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Lasso Regression.

$$L = \text{MSE} + \lambda |w|$$

$$\text{where } |w| = |w_1| + |w_2| + |w_3| + \dots + |w_n|$$

Ridge Regression

$$L = \text{MSE} + \lambda \|w\|^2$$

$$\hookrightarrow \lambda (w_1^2 + w_2^2 + w_3^2 + w_4^2)$$

mean squared error / loss function

$$\hookrightarrow \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

note  $\lambda = \text{alpha}$

on increasing the value of alpha we are minimizing  $\lambda (w)$  - this part

IF  $\lambda = 0$ , lasso reg = normal linear reg

on low value of alpha, lasso reg behave as linear reg or behave under overfitting

thus  $\lambda \geq 0$

$$L = \sum_{i=1}^n (y_i - \hat{y}_i)^2 + \lambda |w|$$

on increasing  $\lambda$  <sup>of the</sup> must weight

ridge reg, coeff  $\neq 0$  (but nearer to 0)  
but lasso = 0 yes (slope hard)

depends on  $\lambda(w)$  or  $w$

lasso - main benefit

$\Rightarrow$  For higher dimension values of  $\lambda$  will do Feature selection  
[will zero the particular column coeff which is not imp or y don't rely on it]

$\Rightarrow$  It also decrease dimension

$\Rightarrow$  mostly use lasso in case of ridge

$x_1$	$\begin{pmatrix} x_2 \\ 1 \end{pmatrix}$	$x_3$	$x_4$	$\mid$	$x_5$	$y$
	coeff				coeff	
	0				0	
	lasso				lasso	
	(say)					

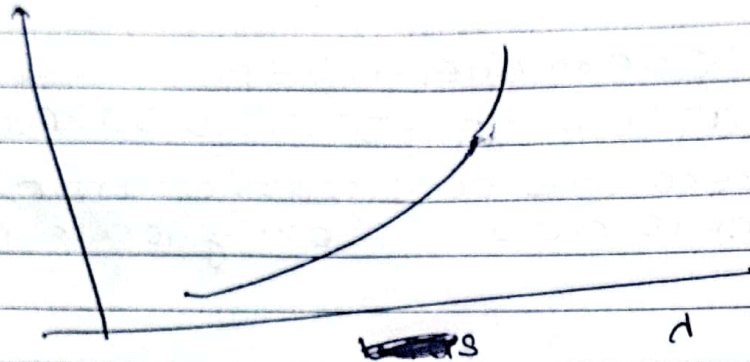
$x_1$   ~~$x_2$~~   $x_3$   $x_4$   $y$  (Feature sel  
decrease dimension)



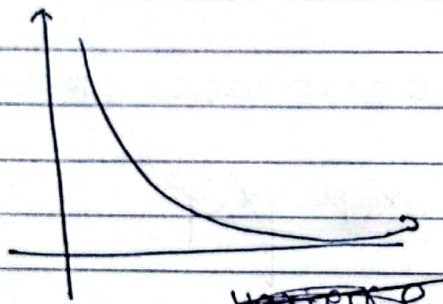
# bias vs variance

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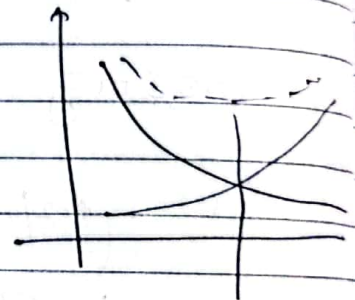
$d \uparrow \rightarrow$  overfitting  $\uparrow \downarrow$   
bias  $\uparrow$   
variance  $\downarrow$



low val of  $d$  low bias



low val of  $d$



Select  
a value

when bias and variance  
is nearest.

# Interview question

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why lasso creates sparsity.  
(i.e.  $d \uparrow$   $w \rightarrow 0$  why)

Solve

For simple linear regression

$$y = mx + b$$

$$b = \bar{y} - m\bar{x}$$

$$m = \frac{\sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

$$\bar{y} \rightarrow \text{mean}(y)$$

$$\bar{x} = \text{mean}(x)$$

For ridge regression

$$m = \frac{\sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2 + \lambda}$$

For Lasso

For <sup>single</sup>  $(x|y)$  ~~with~~

$$L = \sum_{i=1}^n (y_i - \hat{y}_i)^2 + d|m|$$

$$L = \sum_{i=1}^n (y_i - (mx_i + \bar{y} + m\bar{x}))^2 + d|m|$$

(Adding 2 for only calculation)

$$L = \sum_{i=1}^n (y_i - mx_i - \bar{y} - m\bar{x})^2 + 2d|m|$$

we know mod fun is not diff on 0,



so,

case  $\rightarrow I$

$m > 0$

$$L = \sum_{i=1}^n (y_i - mx_i - \bar{y} + m\bar{x})^2 + 2Am$$

$$\frac{dL}{dm} = \sum_{i=1}^n$$

$$\left[ \begin{aligned} \frac{dL}{dm} &= 2 \sum (y_i - mx_i - \bar{y} + m\bar{x}) (-x_i + \bar{x}) \\ 2A &= 0 \end{aligned} \right]$$

