1. Compute mean

Arrange data in ascending order

35, 35, 37, 39, 40, 41, 45, 55, 55, 55, 55, 55, 60, 65

$$\bar{x} = \frac{\Sigma x}{n} = \frac{35+35+37+39+40+41+45+55+55+55+55+55+60+65}{14} = 48$$

$$Md = \frac{n-1}{2}^{th} \text{ item} = \frac{14+1}{2} = 7.5^{th} \text{ item}$$

$$\text{Median value} = \frac{45+55}{2} = 50$$

M_O = Most repeated value = 55

2. The number of telephone calls

No. of calls (x)	0	1	2	3	4	5	6	
Frequency	15	22	28	35	42	34	24	$N = \sum f = 200$
c.f.	15	37	65	100	142	176	200	
fx	0	22	56	105	168	170	144	$\sum fx = 665$

$$\bar{x} = \frac{\sum fx}{N} = \frac{665}{200} = 3.325$$

$$M_d = \frac{N}{2} = \frac{200}{2} = 100^{th} \text{ items} = 3$$

M_0 = No. of calls with maximum frequency = 4

3. The length in hours of 100 VCA cable

Length on inch	3.80 – 3.89	3.90 – 3.99	4.00 – 4.09	4.10 – 4.19	4.20 – 4.29	4.30 – 4.39	4.40 – 4.49	5.50 – 5.59
Frequency (f)	3	8	14	19	28	18	10	8
c.f.	3	11	25	44	72	90	100	108

For mode,

Model class = 4.20 – 4.29

It is inclusive class, hence adjusted model class = (4.195 – 4.295)

$f_0 = 19, f_1 = 28, f_2 = 18$

$$M_0 = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h$$

$$= 4.195 + \frac{f_1 - f_0}{f_1 - f_0 + f_1 - f_2} \times h = 4.195 + \frac{28-19}{28-19+28-18} \times 0.1$$

$$= 4.24$$

For Median $\frac{N}{2} = \frac{108}{2} = 54$

Median class = (4.20 – 4.29)

Adjusted median class = (4.195 – 4.295)

$$M_d = L + \frac{\frac{N}{2} - c.f.}{f} \times h = 4.195 + \frac{45-44}{28} \times 0.1 = 4.23$$

4. Compute the mean, median and mode

IQ	Frequency (f)	Mid-value (x)	$d' = \frac{x - 104.5}{10}$	fd'	c.f.
50 – 59	2	54.5	– 5	– 10	2
60 – 69	4	64.5	– 4	– 16	6
70 – 79	8	74.5	– 3	– 24	14
80 – 89	15	84.5	– 2	– 30	29
90 – 99	21	94.5	– 1	– 21	50
100 – 109	25	104.5	0	0	75
110 – 119	20	114.5	1	20	95
119 – 129	2	124.5	2	4	97
130 – 139	2	134.5	3	6	99
140 – 149	1	144.5	4	4	100
				$\sum fd' = -67$	

$$\bar{x} = A + \frac{\sum fd'}{N} \times h = 104.5 + \frac{(-67)}{100} \times 10 = 97.8$$

$$\frac{N}{2} = \frac{100}{2} = 50,$$

$$Md = L + \frac{\frac{N}{2} - c.f.}{f} \times h = 89.5 + \frac{50 - 25}{21} \times 10 = 99.5$$

Hence M_d class = (90 – 99)

Adjusted M_d class = 89.5 – 99.5

Model class = (100 – 109)

Adjusted model class = (99.5 – 109.5)

$f_0 = 21, f_1 = 25, f_2 = 20, \Delta_1 = f_1 - f_0 = 4, \Delta_2 = f_1 - f_2 = 5$

$$M_0 = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 99.5 + \frac{4}{4 + 5} \times 10 = 103.94$$

5. Compute G.M.

G.M. = $(38 \times 45 \times 50 \times 68 \times 82 \times 40)^{1/6} = 51.68$

6. Compute G.M.

Weight	50 – 54	55 – 59	60 – 64	65 – 69	70 – 74
Mid-weight	52	57	62	67	72
No. of students (f)	30	25	32	20	13
logx	1.716	1.755	1.792	1.826	1.857
f logx	51.48	43.87	57.34	36.52	24.14

$\sum f = 120; \ \sum f \log x = 213.35$

$$G.M. = \text{Antilog}\left(\frac{\sum f \log x}{N}\right)$$

$$= \text{Antilog}\left(\frac{213.35}{120}\right) = \text{Antilog } 1.777 = 59.96\text{kg}$$

7. A boy travels

$n = 5$

$$H.M. = \frac{n}{\sum\frac{1}{x}} = \frac{5}{\frac{1}{40}+\frac{1}{38}+\frac{1}{35}+\frac{1}{36}+\frac{1}{39}} = \frac{5}{0.1333} = 37.509$$

8. The data relating to

Increase in wt	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40
No. of animals (f)	3	7	18	35	20	8	5	4
c.f.	3	10	28	63	83	91	96	100

$\frac{N}{4} = \frac{100}{4} = 25$ $\therefore Q_1$ class = (10 – 15)

$$Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 20 + \frac{25-10}{18} \times 5 = 14.166$$

$\frac{3N}{4} = 3 \times 25 = 75$ $\therefore Q_3$ class = (20 – 25)

$$Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 20 + \frac{75-63}{20} \times 5 = 23$$

$\frac{9N}{10} = \frac{9 \times 100}{10} = 90$ $\therefore D_9$ class = (25 – 30)

$$D_9 = L + \frac{\frac{9N}{10} - c.f.}{f} \times h = 25 + \frac{90-83}{8} \times 5 = 29.375$$

$\frac{15N}{100} = \frac{15 \times 100}{100} = 15$ $\therefore P_{15}$ class = (10 – 15)

$$P_{15} = L + \frac{\frac{15N}{100} - c.f.}{f} \times h = 10 + \frac{15-10}{18} \times 5 = 11.388$$

$\frac{90N}{100} = \frac{90 \times 100}{100} = 90$ $\therefore P_{90}$ class = (25 – 30)

$$P_{90} = L + \frac{\frac{90N}{100} - c.f.}{f} \times h = 25 + \frac{90-83}{8} \times 5 = 29.375$$

9. Random sample

Size of pen drive	No. of user (f)
4 – 5	10
5 – 6	35
6 – 7	70
7 – 8	35
8 – 9	12
9 – 10	6

Measure of central tendency in Mode.

Here modal class is (6 – 7)

$L = 6, f_0 = 35, f_1 = 70, f_2 = 35, \Delta_{12} = f_1 - f_0 = 70 - 35 = 35$

$\Delta_2 = f_1 - f_2 = 70 - 35 = 35$

$$M_O = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 6 + \frac{35}{35 + 35} \times 1 = 6.5$$

10. A software

$x_1 = 50, x_2 = 35, x_3 = 40, x_4 = 25, \bar{x}_1 = 30{,}000; \bar{x}_2 = 35{,}000, \bar{x}_3 = 40{,}000,$
$\bar{x}_4 = 50{,}000$

$$\bar{x}_{1234} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2 + n_3 \bar{x}_3 + n_4 \bar{x}_4}{n_1 + n_2 + n_3 + n_4}$$

$$= \frac{50 \times 30000 + 35 \times 35000 + 40 \times 40000 + 25 \times 50000}{50 + 35 + 40 + 25} = 37166.67$$

11. The percentage age

Age	0–4	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60 and above
Male pop^n (f)	11.8	12.9	12.5	11.2	10.7	8.9	7.2	6.2	4.7	4.0	2.9	2.3	4.7
c.f.	11.8	24.7	37.2	48.4	59.1	68.0	75.2	81.4	86.1	90.1	93.0	95.3	100

$\frac{N}{4} = \frac{100}{4} = 25$; Hence Q_1 class = (10 – 14)

Adjusted Q_1 class = (9.5 – 14.5)

$$Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 9.5 + \frac{25 - 24.7}{12.5} \times 5 = 9.62$$

$\frac{3N}{4} = 3 \times 25 = 75$; Hence Q_3 class = (30 – 34)

Adjusted Q_3 class = (29.5 – 34.5)

$$Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 29.5 + \frac{75 - 68}{7.2} \times 5 = 34.36$$

$\frac{8N}{10} = \frac{8 \times 100}{10} = 80$

D_{80} Class = (35 – 39)

Adj. D_{80} class = (34.5 – 39.5)

$$D_{80} = L + \frac{\frac{8N}{10} - cf}{f} \times h = 34.5 + \frac{80 - 75.2}{6.2} \times 5 = 38.37$$

$\frac{70N}{100} = \frac{70 \times 100}{100} = 70$

P_{70} class (30 – 34)

Adj. P_{70} class = (29.5 – 34.5)

$$P_{70} = L + \frac{\frac{70\,N}{100} - cf}{f} \times h = 29.5 + \frac{70 - 68}{7.2} \times 5 = 30.88$$

2. The percentage of

$$GM = (x_1 \times x_2 \times x_3 \times x_4 \times x_5 \times x_6 \times x_7)^{1/7}$$
$$= (49.19 \times 49.25 \times 50.34 \times 45.122 \times 49.87 \times 58.27 \times 83.58)^{1/7}$$
$$= 48.717$$

3.

Marks	No. of students (f)	c.f.
10 – 20	15	15
20 – 40	20	35
40 – 70	30	65
70 – 90	20	85
90 – 100	15	100
	$N = \Sigma f = 100$	

Highest marks of weakest 30% students is P_{30}

$$P_{30} = \frac{30\,N}{100}^{th} \text{ item} = \frac{30 \times 100}{100} = 30$$

P_{30} class = (20 – 40)

$$P_{30} = L + \frac{\frac{30\,N}{100} - c.f.}{f} \times h = 20 + \frac{30 - 15}{20} \times 20 = 35$$

Lowest marks of top 40% student is P_{60}

$$P_{60} = \frac{60\,N}{100} \text{ item} = \frac{60 \times 100}{100} = 60$$

P_{60} class = (40 – 70)

$$P_{60} = L + \frac{\frac{60\,N}{100} - c.f.}{f} \times h = 40 + \frac{60 - 35}{30} \times 30 = 65$$

Lowest marks of top 20% students P_{80}

$$P_{80} = \frac{80\,N}{100} \text{ item} = \frac{80 \times 100}{100} = 80$$

P_{80} class = (70 – 90)

$$P_{80} = L + \frac{\frac{80\,N}{100} - c.f.}{f} \times h = 70 + \frac{80 - 65}{20} \times 20 = 85$$

Limit of middle 50% student is P_{25} to P_{75}.

$$P_{25} = \frac{25\,N}{100}^{th} \text{ item} = \frac{25 \times 100}{100} = 25$$

P_{25} class = (20 – 40)

$$P_{25} = L + \frac{\frac{25\,N}{100} - c.f.}{f} \times h = 20 + \frac{25 - 15}{20} \times 20 = 30$$

$$P_{75} = \frac{75\,N}{100} = L + \frac{\frac{75\,N}{100} - cf}{f} \times h = 70 + \frac{75 - 65}{20} \times 20 = 80$$

Range = $P_{75} - P_{25} = 80 - 30 = 50$

14. Compute the quartile deviation

Size of screen	No. of laptop (f)	c.f.
9.5	1	1
10.0	8	9
10.5	20	29
11.0	30	59
11.5	50	109
12.0	95	204
12.5	110	314
13.0	150	464
13.5	200	664
14.0	250	914
14.5	280	1194
15.0	245	1439
15.5	80	1519
16.0	40	1559
16.5	35	1594
17.0	5	1599
	$N = \sum f = 1599$	

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$ item $= \frac{1599 + 1}{4} = \frac{16000}{4} = 400$ $\quad \therefore Q_1 = 13,$

$Q_3 = \left(\frac{3\,(N+1)}{4}\right)^{th}$ item $= 3 \times 400 = 1200$ $\quad \therefore Q_3 = 15$

$$\text{Quartile deviation} = \frac{Q_3 - Q_1}{2} = \frac{15 - 13}{2} = 1$$

15. The following frequency

Weight	4–5	5–6	6–7	7–8	8–9	9–10	10–11	11–12
Freq. (f)	20	24	35	48	32	24	8	2
c.f.	20	44	79	127	159	183	191	193

$\frac{N}{4} = \frac{193}{4} = 48.25$ $\quad \therefore Q_1$ class = (6 – 7)

$$Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 6 + \frac{48.25 - 44}{35} \times 1 = 6.12$$

$\frac{2N}{4} = 2 \times 48.25 = 96.5 \quad \therefore Q_2 \text{ class} = (7 - 8)$

$$Q_2 = L + \frac{\frac{2N}{4} - \text{c.f.}}{f} \times h = 7 + \frac{96.5 - 79}{48} \times 1 = 7.36$$

$\frac{3N}{4} = 3 \times 48.25 = 144.75 \quad \therefore Q_3 \text{ class} = (8 - 9)$

$$Q_3 = L + \frac{\frac{3N}{4} - \text{c.f.}}{f} \times h = 8 + \frac{144.75 - 127}{32} \times 1 = 8.55$$

$QD = (Q_3 - Q_1)/2 = (8.55 - 6.12)/2 = 1.215$

16. The scores obtained

$L = 65, S = 35$

Range $= L - S = 65 - 35 = 30$

Score (x)	x^2
55	3025
35	1225
60	3600
55	3025
55	3025
65	4225
40	1600
45	2025
35	1225
42	1764
$\Sigma x = 487$	$\Sigma x^2 = 24739$

$$\text{S.D.} = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{24739}{10} - \left(\frac{487}{10}\right)^2} = 10.109$$

17. Arrange data in ascending order

32, 34, 34, 35, 35, 35, 36

Range $= L - S = 36 - 32 = 4$

$n = 7$

$$Q_1 = \frac{n+1}{4}\text{th item} = \frac{7+1}{4} = 2^{nd} \text{ item} = 34$$

$$Q_3 = \frac{3(n+1)}{4}\text{th item} = 3 \times 2 = 6^{th} \text{ item} = 35$$

$$\text{Q.D.} = \frac{Q_3 - Q_1}{2} = \frac{35 - 34}{1} = 0.5$$

18. The time in minutes

x	10	12	11	15	18	20	24	23	26	16
$\lvert x - \bar{x} \rvert$	7.5	5.5	6.5	2.5	0.5	2.5	6.5	5.5	8.5	1.5
x^2	100	144	121	225	324	400	576	529	676	256

$\sum x = 175, \sum|x - \bar{x}| = 47, \sum x^2 = 3351$

$$\bar{x} = \frac{\sum x}{n} = \frac{175}{10} = 17.5$$

$$\text{M.D. from } \bar{x} = \frac{\sum|x - \bar{x}|}{n} = \frac{47}{10} = 4.7$$

$$\text{S.D.} = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \sqrt{\frac{3351}{10} - (17.5)^2} = 5.37$$

19. To obtain information

Customer service time	fd'^2	No. of customers (f)	Mid-value (x)	$d' = \frac{x - 17.5}{5}$	fd'
0 – 5	18	2	2.5	– 3	– 6
5 – 10	32	3	7.5	– 2	– 16
10 – 15	26	26	12.5	– 1	– 26
15 – 20	0	30	17.5	0	0
20 – 25	28	28	22.5	1	28
25 – 30	24	6	27.5	2	12
	$\sum fd'^2$ = 128	N = $\sum f$= 100			$\sum fd'$ = – 8

$$\sigma = \sqrt{\frac{\sum fd'^2}{N} - \left(\frac{\sum fd'}{N}\right)^2} \times h = \sqrt{\frac{128}{100} - \left(\frac{-8}{100}\right)^2} \times 5 = 5.64$$

20. The frequency distribution

Time in sec	No. of customers (f)	Mid-value (x)	$d' = \frac{x - 17}{5}$	fd'	fd'^2
0 – 4	2	2	– 3	– 6	18
5 – 9	20	7	– 2	– 40	80
10 – 14	35	12	– 1	– 35	35
15 – 19	40	17	0	0	0
20 – 24	28	22	1	28	28
25 – 29	32	27	2	64	128
30 – 34	8	32	3	24	72
35 – 39	5	37	4	20	80
	170			55	441

$N = \sum f = 170, \sum fd' = 55, \sum fd'^2 = 441$

$$\text{S.D. } (\sigma) = \sqrt{\frac{\sum fd'^2}{N} - \left(\frac{fd'}{N}\right)^2} \times h$$

$$= \sqrt{\frac{441}{170} - \left(\frac{55}{70}\right)^2} \times 5 = \sqrt{2.594 - 0.104} \times 5 = 7.88$$

1. The average marks secured by

$\bar{x}_A = 78,\ \bar{x}_B = 80,\ \sigma_A^2 = 100,\ \sigma_B^2 = 81,\ x_A = 100,\ x_B = 150$

$$\bar{x}_{AB} = \frac{n_A \bar{x}_A + n_B \bar{x}_B}{n_A + n_B} = \frac{100 \times 78 + 150 \times 80}{100 + 150} = 79.2$$

$$d_A = \bar{x}_A - \bar{x}_{AB} = 78 - 79.2 = -1.2$$

$$d_B = \bar{x}_B - \bar{x}_{AB} = 80 - 79.2 = 0.8$$

$$\sigma_{AB} = \sqrt{\frac{n_A(\sigma_A^2 + d_B^2) + n_B(\sigma_B^2 + d_B^2)}{n_A + n_B}}$$

$$= \sqrt{\frac{100\,(100 + 1.44) + 150\,(81 + 0.64)}{100 + 150}} = 9.46$$

$$\sigma_{AB}^2 = 89.5$$

22. The number of runs

Group A (x_A)	Group B (x_B)	x_A^2	x_B^2
10	120	100	14400
25	15	625	225
85	30	7225	900
72	35	5184	1225
115	42	13225	1764
80	65	6400	4225
52	80	2704	6400
45	34	2025	1156
30	25	900	625
10	15	100	225
$\sum x_A = 524$	$\sum x_B = 461$	$\sum x_A^2 = 38488$	$\sum x_B^2 = 29929$

$$\bar{x}_A = \frac{\sum x_A}{n} = \frac{524}{10} = 52.4$$

$$\sigma_A = \sqrt{\frac{\sum x_A}{n} - \left(\frac{\sum x_A}{n}\right)^2} = \sqrt{\frac{38488}{10} - \left(\frac{524}{10}\right)^2} = 33.212$$

$$\bar{x}_B = \frac{\sum x_B}{n} = \frac{461}{10} = 46.1$$

$$\sigma_B = \sqrt{\frac{\sum x_B}{n} - \left(\frac{\sum x_B}{n}\right)^2} = \sqrt{\frac{29920}{10} - \left(\frac{5161}{10}\right)^2} = 29.441$$

$$C.V._A = \frac{\sigma_A}{\bar{x}_A} \times 100\% = \frac{33.212}{52.4} \times 100\% = 63.38\%$$

$$C.V._B = \frac{\sigma_B}{\bar{x}_B} \times 100\% = \frac{29.441}{46.1} \times 100\% = 63.86\%$$

$C.V._A < C.V._B$; Hence group A is more consistent.

