Step-Down/ Type-A Chopper - Detailed Analysis

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Buck/ Step-Down Converter

1. Reduce vltg level at op side. $V_o \le V_s$
   $@T_{on}, v_o = V_s$.
   $@T_{off}, v_o = 0$ as load current flow by FD. So load terminals get shorted.
   $V_o = \delta V_s \Rightarrow$ Load vltg is indpdt of load current.
2. So load vltg can be varied by varn of δ.

R-Load

When sw is close, $v_o = V_s$.
When sw is open, $v_o = 0$.
$i_o = \frac{v_o}{R}$
Diode is useless here.

RL Load

When sw is close, $V_o = V_s$. FD is inactive. Indt is charged in this state.
When sw is open, $V_o = 0$. Indt discharge by diode path.

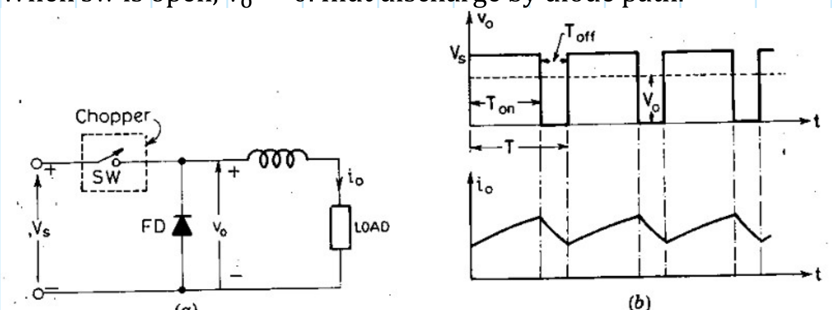


Fig. 7.2 (a) Elementry chopper circuit and (b) output voltage and current waveforms.

[Cont Load Current]

Avg Load Voltage

$$V_o = \frac{1}{T}\int_0^T v_o dt = \frac{1}{T}\int_0^{T_{ON}} V_s dt = \frac{V_s T_{on}}{T} = \delta V_s$$

So varying δ, we can step down op vltg across load.
sw is always OFF $\Rightarrow \delta = 0 \Rightarrow V_o = 0$
sw is always ON $\Rightarrow \delta = 1 \Rightarrow V_o = V_s$
$\Rightarrow V_o \in [0, V_s]$

Avg Load Current

$I_o = \frac{V_o}{R} = \frac{\delta V_s}{R}$
[R-Load]

RMS Op Vltg

$$V_{o(rms)} = \sqrt{\frac{1}{T}\int_0^{T_{on}} V_s^2 dt} = \sqrt{\frac{V_s^2 T_{on}}{T}} = V_s\sqrt{\delta}$$

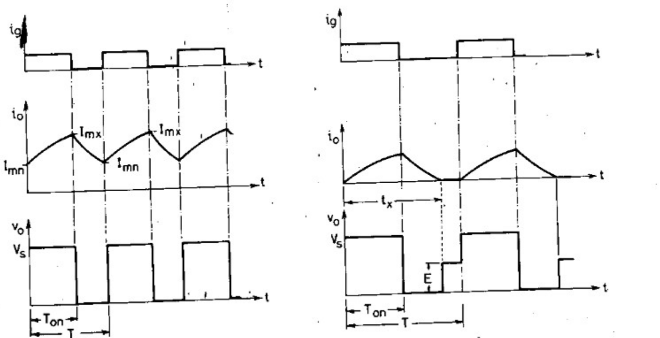
For lossless converter, ip power et op power given by:

$$P_i = \frac{1}{T}\int_0^{\delta T} V_o i dt = \frac{1}{T}\int_0^{\delta T} \frac{V_o^2}{R} dt = \frac{1}{T}\frac{V_s^2}{R}(\delta T) = \frac{\delta V_s^2}{R}$$

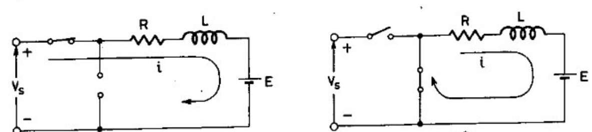
Effective Ip Rst seen by power source P

$$R_i = \frac{V_s}{I_o} = \frac{V_s}{\delta V_s / R} = \frac{R}{\alpha}$$

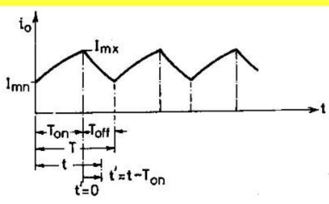
SS Time-Domain Analysis of Type-A Chopper with RLE-Load



(a) (b)
Fig. 7.12. Type-A chopper (a) continuous load current and (b) Discontinuous load current.



