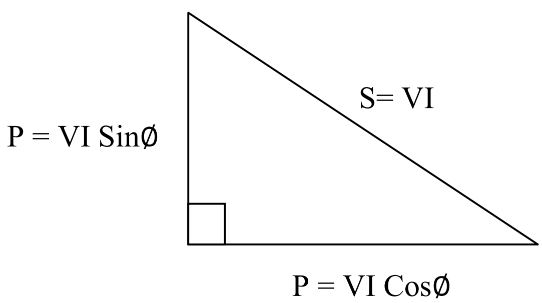
V = RMS value of applied voltage

I = Actual power (P)

$\emptyset$ = angle between voltage V and current I

1. **Apparent power:** It is the product of rms value of applied voltage and current. It is also known as watt less or idle power. S = V.I (volt-amp).
2. **Active power (P):** It is the power which is developed in the circuit resistance.
   P = V.I. cos$\emptyset$
   $= V.I.\ \frac{R}{Z} = \frac{V}{Z}\ .I.R$
3. **Reactive power (Q):** It is the power which is developed in the circuit reactance.
   Q = V I Sin$\emptyset$
   $= V.\ I.\ \frac{X}{Z} = \frac{V}{Z}\ .I.X$
   $= I.\ I.\ X = I^2X$

S = VI; **P = VI cos$\emptyset$;** **Q = VI sin$\emptyset$**



$P^2 + Q^2 = (V\ I\ Cos\emptyset)^2 + (VI\ Sin\emptyset)^2$

$P^2 + Q^2 = (VI)^2\ (Cos^2\emptyset + Sin^2\emptyset)$

$P^2 + Q^2 = (VI)^2\ .1$

$P^2 + Q^2 = (S)^2$

$S = \sqrt{P^2 + Q^2}$

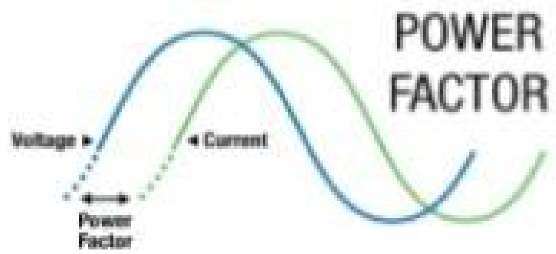
Power factor and significance of power factor:

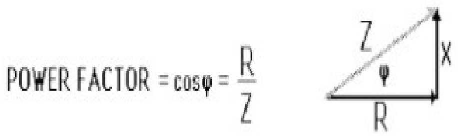
Power factor:

- The cosine of angle between voltage and current is called power factor and is given by $\cos\emptyset$ where $\emptyset$ is angle between voltage and current.

- It is defined as the ratio of resistance of the circuit to impedance of the circuit. i.e. $\cos\emptyset = \frac{R}{Z}$

- For purely resistive circuit, R = Z and power factor = 1.





- Power is ratio of Active power to Apparent power i.e. P.F. = $\frac{\text{KW}}{\text{KVA}} = \frac{\text{KW}}{\text{KVA}}$

What are the causes of low power factor?

- Since power factor is defined as the ratio of KW to KVA so the lower power factor results when KW is small in relation to KVA.
- The causes of low power factor are inductive loads. These inductive loads constitute major portion of power consumed in industrial complexes.
- Reactive power (KVAR) required by inductive loads increase the amount of apparent power (KVA) in our distribution system.
- The increase in reactive power and apparent power results in a larger angle of $\emptyset$. As $\emptyset$ increases, cosine $\emptyset$ (power factor) decreases.

Significance of power factor:

- A power factor of one or "unity power factor" is the goal of any electric utility company since if power factor is less than one, they have to supply more current to the user for a given amount of power use.
- Many practical loads have inductance as a result of their particular function, and it is essential for their proper operation. Examples are transformers, electric motors and speakers.
- A high power factor is an advantage in delivering power more efficiently to load.

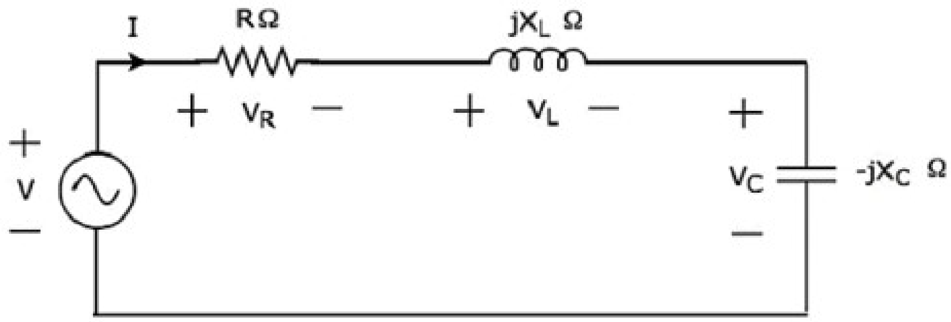
- Capacitors have the opposite effect and can compensate for the inductive motor windings. Some industrial sites will have large banks of capacitors for the purpose of correcting the power factor.