23. The following data

Students	Section A (x_A)	Section B (x_B)	x_A^2	x_B^2
1	9	10	81	100
2	8	8	64	64
3	10	6	100	36
4	6	8	36	64
5	7	9	49	81
6	8	8	64	64
7	5	7	25	49
8	6	8	36	64
9	7	5	49	25
10	8	8	64	64
	$\sum x_A = 74$	$\sum x_B = 77$	$\sum x_A^2 = 568$	$\sum x_B^2 = 611$

$$\bar{x}_A = \frac{\sum x_A}{n} = \frac{74}{10} = 7.4$$

$$\sigma_A = \sqrt{\frac{\sum x_A^2}{n} - \left(\frac{\sum x_A}{n}\right)^2} = \sqrt{\frac{568}{10} - (7.4)^2} = 1.42$$

$$\bar{x}_B = \frac{\sum x_B}{n} = \frac{77}{10} = 7.7$$

$$\sigma_B = \sqrt{\frac{\sum x_B^2}{n} - \left(\frac{\sum x_B}{n}\right)^2} = \sqrt{\frac{611}{10} - (7.7)^2} = 1.34$$

$$C.V._A = \frac{\sigma_A}{\bar{x}_A} \times 100\% = \frac{1.42}{7.4} \times 100\% = 19.18$$

$$C.V._B = \frac{\sigma_B}{\bar{x}_B} \times 100\% = \frac{1.34}{7.7} \times 100\% = 17.4\%$$

$C.V._B < C.V._A$. Hence, group B is more consistent.

24. The following data

Time for male (x_A)	Time for female (x_B)	x_A^2	x_B^2
5.70	7.52	32.49	56.55
6.80	8.20	46.24	67.24
7.25	8.32	52.56	69.22
8.20	6.90	67.24	47.61
8.10	6.80	65.61	46.24
7.20	8.30	51.84	68.89
6.88	7.45	47.33	55.50
7.20	9.00	51.84	81.00
7.35	10.50	54.02	110.25
7.45	7.20	55.50	51.84

6.90	10.20	47.61	104.04
7.22	8.26	52.12	68.22
6.85	8.50	46.92	72.25
6.40	8.32	40.96	69.22
6.20	10.00	38.44	100
$\sum x_A = 105.7$	$\sum x_B = 125.47$	$\sum x_A^2 = 750.72$	$\sum x_B = 1068.07$

$$\bar{x}_A = \frac{\sum x_A}{n} = \frac{105.7}{15} = 7.046$$

$$\sigma_A = \sqrt{\frac{\sum x_A^2}{n} - \left(\frac{\sum x_A}{n}\right)^2} = \sqrt{\frac{750.72}{15} - \left(\frac{105.7}{15}\right)^2} = 0.673$$

$$\bar{x}_B = \frac{\sum x_B}{n} = \frac{125.47}{15} = 8.364$$

$$\sigma_B = \sqrt{\frac{\sum x_B^2}{n} - \left(\frac{\sum x_B}{n}\right)^2} = \sqrt{\frac{1068.07}{15} - \left(\frac{125.47}{15}\right)^2} = 1.117$$

$$C.V._A = \frac{\sigma_A}{\bar{x}_A} \times 100\% = \frac{0.633}{7.046} \times 100\% = 8.98\%$$

$$C.V._B = \frac{\sigma_B}{\bar{x}_B} \times 100\% = \frac{1.117}{8.364} \times 100\% = 13.35\%$$

$CV_A < CV_B$. Hence male students is more consistent.

25. Solution.

$$\bar{x} = \frac{\Sigma x}{n} = \frac{552}{10} = 55.20$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{31700}{10} - \left(\frac{552}{10}\right)^2} = \sqrt{122.96} = 11.08$$

$$S_{KP} = \frac{\bar{x} - M_o}{\sigma} = \frac{55.2 - 46.2}{11.08} = 0.81$$

26. Solution.

$n = 20, \Sigma x = 300, \Sigma x^2 = 5000, Md = 15, \bar{x} = \frac{300}{20} = 15,$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{5000}{20} - (15)^2} = 5$$

$$S_{KP} = 3\frac{\left(\bar{x} - Md\right)}{\sigma} = 3 \times \frac{15 - 15}{5} = 0$$

$$CV = \frac{\sigma}{\bar{x}} \times 100 = \frac{5}{15} \times 100 = 33.3\%$$

27. Solution.

$\bar{x} = 65, Md = 70, S_{KP} = -0.6, M_o = ?, C.V. = ?$

$$M_O = 3Md - 2\bar{x} = 3 \times 70 - 2 \times 65 = 80$$

$$S_{KP} = \frac{\bar{x} - M_O}{\sigma}$$

or $-0.6 = \frac{65 - 80}{\sigma}$

or $-0.6 = \frac{-15}{\sigma}$

or $\sigma = \frac{15}{0.6} = \frac{150}{6} = 25$

$$CV = \frac{\sigma}{\bar{x}} \times 100\% = \frac{25}{65} \times 100\% = 38.40\%$$

28. Solution.

Given, $\bar{x} = 200$, C.V. = 8%, $S_{KP} = 0.3$, M_O = ?, Md = ?

$$C.V. = \frac{\sigma}{\bar{x}} \times 100\%$$

or $8\% = \frac{\sigma}{200} \times 100\%$

or $\sigma = \frac{200 \times 8}{100} = 16$

$$S_{KP} = \frac{\bar{x} - M_O}{\sigma}$$

or $0.3 = \frac{200 - M_O}{16}$

or $4.8 = 200 - M_O$

or $M_O = 200 - 4.8 = 195.20$

$$M_O = 3Md - 2\bar{x}$$

or $195.2 = 3Md - 2 \times 200$

or $3\ Md = 400 + 195.20 = 595.2$

or $Md = \frac{595.2}{3} = 198.40$

29. Solution.

Size (x)	Frequency (f)	fx	fx^2
6	7	42	252
9	12	108	972
12	19	228	2736
15	10	150	2250
18	2	36	648
	N = 50	$\Sigma fx = 564$	$\Sigma fx^2 = 6858$

$$\bar{x} = \frac{\Sigma fx}{N} = \frac{564}{50} = 11.28$$

$$\sigma = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2} = \sqrt{\frac{6858}{50} - \left(\frac{564}{50}\right)^2} = 3.149$$

$M_O = 12$

$$S_{KP} = \frac{\overline{x} - M_O}{\sigma} = \frac{11.28 - 12}{3.149} = -0.23$$

Solution.

Mid Value (x)	No. of workers (f)	fx	fx²
150	80		
250	105		
350	120		
450	165		
550	100		
650	90		
750	60		
850	40		
	N = 50	Σfx = 347000	Σfx² = 187400000

$$\overline{x} = \frac{\Sigma fx}{N} = \frac{347000}{760} = 456.58$$

$$\sigma = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2} = \sqrt{\frac{187400000}{760} - \left(\frac{347000}{760}\right)^2} = 195.23$$

Model class = (400 – 500)

$f_0 = 120,\ f_1 = 165,\ f_2 = 100$

$\Delta_1 = f_1 - f_0 = 165 - 120 = 45$ $\Delta_2 = f_1 - f_2 = 165 - 100 = 65$

$$M_O = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 400 + \frac{45}{45 + 65} \times 100 = 400 + \frac{4500}{110} = 440.9$$

$$S_{KP} = \frac{\overline{x} - M_O}{\sigma} = \frac{456.57 - 440.90}{195.23} = 0.08$$

1. $S_{KB} = 0.6$

$Q_3 + Q_1 = 100$ .... (1)

$Md = 38$

$Q_3 = ?$

$$S_{KB} = \frac{Q_3 + Q_1 - 2\,Md}{Q_3 - Q_1}$$

or $0.6 = \dfrac{100 - 2 \times 38}{Q_3 - Q_1}$

or $Q_3 - Q_1 = \dfrac{100 - 76}{0.6}$

or $Q_3 - Q_1 = 40$ .... (2)

Add (1) and (2)

$2Q_3 = 140$

$\therefore$ $Q_3 = 70$

32. **Solution.**

$S_{KB} = -0.8, Q_1 = 44.10, Q_3 = 56.60, Md = ?$

$$S_{KB} = \frac{Q_3 + Q_1 - 2\,Md}{Q_3 - Q_1}$$

or $-0.8 = \frac{56.6 + 44.1 - 2Md}{56.6 - 44.1}$

or $-0.8 = \frac{100.70 - 2\,Md}{12.5}$

or $-0.8 \times 12.5 = 100.70 - 2\,Md$

or $2Md = = 10 + 100.70$

or $Md = \frac{110.70}{2} = 55.35$

33. **Solution.**

x	f	c.f.
20	3	3
23	6	9
24	10	19
26	12	31
28	11	42
30	9	51
40	4	55
	$N = \Sigma f = 55$	

$\sigma_1 = \frac{N+1}{4}$ th item $= \frac{55+1}{4} = 14^{th}$ item $= 24$

$Md = \frac{2(N+1)}{4}$ th item $= 2 \times 14 = 28^{th}$ item $= 26$

$Q_3 = \frac{3(N+1)}{4}$ th item $= 3 \times 14 = 42^{th}$ item $= 28$

$$S_{KB} = \frac{Q_3 + Q_1 - 2\,Md}{Q_3 - Q_1} = \frac{28 + 24 - 26}{28 - 24} = 0$$

34. **Solution.**

Hourly wage	No. of workers (f)	c.f.
23 – 27	22	22
28 – 32	16	38
33 – 37	9	47
38 – 42	4	51
43 - 47	3	54
48 – 52	1	55
	$N = \Sigma f = 55$	

$\frac{N}{4} = \frac{55}{4} = 13.75$

Q_1 class = (23 – 27)

Adjusted Q_1 class = 22.5 – 27.5

$$Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 22.5 + \frac{13.75 - 0}{22} \times 5 = 25.62$$

$$\frac{2N}{4} = 2 \times 13.75 = 27.5$$

Md class = 28 – 32

Adjusted Md class = 27.5 – 32.5

$$Md = L + \frac{\frac{2N}{4} - c.f.}{f} \times h = 27.5 + \frac{27.5 - 22}{16} \times 5 = 27.5 + 1.718 = 29.218$$

$$\frac{3N}{4} = 3 \times 13.75 = 41.27$$

Q_3 class = (33 – 37) $\therefore$ Adjusted class = 32.5 – 37.5

$$Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 32.5 + \frac{41.27 - 38}{9} \times 5 = 34.305$$

$$S_{KB} = \frac{Q_3 + Q_1 - 2Md}{Q_3 - Q_1} = \frac{34.305 + 25.62 - 2 \times 29.218}{34.305 - 25.62} = 0.17$$

35. **Solution.**

Download speed (Mbps)	Time in min.	c.f.
Below 100	10	10
100 – 150	25	35
150 – 200	145	180
200 – 250	220	400
250 – 300	70	470
300 and above	30	500

$\frac{N}{4} = \frac{500}{4} = 125$ $\therefore$ Q_1 class = (150 – 200)

$$Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 150 + \frac{125 - 35}{145} \times 50 = 181.034$$

$\frac{2N}{4} = 2 \times 125 = 250$ $\therefore$ Md class = (200 – 250)

$$Md = L + \frac{\frac{2N}{4} - c.f.}{f} \times h = 200 + \frac{250 - 180}{220} \times 50 = 215.90$$

$$\frac{3N}{4} = 3 \times 125 = 375$$

Q_3 class = (200 – 250)

$$Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 200 + \frac{375 - 180}{220} \times 50 = 244.32$$

$$S_{KB} = \frac{Q_3 + Q_1 - 2\,Md}{Q_3 - Q_1} = \frac{244.32 + 181.03 - 2 \times 215.90}{244.32 - 181.03} = -0.102$$

36. Solution.

x	f	c.f.
2	1	1
4	5	6
6	10	16
8	3	19
10	1	20
	$N = \Sigma f = 20$	

For P_{10}, $\frac{10(N+1)}{100} = \frac{10 \times 21}{100} = 2.1$ $\therefore P_{10} = 4$

For P_{90}, $\frac{90(N+1)}{100} = \frac{90 \times 21}{100} = 18.9$ $\therefore P_{90} = 8$

For Q_1, $\frac{N+1}{4} = \frac{20+1}{4} = 5.25$ $\therefore Q_1 = 4$

For Q_3, $\frac{3(N+1)}{4} = 3 \times 5.25 = 15.75$ $\therefore Q_3 = 6$

$$K = \frac{Q_3 - Q_1}{2(P_{90} - P_{10})} = \frac{6-4}{2(8-4)} = \frac{2}{2 \times 4} = 0.25$$

37. Solution.

Hourly remuneration	No. of person (f)	c.f.
100 – 110	10	10
110 – 120	14	24
120 – 130	18	42
130 – 140	24	66
140 – 150	16	82
150 – 160	12	94
160 – 170	6	100
	$N = \Sigma f = 100$	

For P_{10}, $\frac{10N}{100} = \frac{10 \times 100}{100} = 10$ $\therefore P_{10}$ class = 100 – 110

$$P_{10} = L + \frac{\frac{10N}{100} - c.f.}{f} \times h = 100 + \frac{10-0}{10} \times 10 = 110$$

For P_{90}, $\frac{90N}{100} = \frac{90 \times 100}{100} = 90$ $\therefore P_{90}$ class = 150 – 160

$$P_{90} = L + \frac{\frac{90N}{100} - c.f.}{f} \times h = 150 + \frac{90-82}{12} \times 10 = 156.66$$

For Q_1, $\frac{N}{4} = \frac{100}{4} = 25$ $\therefore Q_1$ class = (120 – 130)

$$Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 120 + \frac{25-24}{18} \times 10 = 120.55$$

or Q_3, $\frac{3N}{4} = 3 \times 25 = 75$ $\therefore$ Q_3 class = (140 – 150)

$$Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 140 + \frac{75 - 66}{16} \times 10 = 145.625$$

$$K = \frac{145.625 - 120.55}{2\,(156.66 - 110)} = \frac{25.075}{93.32} = 0.268$$

38. Solution.

Class interval	frequency (f)	Mid-value (x)	fx	$(x-\overline{x})$	$f(x-\overline{x})$	$f(x-\overline{x})^2$	$(x-\overline{x})^3$	$f(x-\overline{x})^4$
0 – 10	5	5	25	– 20	– 100	25	– 20	– 100
10 – 20	7	15	105	– 10	– 70	105	– 10	– 70
20 – 30	9	25	225	0	0	225	0	0
30 – 40	7	35	245	10	70	245	10	70
40 – 50	5	45	225	20	100	225	20	100
	33		825		0	5400	0	1740000

$$\overline{x} = \frac{\Sigma fx}{N} = \frac{825}{33} = 25$$

$$\mu_1 = \frac{\Sigma f(x-\overline{x})}{N} = \frac{0}{33} = 0 \qquad \mu_2 = \frac{\Sigma f(x-\overline{x})^2}{N} = \frac{5400}{33} = 163.63$$

$$\mu_3 = \frac{\Sigma f(x-\overline{x})^3}{N} = \frac{0}{33} = 0 \qquad \mu_4 = \frac{\Sigma f(x-\overline{x})^4}{N} = \frac{1740000}{33} = 52727.27$$

39. Solution.

x	f	d = x – 4	fd	fd^2	fd^3	fd^4
2	1	–2	–2	4	– 8	16
3	3	–1	–3	3	–3	3
4	7	0	0	0	0	0
5	2	1	2	2	2	2
6	1	2	2	4	8	16
	N = 14		$\Sigma fd = -1$	$\Sigma fd^2 = 13$	$\Sigma fd^3 = -1$	$\Sigma fd^4 = 37$

$$\mu_1' = \frac{\Sigma fd}{N} = \frac{-1}{14} = -0.071 \qquad \mu_2' = \frac{\Sigma fd^2}{N} = \frac{13}{14} = 0.928$$

$$\mu_3' = \frac{\Sigma fd^3}{N} = \frac{-1}{14} = -0.071 \qquad \mu_4' = \frac{\Sigma fd^4}{N} = \frac{37}{14} = 2.64$$

40. Solution.

Marks	f	x	$d' = \frac{x-45}{10}$	fd'	fd'^2	fd'^3
20 – 30	7	25	–2	–14	28	– 56
30 – 40	10	35	–1	–10	10	– 10
40 – 50	20	45	0	0	0	0
50 – 60	18	55	1	18	18	18
60 – 70	7	65	2	14	28	56
	N = Σf = 62			Σfd' = 8	$fd'^2 = 84$	$fd'^3 = 8$

$\mu_1' = h \times \frac{\Sigma fd}{N} = 10 \times \frac{8}{62} = 1.29$ $\mu_2' = h^2 \times \frac{\Sigma fd^2}{N} = 10^2 \times \frac{84}{62} = 135.483$

$\mu_3' = h^3 \times \frac{\Sigma fd^3}{N} = 10^3 \times \frac{8}{62} = 129.03$

$\mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2 = 135.483 - (1.29)^2 = 133.81$

$\mu_3 = \mu_3' - 3\mu_2' \mu_1' + 2(\mu_1')^3 = 129.03 - 3 \times 135.483 \times 1.29 + 2 \times (1.29)^3 = -390.98$

41. given

First arbitrary moment = $\mu_1' = -2$

Second arbitrary moment = $\mu_2' = 14$

Third arbitrary moment = $\mu_3' = -20$

Fourth arbitrary moment = $\mu_4' = 50$

Now We have

First central moment = $\mu_1 = \mu'_1 - \mu'_1 = 0$

Second central moment = $\mu'_2 = \mu'_2 - \mu'_1{}^2 = 14 - 2^2 = 10$

Third central moment = $\mu_3' - 3\mu_2' \mu_1' + 2(\mu_1')^3$

$= -20 - 3\times 14 - 2 + 2 \times (-2)^3 = -20 + 84 - 16 = 48$

Fourth central moment = $\mu_4 = \mu_4' - 4\mu_3' \mu_1' + 6\mu_2' \mu_1'^2 + 3(\mu_1')^4$

$= 50 - 4 \times 200020 \times (-2) + 6 \times 14 \times (-2)^2 - 3 \times (-2)^4$

$= 50 - 160 + 336 - 48 = 178$

42. Solution.

Given

First arbitrary moment = $\mu_1 = 0$

Second arbitrary moment = $\mu_2 = 2.8$

Third arbitrary moment = $\mu_3 = -2$

Fourth arbitrary moment = $\mu_4 = 24.5$

Now

We have

Coefficient of skewness $(\gamma_1) = \frac{\mu_3}{\mu_2^{3/2}} = \frac{-2}{(2.8)^{3/2}} = -0.427$

$\gamma_1 = -0.427 < 0$, hence the distribution is negatively skewed.

Now, coefficient of kurtosis $(\beta_2) = \frac{\mu_4}{\mu_2^2} = \frac{24.5}{(2.8)^2} = 3.125$

$\beta_2 = 3.125 > 3$, hence the distribution is leptokurtic.

