PW SKILLS

Recursive Tree Method

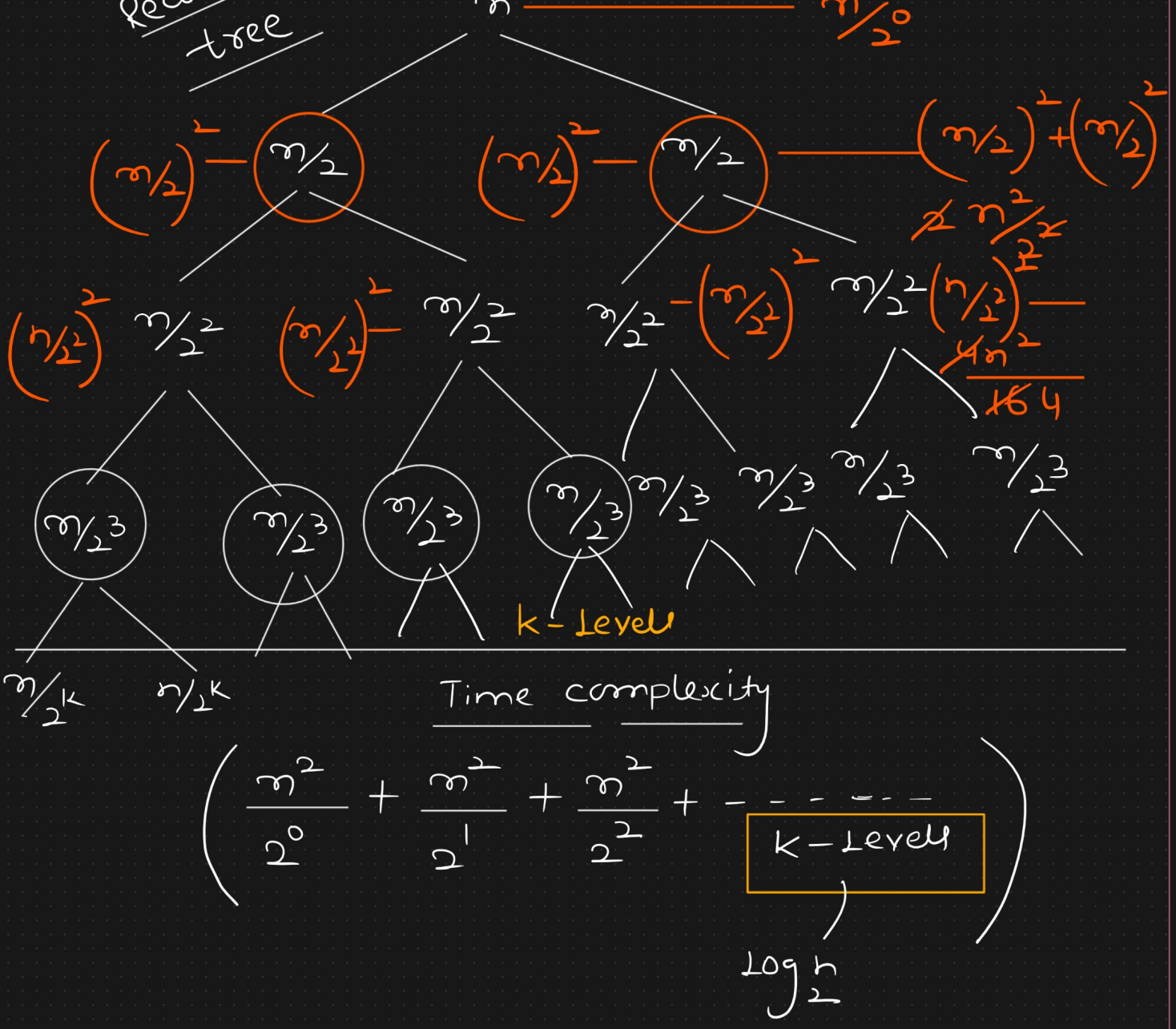
↳ More than one Recursive term in any recurrence relation

