COL 351 : Analysis and Design of Algorithms
Semester I, 2022-23, CSE, IIT Delhi

Assignment - 1 (due on 1st September, 11:00 PM)

Important Guidelines:

- Each assignment must be done in a group of size at most two.
- Handwritten submissions will not be accepted. Solutions must be typed-up (in Latex, Microsoft Word, etc.), and submitted in pdf format. Each solution must start on a new page.
- **Your answer to each question must be formal and have a proper correctness proof.** No marks will be granted for vague answers with intuition or for algorithms without proof. You must be very rigorous in providing mathematical detail in support of your arguments.
- Cheating of any form will lead to strict penalty.

1 Minimum Spanning Tree

Let G be an edge-weighted connected graph with n vertices and m edges.

(a) Prove that G has a unique MST if all edge weights in G are distinct. [5 marks]

(b) A graph $G = (V, E)$ is said to be edge-fault-resilient if the following condition holds: "For each edge $e \in E$, an MST of graph $G - e$ is also an MST of graph G." Design an $O(mn)$ time algorithm to check if a given connected graph G is edge-fault-resilient. [15 marks]

2 Interval Covering

Let X be a set of n intervals on the real line. A subset of intervals $Y \subseteq X$ is called a *covering* if the intervals in Y cover the intervals in X, that is, any real value that is contained in some interval in X is also contained in some interval in Y . Present a polynomial-time *greedy* algorithm to compute the smallest *covering* of X. Prove the correctness using exchange argument. [12 marks]

3 Bridge Edges

Let $G = (V, E)$ be an undirected connected graph with $n \geqslant 3$ vertices and m edges. Define a relation $\mathcal{R}$ on V as follows: $x\mathcal{R}y$ iff there exists an $x - y$ path in G not containing bridge edges.

(a) Prove that $\mathcal{R}$ is a transitive relation. [3 marks]

(b) Show that the equivalence classes induced by $\mathcal{R}$ are computable in $O(m+n)$ time. [5 marks]

(c) A matrix A of $n \times n$ size is called a witness matrix for G if for each $x, y \in V$ unrelated under relation $\mathcal{R}$, $A(x, y)$ stores a bridge edge e satisfying that *all* the $x - y$ paths in G contain e. Design an $O(n^2)$ time algorithm to compute a witness matrix for G. [12 marks]

(d) Describe an $O(m + n)$ time algorithm to compute a set E_0 of vertex-pairs of size at most $n - 1$ such that (i) $E \cap E_0$ is empty, and (ii) graph $G_0 := (V, E \cup E_0)$ contains no bridge edges. [5 marks]