43. Solution.

First arbitrary moment about x=5 = $\mu_1' = 2$

Second arbitrary moment about x=5 = $\mu_2' = 20$

Third arbitrary moment about x=5 = $\mu_3' = 40$

Now, We have

Central Tendency = AM = $\overline{X}$ = $A + \mu_1' = 5+2 = 7$

Dispersion S.D. = $\sqrt{\mu_2} = \sqrt{\mu_2' - \mu_1'^2} = \sqrt{20 - 2^2} = 4$

Third central moment = $\mu_3 = \mu_3' - 3\mu_2' \mu_1' + 2(\mu_1')^3$

$= 40 - 3 \times 20 \times 2 + 2 \times 2^3 = 40 - 120 + 16 = -64$

Coefficient of skewness $(\gamma_1) = \frac{\mu_3}{\mu_2^{3/2}} = \frac{-64}{(16)^{3/2}} = \frac{-64}{64} = -1$

$\gamma_1 = -1 < 0$, hence the distribution is negatively skewed.

44. Solution

Here, $\sigma = 11$

Or, $\mu_2 = \sigma^2 = 11^2 = 121$

i) For leptokurtic distribution $\beta_2 > 3$

$\Rightarrow \frac{\mu_4}{\mu_2^2} > 3$

$\Rightarrow \frac{\mu_4}{121^2} > 3$

$\Rightarrow \mu_4 > 14641 \times 3$

$\Rightarrow \mu_4 > 43923$

ii) For mesokurtic distribution, $\beta_2 = 3$

$\Rightarrow \frac{\mu_4}{\mu_2^2} = 3$

$\Rightarrow \frac{\mu_4}{121^2} = 3$

$\Rightarrow \mu_4 = 16641 \times 3$

$\Rightarrow \mu_4 = 43923$

(iii) For platykurtic distribution, $\beta_2 < 3$

$\Rightarrow \frac{\mu_4}{\mu_2^2} < 3$

$\Rightarrow \frac{\mu_4}{121^2} < 3$

$\Rightarrow \mu_4 < 16641 \times 3$

$\Rightarrow \mu_4 < 43923$

45. Solution.

x	f	fx	$d=x-\overline{x}$	fd	fd^2	fd^3	fd^4
0	5	0	-5	-25	125	-625	3125
1	10	10	-4	-40	160	-640	2560
2	30	60	-3	-90	270	-810	2430
3	70	210	-2	-140	280	-560	1120
4	140	560	-1	-140	140	-140	140
5	200	1000	0	0	0	0	0
6	140	840	1	140	140	140	140
7	70	490	2	140	280	560	1120
8	30	240	3	90	270	810	2430
9	10	90	4	40	160	640	2560
10	5	50	5	25	125	625	3125
	N = 710	Σfx=3550		Σfd= 0	Σfd^2= 1950	Σfd^3=0	Σfd^4= 18750

Mean $= \frac{\Sigma fm}{N} = \frac{3550}{710} = 5$

First central moment $= \mu_1 = 0$

Second central moment $= \mu_2 = \frac{\Sigma fd^2}{N} = \frac{1950}{710} = 2.75$

Third central moment $= \mu_3 = \frac{\Sigma fd^3}{N} = \frac{0}{710} = 0$

Fourth central moment $= \mu_4 = \frac{\Sigma fd^4}{N} = \frac{18750}{710} = 26.41$

Now, we have

Coefficient of skewness $(\gamma_1) = \frac{\mu_3}{\mu_2^{3/2}} = \frac{0}{(2.75)^{3/2}} = 0$

$\gamma_1 = 0$, hence the distribution is symmetrical.

Now, coefficient of kurtosis $(\beta_2) = \frac{\mu_4}{\mu_2^2} = \frac{26.41}{(2.75)^2} = 3.49$

$\beta_2 = 3.49 > 3$, hence the distribution is leptokurtic.

46. Solution.

For B Sc CSIT Students

Project conducted (x)	No of Students (f)	fX	$(X-\bar{X})$	$f(X-\bar{X})$	$f(X-\bar{X})^2$	$f(X-\bar{X})^3$	$f(X-\bar{X})^4$
20	5	100	-3.26	-16.3	53.14	-173.23	564.73
22	7	154	-1.26	-8.82	11.11	-14.00	17.64
23	10	230	-0.26	-2.6	0.68	-0.18	0.05
25	8	200	1.74	13.92	24.22	42.14	73.33
26	5	130	2.74	13.7	37.54	102.85	281.82
	$N = \Sigma f = 35$	$\Sigma fX = 818$		$\Sigma f(X-\bar{X}) = 0$	$\Sigma f(X-\bar{X})^2 = 126.69$	$\Sigma f(X-\bar{X})^4 = -42.41$	$\Sigma f(X-\bar{X})^4 = 937.57$

Mean Value $(\bar{X}) = \frac{\Sigma fx}{N} = \frac{818}{35} = 23.26$

Variance $= \mu_2 = \frac{\Sigma f(X-\bar{X})^2}{N} = \frac{126.69}{35} = 3.62$

Standard Deviation $= \sqrt{\mu^2} = \sqrt{3.62} = 1.9$

Coefficient of Variation $= \frac{SD}{\bar{X}} \times 100 = \frac{1.9}{23.26} \times 100 = 8.18$

Third Central Moment $\mu_3 = \frac{\Sigma f(X-\bar{X})^3}{N} = \frac{-42.41}{35} = -1.21$

Fourth Central Moment $\mu_4 = \frac{\Sigma f(X-\bar{X})^4}{N} = \frac{937.57}{35} = 26.79$

oefficient of skewness $(\gamma_1) = \frac{\mu_3}{\mu_2^{3/2}} = \frac{-1.21}{(3.62)^{3/2}} = \frac{-1.21}{6.89} = -0.18$

ow, coefficient of kurtosis $(\beta_2) = \frac{\mu_4}{\mu_2^2} = \frac{26.79}{(3.62)^2} = 2.04$

< 0, hence the distribution is negatively skewed.

$= 2.04 > 3$, hence the distribution is platykurtic.

or BCA Students

Project conducted (x)	No of Students (f)	fX	$(X-\bar{X})$	$f(X-\bar{X})$	$f(X-\bar{X})^2$	$f(X-\bar{X})^3$	$f(X-\bar{X})^4$
20	3	60	-3.17	-9.51	30.15	-95.57	302.94
22	7	154	-1.17	-8.19	9.58	-11.21	13.12
23	15	345	-0.17	-2.55	0.43	-0.07	0.01
25	8	200	1.83	14.64	26.79	49.03	89.72
26	2	52	2.83	5.66	16.02	45.33	128.28
	$N = \Sigma f = 35$	$\Sigma fX = 811$		$\Sigma f(X-\bar{X}) = 0$	$\Sigma f(X-\bar{X})^2 = 82.97$	$\Sigma f(X-\bar{X})^3 = -12.49$	$\Sigma f(X-\bar{X})^4 = 534.08$

Mean Value $(\bar{X}) = \frac{\Sigma fx}{N} = \frac{811}{35} = 23.17$

Variance $= \mu_2 = \frac{\Sigma f(X-\bar{X})^2}{N} = \frac{82.97}{35} = 2.37$

Standard Deviation $= \sqrt{\mu^2} = \sqrt{2.37} = 1.54$

Coefficient of Variation $= \frac{S.D.}{\bar{X}} \times 100 = \frac{1.54}{23.17} \times 100 = 6.65$

Third Central Moment $\mu_3 = \frac{\Sigma f(X-\bar{X})^3}{N} = \frac{-12.49}{35} = -0.36$

Fourth Central Moment $\mu_4 = \frac{\Sigma f(X-\bar{X})^4}{N} = \frac{534.08}{35} = 15.26$

Coefficient of skewness $(\gamma_1) = \frac{\mu_3}{\mu_2^{3/2}} = \frac{-0.36}{(2.37)^{3/2}} = -0.10$

Now, coefficient of kurtosis $(\beta_2) = \frac{\mu^4}{\mu_2^2} = \frac{15.26}{(2.37)^2} = 2.72$

$\gamma_1 < 0$, hence the distribution is negatively skewed.

$\beta_2 = 2.04 > 3$, hence the distribution is platykurtic.

47. Solution.

Wage (Rs)	Mid value(m)	No of Students (f)	Cf
Below 97	95	2	2
98 – 102	100	5	7
103 – 107	105	12	19
108 – 112	110	17	36
113 – 117	115	14	50

118 – 122	120	6	56
123 – 127	125	3	59
128 above	130	1	60
		$N = \Sigma f = 60$	

Here,

It is open end class; hence we have to use median to find out central tendency

Position of Median = $N/2^{th}$ item = 60/2 = 30th item

Cf just greater than N/2 is 30 for which corresponding class is (108-112), hence corrected median class is (107.5 – 112.5)

L = 107.5, h = 5, f = 17 , cf =19

$$\text{Median} = L + \frac{\frac{N}{2} - c.f.}{f} \times h = 107.5 + \frac{30 - 19}{17} \times 5 = 110.74$$

$$P_{25} = 25N / 100^{th} \text{ item} = 25 \times \frac{60}{100} = 15$$

Cf just greater than 25N/100 is 19 for which corresponding class is (103 – 107), hence corrected P_{25} class is (102.5 – 107.5)

L = 102.5, h = 5 f = 12 , cf =7

$$Q_1 = P_{25} = L + \frac{\frac{25\,N}{100} - c.f.}{f} \times h = 102.5 + \frac{15 - 7}{12} \times 5 = 105.83$$

$$P_{75} = 75N/100^{th} \text{ item} = 75 \times \frac{60}{100} = 45$$

Cf just greater than 75N/100 is 50 for which corresponding class is (113 – 117), hence correct P_{75} class is (112.5 – 117.5)

L = 112.50, h = 5 f = 14, cf = 36

$$Q_3 = P_{75} = L + \frac{\frac{75N}{100} - c.f.}{f} \times h = 112.5 + \frac{45 - 36}{14} \times 5 = 115.71$$

$$\text{Quartile Deviation} = QD = \frac{Q_3 - Q_1}{2} = \frac{115.71 - 105.83}{2} = 4.94$$

$$\text{Now; } S_{KB} = \frac{Q_3 + Q_1 - 2Md}{Q_3 - Q_1} = \frac{115.71 + 105.83 - 2 \times 110.74}{115.71 - 105.83} = 0.006$$

S_{KB} = 0.006, hence the distribution is positively skewed.

Here we have to use the percentile coefficient of kurtosis.

Here,

$$P_{10} = 10N/100^{th} \text{ item} = 10 \times \frac{60}{100} = 6$$

Cf just greater than 10N/100 is 6 for which corresponding class is (98 – 102), hence P_{10} class is (97.5 – 102.5)

L = 97.5, h = 5, f = 5, cf = 2

$$P_{10} = L + \frac{\frac{10\,N}{100} - c.f.}{f} \times h = 97.5 + \frac{6 - 2}{5} \times 5 = 101.5$$

$P_{90} = 90N/100^{th}$ item $= 90 \times \frac{60}{100} = 54$

Cf just greater than 90N/100 is 56 for which corresponding class is (118 – 122), hence P_{75} class is (117.5 – 122.5)

L = 117.5, h = 5 f = 6 , cf =50

$$P_{90} = L + \frac{\frac{90N}{100} - c.f.}{f} \times h = 117.5 + \frac{54-50}{6} \times 5 = 120.83$$

Percentile coefficient of kurtosis (K) $= \frac{P_{75} - P_{25}}{2(P_{90} - P_{10})} = \frac{115.71 - 105.83}{2(120.83 - 101.50)} = 0.255$

K = 0.255< 0.263, hence the distribution is leptokurtic.

48. Solution.

Arranging the data in ascending order of magnitude

0	0	1	1	1	2	2	2	2	2	3	3	3	3	3	3

3	4	4	4	4	4	5	5	6

There are a total of twenty five points in the data set. The median is thus the 13th data values or:

Here size of median $= \left(\frac{N+1}{2}\right)^{th}$ item$= \left(\frac{26}{2}\right)^{th}$ item = 13th item

Or, Median = 3

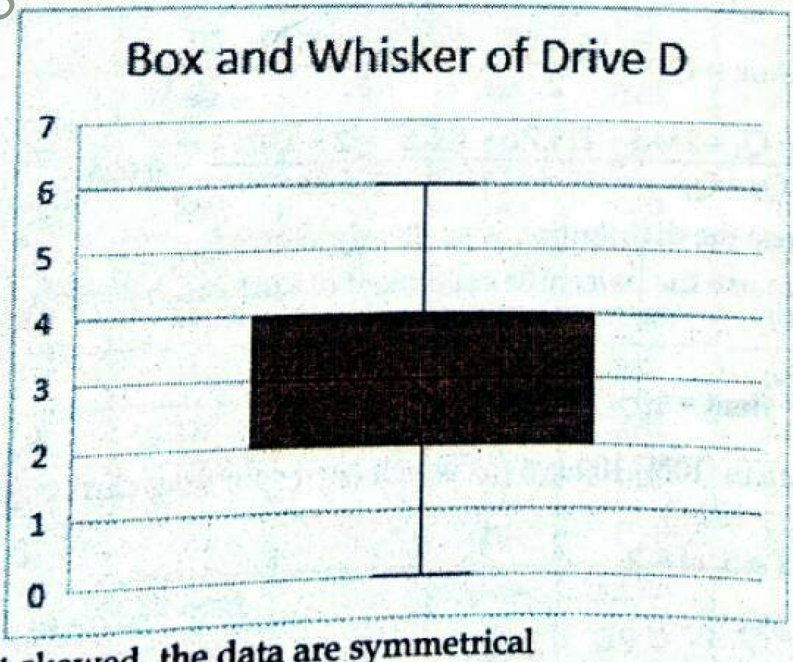
Similarly, size of $Q_1 = \left(\frac{N+1}{2}\right)^{th}$ item$= \left(\frac{26}{2}\right)^{th}$ item = 6.5th item

$Q_1 = 6^{th} + (7^{th} - 6^{th}$ item$) \times 0.5 = 2 + (2-2) \times 0.5 = 2$

And, size of $Q_3 = \left(3\frac{N+1}{4}\right)^{th} = \left(\frac{78}{4}\right)^{th} = 19.5^{th}$ item

$Q_3 = 19^{th} + (20^{th} - 19^{th}$ item$) \times 0.5 = 4 + (4-4) \times 0.5 = 4$

We assemble all of the above results together, and report that the five number summaries for the above set of data is 0, 2, 3, 4, 6.



The data are not skewed, the data are symmetrical

49. Solution

Marks, x	No. of students, f	c f
35	3	3
38	5	8
45	10	18
60	8	26
72	4	30
80	2	32
85	1	33

Here size of median = $\left(\frac{N+1}{2}\right)^{th}$ item= $\left(\frac{34}{2}\right)^{th}$ item = 17^{th} item

The cf just greater than 17 is 18 and the corresponding value is 45

Therefore, Median = 45

Similarly, size of $Q_1 = \left(\frac{N+1}{4}\right)^{th} = \left(\frac{34}{4}\right)^{th} = 8.5^{th}$ item

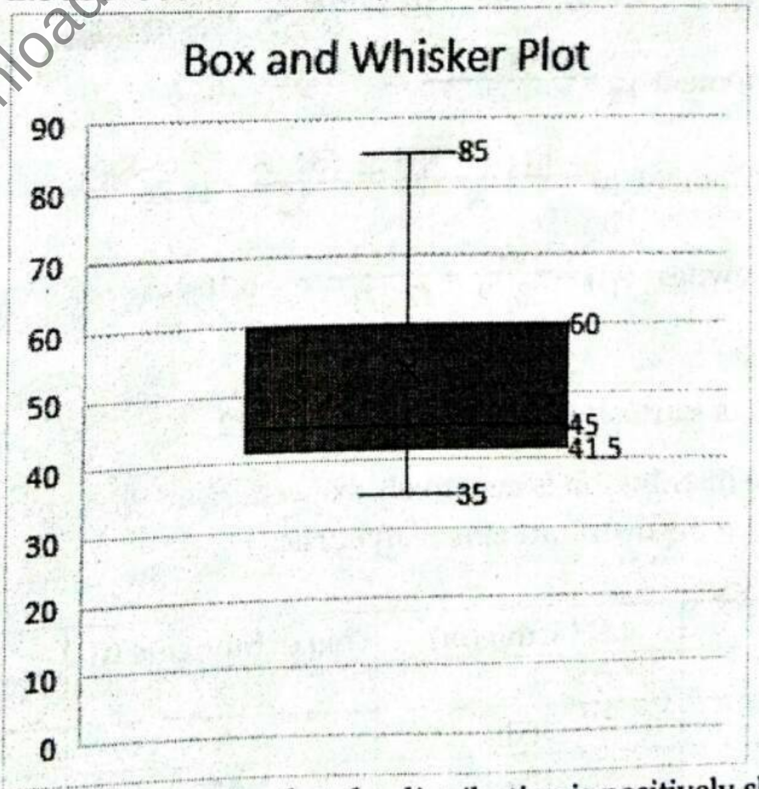
$Q_1 = 8^{th} + (9^{th} - 8^{th}$ item$) \times 0.5 = 38 + (45-38) \times 0.5 = 41.5$

And, size of $Q_3 = \left(3\,\frac{N+1}{4}\right)^{th} = \left(\frac{102}{4}\right)^{th} = 25.5^{th}$ item

The cf just greater than 25.5 is 26 and the corresponding value is 60

$Q_3 = 60$

We assemble all of the above results together, and report that the five number summaries for the above set of data is 35, 41.5, 45, 60, 85.



From the above figure we can say that the distribution is positively skewed.

. Solution.

vehicles		No of days (f)	cf
0	10	3	3
10	20	14	17
20	30	53	70
30	40	20	90
40	50	10	100
		100	

ze of $M_d = \frac{N}{2} = \frac{100}{2} = 50^{th}$ item

he c.f. just greater than 50 is 70. Hence the corresponding class 20 - 30 is the edian class.

$$M_d = L + \frac{\frac{N}{2} - c.f.}{f} \times h = 20 + \frac{50 - 17}{53} \times 10 = 20 + 6.23 = 26.23$$

ize of $Q_1 = \frac{N}{4} = \frac{100}{4} = 25$

he c.f. just greater than 25 is 70. Hence, Q_1 lies in class 20 - 30

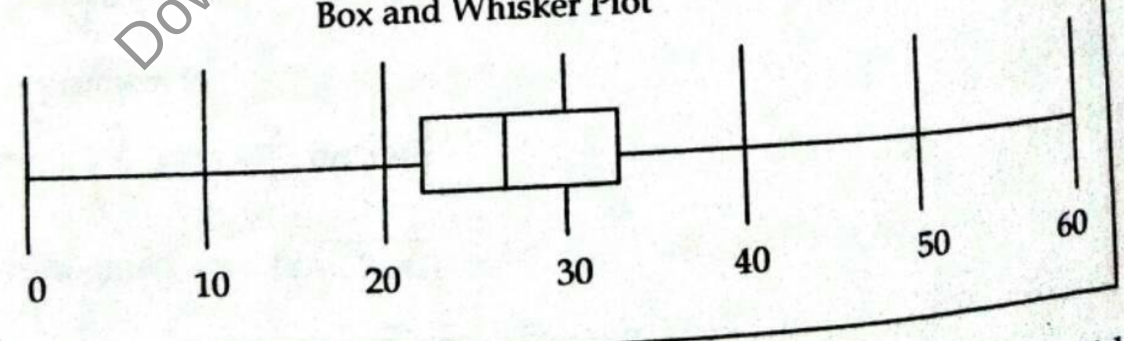
$$Q_1 = L + \frac{\frac{N}{4} - c.f.}{f} \times h = 20 + \frac{25 - 17}{53} \times 10 = 20 + 1.51 = 21.51$$

Size of $Q_3 = \frac{3N}{4} = \frac{3 \times 100}{4} = 75$

The c.f. just greater than 75 is 90. Hence, Q_3 lies in class 30-40

$$Q_3 = L + \frac{\frac{3N}{4} - c.f.}{f} \times h = 30 + \frac{75 - 70}{20} \times 10 = 30 + 2.5 = 32.5$$

Box and Whisker Plot



The five number summary is {0, 21.51, 26.23, 32.5, 50}, since box and whisker of right side is larger than left so the distribution is positively skewed.