Resonance in series and parallel RLC circuit

Resonance in series RLC circuit:

Resonance occurs in electric circuits due to the presence of energy storing elements like inductor and capacitor**.**

If the resonance occurs in series RLC circuit, then it is called as **Series Resonance**. Consider the following **series RLC circuit**, which is represented in phasor domain.



Here, the passive elements such as resistor, inductor and capacitor are connected in series. This entire combination is in **series** with the input sinusoidal voltage source.

Apply **KVL** around the loop.

$V - V_R - V_L - V_C = 0$

$\Rightarrow V - IR - I(jX_L) - I(-jX_C) = 0$

$\Rightarrow V = IR + I(jX_L) + I(-jX_C)$

$\Rightarrow V = I[R + j(X_L - X_C)]$............................(i)

The above equation is in the form of $\boldsymbol{V = IZ}$.

Therefore, the **impedance Z** of series RLC circuit will be

$Z = R + j(X_L - X_C)$

Now, let us derive the values of parameters and electrical quantities at resonance of series RLC circuit.

Resonant Frequency

The frequency at which resonance occurs is called as **resonant frequency** $\boldsymbol{f_r}$. In series RLC circuit resonance occurs, when the imaginary term of impedance Z is zero, i.e., the value of $X_L - X_C$ should be equal to zero.

$\Rightarrow X_L = X_C$

Substitute $X_L = 2\pi fL$ and $XC = 1/2\pi fC$ in the above equation.

$2\pi fL = 1/2\pi fC$

or, $4\pi^2 f^2 LC = 1$

or, $f^2 = \frac{1}{4\pi^2 LC}$

or $f = \frac{1}{2\pi\sqrt{LC}}$

Therefore, the **resonant frequency** f_r of series RLC circuit is

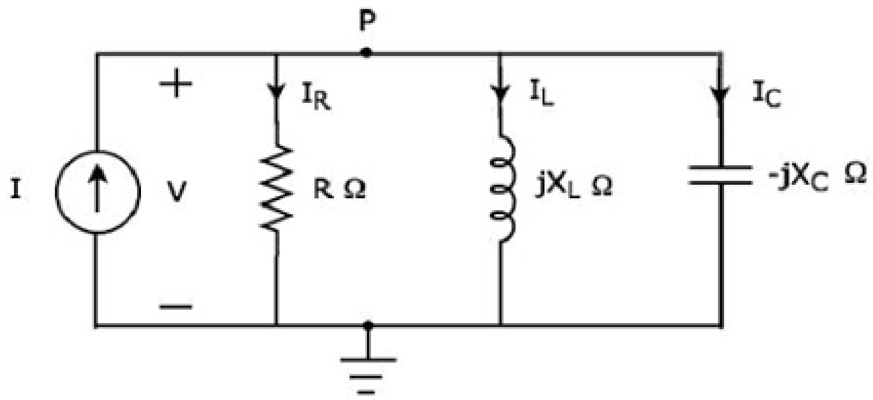
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Where, ***L*** is the inductance of an inductor and ***C*** is the capacitance of a capacitor.

The **resonant frequency** f_r of series RLC circuit depends only on the inductance ***L*** and capacitance ***C***. But, it is independent of resistance ***R***.

Resonance in parallel RLC circuit

If the resonance occurs in parallel RLC circuit, then it is called as **Parallel Resonance**. Consider the following **parallel RLC circuit**.



Here, the passive elements such as resistor, inductor and capacitor are connected in parallel. This entire combination is in **parallel** with the input sinusoidal current source.

Write **nodal equation** at node P.

$I - I_R - I_L - I_C = 0$

$$I - \frac{V}{R} - \frac{V}{jX_L} - \frac{V}{-jX_C} = 0$$

$$\Rightarrow I = \frac{V}{R} + \frac{V}{jX_L} - \frac{V}{jX_C}$$

$$\Rightarrow I = \frac{V}{R} - \frac{jV}{X_L} + \frac{jV}{X_C}$$

$$\Rightarrow I = V\left[\frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)\right] \dots\dots(i)$$

Above equation is in the form of I = VY

Therefore, the admittance Y of the parallel RLC circuit will be

$$Y = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

Now, let us derive the values of parameters and electrical quantities at resonance of parallel RLC circuit

Resonant Frequency

We know that the **resonant frequency,** f_r is the frequency at which, resonance occurs. In parallel RLC circuit resonance occurs, when the imaginary term of admittance, Y is zero. i.e., the value of $\frac{1}{X_C} - \frac{1}{X_L}$ should be equal to zero

$$\Rightarrow \frac{1}{X_C} = \frac{1}{X_L}$$

$\Rightarrow X_L = X_C$

The above resonance condition is same as that of series RLC circuit. So, the **resonant frequency,** f_r will be same in both series RLC circuit and parallel RLC circuit.

