

Recursion

↳ when any function calls itself

Method name

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Base $\text{fact}(n)$:
condition

if $n \leq 1$
↳ $0!$ ↳ 1

$n = 0$

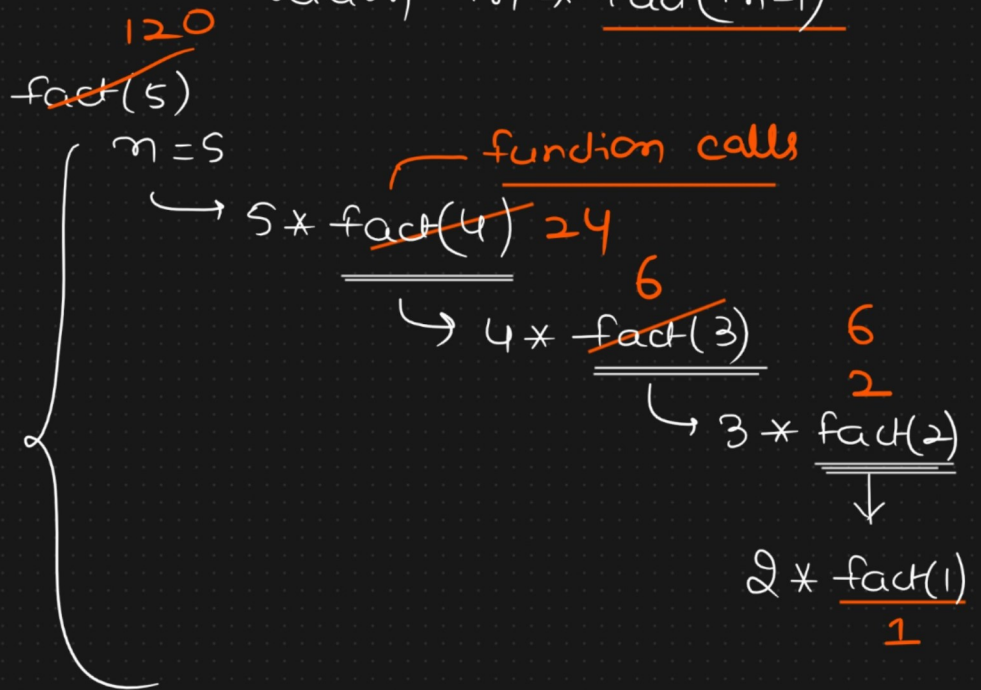
$$\underline{1! = 1, 0! = 1}$$

else: $n > 1$

Recursion

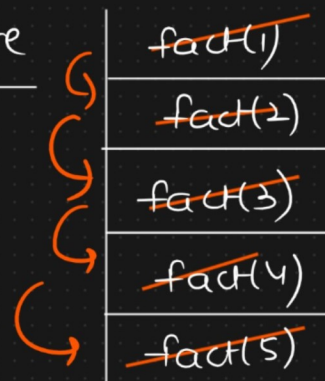
return $n \times \text{fact}(n-1)$

Recursive Tree



Stack Data Structure

$O(n)$



Note: for every recursive code, there exists a
recurrence relation

- ↳ 1) Substitution Method
2) Recursive tree Method
3) Master's Theorem

Substitution Method

Example 1

$$\underline{\underline{T(n)}} = 2 T\left(\frac{n}{2}\right) + 4n \quad \text{--- (1)}$$

$$T\left(\frac{n}{2}\right) = 2 T\left(\frac{n}{2^2}\right) + 4\left(\frac{n}{2}\right)$$

$$T(n) = 2 \left(2 T\left(\frac{n}{2^2}\right) + 4\left(\frac{n}{2}\right) \right) + 4n$$

$$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + 4n + 4n$$

$$T(n) = 2^2 \underline{T\left(\frac{n}{2^2}\right)} + 2 \times 4n \quad \text{--- (2)}$$

$$T\left(\frac{n}{2^2}\right) = 2 T\left(\frac{n}{2^3}\right) + 4\left(\frac{n}{2^2}\right)$$

$$T(n) = 2^2 \left(2 T\left(\frac{n}{2^3}\right) + 4 \left(\frac{n}{2^2}\right) \right) + 2 * (4n)$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 4n + 2 * (4n)$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 3 * (4n) \quad \text{--- (3)}$$

\downarrow
k times
 $T(1) = 1$

$$T(n) = 2^k \left[T\left(\frac{n}{2^k}\right) \right] + k * (4n)$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$k = \log_2 n$$

$$\log_2 n = k \log_2 2$$

$$T(n) = 2^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + (4n) * \log_2 n$$

$$a^{\log_2 b} = b^{\log_2 a}$$

$$T(n) = n^{\cancel{\log_2}} T\left(\frac{n}{\cancel{n^{\log_2}}}\right) + (4n) * \log_2 n$$

$$T(n) = \underline{n} + \overset{c}{(4n) * \log_2 n}$$

Time complexity \Rightarrow

$$T(n) = O(n \log_2 n)$$

Loop \rightarrow Iterative approach

Recursion \rightarrow Recursive approach

\hookrightarrow Substitution
Method

Example 2

✓ 1, 7, 31, 127, 511
1 2 3 4 5

$$T(1) = 1$$

$$\begin{aligned} T(2) &= 4 \times T(n-1) + 3 \\ &= 4 \times T(1) + 3 \\ &= 4 \times 1 + 3 \\ &= 7 \end{aligned}$$

$$\begin{aligned} T(3) &= 4 \times T(n-1) + 3 \\ &= 4 \times T(2) + 3 \\ &= 4 \times 7 + 3 \\ &= 31 \end{aligned}$$

$$\begin{aligned} T(4) &= 4 \times T(n-1) + 3 \\ &= 4 \times T(3) + 3 \\ &= 4 \times 31 + 3 \\ &= 124 + 3 \\ &= \underline{\underline{127}} \end{aligned}$$

$$\begin{aligned} T(5) &= 4 \times T(n-1) + 3 \\ &= 4 \times T(4) + 3 \\ &= 4 \times 127 + 3 \\ &= 508 + 3 \\ &= 511 \end{aligned}$$

$$T(n) = \begin{cases} 1 & n=1 \\ 4 \times T(n-1) + 3 & n \geq 1 \end{cases}$$

↳ Recurrence
Relation