51. **Solution.** Stem leaf display of above data are as follows

First week	stem	Second week
	2	8
	3	9 9
7 7 2 1	4	0 2 3 4 5 6 6
8 4 4 3 1	5	0 1 2 3 5 6
4 3 2 0 0	6	0 0

52. **Solution.**

Stem	leaf
11	9
12	0
13	1 3 4 5
14	8 8
15	0 2 4 8 8 9
16	1 2 2 3 7 9
17	1 1 2 2 5 6 7
18	5 6 7 8
19	0 3 4 5 7 8 9
20	2 7
21	6 7 7 9
22	1 2 8
23	9 9
24	1

Practical work

1. **Solution**

Here Mean $= \frac{\text{Sum of all observation}}{\text{Number of observation}} = \frac{2152}{50} = 43.04$

Arranging the data in descending order of magnitude

1	2	4	4	5	5	5	10	11	12	13	13	14	19	20
29	29	29	30	31	31	32	33	35	35	36	52	54	61	68

2 2	2 3	2 5	2 6	2 6	2 6	2 7	2 7	2 7	2 8
7 4	8 1	9 4	1 1 0	1 1 0	1 2 3	1 2 6	1 3 7	1 5 2	1 6 5

Median = 25.5^{th} item = average of 25^{th} and 26^{th} item $= \frac{28+29}{2} = 28.5$

Mode = Since, mode is ill defined because items 5,26,27,29 are repeated 3 times

First quartile $= Q_1 = 51/4^{\text{ th}}$ item $= 12.75^{th}$ item

$= 12^{th}$ item $+ (13\text{-}12)^{th}$ item $\times 0.75 = 13+(14\text{-}13)*.75=13.75$

Third quartile $= Q_3 = 3\times 51/4^{\text{ th}}$ item $= 38.25^{th}$ item

$= 38^{th}$ item $+ (39\text{-}38)^{th}$ item $\times 0.25 = 54+(61\text{-}54) \times 0.25=55.75$

Range = Largest item – Smallest item = 165-1 = 164

Inter-quartile range $= Q_3 - Q_1 = 55.75 - 13.75 = 42$

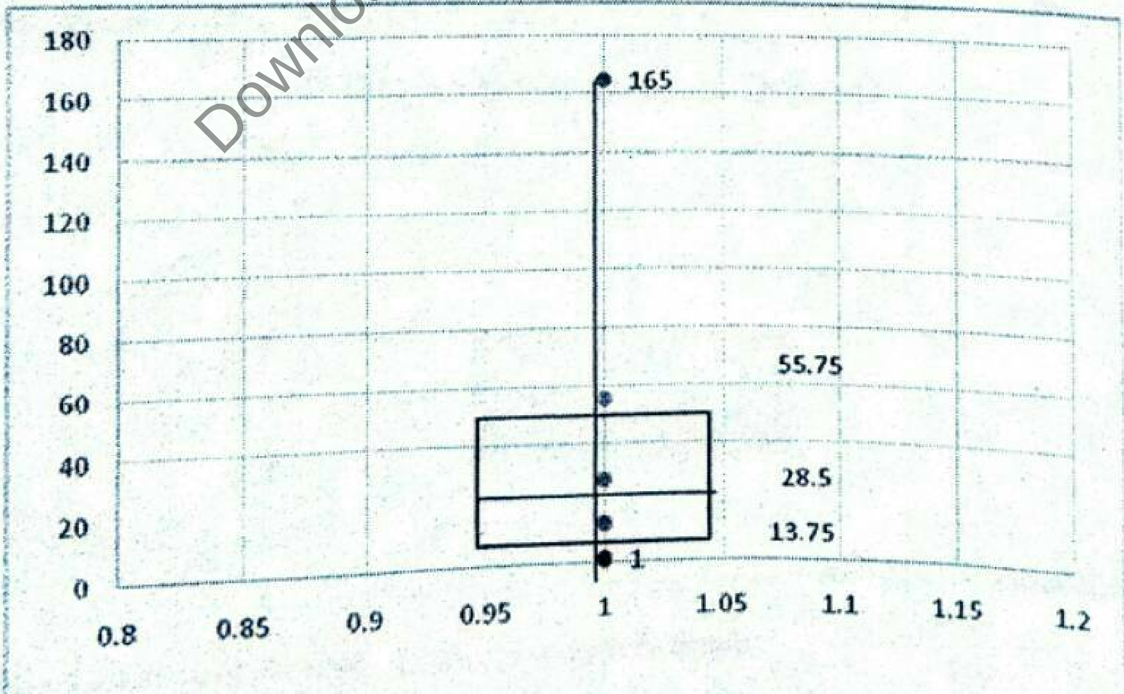
Variance $= \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2 = \frac{178754}{50} - \left(\frac{2152}{50}\right)^2 = 3575.08 - 1852.44 = 1722.64$

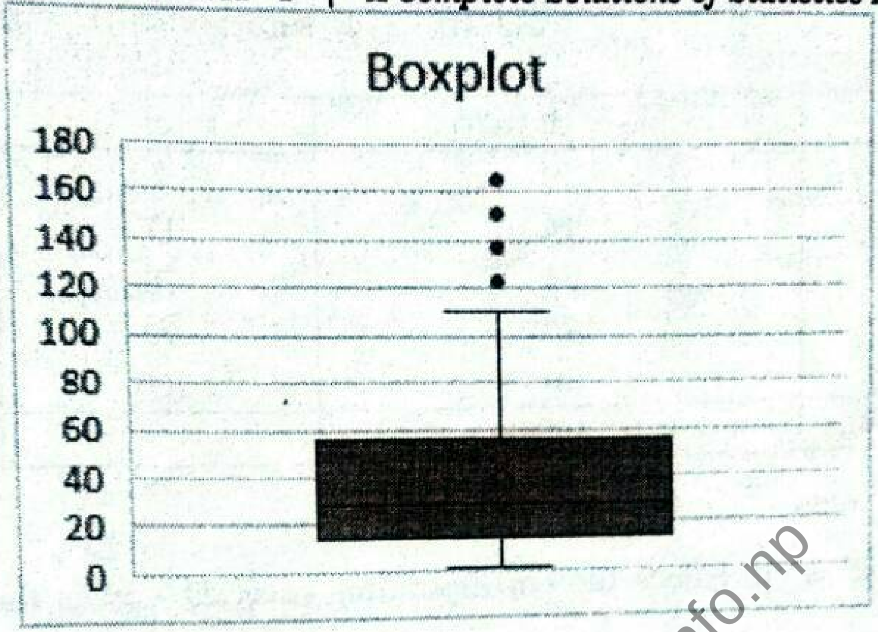
Standard deviation = SD = $\sqrt{1722.64} = 41.5$

Coefficient of variation = CV $= \frac{SD}{Mean} \times 100 = \frac{41.5}{43.04} \times 100 = 96.42$

Boxplot

(1, 13.75, 28.5, 55.75, 165)




Boxplot
180
160
140
120
100
80
60
40
20
0

On throwing dice

Sample space {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Let A = sum greater than 9, B = Neither 8 nor 10

N = 3C

m(A) = 6

m(B) = 36 – 7 = 28

$$P(A) = \frac{m(A)}{N} = \frac{6}{36} = \frac{1}{6}; \quad P(B) = \frac{m(B)}{N} = \frac{28}{36} = \frac{7}{9}$$

A secretary

$N = {}^5P_5 = 5! = 120$

Let event of putting letters in correct envelop = A

m(A) = 1

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{m(A)}{N} = 1 - \frac{1}{120} = \frac{119}{120}$$

Solution.

Leap year contains 366 days

A year has 52 complete weeks = 52 × 7 = 364 days

Remaining days = 2

Sample space of remaining 2 days

(Sun – Mon, Mon– Tue, Tue – Wed, Wed – Thu, Thu – Fri, Fri – Sat, Sat – Sun)

Favourable no. of cases for Saturday (m) = 2

Total no. of cases (N) = 7

$$P = \frac{m}{N} = \frac{2}{7}$$

A problem in

$$P(A) = \frac{1}{5}, P(B) = \frac{2}{5}, P(C) = \frac{3}{5}$$

$$\begin{aligned} P(A \cup B \cup C) &= 1 - P(\overline{A \cup B \cup C}) \\ &= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C}) \\ &= 1 - P(\bar{A})\, P(\bar{B})\, P(\bar{C}) \\ &= 1 - [1 - P(A)]\,[1 - P(B)]\,[1 - P(C)] \\ &= 1 - \left[1 - \frac{1}{5}\right]\left[1 - \frac{2}{5}\right]\left[1 - \frac{3}{5}\right] \\ &= 1 - \frac{4}{5} \times \frac{3}{5} \times \frac{2}{5} = 1 - \frac{24}{125} = \frac{121}{125} \end{aligned}$$

5. The odds against

For B

$$\frac{N-m}{m} = \frac{4}{3}$$

Or, $P(B) = \frac{m}{N} = \frac{3}{3+4} = \frac{3}{7}$

For A

$$\frac{m}{N-m} = \frac{7}{6}$$

$$P(A) = \frac{m}{N} = \frac{7}{7+6} = \frac{7}{13}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{7} + \frac{7}{13} - \frac{3}{7} \times \frac{7}{13} = \frac{39+49-21}{91} = \frac{67}{91}$$

6. From a pack of

Total cards = 52, selected = 2

Let king = K, Queen = Q, Face card = F, Spade = S, Club = C

$N = {}^{52}C_2 = 1326$

$$P(KQ) = \frac{m(KQ)}{N} = \frac{{}^4C_1 \times {}^4C_1}{1326} = \frac{4 \times 4}{1326} = 0.012$$

$$P(2F) = \frac{m(2F)}{N} = \frac{{}^{12}C_2}{1326} = \frac{66}{1326} = 0.049$$

$$P(SC) = \frac{m(SC)}{N} = \frac{{}^{13}C_1 \times {}^{13}C_1}{1326} = \frac{13 \times 13}{1326} = 0.127$$

7. Probability that a

Let Numerical method = M, Computer graphic = S

$P(M) = 0.75$, $P(S) = 0.85$

$P(M \cap S) = P(M)\, P(S) = 0.75 \times 0.85 = 0.6375$

$P(M \cup S) = P(M) + P(S) - P(M \cap S) = 0.75 + 0.85 - 0.6375 = 0.962$

8.

$P(A) = 0.4$, $P(B) = 0.3$, $P(A \cup B) = 0.58$

Now,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Or, $0.58 = 0.4 + 0.3 - P(A \cap B)$

Or, $P(A \cap B) = 0.7 - 0.58 = 0.12$

Here, $P(A)\, P(B) = 0.4 \times 0.3 = 0.12$

$P(A \cap B) = P(A)\, P(B)$

Hence, A and B are independent.

9. A and B play

Prob. of A's getting 2, $P(A) = \frac{1}{6}$

Prob. not getting 2, $P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{6} = \frac{5}{6}$

Prob. of B's getting 2, $P(B) = \frac{1}{6}$

Prob. of not getting 2, $P(\bar{B}) = \frac{5}{6}$

If A begins the game then prob. of A's winning

$= P(A \cup \bar{A}\bar{B} A \cup \bar{A}\bar{B}\bar{A}\bar{B}A \cup ....)$

$= P(A) + P(\bar{A})\ P(\bar{B})\ P(A) + P(\bar{A})\ P(\bar{B})\ P(\bar{A})P(\bar{B})\ P(A) + ...$

$= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + ...$

$= \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$

Prob. of B's winning = 1 – Prob. of A's winning $= 1 - \frac{6}{11} = \frac{5}{11}$

10. The probability that

Let defective iteching = A, crack defect = B

$P(A) = 0.12, P(B) = 0.29, P(A \cap B) = 0.07$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.12 + 0.29 - 0.07 = 0.34$

11. The probability that

Let 50 yrs old man alive at 60 = A

45 yrs woman alive at 55 = B

$P(A) = 0.83, P(B) = 0.87$

i. $P(A \cap B) = P(A)\ P(B) = 0.83 \times 0.87 = 0.722$

ii. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.83 + 0.87 - 0.722 = 0.978$

12. A card is drawn

Total cards = 52, Cards selected = 1

Let, King = k, Queen = Q, Jack = J, Club = C

$N = 52, m(k) = 4, m(Q) = 4, m(J) = 4, m(C) = 13, m(J \cap C) = 1$

i. $P(k \cup Q) = P(k) + P(Q)$

$= \frac{m(k)}{N} + \frac{m(Q)}{N} = \frac{4}{52} + \frac{4}{52} = \frac{2}{13}$

ii. $P(J \cup C) = P(J) + P(C) - P(J \cap C)$

$= \frac{m(J)}{N} + \frac{m(C)}{N} - \frac{m(J \cap C)}{N} = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

13. Two cards are

Total cards = 52, Black cards = 26, Red cards = 26, Ace card = 4

Selected cards = 2 in which are after other

Let Black card = B, red card = R, Ace = A

i. $P((B_I \cap R_{II}) = P(B_I)\,P(R_{II}/B_I) = \frac{26}{52} \times \frac{26}{51} = \frac{13}{51}$

ii. $P(A_I \cap A_{II}) = P(A_I)\,P(A_{II}/A_I) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

14. A book case contains

Total books = 6 DL + 9 MP = 15 books

Books selected = 4

i. $P(4DL_I \cap 4MP_{II}) = P(4\ DL_I)\,P(4\ MP_{II}) = \frac{{}^6C_4}{{}^{15}C_4} \times \frac{{}^9C_4}{{}^{15}C_4} = \frac{15 \times 126}{1365 \times 1365} = 0.001$

ii. $P(4DL_I \cap 4MP_{II}) = P(4DL_I) \times P(4MP_{II}/4DL_I) = \frac{{}^6C_4}{{}^{15}C_4} \times \frac{{}^9C_4}{{}^{11}C_4} = \frac{15 \times 125}{1365 \times 330} = 0.004$

15. The student body

Let girls = G, Boys = B, Interest in sports = S

$P(G) = 60\% = 0.6,\ P(B) = 40\% = 0.4$

$P(G \cap S) = 40\%$ of $60\% = 0.4 \times 0.6 = 0.24$

$P(B \cap S) = 60\%$ of $40\% = 0.6 \times 0.4 = 0.24$

$P(S/G) = P(GS)/P(G) = \frac{0.24}{0.6} = \frac{2}{5}$

16. A and B toss

Prob. of A's getting head $P(A) = \frac{1}{2}$

Prob. of not getting head $P(\bar{A}) = 1 - P(A) = \frac{1}{2}$

Prob. of B's getting head $P(B) = \frac{1}{2}$

Prob. of not getting $P(\bar{B}) = 1 - P(B) = \frac{1}{2}$

If A starts game

Prob. of A's winning $= P(A \cup \bar{A}\,\bar{B}\,A \cup \bar{A}\bar{B}\bar{A}\bar{B}A \cup \ldots)$

$= P(A) + P(\bar{A})\,P(\bar{B})\,P(A) + P(\bar{A})\,P(\bar{B})\,P(\bar{A})P(\bar{B})\,P(A) + \ldots$

$= \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \ldots\ldots$

$= \frac{\frac{1}{2}}{1 - \frac{1}{2} \times \frac{1}{2}} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$

Prob. of B's winning = 1 – Prob. of A's winning $= 1 - \frac{2}{3} = \frac{1}{3}$

A person is known
Let a person A and another B
Here $P(A) = \frac{4}{5}$, $P(B) = \frac{3}{4}$

$P(A \cap B) = P(A) \times P(B) = \frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$

A six faced dice
Let Odd = O and even = E
$P(E) = 2/3$, $P(O) = 1/3$
$P(\text{Even}) = P(\text{EE or OO}) = P(E)P(E) + P(O)P(O) = \frac{2}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{4}{9} + \frac{1}{9} = \frac{5}{9}$

In tossing a coin
Let head = H, tail = T
Sample space = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
N = 8

$P(\text{No. of heads}) = \frac{1}{8}$

$P(\text{Two heads}) = \frac{3}{8}$

$P(\text{At least two heads}) = \frac{4}{8} = \frac{1}{2}$

There are three traffic
$P(G) = 0.7$, $P(R) = 1 - P(G) = 1 - 0.7 = 0.3$
Sample space = {RRR, RRG, GRR, GRG, RGG, GGR, GGG}
$P(\text{stop no. more than one}) = P(G\,R\,G \cup R\,GG \cup GG\,R \cup GGG)$
$= P(G)\,P(R)\,P(G) + P(R)\,P(G)\,P(G) + P(G)\,P(G)\,P(R) + P(G)\,P(G)\,P(G)$
$= 0.7 \times 0.3 \times 0.7 + 0.3 \times 0.7 \times 0.7 + 0.7 \times 0.7 \times 0.3 + 0.7 \times 0.7 \times 0.7$
$= 0.441 + 0.343 = 0.784$

21. The odds that
Let first critics A, second B and third C.

For A, $\frac{m}{N-m} = \frac{3}{2}$

$P(A) = \frac{m}{N} = \frac{3}{3+2} = \frac{3}{5}$

For B, $\frac{m}{N-m} = \frac{4}{3}$

$P(A) = \frac{m}{N} = \frac{4}{3} = \frac{4}{7}$

For C, $\frac{m}{N-m} = \frac{2}{3}$

$P(C) = \frac{m}{N} = \frac{2}{2+3} = \frac{2}{15}$

$P(\text{majority will be favourable}) = P(AB\bar{C} \cup \bar{A}\,B\,C \cup A\,\bar{B}\,C \cup ABC)$
$= P(A)\,P(B)\,P(\bar{C}) + P(\bar{A})\,P(B)\,P(C) + P(A)\,P(\bar{B})\,P(C) + P(A)\,P(B)\,P(C)$
$= \frac{3}{5} \times \frac{4}{7} \times \left(1 - \frac{2}{5}\right) + \left(1 - \frac{3}{5}\right) \times \frac{4}{7} \times \frac{2}{5} + \frac{3}{5} \times \left(1 - \frac{4}{7}\right) \times \frac{2}{5} + \frac{3}{5} \times \frac{4}{7} \times \frac{2}{5}$

$= \frac{36}{175} + \frac{16}{175} + \frac{18}{175} + \frac{24}{175} = \frac{94}{175} = 0.537$

22. **Solution.**

Bank Credit card	Travel & Entertainment Credit card		
	Yes	No	Total
Yes	60	60	120
No	15	65	80
Total	75	125	200

a. P(the student has a bank credit card) $= \frac{120}{200} = \frac{3}{5} = 0.6$

b. P(the student has a bank credit card and a travel and entertainment card)

$= \frac{60}{200} = 0.3$

c. P(the student has a bank credit card or a travel and entertainment card)

$= P(B \text{ or } T) = P(B) + P(T) - P(B\&T) = \frac{120}{200} + \frac{75}{200} - \frac{60}{200} = \frac{135}{200} = 0.675$

d. P(he or she has a travel and entertainment card) $= \frac{60}{120} = 0.5$

23. **Solution**

a.

