COL 351 : Analysis and Design of Algorithms
Semester I, 2022-23, CSE, IIT Delhi

Assignment - 4 (due on 15th November, 11:00 PM)

Important Guidelines:

- Each assignment must be done in a group of size at most two.
- Handwritten submissions will not be accepted. Solutions must be typed-up (in Latex, Microsoft Word, etc.), and submitted in pdf format. Each solution must start on a new page.
- **Your answer to each question must be formal, have a proper correctness proof, and should clearly state all the intermediate claims**. No marks will be granted for vague answers with intuition or for algorithms without proof. You must be very rigorous in providing mathematical detail in support of your arguments.
- Cheating of any form will lead to strict penalty.

1 Hitting Set

Consider a set $U = \{u_1, \dots, u_n\}$ of n elements and a collection $A_1, A_2, \dots, A_m$ of subsets of U. That is, $A_i \subseteq U$, for $i \in [1, m]$. We say that a set $S \subseteq U$ is a hitting-set for the collection $A_1, A_2, \dots, A_m$ if $S \cap A_i$ is non-empty for each i.

The *Hitting-Set Problem* (HS) for the input $(U, A_1, \dots, A_m)$ is to decide if there exists a hitting-set $S \subseteq U$ of size at most k.

1. Prove that Hitting-Set problem is in NP class. [5 marks]
2. Prove that Hitting Set is NP-complete by reducing Vertex-cover to Hitting Set. [12 marks]

2 Tracking Shortest Paths

Let $G = (V, E)$ be an undirected graph , s be a source, and t be a destination. A set T of vertices is said to be *tracking set* if for any two distinct *shortest* $s - t$ paths P_1 and P_2, we have

$T \cap V(P_1) \neq T \cap V(P_2)$.[1] The *Tracking Shortest Path Problem* (TSPP) asks: Given G and k, does there exist a tracking set of size at most k.

(i) Show that the Tracking Shortest Path Problem lies in NP class. [10 marks]

(ii) Prove that that the Tracking Shortest Path Problem is NP-complete. [18 marks]

3 Flows and Cuts

Let $G = (V, E)$ be an undirected graph. For any $S \subseteq V$, let $E(S)$ denote the set of edges in G that have both endpoints in S. We define the density of S as $|E(S)|/|S|$.

(i) Design a polynomial time algorithm that given a rational number α determines if there exists a set S with density at least α. [10 marks]

(ii) Present a polynomial time algorithm to find a set S of vertices with maximum density. [5 marks]

[1] $V(P)$ denotes the set of vertices lying in path P.