(c) (d)
Fig. 7.12. Equivalent circuit for type-A chopper with (c) CH1 on and (d) CH1 off.



(e)
Fig. 7.12 (e) Pertaining to t'.

For RLE-load,

$@\ t \in [0, T_{on}]$, sw is on $\Rightarrow V_s = Ri + \frac{Ldi}{dt} + E$	$@\ t \in [T_{on}, T]$ or $t' \in [0, T - T_{on} = T_{off}] \Rightarrow t' = t - T_{on}$ sw is off, load current flow by FD. $\Rightarrow 0 = Ri + \frac{Ldi}{dt} + E$

2 boundary values are I_{mn}, I_{mx}.

$RI + L[sI - I_{mn}] = \frac{V_s - E}{s}$ $I(s) = \frac{V_s - E}{s(R + Ls)} + \frac{LI_{mn}}{R + Ls}$ $= \frac{V_s - E}{Ls\left(s + \frac{R}{L}\right)} + \frac{I_{mn}}{s + \frac{R}{L}}$ $\Rightarrow i(t) = \frac{V_s - E}{R}\left(1 - e^{-\frac{Rt}{L}}\right) + I_{mn}e^{-Rt/L}$	$RI + L[sI - I_{mx}] = -\frac{E}{s}$ ... $\Rightarrow i(t') = -\frac{E}{R}\left(1 - e^{-\frac{Rt'}{L}}\right) + I_{mx}e^{-\frac{Rt'}{L}}, t \in [T_{on}, T]$ just replace t by t', $V_s = 0$ and I_{mn} by I_{mx}

$$\text{Let}\left[e^{-\frac{T_{on}}{\tau}} = m, e^{-\frac{T_{off}}{\tau}} = n, \tau = \frac{R}{L} \Rightarrow mn = e^{-\frac{T}{\tau}}\right]$$

$@t = T_{on}, i(t) = I_{mx}$ $\Rightarrow I_{mx} = \frac{V_s - E}{R}\left(1 - e^{-\frac{R}{L}T_{on}}\right) + I_{mn}e^{-\frac{R}{L}T_{on}}$	$@t' = T_{off}, i(t) = I_{mn}$ $\Rightarrow I_{mn} = \frac{-E}{R}\left(1 - e^{-\frac{R}{L}T_{off}}\right) + I_{mx}e^{-\frac{R}{L}T_{off}}$

mn mx

$= \frac{V_s - E}{R}(1-m) + mI_{mn}$	$= -\frac{E}{R}(1-n) + I_{mx}n$

Put I_{mn} in I_{mx} and solve for I_{mx}, then find I_{mn} from obtd eqn:

$$\Rightarrow I_{mx} = \frac{V_s - E}{R}(1-m) + m\left[-\frac{E}{R}(1-n) + I_{mx}n\right]$$

$$\Rightarrow I_{mx} = -\frac{E}{R} + \frac{V_s}{R}\cdot\frac{(1-m)}{1-mn} = -\frac{E}{R} + \frac{1-e^{-\frac{T_{on}}{\tau}}}{1-e^{-\frac{T}{\tau}}}$$

and

$$\Rightarrow I_{mn} = -\frac{E}{R}(1-n) + \left[-\frac{E}{R} + \frac{V_s}{R}\cdot\frac{(1-m)}{1-mn}\right]n$$

$$\Rightarrow I_{mn} = -\frac{E}{R} + \frac{n(1-m)}{1-mn} = -\frac{E}{R} + \frac{1-\frac{1}{m}}{1-\frac{1}{mn}} = -\frac{E}{R} + \frac{1-e^{\frac{T_{on}}{\tau}}}{1-e^{\frac{T}{\tau}}}$$