Gender	Health Club Facility		
	Used	Not used	Total
Male	65	105	170
Female	45	35	80
total	110	140	250

b. P(an employee chosen at random is a female and has used the health club facilities) $= \frac{45}{250} = 0.18$

c. P(an employee chosen at random is a male) $= \frac{170}{250} = 0.68$

d. P(an employee chosen at random is a male or has not used the health club facilities)

$P(M \text{ or } NH) = P(M) + P(NH) - P(M\&NH)$

$$= \frac{170}{250} + \frac{140}{250} - \frac{105}{250} = \frac{100}{250} = 0.82$$

24. **Solution.**

Total IC chips = 5000

IC chips manufactured by company x = 1000,

y = 5000 – 1000 = 4000

Let A_1 = IC chips manufactured by company x

A_2 -= IC chips manufactured by company y

$P(A_1) = \frac{1000}{5000} = 0.2$ $P(A_2) = \frac{4000}{5000} = 0.8$

Let D = Defective IC chips

$P(D/A_1) = 10\% = 0.1$

$P(D/A_2) = 5\% = 0.05$

$P(A_1/D) = ?$

$$P(A_1/D) = \frac{P(A_1)\,P(D/A_1)}{P(A_1)\,P(D/A_1) + P(A_2)\,P(D/A_2)}$$

$$= \frac{0.2 \times 0.1}{0.2 \times 0.1 + 0.8 \times 0.05} = \frac{0.2 \times 0.1}{0.2 \times 0.1 + 0.8 \times 0.05} = 0.33$$

25. **Solution.**

In a certain assembly

Let D = defective,

$P(A_1) = 30\% = 0.3$, $P(A_2) = 45\% = 0.45$, $P(A_3) = 25\% = 0.25$

$P(D/A_1) = 2\% = 0.02$, $P(D/A_2) = 3\% = 0.03$, $P(D/A_3) = 2\% = 0.02$

$$P(A_3/D) = \frac{P(A_3)\,P(D_1/A_3)}{P(D)}$$

$$= \frac{P(A_3)\,P(D/A_3)}{P(A_1)\,P(D/A_1) + P(A_2)\,P(D/A_2) + P(A_3)\,P(D/A_3)}$$

$$= \frac{0.25 \times 0.02}{0.3 \times 0.02 + 0.45 \times 0.03 + 0.25 \times 0.02} = \frac{0.005}{0.0245} = 0.204$$

26. **Solution.**

In a college the

Let central region $= A_2$, Eastern $= A_1$, Western $= A_3$, Mid-western $= A_4$, Far western $= A_5$

Let Black hair = D

$P(A_1) = 30\% = 0.3$, $P(A_2) = 50\% = 0.5$, $P(A_3) = 10\% = 0.1$,

$P(A_4) = 8\% = 0.08$, $P(A_5) = 2\% = 0.02$

$P(D/A_1) = 70\% = 0.7$, $P(D/A_2) = 80\% = 0.8$, $P(D/A_3) = 60\% = 0.6$,

$P(D/A_4) = 65\% = 0.65$, $P(D/A_5) = 75\% = 0.75$

$$P(A_2/D) = \frac{P(A_2)\,P(D/A_2)}{P(A_1)\,P(D/A_1) + P(A_2)\,P(D/A_2) + P(A_2)\,P(D/A_3)}$$

$$= \frac{0.5 \times 0.8}{0.3 \times 0.7 + 0.5 \times 0.8 + 0.1 \times 0.6 + 0.08 \times 0.65 + 0.02 \times 0.75}$$

$= 0.542$

27. **Solution.**

A consulting firm

Let agency $D = A_1$, agency $E = A_2$ and agency $E = A_3$, Bad tyres = D

$P(A_1) = 20\% = 0.2$, $P(A_2) = 20\% = 0.2$, $P(A_3) = 60\% = 0.67$

$P(D/A_1) = 10\% = 0.1$, $P(D/A_2) = 12\% = 0.12$, $P(D/A_3) = 4\% = 0.04$

$P(D) = P(A_1)\,P(D/A_1) + P(A_2)\,P(D/A_2) + P(A_3)\,P(D/A_3)$

$= 0.2 \times 0.1 + 0.2 \times 0.12 + 0.6 \times 0.04$

$= 0.068$

28. Solution.

Urn A contains

Let Urn A = A_1, Urn B = A_2, Urn C = A_3, Red and black ball = D

$$P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$$

$$P(D/A_1) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{3}{15} = \frac{1}{5}$$

$$P(D/A_2) = \frac{{}^4C_1 \times {}^2C_1}{{}^9C_2} = \frac{8}{36} = \frac{2}{9}$$

$$P(D/A_3) = \frac{{}^2C_1 \times {}^3C_1}{{}^9C_2} = \frac{6}{36} = \frac{1}{6}$$

$$P(A_2/D) = \frac{P(A_2)\,P(D/A_2)}{P(A_1)\,P(D/A_1) + P(A_2)\,P(D/A_2) + P(A_3)\,P(D/A_3)}$$

$$= \frac{\frac{1}{3} \times \frac{2}{9}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{2}{9} + \frac{1}{3} \times \frac{1}{6}} = \frac{\frac{2}{27}}{\frac{1}{15} + \frac{2}{27} + \frac{1}{18}} = \frac{0.074}{0.196} = 0.377$$

29. Solution.

The chances of X,

Let x = A_1, y = A_2, z = A_3

Bonus scheme = D

$$P(A_1) = \frac{4}{4+2+3} = \frac{4}{9}$$

$$P(A_2) = \frac{2}{4+2+3} = \frac{2}{9}$$

$$P(A_3) = \frac{3}{4+2+3} = \frac{3}{9}$$

$P(D/A_1) = 0.3 \quad P(D/A_2) = 0.5 \quad P(D/A_3) = 0.8$

$$P(A_1/D) = \frac{P(A_1)\,P(D/A_1)}{P(A_1)\,P(D/A_1) + P(A_2)\,P(D/A_2) + P(A_3)\,P(D/A_3)}$$

$$= \frac{\frac{4}{9} \times 0.3}{\frac{4}{9} \times 0.3 + \frac{2}{9} \times 0.5 + \frac{3}{9} \times 0.8} = \frac{0.1333}{0.511} = 0.26$$

30. Solution.

In a television manufacturing factory.

Let factory A = A_1, B = A_2, C = A_3

Defective = D

$P(A_1) = 25\% = 0.25$, $P(A_2) = 35\% = 0.35$, $P(A_3) = 40\% = 0.4$

$P(D/A_1) = 5\% = 0.05$, $P(D/A_2) = 4\% = 0.04$, $P(D/A_3) = 2\% = 0.02$

$$P(A_1/D) = \frac{P(A_1)\,P(D/A_1)}{P(A_1)\,P(D/A_1) + P(A_2)\,P(D/A_2) + P(A_3)\,P(D/A_3)}$$

$$= \frac{0.25 \times 0.05}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.4 \times 0.02} = \frac{0.0125}{0.0345} = 0.36$$

$$P(A_2/D) = \frac{P(A_2)\,P(D/A_2)}{P(A_1)\,P(D/A_1) + P(A_2)\,P(D/A_2) + P(A_3)\,P(D/A_3)}$$

$$= \frac{0.35 \times 0.04}{0.0345} = 0.405$$

$$P(A_3/D) = \frac{P(A_3)\,P(D/A_3)}{P(A_1)\,P(D/A_1) + P(A_2)\,P(D/A_2) + P(A_3)\,P(D/A_3)}$$

$$= \frac{0.4 \times 0.02}{0.0345} = \frac{0.008}{0.0345} = 0.231$$

Solution.

Denote the events,

$A = \{\text{errors in module I}\}$, $B = \{\text{errors in module II}\}$, $C = \{\text{crash}\}$

Further, $\{\text{errors in module I alone}\} = A\setminus B = A\setminus(A\cap B)$

$\{\text{errors in module II alone}\} = B\setminus A = B\setminus(A\cap B)$.

It is given that $P\{A\} = 0.2$, $P\{B\} = 0.4$, $P(A\cap B) = (0.2)\,(0.4) = 0.08$,

By independence,

$P\{C \mid A\setminus B\} = 0.5$, $P\{C \mid B\setminus A\} = 0.8$, and $P\{C \mid A\cap B\} = 0.9$

We need to compute $P\{A\cap B \mid C\}$. Since A is a union of disjoint events $A\setminus B$ and $A\cap B$, we compute

$P\{A\setminus B\} = P\{A\} - P\{A\cap B\} = 0.2 - 0.08 = 0.12$.

Similarly,

$$P\{B\setminus A\} = 0.4 - 0.08 = 0.32$$

Events $(A\setminus B)$, $(B\setminus A)$, $A\cap B$, and $(\overline{A\cup B})$ form a partition of Ω, because they are mutually exclusive and exhaustive. The last of them is the event of no errors in the entire program. Given this event, the probability of a crash is 0. Notice that A, B, and $(A\cap B)$ are neither mutually exclusive nor exhaustive, so they cannot be used for the Bayes Rule. Now organize the data.

Location of errors	Probability of a crash
$P\{A\setminus B\} = 0.12$	$P\{C \mid A\setminus B\} = 0.5$
$P\{B\setminus A\} = 0.32$	$P\{C \mid B\setminus A\} = 0.8$
$P(A\cap B) = 0.08$	$P(C \mid A\cap B\} = 0.9$
$P(\overline{A\cup B}) = 0.48$	$P\,(C \mid \overline{A\cup B}) = 0$

Combining the Bayes Rule and the Law of Total Probability,

$$P\{A\cap B \mid C\} = \frac{P\{A \mid A\cap B\}\,P\{A\cap B\}}{P\{C\}},$$

Where

$$P\{C\} = P\{C \mid A\setminus B\}\,P\{A\setminus B\} + P\{C \mid B\setminus A\}\,P\{B\setminus A\} + P\{C \mid A\cap B\}\,P\{A\cap B\} + P\{C \mid \overline{A\cup B}\}\,P\{\overline{A\cup B}\}$$

Then

$$P\{A\cap B \mid C\} = \frac{(0.9)\,(0.08)}{(0.5)\,(0.12) + (0.8)\,(0.32) + (0.9)\,(0.08) + 0} = 0.1856$$

32. **Solution.**

Let urn I = A_1, urn II = A_2, urn III = A_3

$$P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$$

Let R = Real chips

$$P(R/A_1) = \frac{6}{10};\quad P(R/A_2) = \frac{2}{8};\quad P(R/A_3) = \frac{1}{9}$$

$P(A_1/R) = ?$

$$P(A_1/R) = \frac{P(A_1)\,P(R/A_1)}{P(A_1)\,P(R/A_1) + P(A_2)\,P(R/A_2) + P(A_3)\,P(R/A_3)}$$

$$= \frac{\frac{1}{3}\times\frac{6}{10}}{\frac{1}{3}\times\frac{6}{10}+\frac{1}{3}\times\frac{2}{8}+\frac{1}{3}\times\frac{1}{9}} = 0.624$$

Let the chips drawn are red without replacement = Rw

$$P(Rw/A_1) = \frac{6}{10}\times\frac{5}{9} = \frac{1}{3} \qquad P(Rw/A_2) = \frac{2}{8}\times\frac{1}{7} = \frac{1}{28}$$

$$P(Rw/A_3) = \frac{1}{9}\times\frac{0}{8} = 0 \qquad P(A_1/Rw) = ?$$

$$P(A_1/Rw) = \frac{P(A_1)\,P(Rw/A_1)}{P(A_1)\,P(Rw/A_1) + P(A_2)\,P(Rw/A_2) + P(A_3) + P(Rw/A_3)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{3}}{\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{28}+\frac{1}{3}\times 0} = 0.903$$

$$P(A_3/Rw) = \frac{P(A_3)\,P(Rw/A_3)}{P(A_1)\,P(Rw/A_1) + P(A_2)\,P(Rw/A_2) + P(A_3) + P(Rw/A_3)}$$

$$= \frac{\frac{1}{3}\times 0}{\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{28}+\frac{1}{3}\times 0} = 0$$

1. Solution

a. Here for Population Mean $\bar{Y} = \frac{\sum Y}{N} = \frac{0+1+3+8+9}{5} = 4.2$

Population Variance, $S^2 = \frac{1}{N-1}\sum(Y_i - \bar{Y})^2$

$$= \frac{1}{N-1}\{\sum Y_i^2 - N\bar{Y}^2\}$$

$$= \frac{1}{4}\{0^2 + 1^2 + 3^2 + 8^2 + 9^2 - 5(4.2)^2\} = 16.7$$

b. Here the sample of size 2 from population of size 6 can be drawn in C(5,2) ways = 10 ways

Hence the samples are;(0,1) (0,3) (0,8) (0,9) (1,3) (1,8) (1,9) (3,8) (3,9) (8,9)

Sample No.	Sample values (y_i)	Sample mean ($\bar{y}_i$)	$\{\bar{y}_i . E(\bar{y})\}^2$	$s_i^2 = \frac{1}{n-1}\sum(y_i^2 - \bar{y})^2$
1	(0,1)	0.5	13.69	0.5
2	(0,3)	1.5	7.29	4.5
3	(0,8)	4	0.04	32
4	(0,9)	4.5	0.09	40.5
5	(1,3)	2	4.84	2
6	(1,8)	4.5	0.09	24.5
7	(1,9)	5	0.64	32
8	(3,8)	5.5	1.69	12.5
9	(3,9)	6	3.24	18
10	(8,9)	8.5	18.49	0.5
Total		42	50.1	167

c. $E(\bar{y}) = \frac{1}{C(N,n)}\sum_i y_1 = \frac{42}{10} = 4.2.$

d. $V(\bar{y}) = \frac{1}{C(N,n)}\sum_i \bar{y}_i - E(\bar{y})^2\} = \frac{50.1}{10} = 5.01$

e. Yes, $E(\bar{y}) = \bar{Y}$ verified

Here, sample mean is unbiased estimate of the population mean

$E(s^2) = \frac{1}{C(N,n)}\sum_i S_i^2 = \frac{167}{10} = 16.7$. Hence, $E(s^2) = S^2$

f. Now, $V(\bar{y}) = \frac{(1-f)S^2}{n} = \left(1 - \frac{2}{5}\right) \times \frac{16.7}{2} = 5.01$ Hence, Formula verified.

2. **Solution.**

Here, Population size (N) = 5, Sample size (n) = 2.

a. Calculation of population mean and population variance:

Y_i	$Y_i - \bar{Y}$	$(Y - \bar{Y})^2$
0	-3.2	10.24
1	-2.2	4.84
3	-0.2	0.04
5	1.8	3.24
7	3.8	14.44
$\Sigma Y = 16$		$\Sigma(Y - \bar{Y})^2 = 32.8$

Population mean (μ or $\bar{Y}$) $= \frac{\Sigma Y_i}{N} = \frac{16}{5} = 3.2$

Population variance (σ^2) $= \frac{\Sigma(Y_i - \bar{Y})^2}{N} = \frac{32.8}{5} = 6.56$

b. Number of possible samples of size 2 that can be drawn from the populatio[n] of size N = 5 by using with replacement technique is: Number of possib[le] samples = $N^n = 5^2 = 25$

Thus the possible samples are: (0,0), (0,1), (0,3), (0,5), (0,7), (1,0), (1,1), (1,3), (1,5), (1,7), (3,0), (3,1), (3,3), (3,5), (3,7), (5,0), (5,1), (5,3), (5,5), (5,7), (7,0), (7,1), (7,3), (7,5), (7,7).

c. Calculation of sample means and variance of the sampling distribution o[f] means:

S. No.	Sample Values (Y_i)	Sample Means ($\bar{y}_i$)	$(\bar{y}_i - \bar{y})$	$(\bar{y}_i - \bar{y})^2$	$s_i^2 = \frac{1}{n-1}\{\Sigma y_i^2 - n\bar{y}^2\}$
1	(0,0)	0	-3.2	10.2	0
2	(0,1)	0.5	-2.7	7.29	0.5
3	(0,3)	1.5	-1.7	2.89	4.5
4	(0,5)	2.5	-0.7	0.49	12.5
5	(0,7)	3.5	0.3	0.09	24.5
6	(1,0)	0.5	-2.7	7.29	0.5
7	(1,1)	1	-2.2	4.84	0
8	(1,3)	2	-1.2	1.44	2
9	(1,5)	3	-0.2	0.04	8
10	(1,7)	4	0.8	0.64	18
11	(3,0)	1.5	-1.7	2.89	4.5
12	(3,1)	2	-1.2	1.44	2
13	(3,3)	3	-0.2	0.04	0
14	(3,5)	4	0.8	0.64	2
15	(3,7)	5	1.8	3.24	8
16	(5,0)	2.5	-0.7	0.49	12.5

17	(5,1)	3	-0.2	0.04	8
18	(5,3)	4	0.8	0.64	2
19	(5,5)	5	1.8	3.24	0
20	(5,7)	6	2.8	7.84	2
21	(7,0)	3.5	0.3	0.09	24.5
22	(7,1)	4	0.8	0.64	18
23	(7,3)	5	1.8	3.24	8
24	(7,5)	6	2.8	7.84	2
25	(7,7)	7	3.8	14.4	0
		80		82	

Mean of the sample means $(\bar{y}) = \dfrac{\Sigma\bar{y}_i}{\text{No. of samples } (N^n)} = \dfrac{80}{25} = 3.2$

Since the mean of the sample means $(\bar{y}) = 3.2$ is equal to the population mean $(\bar{Y}) = 3.2$, so we can conclude that the mean of the sampling distribution of the sample means is equal to the population mean.