Example 1

$$T(n) = 2T(n/2) + n^2$$

$$T(n) = T(n/2) + T(n/2) + n^2$$

Recursive tree



Time complexity

$$\left(\frac{n^2}{2^0} + \frac{n^2}{2^1} + \frac{n^2}{2^2} + \cdots \text{(k-Level)} \right)$$

$\log_2 n$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log_2 n = k \log_2 2$$

$$\boxed{k = \log_2 n}$$

GP series

$$n^2\left(\underbrace{\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \cdots}_{\log_2 n \text{ times}}\right)$$

$r = \frac{1}{2}$

$$\text{Sum of GP series} = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = 2$$

$$n^2\left(2\right) = O(n^2)$$

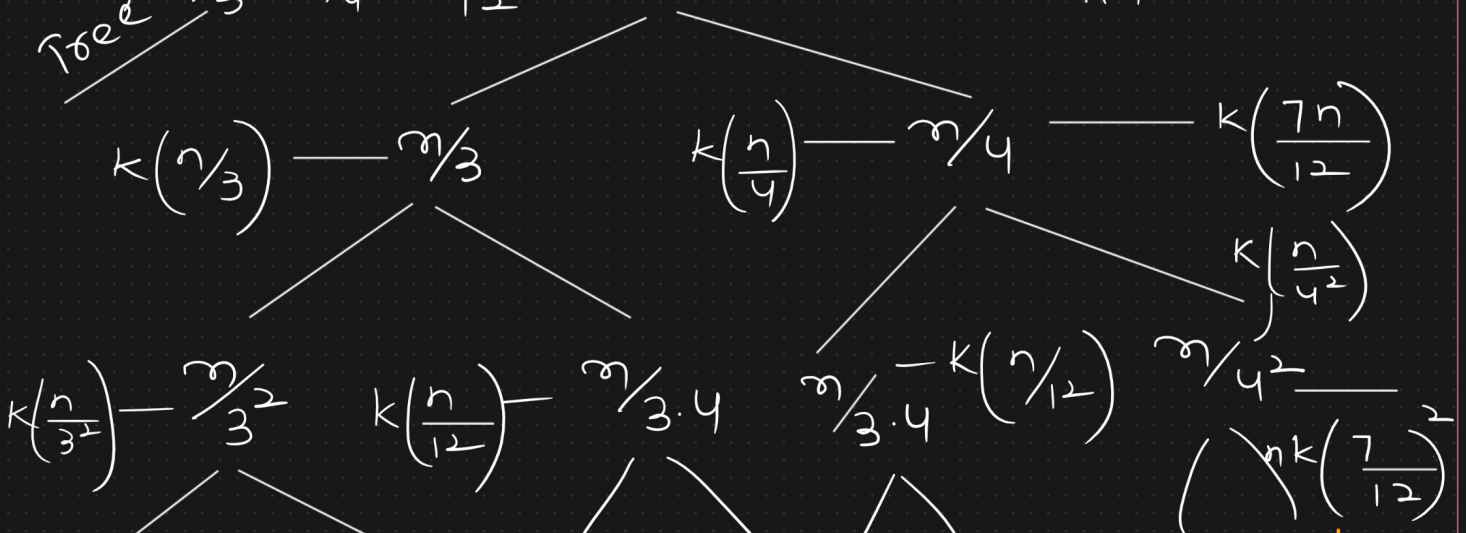
Example 2

Recursive

Recursive Terms

$$T(n) = T(n/3) + T(n/4) + kn$$

Tree $n/3 + n/4 = \frac{4n+3n}{12}$



Left Part

$\frac{n}{3^c} = 1$ Higher Level

$c = \log_3 n$

Right Part

$n/4^c = 1$

$c = \log_4 n$

Lower the base, higher the value

$$kn\left(\frac{7}{12}\right)^0 + kn\left(\frac{7}{12}\right)^1 + kn\left(\frac{7}{12}\right)^2 + \text{---} \quad \text{c levels}$$

$\hookrightarrow \log_3 n$

$$kn\left(\left(\frac{7}{12}\right)^0 + \left(\frac{7}{12}\right)^1 + \left(\frac{7}{12}\right)^2 + \text{----}\right)$$

$$kn\left(\frac{1}{1 - 7/12}\right) = O(n)$$