Therefore, the **resonant frequency** f_r of series RLC circuit is

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Where, L is the inductance of an inductor and C is the capacitance of a capacitor.

The **resonant frequency,** f_r of parallel RLC circuit depends only on the inductance ***L*** and capacitance ***C***. But, it is independent of resistance ***R***.

Admittance

We got the **admittance Y** of parallel RLC circuit as

$$Y = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

Substitute, $X_L = X_C$.

$$Y = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_C}\right)$$

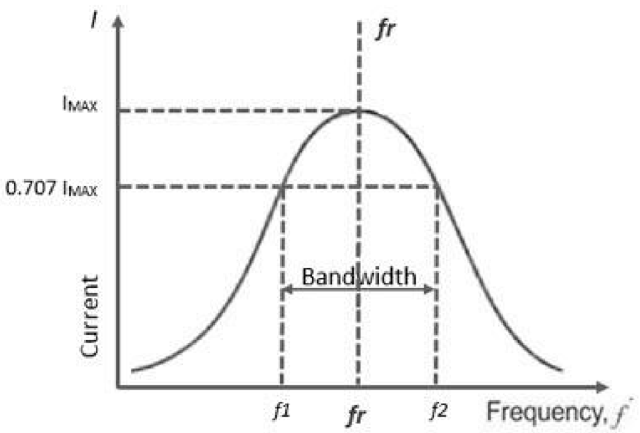
$$Y = \frac{1}{R} + j(0)$$

$$Y = \frac{1}{R}$$

At resonance, the **admittance**, Y of parallel RLC circuit is equal to the reciprocal of the resistance, R. i.e. $Y = \frac{1}{R}$

Bandwidth

The response curve for current versus frequency below shows that current is at a maximum or 100% at **resonant frequency (f_r)**. The **bandwidth (BW)** of a resonant circuit is defined as the total number of cycles below and above the resonant frequency for which the current is equal to or greater than 70.7% of its resonant value. The two frequencies in the curve that are at 0.707 of the maximum current are called band, or half-power frequencies. These frequencies are identified on the curve as f1 and f2, and are often referred to as the critical frequencies, or cutoff frequencies, of a resonant circuit.



Bandwidth of a Series Resonant Circuit

The resonant frequency can be determined by the following equation:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Bandwidth can be expressed mathematically as $\mathbf{BW = f_2 - f_1}$

Other formula used to calculate bandwidth is: BW $= \frac{\mathbf{f_r}}{\mathbf{Q}}$ where the ***Q factor*** is a measure of the quality of a resonance circuit represented by the letter ***Q***.

Q factor is calculated using the formula:

$$Q_{\text{factor}} = \frac{voltage\ across\ capacitor\ or\ inductor}{Voltage\ across\ capacitor} = \frac{IX_{L\ or}\ IX_C}{IR} = \frac{X_L}{R} \text{ or } \frac{X_C}{R}$$

$$\text{Now, } Q_{\text{factor}} = \frac{X_L}{R} = \frac{wL}{R} = \frac{L}{\sqrt{LC}} \times \frac{1}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Effect of Q-factor in resonance:

The Q, or quality, factor of a resonant circuit is a measure of the "goodness" or quality of a resonant circuit. A higher value for this figure of merit corresponds to a more narrow bandwidth, which is desirable in many applications. More formally, Q is the ratio of power stored to power dissipated in the circuit reactance and resistance, respectively:

$$Q = P_{stored} / P_{dissipated} = I^2X/I^2R$$

Q = X/R where: X = Capacitive or Inductive reactance at resonance R = Series resistance.

This formula is applicable to series resonant circuits, and also parallel resonant circuits if the resistance is in series with the inductor. This is the case in practical applications, as we are mostly concerned with the resistance of the inductor limiting the Q.

The sharpness of resonance is defined using the Q factor which explains how fast energy decay in an oscillating system. The sharpness of resonance depends upon:

- Damping: Effect due to which there is a reduction in amplitude of vibrations
- Amplitude: Maximum displacement of a point on a vibrating body which is measured from its equilibrium position.

The sharpness of resonance increases or decreases with an increase or decrease in damping and as the amplitude increases, the sharpness of resonance decreases.