Variance of the sample means is

$$V(\bar{y}) = \frac{\Sigma(\bar{y}_i - \bar{y})^2}{\text{No. of samples} = N^n} = \frac{82}{25} = 3.28 \text{ or}$$

$$V(\bar{y}) = \frac{\sigma^2}{n} = \frac{6.56}{2} = 3.28$$

variance of sample means gives the same value as calculated from (d)

Solution:

Here N =800, n = 120

$$\text{Var}(\bar{y}_{st})_{prop} = \left(\frac{1}{n} - \frac{1}{N}\right)\Sigma W_i S_i^2$$

$$= \left(\frac{1}{n\,N} - \frac{1}{N^2}\right)\Sigma N_i S_i^2 \quad \left\{W_i = \frac{N_i}{N}\right\}$$

$$= \left(\frac{1}{120 \times 800} - \frac{1}{800^2}\right)\{200\times36 + 300\times64 + 300\times144\}$$

$$= 0.616$$

$$\text{Var}(\bar{y}_{st})_{opt} = \frac{1}{n}(\Sigma W_i S_i)^2 - \frac{1}{N}\Sigma W_i S_i^2$$

$$= \frac{1}{n\,N^2}(\Sigma N_i S_i)^2 - \frac{1}{N^2}\Sigma N_i S_i^2$$

$$= \frac{1}{120 \times 800^2}(200 \times 6 + 300 \times 8 + 300 \times 12)^2 - \frac{1}{800^2}(200 \times 36 + 300 \times 64 + 300 \times 144)$$

$$= 0.675 - 0.1087$$

$$= 0.566$$

4. **Solution.**

Here, $N = 800$, $N_1 = 200$, $N_2 = 300$, $N_3 = 300$ $S_1 = 6$, $S_2 = 8$, $S_3 = 12$, $n = 120$

$N_1S_1 = 200 \times 6 = 1200$, $N_2S_2 = 300 \times 8 = 2400$, $N_3S_3 = 300 \times 12 = 3600$,

$\sum N_iS_i = 1200 + 2400 + 3600 = 7200$

(i) Under proportional allocation

$$\frac{n_i}{N_i} = \frac{n}{N}$$

$$n_i = \frac{n}{N} N_i, \qquad i = 1,2,3$$

$$n_1 = \frac{120}{800} \times 200 = 30$$

$$n_2 = \frac{120}{800} \times 300 = 45$$

$$n_3 = \frac{120}{800} \times 300 = 45$$

ii) Under Neyman's optimum allocation

$$\frac{n_i}{n} = \frac{N_iS_i}{\sum N_iS_i}$$

$$n_i = \frac{n\, N_iS_i}{\sum N_iS_i}$$

$$n_1 = \frac{120 \times 1200}{7200} = 20$$

$$n_2 = \frac{120 \times 240}{7200} = 40$$

$$n_2 = \frac{120 \times 3600}{7200} = 60$$

In the proportional allocation the sample fraction n_i/N_i is constant for each stratum

$$\frac{n_i}{N_i} = \frac{n}{N} = C$$

n_i proportional to N_i

In Neyman's optimum allocation $V(\bar{y}_{st})$ is minimum for fixed n if n_i proportional to N_iS_i.

Hence in former one n_i depends on N_i but in later one n_i also depends on S_i

1. Solution.

x	P(x)
0	a
1	3a
2	5a
3	7a
4	9a
5	11a
6	13a
7	15a
8	17a
	$\Sigma P(x) = 81$ a

Since, $\Sigma P(x) = 1$

$\Rightarrow \quad 81a = 1 \qquad \Rightarrow a = \frac{1}{81}$

$P(x < 3) = P(x = 0) + P(x = 1) + P(x = 2)$

$= a + 3a + 5a = 9a = 9 \times \frac{1}{81} = \frac{1}{9}$

$P(x \geq 3) = P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6) + P(x = 7) + P(x = 8)$

$= 7a + 9a + 11a + 13\,a + 15a + 17a$

$= 72a$

$= 72 \times \frac{1}{81} = \frac{8}{9}$

$P(0 < x < 5) = P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4)$

$= 3a + 5a + 7a + 9a$

$= 24a = 24 \times \frac{1}{81} = \frac{8}{27}$

2. Let x be the number of toss required to get tail.

Let T denotes out tail and H denotes outcome head.

Outcomes	No. of TBS	P(x)	x P(x)
T	1	1/2	1/2
HT	2	1/4	2/4
HHT	3	1/8	3/8
HHHT	4	1/16	4/16
HHHHT	5	1/32	5/32

$E(x) = \Sigma\, x\, P(x) = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + \ldots\ldots$

Let $S = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} \ldots\ldots\ldots\ldots\ldots$

$$\frac{S}{2} = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} \ldots\ldots\ldots\ldots\ldots$$

$$5 - \frac{S}{2} = \frac{1}{2} + \left(\frac{2}{4} - \frac{1}{4}\right) + \left(\frac{4}{16} - \frac{3}{16}\right) + \ldots\ldots$$

$$\frac{S}{2} = \frac{\frac{1}{2}}{1 - \frac{1}{2}}$$

$$\frac{S}{2} = 1$$

$\Rightarrow$ $S = 2$

$\therefore E(x) = 2$

3. **Solution.**

On throwing two dice possible outcomes are (1,1) (1,2), (1,3), (1,4), (1,5), (1,6 (2,1), (2, 2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2 (4,3), (4,4), (4,5), (4, 6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4 (6,5), (6,6).

Let x = Sum of numbers

x	P(x)	x P(x)
2	1/36	2/36
3	2/36	6/36
4	3/36	12/36
5	4/36	20/36
6	5/36	30/36
7	6/36	42/36
8	5/36	40/36
9	4/36	36/36
10	3/36	30/36
11	2/36	22/36
12	1/36	12/36
		$\Sigma x\, P(x) = \frac{252}{36} = 7$

$E(x) = \Sigma x\, P(x) = 7$

4. **Solution.**

x	P(x)	x P(x)
–2	0.2	– 0.4
–1	k	– k
0	0.4	0
1	2k	2k
2	k	2k
	$\Sigma P(x) = 4k + 0.6$	$\Sigma x\, P(x) = 3k - 0.4$

Since, $\sum_x x\,P(x) = 1$

$4k + 0.6 = 1$

$4k = 0.4 \Rightarrow k = 0.1$

$P(x \geq 0) = P(x = 0) + P(x = 1) + P(x = 2) = 0.4 + 2k + k = 0.4 + 3 \times 0.1 = 0.7$

$E(x) = 3k - 0.4 = 3 \times 0.1 - 0.4 = -0.1$

$E(4x + 5) = 4\,E(x) + 5 = 4 \times (-0.1) + 5 = 4.6$

Solution.

x	P(x)	x P(x)	x^2 P(x)
0	0.05	0	0
1	0.35	0.35	0.35
2	0.2	0.40	0.80
3	0.25	0.75	2.25
4	0.15	0.60	2.40
		$\Sigma x\,P(x) = 2.1$	$\Sigma x^2\,P(x) = 5.8$

$E(x) = \Sigma x\,P(x) = 2.1$

$E(x^2) = \Sigma x^2\,P(x) = 5.8$

$V(x) = E(x^2) - [E(x)]^2 = 5.8 - (2.1)^2 = 1.39$

$S.D. = \sqrt{V(x)} = \sqrt{1.39} = 1.178$

Solution.

x	P(x)	x P(x)
700	1/12	700/12
900	1/12	900/12
1100	3/12	3300/12
1300	3/12	3900/12
1500	2/12	3000/12
1700	2/12	3400/12
		$\Sigma x\,P(x) = 1266.67$

Expected earning = Rs. 1266.67

Solution.

Let x = Profit

x	P(x)	x P(x)
180,000 – 140,000 = 40,000	0.32	12800
170,000 – 140,000 = 30,000	0.55	16500
150,000 – 140,000 = 10,000	0.13	1300
		$\Sigma x\,P(x) = 30600$

$E(x) = \Sigma x\,P(x) = 30{,}600$

8. Let x = Rupees

x	P(x)	x P(x)
5000	0.001	5
3000	0.003	9
1000	0.005	5
		$\Sigma x P(x) = 19$

Fairy price = $\Sigma x P(x) = 19$

9. **Solution.**

Let x = No. of Video card, V = Video card, N = Network card from each urn selected as $N_I N_{II} N_{III}$, $V_I N_{II} N_{III}$, $N_I V_{II} N_{III}$, $N_I N_{II} V_{III}$, $V_I V_{II} N_{III}$, $V_I N_{II} V_{III}$, $N_I V_{II} V_{III}$, $V_I V_{II} V_{III}$.

I: 3N + 2V, II: 5N + 6V, III : 2N + 4V

x	P(x)	x P(x)
0	$\frac{3}{5} \times \frac{5}{11} \times \frac{2}{6} = \frac{30}{330}$	0
1	$\frac{2}{5} \times \frac{5}{11} \times \frac{2}{6} + \frac{3}{5} \times \frac{6}{11} \times \frac{2}{6} + \frac{3}{5} \times \frac{5}{11} \times \frac{4}{6} = \frac{116}{330}$	$\frac{116}{330}$
2	$\frac{2}{5} \times \frac{6}{11} \times \frac{2}{6} + \frac{2}{5} \times \frac{5}{11} \times \frac{4}{6} + \frac{3}{5} \times \frac{6}{11} \times \frac{4}{6} = \frac{136}{330}$	$\frac{272}{330}$
3.	$\frac{2}{5} \times \frac{6}{11} \times \frac{4}{6} = \frac{48}{330}$	$\frac{144}{330}$
		$\Sigma x P(x) = 532/330$

$E(x) = \Sigma x P(x) = 532/330 = 1.61$

10. **Solution.** Let x = Profit

x	P(x)	x P(x)
5000	0.18	900
8000	0.22	1760
0	0.33	0
- 3000	0.27	- 810
		$\Sigma x P(x) = 1850$

$E(x) = \Sigma x P(x) = 1850$

11. **Solution.**

Since, $\int_{-\infty}^{+\infty} f(x)\, dx = 1$

$$\Rightarrow \int_0^1 k(1-x)^2\, dx = 1$$

$$\Rightarrow k\left[\frac{(1-x)^{2+1}}{(2+1)(-1)}\right]_0^1 = 1$$

$$\Rightarrow k\left[\frac{(1-x)^3}{-3}\right]_0^1 = 1$$

$\Rightarrow \quad k\left[0+\frac{1}{3}\right]=1$

$\Rightarrow \quad k=3$

$$P(0<x<0.5)=\int_0^{0.5} f(x)\,dx=\int_0^{0.5} 3\,(1-x)^2\,dx$$
$$=3\left[\frac{(1-x)^3}{-3}\right]_0^{0.5}$$
$$=-\left[(1-x)^3\right]_0^{0.5}$$
$$=-\left[(1-0.5)^3-(1-0)^3\right]=0.875$$

$$E(x)=\int_{-\infty}^{+\infty} x\,f(x)\,dx$$
$$=\int_0^1 x\,3\,(1-x)^2\,dx$$
$$=3\int_0^1 x(1-2x+x^2)\,dx=3\int_0^1 (x-2x^2+x^3)\,dx$$
$$=3\left[\frac{x^2}{2}-2\frac{x^3}{3}+\frac{x^4}{4}\right]_0^1$$
$$=3\left[\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right]=3\,\frac{6-8+3}{12}=\frac{1}{4}$$

12. Solution.

$$\int_{-\infty}^{+\infty} f(x)\,dx=1$$

$\Rightarrow \quad \int_0^1 k\,x^2\,dx=1$

$\Rightarrow \quad k\left[\frac{x^3}{3}\right]_0^1=1$

$\Rightarrow \quad \frac{k}{3}=1$

$\Rightarrow \quad k=3$

$$P(0<x<0.5)=\int_0^{0.5} f(x)\,dx=\int_0^{0.5} 3x^2\,dx=3\left[\frac{x^3}{3}\right]_0^{0.5}=\frac{3}{3}\left[(0.5)^3\right]=0.125$$

$$E(x)=\int_0^1 x\,f(x)\,dx=\int_0^1 x\,f(x)\,dx$$
$$=\int_0^1 x\,3x^2\,dx=3\int_0^1 x\,3\,dx=3\left[\frac{x^4}{4}\right]_0^1=\frac{3}{4}\left[1^4\right]=0.75$$

13. Solution.

$$f(x)=\begin{cases} c\,(10-x)^2 & 0<x<10 \\ 0 & \text{otherwise}\end{cases}$$

$P(1<x<2)=?$

Since, $\int_{-\infty}^{+\infty} f(x)\,dx=1$

Or $\int_0^{10} C(10-x)^2\,dx = 1$

Or $C\left[\dfrac{(10-x)^3}{3\,(-1)}\right]_0^{10} = 1$

Or $C\left(0+\dfrac{10^3}{3}\right) = 1$

Or $C = \dfrac{3}{1000}$

$P(1 < x < 2)$

$= \int_1^2 \dfrac{3}{1000}(10-x)^2\,dx$

$= \dfrac{3}{1000}\left[\dfrac{(10-x)^3}{3\,(-1)}\right]_0^2$

$= 0.217$

14. Solution.

$f(x) = x\,e^{-x},\ x \geq 0$

$E(x) = \int_{-\infty}^{+\infty} x\,f(x)\,dx$

$= \int_0^{\infty} x\,x\,e^{-x}\,dx$

$= \int_0^{\infty} e^{-x}x^2\,dx$

$= \int_0^{\infty} e^{-x}x^{3-1}\,dx$

$= \sqrt{3} = (3-1)! = 2! = 2$

15. Solution.

$$f(x) = \begin{cases} 12x\,(1-x)^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$E(x) = \int_{-\infty}^{+\infty} x\,f(x)\,dx$

$= \int_0^1 12x\,(1-x^2)\,dx$

$= 12\int_0^1 x^2(1-2x+x^2)\,dx$

$= 12\int_0^1 (x^2-2x^3+x^4)\,dx$

$= 12\left[\dfrac{x^3}{3}-\dfrac{2x^4}{4}+\dfrac{x^5}{5}\right]_0^1$

$= 12\left[\dfrac{1}{3}-\dfrac{1}{2}+\dfrac{1}{5}\right] = 12 \times \dfrac{10-15+6}{30} = 2/5$

$E(x^2) = \int x^2 f(x)\,dx$

$$= \int_0^1 x^2\, 12x(1-x)^2\, dx$$

$$= 12 \int x^3 (1 - 2x + x^2)\, dx$$

$$= 12\left[\frac{x^4}{4} - \frac{2x^5}{5} + \frac{x^6}{6}\right]_0^1 = 1/5$$

Solution.

$$f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$E(x) = x\, f(x)\, dx$$

$$= \int_0^1 x\, 2(1-x)\, dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\,\frac{3-2}{6} = \frac{1}{3}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_0^1 x^2\, 2(1-x)\, dx$$

$$= 2\int_0^1 (x^2 - x^3)\, dx$$

$$= 2\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= 2\left[\frac{1}{3} - \frac{1}{4}\right] = 2 \times \frac{4-3}{12} = \frac{1}{6}$$

$$V(x) = E(x^2) - \int [E(x)]^2$$

$$= \frac{1}{6} - \left(\frac{1}{3}\right)^2 = \frac{3-2}{18} = \frac{1}{18}$$

$$S.D. = \sqrt{V(x)} = \sqrt{\frac{1}{18}} = 0.235$$

7. **Solution.**

$f(x) = 3x^2;\ 0 \le x \le 1$

$a = ?,\ b = ?$

) $P(x \le a) = P(x > a)$

$\Rightarrow \int_0^a f(x)\, dx = \int_1^1 f(x)\, dx$

$\Rightarrow \int_0^a 3x^2\, dx = \int_a^1 3x^2\, dx$

$\Rightarrow 3\left[\frac{x^3}{3}\right]_0^a = \left[\frac{x^3}{3}\right]_a^1$

$\Rightarrow a^3 = 1 - a^3$

$\Rightarrow 2a^3 = 1$

$\Rightarrow \quad a^3 = \frac{1}{2}$

$\Rightarrow \quad a = \sqrt[3]{0.5} = 0.793$

(ii) $P(x > b) = 0.05$

$\Rightarrow \quad \int_b^1 f(x)\,dx = 0.05$

$\Rightarrow \quad \int_b^1 3x^2\,dx = 0.05$

$\Rightarrow \quad \left[3\frac{x^3}{3}\right]_b^1 = 0.05$

$\Rightarrow \quad 1 - b^3 = 0.05$

$\Rightarrow \quad b^3 = 0.95$

$\Rightarrow \quad b = \sqrt[3]{0.95} = 0.98$

18. Solution.

$f(x) = ke^{-x/\sigma} \quad 0 < x < \infty, \sigma > 0$

Since, $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$\Rightarrow \quad \int_0^{\infty} k\,e^{-x/\sigma}\,dx = 1$

Let $\frac{x}{0} = y$ then $dx = \sigma\,dy$

Where $x = 0, y = 0$ when $x = \infty, y = \infty$

$\therefore \quad \int_0^{\infty} k\,e^{-x/\sigma}\,dx = 1$

$\Rightarrow \quad k\sigma \int_0^{\infty} e^{-y}\,y^{1-1}\,dy = 1$

$\Rightarrow \quad k\sigma \times 1 = 1$

$\Rightarrow \quad k = \frac{1}{\sigma}$

$\therefore \quad f(x) = \frac{1}{\sigma} e^{-x/\sigma}$

$E(x) = \int x\,f(x)\,dx$

$= \int_0^{\infty} x \frac{1}{\sigma} e^{-x/\sigma}\,dx$

$= \int_0^{\infty} y\,e^{-y}\,\sigma\,dy$

$= \sigma \int_0^{\infty} e^{-y}\,y^{2-1}\,dy$

$= \sigma\sqrt{2}$

$= \sigma\,1\,1$

$= \sigma$

$E(x^2) = \int x^2\,f(x)\,dx$

$= \int_0^\infty x^2 \frac{1}{\sigma} e^{-x/\sigma} dx$

$= \int_0^\infty x \frac{x}{\sigma} e^{-x/\sigma} \sigma dy$

$= \int_0^\infty \sigma_y . y e^{-y} \sigma dy$

$= \sigma^2 \int_0^\infty e^{-y} y^2 dy$

$= \sigma^2 \int_0^\infty e^{-y} y^{3-1} dy$

$= \sigma^2 \sqrt{3}$

$= \sigma^2 2 1$

$= 2 \sigma^2$

$v(x) = E(x^2) = - [E(x)]^2$

$= 2\sigma^2 - \sigma^2 = \sigma^2$

19. Solution.

x	P(x)	x P(x)	x^2 P(x)	x^3 P(x)	x^4 P(x)
–3	1/9	–3/9	9/9	–27/9	81/9
–2	1/9	–2/9	4/9	–8/9	16/9
–1	1/9	–1/9	1/9	–1/9	1/9
0	1/3	0	0	0	0
1	1/9	1/9	1/9	1/9	1/9
2	1/9	2/9	4/9	8/9	16/9
3	1/9	3/9	9/9	27/9	81/9
	$\Sigma P(x) = 0$	$\Sigma x P(x) =$ 28/9	$\Sigma x^2 P(x) =$ 28/9	$\Sigma x^3 P(x) =$ 0	$\Sigma x^4 P(x) =$ 196/9

First four moments about origin,

$\mu_1' = E(x) = \Sigma x P(x) = 0$

$\mu_2' = E(x^2) = \Sigma x^2 P(x) = 28/9 = 3.11$

$\mu_3' = E(x^3) = \Sigma x^3 P(x) = 0$

$\mu_4' = E(x^4) = \Sigma x^4 P(x) = 196/9 = 21.77$

First four moments about mean

$\mu_1 = 0$

$\mu_2 = \mu_2' - \mu_2'^2 = 3.11 - 0^2 = 3.11$

$\mu_3 = \mu_3' - 3\mu_2' \mu_1' + 2\mu_3'^3 = 0 - 3 \times 3.11 \times 0 + 2 \times 0^3 = 0$