Rearrangement

$$\frac{dL}{dm} = -2 \sum [(y_i - \bar{y}) - m(x_i - \bar{x})] [-x_i + \bar{x}] + 2A = 0$$

$$= - \sum [(y_i - \bar{y})(x_i - \bar{x}) - m(x_i - \bar{x})^2] + A = 0$$

$$= - \sum (y_i - \bar{y})(x_i - \bar{x}) - m \sum (x_i - \bar{x})^2 + A = 0$$

$$\text{or, } m \sum (x_i - \bar{x})^2 = \sum (y_i - \bar{y})(x_i - \bar{x})$$

$$m = \frac{\sum (y_i - \bar{y})(x_i - \bar{x}) - A}{\sum (x_i - \bar{x})^2}$$

For  $m = +ve$

laso

For  $m > 0$

$$m = \frac{\sum (y_i - \bar{y})(x_i - \bar{x}) - 1}{\sum (x_i - \bar{x})^2}$$

~~For  $m = 0$~~

$$0 = \sum (y_i - \bar{y})(x_i - \bar{x}) - 1$$

For  $m = 0$

$$L = \sum_{i=1}^n (y_i - \bar{y})^2 + 1|m|$$

$$L = \sum_{i=1}^n (y_i - \bar{y})^2$$

$$L = \sum_{i=1}^n [y_i - (\bar{y} + m(x_i - \bar{x}))]^2$$

$$L = \sum_{i=1}^n (y_i - \bar{y} - m x_i + m \bar{x})^2$$

$$\frac{dL}{dm} = 2 \sum_{i=1}^n (y_i - \bar{y} - m x_i + m \bar{x}) (-x_i + \bar{x})$$

$$\frac{dL}{dm} = 2 \sum_{i=1}^n (y_i - \bar{y} - m x_i + m \bar{x}) (\bar{x} - x_i)$$

$$= -2 \sum_{i=1}^n [(y_i - \bar{y}) - m(x_i - \bar{x})] (x_i - \bar{x})$$



$$\frac{dL}{dm} = -2 \sum_{i=1}^n (y_i - \bar{y}) \frac{(x_i - \bar{x})}{n} - m \sum_{i=1}^n (x_i - \bar{x})^2$$

$$0 = -2 \sum_{i=1}^n (y_i - \bar{y}) (x_i - \bar{x}) - m \sum_{i=1}^n (x_i - \bar{x})^2$$

$$m \sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n (y_i - \bar{y}) (x_i - \bar{x})$$

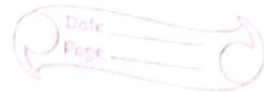
$$m = \frac{\sum_{i=1}^n (y_i - \bar{y}) (x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

Same as linear regression

For  $m < 0$

## why sparsity

Three case



For  $m > 0$

$$m = \frac{\sum (y_i - \bar{y})(x_i - \bar{x}) - 1}{\sum (x_i - \bar{x})^2} \quad \text{--- (i)}$$

for  $m = 0$

$$m = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sum (x_i - \bar{x})^2} \quad \text{--- (ii)}$$

For  $m < 0$

$$m = \frac{\sum (y_i - \bar{y})(x_i - \bar{x}) + 1}{\sum (x_i - \bar{x})^2} \quad \text{--- (iii)}$$

From eq (i)

$$\text{let } y = (y_i - \bar{y}), \text{ and } x = (x_i - \bar{x})$$

and

$$\text{let } y \cdot x = 100, x^2 = 50$$

or,

$$m = \frac{100 - 1}{50}$$

$$d = 0$$

$$m = \frac{100 - 0}{50}$$

$$m = 2$$

$$d = 2$$

$$m = \frac{9}{5}$$

$$d = 100$$

$$m = 0$$

$$d > 100$$

$$m = -1$$



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we need to use all iii

$$\begin{array}{r} 100 + 150 \\ 50 \\ \hline \end{array}$$

$$m = \frac{yx+d}{x^2}$$
$$= \frac{100+d}{50}$$
$$= \frac{100+150}{50}$$
$$m = 5$$

3  
"  
2  
1  
0  
1  
2

A hand-drawn diagram of a cell. It shows an oval shape with a thick outer boundary labeled 'Cell wall' and a thinner inner boundary labeled 'Cell membrane'. The space between these boundaries is labeled 'Cytoplasm'. In the center is a large, dark, oval-shaped structure labeled 'Nucleus'.

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$$yx = -100, x^2 = 50$$

$d = 150, m = 2$

$$\begin{array}{r} 2 \\ -100 - 150 \\ \hline 50 \end{array}$$

o tira ine hudaixu bux

So So is  
Jan puto



For rigid

$$m = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sum (x_i - \bar{x})^2 + \lambda}$$

IF  $\sum (y_i - \bar{y})(x_i - \bar{x}) \neq 0$   
then on increasing any value of  $\lambda$   
 $m \neq 0$

because, as  $\lambda \rightarrow \infty$  in numerator  
rigid,  $\lambda \rightarrow$  denominator  
and  $m \rightarrow 0$  depends on value of num  
erator so.

IF the input columns are imp then rigid  
or IF you know, all columns are not  
important then.

what if you have too columns or more  
more datasets, you don't have idea abt  
columns,

then use elastic regression which  
is combination of both.

$$L = \sum (y_i - \hat{y}_i)^2 + a||w||^2 + b||w||$$

note

$$\lambda = a + b ; \text{L-ratio} = \frac{a}{a+b} \rightarrow \text{trade}$$

by default

$$\lambda = 1, \text{L-ratio} = 0.5$$

$$1 = a + b, \quad 0.5 = \frac{a}{a+b}$$

$$; \quad 0.5 = \frac{a}{1} ; \quad a = 0.5$$

$$\text{also } \lambda = \frac{a}{\lambda}$$

$$a = \lambda \times \lambda$$

$$b = \lambda - a$$

- also use this on multicollinearity (if)  
then,  
multicollinearity: 2 depend on  
each others,  
negate and weight datasets are multi  
collinearity (hr, wpr) (h, w, b)

L2 ratio  $\rightarrow 0.9$

90% use ridge  $\rightarrow$  10% use lasso