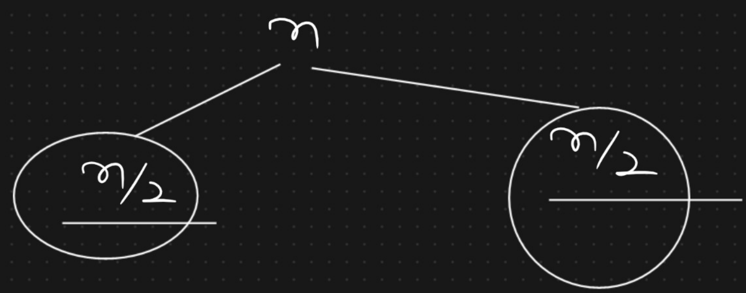
PW SKILLS

Master's Theorem

$$\boxed{T(n) = aT\left(\frac{n}{b}\right) + \Theta(n^k \log^p n)}$$

Constraints

- $n \rightarrow$ size of the problem
- $a \rightarrow$ number of subproblems
- $n/b \rightarrow$ size of subproblem
- $a >= 1$, $b > 1$, $k >= 0$
- $p \rightarrow$ Real number



Example 1

$$T(n) = 3T\left(\frac{n}{5}\right) + 5n^2$$

$a = 3$

$b = 5$

$k = 2$

$p = 0$

$a = 3 \qquad b^k = 5^2 = 25 \longrightarrow a < b^k$

$p >= 0 \qquad O(n^k \log^p n)$

$O(n^2)$

case 1

$$a > b^k$$

$$T(n) = O(n^{\log_b a})$$

case 2 $\quad a = b^k$

2.1 $\quad p > -1 \quad T(n) = O(n^{\log_b a} \log^{p+1} n)$

2.2 $\quad p = -1 \quad T(n) = O(n^{\log_b a} \log\log n)$

2.3 $\quad p < -1 \quad T(n) = O(n^{\log_b a})$

Case 3 $\quad a < b^k$

3.1 $\quad p >= 0 \quad T(n) = O(n^k \log^p n)$

3.2 $\quad p < 0 \quad T(n) = O(n^k)$

Example 2

$$T(n) = 2T\left(\frac{2n}{3}\right) + 1$$

constraints follow $\longrightarrow$ $\begin{cases} a = 2 & K = 0 \\ b = 3/2 & P = 0 \end{cases}$

$$b^K = \left(3/2\right)^0 = 1$$

$\begin{cases} a > b^K \\ 2 > 1 \end{cases}$ $\longrightarrow$ case 1

$$O\left(n^{\log_b a}\right) = O\left(n^{\log_{3/2} 2}\right)$$

$$= O\left(n^{\log_{1.5} 2}\right)$$

$a = 2$ $\quad T(n) = 2T\left(\frac{n}{2}\right) + \underline{\underline{n^2}}_{f(n)}$ $\quad \underline{\underline{O(n^2)}}$

$b = 2$

$n^{\log_b a}$ $\quad$ $\underline{\underline{n^2}}$

$n^{\log_2 \cancel{2}} = \underline{\underline{n}}$

$$\boxed{T(n) = O(n^2)}$$