$\mu_4 = \mu_4' - 4\mu_3' \mu_1' + 6 \mu_2' \mu_1'^2 + 3\mu_1'^4 = 21.77 - 0 + 0 - 0 = 21.77$

Measure of central tendency, E(x) = 0

Measure of skewness

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{0^2}{(3.11)^3} = 0$$

Measure of kurtosis

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{21.77}{(3.11)^2} = 2.25$$

20. Solution.

$f(x) = kx\,(2 - x)\ 0 \le x \le 2$

Since, $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$$\Rightarrow \int_0^2 kx\,(2 - x)\,dx = 1$$

$$\Rightarrow k\int_0^2 (2x - x^2)\,dx = 1$$

$$\Rightarrow k\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$$

$$\Rightarrow k\left[2^2 - \frac{2^3}{3}\right] = 1$$

$$\Rightarrow k\left(4 - \frac{8}{3}\right) = 1$$

$$\Rightarrow k = 3/4$$

$$\therefore f(x) = \frac{3}{4}x\,(2 - x)$$

$$\mu_2' = \int_{-\infty}^{+\infty} x\,f(x)\,dx = \int_0^2 x\,\frac{3}{4}x\,(2 - x)\,dx$$

$$= \frac{3}{4}\int_0^2 (2x^2 - x^3)\,dx$$

$$= \frac{3}{4}\left[2\frac{x^3}{3} - \frac{x^4}{4}\right]_0^2$$

$$= \frac{3}{4}\left[2 \times \frac{8}{3} - \frac{16}{4}\right] = \frac{3}{4}\left[\frac{64 - 48}{12}\right] = 1$$

$$\mu_2' = E(x^2) = \int_{-\infty}^{+\infty} x^2 f(x)\,dx = \int_0^2 x^2\,\frac{3}{4}x\,(2 - x)\,dx$$

$$= \frac{3}{4}\int_0^2 (2x^2 - x^3)\,dx$$

$$= \frac{3}{4}\left[\frac{2x^4}{4} - \frac{x^5}{5}\right]_0^2$$

$$= \frac{3}{4}\left[\frac{2^4}{2} - \frac{2^5}{5}\right] = \frac{6}{5} = 1.2$$

$$\mu_3' = E(x^3) = \int_{-\infty}^{+\infty} x^3 f(x)\,dx = \int_0^2 x^3\,\frac{3}{4}x\,(2 - x)\,dx$$

$$= \frac{3}{4}\int_0^2 (2x^4 - x^5)\,dx$$

$$= \frac{3}{4}\left[2\frac{x^5}{5} - \frac{x^6}{6}\right]_0^2$$

$$= \frac{3}{4}\left[2 \times \frac{2^5}{5} - \frac{2^6}{6}\right]$$

$$= \frac{3}{4} \times \frac{384 - 320}{30} = \frac{3}{4} \times \frac{64}{30}$$

$$\mu_4' = E(x^4) = \int_{-\infty}^{+\infty} x^4 f(x)\, dx = \int_0^2 x^4 \frac{3}{4} x (2 - x)\, dx$$

$$= \frac{3}{4} \int_0^2 (2x^5 - x^6)\, dx$$

$$= \frac{3}{4}\left[\frac{2x^6}{6} - \frac{x^7}{7}\right]_0^2$$

$$= \frac{3}{4}\left[2 \times \frac{2^6}{6} - \frac{2^7}{7}\right] = \frac{3}{4}\left[\frac{128}{6} - \frac{128}{7}\right] = \frac{3}{4} \times \frac{128}{42} = 2.28$$

$\mu_2 = \mu_2' - \mu_1'^2 = 1.2 - 1^2 = 0.2$

$\mu_3 = \mu_3' - 3\, \mu_2'\, \mu_1' + 2\, \mu_1'^3 = 1.6 - 3 \times 1.2 \times 1 + 2 \times 1^3 = 1.6 - 3.6 + 2 = 0$

$\mu_4 = \mu_4' - 3\, \mu_3'\, \mu_1' + 6\, \mu_2'\, \mu_1'^2 - 3\, \mu_1'^4 = 2.28 - 4 \times 1.6 \times 1 + 6 \times 1.2 \times 1^2 - 3 \times 1^4 = 0.08$

Measures

Central tendency $= E(x) = \mu_1' = 1$

Dispersion $= \sigma = \sqrt{V(x)} = \sqrt{\mu_2} = \sqrt{0.2} = 0.44$

Skewness $= \beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{0}{(0.2)^3} = 0$

Kurtosis $= \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{0.08}{(0.2)^2} = 2$

. $f(x) = k\, x^2 e^{-x} \quad 0 < x < \infty$

Since, $\int_{-\infty}^{+\infty} f(x)\, dx = 1$

$\Rightarrow \int_0^{\infty} kx^2 e^{-x}\, dx = 1$

$\Rightarrow k \int_0^{\infty} x^2 e^{-x} x^{3-1}\, dx = 1$

$\Rightarrow k \sqrt{3} = 1$

$\Rightarrow k\, 2! = 1$

$\Rightarrow k = \frac{1}{2}$

$\therefore f(x) = \frac{1}{2} x^2 e^{-x}\, dx$

Now,

$$\mu_1' = E(x) = \int_{-\infty}^{+\infty} x\, f(x)\, dx = \int_{-\infty}^{+\infty} x \frac{1}{2} x^2 e^{-x} = \frac{1}{2} \int_0^{\infty} e^{-x} x^3\, dx = \frac{3\,!}{2} = 3$$

$$\mu_2' = E(x^2) = \int_0^{\infty} x^2 e^{-x}\, dx = \frac{1}{2} \int_0^{\infty} e^{-x} x^4\, dx = \frac{4\,!}{2} = 12$$

$$\mu_3' = E(x^2) = \int_0^{\infty} x^3 f(x)\, dx = \int_0^{\infty} x^3 \frac{1}{2} x^2 e^{-x}\, dx = \frac{1}{2} \int_0^{\infty} e^{-x} x^5\, dx = \frac{5!}{2} = 60$$

$$\mu_4' = E(x^4) = \int_0^{\infty} x^4 f(x)\, dx = \int_0^{\infty} x^4 \frac{1}{2} x^2 e^{-x}\, dx = \frac{1}{2} \int_0^{\infty} e^{-x} x^6\, dx = \frac{6!}{2} = 360$$

$\mu_2 = \mu_2' - \mu_1'^2 = 12 - 3^2 = 3$

$\mu_3 = \mu_3' - 3\, \mu_2'\, \mu_1' + 2\, \mu_1'^3 = 60 - 2 \times 12 \times 3 + 2 \times 3^3 = 6$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4 = 360 - 4 \times 60 \times 3 + 6 \times 12 \times 3^2 - 3 \times 3^4 = 45$

Mean $= E(x) = \mu_1' = 3$

S.D. $= \sqrt{V(x)} = \sqrt{\mu_2} = \sqrt{3} = 1.73$

Skewness $= \beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{6^2}{3^3} = \frac{36}{27} = 1$

Kurtosis $= \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{45}{3^2} = 5$

22. $P(x, y) = \frac{2x + y}{28}$ $\begin{matrix} x = 1, 3 \\ y = 2, 4 \end{matrix}$

$P(y) = \frac{y + 4}{14}$

$$E(x(y = 2) = \sum_x x\, P(x/y=2)$$

$$= \sum_x x \frac{\frac{2x+2}{28}}{\frac{2+4}{14}}$$

$$= \sum x \frac{2x + 2}{28} \times \frac{14}{6}$$

$$= \frac{1}{12}\sum_x 2x^2 + 2x = \frac{1}{12}[(2 \times 1^2 + 2 \times 1) + (2 \times 3^2 + 2 \times 3)] = \frac{7}{3}$$

$$E(x^2/y = 2) = \sum x^2\, P(x/y = 2)$$

$$= \sum x^2 \frac{P(x, y)}{P(y)}$$

$$= \sum_x x^2 \frac{\frac{2x+2}{28}}{\frac{2+4}{14}}$$

$$= \sum_x x^2 \frac{2x + 2}{28} \times \frac{14}{6}$$

$$= \frac{1}{12}\sum_x 2x^3 + 2x^2 = \frac{1}{12}[(2 \times 1^3 + 2 \times 1^2) + (2 \times 3^3 + 2 \times 3^2)]$$

$$= \frac{1}{12}[4 + 54 + 18] = \frac{76}{12}$$

$$V(x/y = 2) = E(x^2/y = 2) - [E(x/y = 2)]^2$$

$$= \frac{76}{12} - \left(\frac{7}{3}\right)^2 = \frac{8}{9}$$

1. $f(x, y) = \frac{6}{5}(x + y^2) \quad 0 < x < 1, 0 < y < 1$

(i) $P(0.2 < x < 0.5, 0.4 < y < 0.6)$

$$= \int_{0.2}^{0.5}\left\{\int_{0.4}^{0.6} f(x, y)\, dy\right\} dx$$

$$= \int_{0.2}^{0.5}\left\{\int_{0.4}^{0.6} \frac{6}{5}(x + y^2)\, dy\right\} dx$$

$$= \frac{6}{5}\int_{0.2}^{0.5}\left[xy + \frac{y^3}{3}\right]_{0.4}^{0.6} dx$$

$$= \frac{6}{5}\int_{0.2}^{0.5}\left[0.6x + \frac{(0.6)^3}{3} - 0.4x - \frac{(0.4)^3}{3}\right] dx$$

$$= \frac{6}{5}\int_{0.2}^{0.5}\left(0.2x + \frac{0.152}{3}\right) dx$$

$$= \frac{6}{5}\int_{0.2}^{0.5}\left[0.2\frac{x^2}{2} + \frac{0.152}{3}x\right]_{0.2}^{0.5}$$

$$= \frac{6}{5}\left[\frac{0.2}{2} \times (0.5)^2 + \frac{0.152}{3} \times 0.5 - \frac{0.2}{2} \times (0.2)^2 - \frac{0.152}{3} \times (0.2)\right]$$

$$= \frac{6}{5}[0.025 + 0.0253 - 0.004 - 0.0103] = 0.043$$

$$P(x > 0.4, y < 0.5) = \int_{0.4}^{1}\left[\int_{0}^{0.5} f(x, y)\right] dy\, dx$$

$$= \int_{0.4}^{1}\left[\int_{0}^{0.5} \frac{6}{5}(x + y^2)\, dy\right] dx$$

$$= \frac{6}{5}\int_{0.4}^{1}\left[xy + \frac{y^3}{3}\right]_{0}^{0.5} dx$$

$$= \frac{6}{5}\int_{0.4}^{1}\left[0.5x + \frac{(0.5)^3}{3}\right] dx$$

$$= \frac{6}{5}\int_{0.4}^{1}\left[0.5x + \frac{0.125}{3}\right] dx$$

$$= \frac{6}{5}\left[0.5\frac{x^2}{2} + \frac{0.125}{3}x\right]_{0.4}^{1}$$

$$= \frac{6}{5}\left[\frac{0.5}{2}(1)^2 + \frac{0.125}{3}(1) - \frac{0.5}{2}(0.4)^2 - \frac{0.125}{3}(0.4)\right] = 0.28$$

2. **Solution.**

$$f(x, y) = \begin{cases} x + y & 0 < x, y < 1 \\ 0 & \text{Otherwise} \end{cases}$$

$$f(x) = \int_0^1 f(x, y)\, dy = \int_0^1 (x + y)\, dy = \left[xy + \frac{y^2}{2}\right]_0^1 = x + \frac{1}{2}$$

$$f(y) = \int_0^1 f(x, y)\, dx = \int_0^1 (x + y)\, dx = \left[\frac{x^2}{2} + xy\right]_0^1 = \frac{1}{2} + y$$

$$E(xy) = \int_0^1 \int_0^1 xu\, f(x, y)\, dx\, dy$$

$$= \int_0^1 x\left[\int_0^1 xy\, f(x, y)\, dy\right]$$

$$= \int_0^1 x\left[x\frac{y^2}{2} + \frac{y^3}{3}\right]_0^1 dx$$

$$= \int_0^1 x\left(\frac{x}{2} + \frac{1}{3}\right) dx$$

$$= \int_0^1 \left(\frac{x^2}{2} + \frac{x}{3}\right) dx$$

$$= \left[\frac{1}{2}\frac{x^3}{3} + \frac{1}{3}\frac{x^2}{2}\right]_0^1 = \left[\frac{x^3}{6} + \frac{x^2}{6}\right]_0^1 = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$E(x) = \int_0^1 x\, f(x)\, dx = \int_0^1 x\left(x + \frac{1}{2}\right) dx = \left[\frac{x^3}{3} + \frac{1}{2} \cdot \frac{x^2}{2}\right]_0^1 = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

$$E(y) = \int_0^1 y\, f(y)\, dy = \frac{7}{12}$$

$$Cov(x, y) = E(xy) - E(x)\, E(y) = \frac{1}{3} - \frac{7}{12} \times \frac{7}{12} = -0.0069$$

3. **Solution.**

$$f(x_1, x_2) = \frac{2}{3}(x_1 + 2x_2)\ 0 < x_1 < 1,\ 0 < x_2 < 1$$

$$f(x_1) = \int_0^1 f(x_1, x_2)\, dx_2 = \int_0^1 \frac{2}{3}(x_1 + 2x_2)\, dx_2 = \frac{2}{3}\left[x_1 x_2 + \frac{2x_2^2}{2}\right]_0^1 = \frac{2}{3}(x_1 + 1)$$

$$f(x_2) = \int_0^1 f(x_1, x_2)\, dx_1 = \int_0^1 \frac{2}{3}(x_1 + 2x_2)\, dx_1 = \frac{2}{3}\left[\frac{x_1^2}{2} + 2x_1 x_2\right]_0^1 = \frac{1 + 4x_2}{3}$$

$$f(x_1/x_2) = \frac{f(x_1, x_2)}{f(x_2)} = \frac{\frac{2}{3}(x_1 + 2x_2)}{\frac{1 + 4x_2}{3}} = \frac{2(x_1 + 2x_2)}{1 + 4x_2}$$

$$f(x_2/x_1) = \frac{f(x_1, x_2)}{f(x_1)} = \frac{\frac{2}{3}(x_1 + 2x_2)}{\frac{2}{3}(x_1 + 1)} = \frac{x_1 + 2x_2}{x_1 + 1}$$

4. **Solution.**

$$P(x, y) = \frac{x - y + 3}{48},\ x = 0, 1, 2, 3,\ y = 0, 1, 2, 3$$

$$P(x) = \sum_y P(x, y) = \sum_{y=0}^{3} \frac{x - y + 3}{48} = \frac{x - 0 + 3}{48} + \frac{x - 1 + 3}{48} + \frac{x - 2 + 3}{48} + \frac{x - 3 + 3}{48}$$

$$= \frac{4x - 6 + 12}{48} = \frac{4x + 6}{48} = \frac{2x + 3}{24}$$

$$P(y) = \sum_x P(x, y) = \sum_{x=0}^{3} \frac{x - y + 3}{48}$$

$$= \frac{0 - y + 3}{48} + \frac{1 - y + 3}{48} + \frac{2 - y + 3}{48} + \frac{3 - y + 3}{48} = \frac{9 - 2y}{24}$$

$$P(x/y = 1) = \frac{P(x, y = 1)}{P(y = 1)} = \frac{\frac{x - 1 + 3}{48}}{\frac{9 - 2}{24}} = \frac{x + 2}{14}$$

$$P(y/x = 1) = \frac{P(x = 2, y)}{P(x = x)} = \frac{\frac{2 - y + 3}{48}}{\frac{4 + 3}{24}} = \frac{5 - y}{14}$$

Solution.

y \ x	–1	0	1
0	1/15	2/15	1/15
1	3/15	2/15	1/15
2	2/15	1/15	2/15

$$P(x = 1, y \le 1) = P(x > 1, y = 0) + P(x = 1, y = 1) = \frac{1}{15} + \frac{1}{15} = \frac{2}{15}$$

$$P(y \le 1) = P(y = 0) + P(y = 1) = \left(\frac{1}{15} + \frac{2}{15} + \frac{1}{15}\right) + \left(\frac{3}{15} + \frac{2}{15} + \frac{1}{15}\right) = \frac{4}{15} + \frac{6}{15} = \frac{2}{3}$$

i) $P(x = -1) = \frac{1}{15} + \frac{3}{15} + \frac{2}{15} = \frac{2}{15}$

Solution.

Let hard ware = H and soft ware = S

Total = 3H + 4S = 7

Selected = 2

Here, x = Prob. of Hardware, y = Prob. of Software

$P(x = 0, y = 0) = 0$

$P(x = 0, y = 1) = 0$

$P(x = 0, y = 2) = \frac{{}^4C_2}{{}^7C_2} = \frac{6}{21}$

$P(x = 1, y = 0) = 0$

$P(x = 1, y = 1) = \frac{{}^3C_1 \times {}^4C_1}{{}^7C_2} = \frac{12}{21}$

$P(x = 1, y = 2) = 0$

$P(x = 2, y = 0) = \frac{{}^3C_2}{{}^7C_2} = \frac{3}{21}$

$P(x = 2, y = 1) = 0$

$P(x = 2, y = 2) = 0$

y \ x	0	1	2	P(4)
0	0	0	3/21	3/21
1	0	12/21	0	12/21
2	6/21	0	0	6/21

$P(x = 0/y = 0) = 0, P(x = 0 / y = 1) = 0$

$$P(x = 0/y = 2) = \frac{P(x = 0, y = 2)}{P(y = 2)} = \frac{6/21}{6/21} = 1$$

$$P(x = 1/y = 0) = 0; \; P(x = 1/y = 1) = \frac{P(x = 1, y = 1)}{P(y = 1)} = \frac{\frac{12}{21}}{\frac{12}{21}} = 1$$

$P(x = 1/y = 2) = 0$

$$P(x = 2/y = 0) = \frac{P(x = 2, y = 1)}{P(y = 0)} = \frac{3/21}{3/21} = 1$$

$P(y = 2/y = 1) = 0 \quad P(x = 2/y = 2) = 0$

7. **Solution.**

y \ x	0	1	2	P(4)
–1	0.03	0.02	0.05	0.1
0	0.01	0.25	0.45	0.72
1	0	0.03	0.05	0.18

$$P(x = 0/y = -1) = \frac{0.03}{0.1} = 0.3$$

$$P(x = 0/y = 0) = \frac{0.01}{0.72} = 0.013$$

$$P(x = 0/y = 1) = \frac{0}{0.18} = 0$$

$$P(x = 1/y = -1) = \frac{0.02}{0.1} = 0.2$$

$$P(x = 1/y = 0) = \frac{0.25}{0.72} = 0.347$$

$$P(x = 1/y = 1) = \frac{0.03}{0.18} = 0.166$$

$$P(x = 2/y = -1) = \frac{0.05}{0.1} = 0.5$$

$$P(x = 2/y = 1) = \frac{0.45}{0.72} = 0.625$$

$$P(x = 2/y = 1) = \frac{0.05}{0.18} = 0.277$$

$P(x = 3/y = -1) = \frac{0}{0.1} = 0$

$P(x = 3/y = 0) = \frac{0.01}{0.72} = 0.013$

$P(x = 3/y = 1) = \frac{0.1}{0.18} = 0.555$

$P(y = -1/x = 0) = \frac{0.03}{0.04} = 0.75$

$P(y = -1/x = 1) = \frac{0.02}{0.3} = 0.067$

$P(y = -1/x = 2) = \frac{0.05}{0.55} = 0.109$

$P(y = -1/x = 3) = \frac{0}{0.11} = 0$

$P(y = 0/x = 0) = \frac{0.01}{0.04} = 0.25$

$P(y = 0/x = 1) = \frac{0.25}{0.3} = 0.833$

$P(y = 0/x = 2) = \frac{0.45}{0.55} = 0.818$

$P(y = 0/x = 3) = \frac{0.01}{0.11} = 0.09$

$P(y = 1/x = 0) = \frac{0}{0.04} = 0$

$P(y = 1/x = 1) = \frac{0.03}{0.3} = 0.1$

$P(y = 1/x = 2) = \frac{0.05}{0.55} = 0.09$

$P(y = 1/x = 3) = \frac{0.1}{0.11} = 0.909$

$$\begin{aligned} P(x = 0, y \le 1) &= P(x = 0, y = -1) + P(x = 0, y = 0) + P(x = 0, y = 1) \\ &= 0.03 + 0.01 + 0 \\ &= 0.04 \end{aligned}$$

$P(x \ge 0, y < 1) = P(y = 1) + P(y = 0) = 0.1 + 0.72 = 0.82$

$P(x \le 0/y = 1) = \frac{P(x \le 0, y = 1)}{P(y = 1)} = \frac{0}{0.18} = 0$

8. **Solution.**

$$f(x, y) = \begin{cases} x^2 + kxy & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$= \int_{-\infty}^{+\infty} f(x, y)\, dy = \int_0^1 2x\, dy = [2xy]_0^1 = 2x$$

$$f(y) = \int_{-\infty}^{+\infty} f(x, y)\, dx = \int_0^1 2x\, dx = \left[2\frac{x^2}{2}\right]_0^1 = 1$$

Here, $f(x, y) = 2x$

$f(x)\, f(y) = 2x - 1 = 27$

$f(x, y) = f(x)\, f(y)$

Hence, x and y are independent.

