COL 351: Analysis and Design of Algorithms Semester I, 2022-23, CSE, IIT Delhi

Assignment - 2 (due on 20th September, 11:00 PM)

Important Guidelines:

- Each assignment must be done in a group of size at most two.
- Handwritten submissions will not be accepted. Solutions must be typed-up (in Latex, Microsoft Word, etc.), and submitted in pdf format. Each solution must start on a new page.
- Your answer to each question must be formal and have a proper correctness proof. No marks will be granted for vague answers with intuition or for algorithms without proof. You must be very rigorous in providing mathematical detail in support of your arguments.
- Cheating of any form will lead to strict penalty.

1 Maximum sum

Let $D = (d_1, \ldots, d_n)$ be a disc with n sectors such that sector d_i in disc stores an integer p_i , for $i \in [1, n]$. Design an O(n) time algorithm to compute a contiguous collections of sectors in D such that the sum of integers associated with the chosen sectors is maximum. [18 marks]

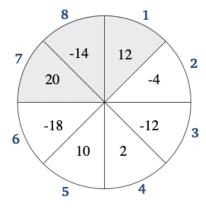


Figure 1: Depiction of a disc with n = 8 sectors. The contiguous collection of sectors that has maximum associated sum is $\{1, 7, 8\}$.

2 Forex Trading

Suppose you are a trader aiming to make money by taking advantage of price differences between different currencies. You model the currency exchange rates as a weighted network, wherein, the nodes correspond to n currencies - c_1, \ldots, c_n , and the edge weights correspond to exchange rates between these currencies. In particular, for a pair (i, j), the weight of edge (i, j), say R(i, j), corresponds to total units of currency c_i received on selling 1 unit of currency c_i .

- 1. Design an algorithm to verify whether or not there exists a cycle $(c_{i_1}, \ldots, c_{i_k}, c_{i_{k+1}} = c_{i_1})$ such that exchanging money over this cycle results in positive gain, or equivalently, the product $R[i_1, i_2] \cdot R[i_2, i_3] \cdots R[i_{k-1}, i_k] \cdot R[i_k, i_1]$ is larger than 1. [10 marks] (Hint: Use the fact that a number x is strictly larger than 1 if and only if $\log(1/x) < 0$).
- 2. Present a cubic time algorithm to print out such a cyclic sequence if it exists. [7 marks]

3 Disjoint collection of Paths

Let T be a binary tree, and S be a set of paths in the tree. Two paths $P,Q \in S$ are said to be disjoint if they do not have a common edge. Give an efficient algorithm which finds a maximum cardinality subset of paths such that each pair of paths in this subset is disjoint. [25 marks]