If sw cont conduct, $T_{on} = T \Rightarrow I_{mx} = I_{mn} = \frac{V_s - E}{R}$

Steady State Ripple

$$\text{Ripple Current} = \Delta I = I_{mx} - I_{mn} = \frac{V_s}{R}\cdot\frac{(1-m)}{1-mn} - \frac{E}{R} - \left[\frac{V_s}{R}\cdot\frac{n(1-m)}{1-mn} - \frac{E}{R}\right]$$

$$= \frac{V_s}{R}\cdot\frac{(1-n)(1-m)}{1-mn} = \frac{V_s}{R}\left[\frac{\left(1-e^{-\frac{T_{on}}{\tau}}\right)\left(1-e^{-\frac{Toff}{\tau}}\right)}{1-e^{-\frac{T}{\tau}}}\right]$$

$$= \frac{V_s}{R}\left[\frac{\left(1-e^{-\frac{\delta T}{\tau}}\right)\left(1-e^{-(1-\delta)\frac{T}{\tau}}\right)}{1-e^{-\frac{T}{\tau}}}\right]$$

$$\text{PU Ripple Crnt} = \frac{I_{mx} - I_{mn}}{V_s/R} = \frac{\left(1-e^{-\frac{\delta T}{\tau}}\right)\left(1-e^{-(1-\delta)\frac{T}{\tau}}\right)}{1-e^{-\frac{T}{\tau}}}$$

This ripple current is indpdt of E.

With L inc, $\tau = \frac{L}{R}$ inc and $\frac{T}{\tau}$ dec, so pu ripple current dec.

Ripple current has max value ΔI_{mx} @ $\delta = 0.5$. Put $\frac{T}{\tau} = x$

$$\Delta I_{mx} = \frac{V_s}{R}\left[\frac{(1-e^{-0.5x})(1-e^{-0.5x})}{1-e^{-x}}\right] = \frac{V_s}{R}\left[\frac{(1-e^{-0.5x})(1-e^{-0.5x})}{(1-e^{-0.5x})(1+e^{-0.5x})}\right]$$

$$= \frac{V_s}{R}\left[\frac{(1-e^{-0.5x})}{(1+e^{-0.5x})}\right] = \frac{V_s}{R}\tanh\frac{x}{4} = \frac{V_s}{R}\tanh\frac{T}{4\tau} = \frac{V_s}{R}\tanh\frac{R}{4fL}$$

$$\Rightarrow \Delta I_{mx} = \frac{V_s}{R}\tanh\frac{R}{4fL}$$

If $4fL \gg R \Rightarrow \tanh\frac{R}{4fL} = \frac{R}{4fL} \Rightarrow \Delta I_{mx} = \frac{V_s}{4fL}$

$\Rightarrow \Delta I_{mx}$ is iprop to chopping freq and L

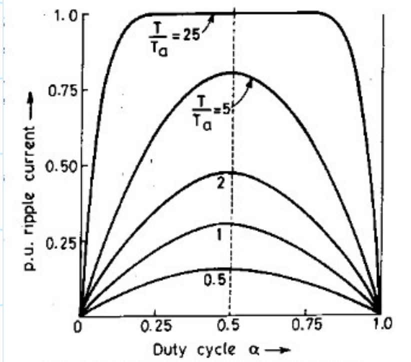


Fig. 7.13. Per unit ripple current as a function of α and T/T_a.

Limit of Continuous Conduction/ Critical Current

$$I_{mn} = 0 \Rightarrow \frac{V_s}{R} \cdot \left[\frac{e^{\frac{T_{on}}{\tau}} - 1}{e^{\frac{T}{\tau}} - 1}\right] - \frac{E}{R} = 0 \Rightarrow \frac{T_{on}}{\tau} = \ln\left(1 + \frac{E}{V_s}\left[e^{\frac{T}{\tau}} - 1\right]\right)$$

$$\Rightarrow \frac{T_{on}}{T} = \delta = \frac{\tau}{T}\ln\left(1 + \frac{E}{V_s}\left[e^{\frac{T}{\tau}} - 1\right]\right)$$

Formula List

$$m = e^{-\frac{T_{on}}{\tau}}, n = e^{-\frac{T_{off}}{\tau}}$$

$$mn = e^{-\frac{T}{\tau}}$$

$$I_{mx} = \frac{V_s}{R} \cdot \frac{1-m}{1-nn} - \frac{E}{R}$$

$$I_{mn} = \frac{V_s}{R} \cdot \frac{n(1-m)}{1-mn} - \frac{E}{R}$$

$$I_{mx} - I_{mn} = \frac{V_s}{R} \cdot \frac{(1-n)(1-m)}{1-mn}$$

Extinction Time (t_x)

Above load crnt expressions apply for cont load currents.
Crnt can be discont due to larger Toff.

$t_x \in [T_{on}, T]$

$@t = t_x, I_{mn} = 0$

$$I_{mx} = \frac{V_s - E}{R}\left(1 - e^{-\frac{R}{L}T_{on}}\right) + [0]e^{-\frac{R}{L}T_{on}} = \frac{V_s - E}{R}\left(1 - e^{-\frac{T_{on}}{\tau}}\right)$$

$$I_{mn} = 0 = -\frac{E}{R}\left(1 - e^{-\frac{t_x - T_{on}}{\tau}}\right) + I_{mx}e^{-\frac{t_x - T_{on}}{\tau}}$$

$$= -\frac{E}{R}\left(1 - e^{-\frac{t_x - T_{on}}{\tau}}\right) + \left[\frac{V_s - E}{R}\left(1 - e^{-\frac{T_{on}}{\tau}}\right)\right]e^{-\frac{t_x - T_{on}}{\tau}}$$

$$\Rightarrow t_x = T_{on} + \tau\ln\left[1 + \frac{V_s - E}{E}\left(1 - e^{-\frac{T_{on}}{\tau}}\right)\right]$$

$$\text{Avg Op Vltg for RLE discont mode} = V_o = \frac{\int_0^{T_{on}} V_s dt + 0 + \int_{t_x}^{T} E dt}{T} = \frac{V_s T_{on} + E(T - t_x)}{T}$$

Fourier Analysis of Op Voltage

Refer vltg wf for cont load current. Vltg is indpdt of load ckt.

$$v_o = V_o + \sum_{n=1}^{\infty} v_n$$

$v_n = A_n \sin(nwt + \theta_n) = \frac{2V_s}{n\pi} \sin n\pi\delta \sin(nwt + \theta_n) \rightarrow$ value of nth harmonic vltg.

$V_o = \delta V_s \rightarrow$ DC Level of Vltg

$\theta_n = \tan^{-1}\left(\frac{\sin 2\pi n\delta}{1 - \cos 2\pi n\delta}\right)$

$A_n \rightarrow$ Ampt of nth harmonic vltg.

@ $\sin n\pi\delta = 1, A_{n(mx)} = \frac{2V_s}{n\pi} = \frac{0.636V_s}{n}$ [Volts]

Its rms value $= A_{n(mx)rms} = \frac{2V_s}{\sqrt{2}n\pi} = \frac{0.45V_s}{n}$ [Volts]

Harmonic crnt in load $= i_n = \frac{v_n}{Z_n}$

$Z_n = \sqrt{R^2 + (nwL)^2} \rightarrow$ Load impd at harmonic freq nf Hz

For nglb load rst R, $i_n = \frac{v_n}{nwL} \Rightarrow i_n \propto \frac{V_s}{n^2}$

$\Rightarrow$ Harmonic vltg dec with n inc.

Det of harmonic component without calculating harmonics.

AC ripple vltg $= V_r = \sqrt{V_{rms}^2 - V_0^2}$

$V_{rms} = \sqrt{\delta}V_s, V_o = \delta V_s \Rightarrow V_r = V_s\sqrt{\delta(1-\delta)}$

RF is ratio of ac ripple vltg to avg vltg $= \frac{V_r}{V_o} = \sqrt{\frac{1}{\delta} - 1}$