9. $f(x, y) = 2x$

$$f(x) = \int_0^1 f(x, y)\, dy = \int_0^1 2x\, dy = 2x\, [y]_0^1 = 2x$$

$$f(y) = \int_0^1 f(x, y)\, dx = \int_0^1 2x\, dx = 2\left[\frac{x^2}{2}\right]_0^1 = 1$$

$f(x)\, f(y) = 2x \,.\, 1 = 2x$

Hence, x and y are independent.

10. $f(x, y) = \begin{cases} k\,(6 - x - y) & 0 \le x \le 2, 0 \le y \le 4 \\ 0 & \text{elsewhere} \end{cases}$

Since, $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x, y)\, dy\, dx = 1$

$$\Rightarrow \int_0^2 \left[\int_2^4 k\,(6 - x - y)\, dy\right] dx = 1$$

$$\Rightarrow k \int_0^2 \left[6y - xy - \frac{y^2}{2}\right]_2^4 dx = 1$$

$$\Rightarrow k \int_0^2 \left(6\times4 - 4x - \frac{16}{2} - 6\times2 + 2x + \frac{4}{2}\right) dx = 1$$

$$\Rightarrow k \int_0^2 (6 - 2x)\, dx = 1$$

$$\Rightarrow k\left[6x - \frac{2x^2}{2}\right]_0^2 = 1$$

$$\Rightarrow k\,(12 - 4) = 1$$

$$\Rightarrow k = \frac{1}{8}$$

$$\therefore f(x, y) = \frac{6 - x - y}{8}$$

$$P(x \le 1 \cap y < 3) = \int_0^1 \left[\int_2^3 \frac{6 - x - y}{8}\, dy\right] dx$$

$$= \frac{1}{8}\int_0^1 \left(18 - 3x - \frac{9}{2} - 12 + 2x + \frac{4}{2}\right) dx$$

$$= \frac{1}{8}\int_0^1 (3.5 - x)\, dx = \frac{1}{8}\left[3.5\,x - \frac{x^2}{2}\right]_0^1 = \frac{1}{8}\left(3.5\times1 - \frac{1}{2}\right) = \frac{3}{8}$$

$$f(y) = \int_{-\infty}^{+\infty} f(x, y)\, dx = \int_0^2 \frac{6 - x - y}{8}\, dx = \frac{1}{8}\left[6x - \frac{x^2}{2} - xy\right]_0^2$$

$$= \frac{1}{8}[12 - 2 - 2y] = \frac{5-y}{4}$$

$$P(y \le 3) = \int_2^3 f(y)\, dy = \int_2^3 \frac{5 - y}{4}\, dy = \frac{1}{4}\left[5y - \frac{y^2}{2}\right]_2^3 = \frac{1}{4}\left[15 - \frac{9}{2} - 10 + \frac{4}{2}\right] = \frac{5}{8}$$

$$P(x \le 1 / y \le 3) = \frac{P(x \le 1, y \le 3)}{P(y \le 3)} = \frac{\int_0^1\left[\int_2^3 f(x, y)\, dy\right] dx}{\int_2^3 f(y)\, dy} = \frac{3/8}{5/8} = \frac{3}{5}$$

1. $f(x, y) = 4xy\, e^{-(x^2 + y^2)}$ $x \ge 0, y \ge 0$

$$f(x) = \int_0^\infty f(x, y)\, dy = \int_0^\infty 4xy\, e^{-(x^2 + y^2)}\, dy = 4x\, e^{-x^2} \int_0^\infty y\, e^{-y^2}\, dy$$

Let $y^2 = z$;

$2y\, dy = dz$

When $y = 0$, $z = 0$ and when $y = \infty$, $z = \infty$

$$= 4\, x\, e^{-x^2} \int_0^\infty e^{-z} \frac{dz}{z}$$

$$= \frac{4x\, e^{-x^2}}{2} \int_0^z e^{-z} z^{1-1}\, dz$$

$$= z\, x\, e^{-x^2}$$

$$f(y) = \int_0^\infty f(x, y)\, dx = 2y\, e^{-y^2}$$

$$f(x/y) = \frac{f(x, y)}{f(y)} = \frac{4xy\, e^{-(x^2 + y^2)}}{2y\, e^{-y^2}} = 2x\, e^{-x^2}$$

Here, $f(x)\, f(y) = 2x\, e^{-x^2}\, 2y\, e^{-y^2} = 4xy\, e^{-(x^2 + y^2)} = f(x, y)$

Hence, x and y are independent.

12. $f(x, y) = k\,(2x + 3y)$ $0 \le x \le 1, 0 \le y \le 1$

Since,

$$\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} f(x, y)\, dy\right] dx = 1$$

$$\Rightarrow \int_0^1\left[\int_0^1 k\,(2x + 3y)\, dy\right] dx = 1$$

$$\Rightarrow k \int_0^1 \left[2xy + \frac{3y^2}{2}\right]_0^1 dx = 1$$

$$\Rightarrow k \int_0^1 \left(2x + \frac{3}{2}\right) dx = 1$$

$$\Rightarrow k\left[\frac{2x^2}{2} + \frac{3}{x}x\right]_0^1 = 1$$

$$\Rightarrow k\left[1 + \frac{3}{2}\right] = 1$$

$\Rightarrow \quad k = \frac{2}{5} \qquad \therefore f(x, y) = \frac{2}{5}(2x + 3y)$

$$f(x) = \int_{-\infty}^{+\infty} f(x, y)\, dy = \int_0^1 \frac{2}{5}(2x + 3y)\, dy$$

$$= \frac{2}{5}\left[2xy + \frac{3y^2}{2}\right]_0^1 = \frac{2}{5}\left[2x + \frac{3}{2}\right] = \frac{4x + 3}{5}$$

$$f(y) = \int_{-\infty}^{+\infty} f(x, y)\, dy = \int_0^1 \frac{2}{5}(2x + 3y)\, dy$$

$$= \frac{2}{5}\left[\frac{2x^2}{2} + 3xy\right]_0^1 = \frac{2}{5}[1 + 3y]$$

$$f(x/y) = \frac{f(x, y)}{f(y)} = \frac{\frac{2}{5}(2x + 3y)}{\frac{2}{5}(1 + 3y)} = \frac{2x + 3y}{1 + 3y}$$

$$f(y/x) = \frac{f(x, y)}{f(x)} = \frac{\frac{2}{5}(2x + 3y)}{\frac{4x + 3}{5}} = \frac{4x + 6y}{4x + 3}$$

$$f(x)\, f(y) = \frac{f(x, y)}{f(x)} = \frac{4x + 3}{5} \times \frac{2}{5}(1 + 3y) = \frac{2}{25}(4x + 12xy + 3 + 9y) \neq f(x, y)$$

Hence, x and y are dependent.

$$E(x) = \int_0^1 x\, f(x)\, dx = \int_0^1 x\, \frac{(4x + 3)}{5}\, dx$$

$$= \frac{1}{5}\int_0^1 (4x^2 + 3x)\, dx$$

$$= \frac{1}{5}\left[\frac{4x^3}{3} + \frac{3x^2}{2}\right]_0^1 = \frac{1}{5}\left[\frac{4}{3} + \frac{3}{2}\right] = \frac{17}{30}$$

$$E(x^2) = \int_0^1 x^2 f(x)\, dx = \int_0^1 x^2 \frac{4x + 3}{5}$$

$$= \frac{1}{5}\int_0^1 (4x^3 + 3x^2)\, dx = \frac{1}{5}\left[\frac{4x^4}{4} + \frac{3x^3}{3}\right]_0^1 = \frac{1}{5}[1+1] = \frac{2}{5}$$

$$E(y) = \int_0^1 y\, f(y)\, dy = \int_0^1 y\, \frac{2}{5}(1 + 3y)\, dy$$

$$= \frac{2}{5}\int_0^1 (y + 3y^2)\, dy$$

$$= \frac{2}{5}\left[\frac{y^2}{2} + \frac{3y^3}{3}\right]_0^1 = \frac{2}{5}\left[\frac{1}{2} + 1\right] = \frac{3}{5}$$

$$E(y)^2 = \int_0^1 y^2 f(y)\, dy = \int_0^1 y^2 \frac{2}{5}(1 + 3y)\, dy$$

$$= \frac{2}{5}\int_0^1 (y^2 + 3y^3)\, dy = \frac{2}{5}\left[\frac{y^3}{3} + \frac{3y^4}{4}\right]_0^1 = \frac{2}{5}\left[\frac{1}{3} + \frac{3}{4}\right] = \frac{13}{30}$$

$$E(x_4) = \int_0^1 \left[\int_0^1 xy\, f(x, y)\, dy \right] dx = \int_0^1 \left[\int_0^1 xy \frac{2}{5}(2x + 3y)\, dy \right] dx$$

$$= \frac{2}{5} \int_0^1 \left[\int_0^1 (2x^2y + 3xy^2)\, dy \right] dx$$

$$= \frac{2}{5} \int_0^1 \left[\frac{2x^2 y^2}{2} + \frac{3xy^3}{3} \right]_0^1 dx$$

$$= \frac{2}{5} \int_0^1 (x^2 + x)\, dx = \frac{2}{5} \left[\frac{x^3}{3} + \frac{x^2}{2} \right]_0^1 = \frac{2}{5} \left[\frac{1}{3} + \frac{1}{2} \right] = \frac{1}{3}$$

$$E(x^2y^2) = \int_0^1 \left[\int_0^1 x^2y^2 f(x, y)\, dy \right] dx = \int_0^1 \left[\int_0^1 x^2y^2 \frac{2}{5}(2x + 3y)\, dy \right] dx$$

$$= \frac{2}{5} \int_0^1 \left[\int_0^1 (2x^3 y^2 + 3x^2 y^3)\, dy \right] dx$$

$$= \frac{2}{5} \int_0^1 \left[\frac{2x^3y^3}{3} + \frac{3x^2 y^4}{4} \right]_0^1 dx$$

$$= \frac{2}{5} \int_0^1 \left(\frac{2}{3} x^3 + \frac{3}{4} x^2 \right) dx$$

$$= \frac{2}{5} \left[\frac{2}{3}\frac{x^4}{4} + \frac{3}{4}\frac{x^3}{3} \right]_0^1 = \frac{2}{5} \left[\frac{1}{6} + \frac{1}{4} \right] = \frac{1}{6}$$

$$V(x) = E(x^2) = [E(x)]^2 = \frac{2}{5} - \left(\frac{17}{30} \right)^2 = 0.4 - (0.566)^2 = 0.079$$

$$V(y) = E(y^2) - [E(y)]^2 = \frac{1}{6} - \left(\frac{1}{3} \right)^2 = \frac{1}{6} - \frac{1}{9} = 0.055$$

$$Cov\,(xy) = E(xy) - E(x) \in E(y) = \frac{1}{3} - \frac{17}{30} \times \frac{3}{5} = -\frac{1}{50}$$

$$r(x, y) = \frac{Cov(x, y)}{\sqrt{V(x)}\,\sqrt{V(y)}} = \frac{-\frac{1}{50}}{\sqrt{0.079}\,\sqrt{0.073}} = -0.087$$

$$E(zx + 3y) = Z\,E(x) + 3\,E(y) = 2 \times \frac{17}{30} + 3 \times \frac{3}{5} = 2.9$$

$$V(3x - 2y) = 9\,V(x) + 4\,V(y) - 12\,Cov(xy)$$

$$= 9 \times 0.079 + 4 \times 0.073 - 12\,(-0.0066) = 1.08$$

13. $P(x, y) = k(x + y), \begin{matrix} x \le 1, 2, 3 \\ y = 1, 2 \end{matrix}$

Since,

$$\sum_x \sum_y P(x, y) = 1$$

$$\Rightarrow \sum_{x=1}^{3} \sum_{y=1}^{2} k\,(x + y) = 1$$

$\Rightarrow$ $k[(1+1)+(1+2)+(2+1)+(2+2)+(3+1)+(3+2)] = 1$

$\Rightarrow$ $k = \frac{1}{21}$

$\therefore$ $P(x, y) = \frac{1}{21} P(x, y)$

$$P(x) = \sum_{y} P(x, y)$$

$$= \sum_{y=1}^{2} \frac{1}{21}(x+y) = \frac{1}{21}[(x+1)+(x+2)] = \frac{2x+3}{21}$$

$$P(y) = \sum_{x} P(x, y)$$

$$= \sum_{x=1}^{2} \frac{1}{21}(x+y) = \frac{1}{21}(1+y+2+y+3+y) = \frac{6+3y}{21}$$

$$P(x, y) = \frac{P(x, y)}{P(y)} = \frac{\frac{1}{21}(x+y)}{\frac{6+3y}{21}} = \frac{x+y}{6+3y} = \frac{x+y}{3y+6}$$

$$P(y/x) = \frac{P(x, y)}{P(x)} = \frac{\frac{1}{21}(x+y)}{\frac{2x+3}{21}} = \frac{x+y}{2x+3}$$

$$P(x, y) = \frac{1}{21}(x+y)$$

$$P(x)\,P(y) = \frac{2x+3}{21} \times \frac{3y+6}{21} = \frac{6xy+12x+9y+18}{441}$$

$P(x, y) \neq P(x), P(y)$

Hence, x and y are dependent

$$E(x) = \sum_{x=1}^{3} 2x^2 + 3x = \frac{1}{21}[(2 \times 1^2 + 3 \times 1) + (2 \times 2^2 + 3 \times 2) + 2 \times 3^2 + 3 \times 3] = \frac{46}{21}$$

$$E(x^2) = \sum_{x} x^2 P(x) = \sum_{x=1}^{3} x^2 \frac{2x+3}{21}$$

$$= \frac{1}{21} \sum_{x=1}^{3} (2x^3 + 3x^2)$$

$$= \frac{1}{21}[(2 \times 1^3 + 3 \times 1^2) + (2 \times 2^3 + 3 \times 2^2) + (2 \times 3^3 + 3 \times 3^2)]$$

$$= \frac{114}{21}$$

$$E(y) = \sum_{x} y\, P(y) = \sum_{y=1}^{2} y \frac{6+3y}{21}$$

$$= \frac{1}{21} \sum_{y=1}^{2} 6y + 3y^2$$

$$= \frac{1}{21} [(6 \times 1 + 3 \times 1^2) + (6 \times 2 \times 3 \times 2^2)] = \frac{33}{21}$$

$$E(y^2) = \sum_{y} y^2 P(y) = \sum_{y=1}^{2} y^2 \frac{(6+3y)}{21}$$

$$= \frac{1}{21} \sum_{y=1}^{2} 6y^2 + 3y^3$$

$$= \frac{1}{21} [(6 \times 1^2 + 3 \times 1^3) + (6 \times 2^2 + 3 \times 2^3)] = \frac{57}{21}$$

$$E(xy) = \sum_{x}\sum_{y} xy\, P(x, y) = \sum_{x=1}^{3} \sum_{y=1}^{2} xy \frac{x+y}{21}$$

$$= \frac{1}{21} \sum_{x=1}^{3} \sum_{y=1}^{2} x^2y + xy^2$$

$$= \frac{1}{21} \sum_{x=1}^{3} (x^2 \times 1 + x \times 1^2) + (x^2 \times 2 + x \times 2^2)$$

$$= \frac{1}{21} \sum_{x=1}^{3} (x^2 + x + 2x^2 + 4x)$$

$$= \frac{1}{21} \sum_{x=1}^{3} (3x^2 + 5x)$$

$$= \frac{1}{21} [(3 \times 1^2 + 5 \times 1) + (3 \times 2^2 + 5 \times 2) + (3 \times 3^2 + 5 \times 3)]$$

$$= \frac{306}{21}$$

$$V(x) = E(x^2) - [E(x)]^2 = \frac{114}{21} - \left(\frac{46}{21}\right)^2 = 5.42 - 4.79 = 0.63$$

$$V(y) = E(y^2) - [E(y)]^2 = \frac{57}{21} - \left(\frac{33}{21}\right)^2 = 2.71 - 2.46 = 0.25$$

$$V(xy) = E(x^2y^2) - [E(xy)]^2 = \frac{306}{21} - \left(\frac{72}{21}\right)^2 = 14.57 - 11.75 = 2.82$$

$$Cov\,(xy) = E(xy) - E(x)\,E(y) = \frac{72}{21} - \frac{46}{21} \times \frac{33}{21} = 3.428 - 3.442 = -0.017$$

$$r = \frac{Cov(x, y)}{\sqrt{V(x)}\sqrt{V(y)}} = \frac{-0.17}{\sqrt{0.63}\sqrt{0.25}} = -0.043$$

$$E(2x - 3y) = 2E(x) - 3E(y) = 2 \times \frac{46}{21} - 3 \times \frac{33}{21} = \frac{92-99}{21} = -0.33$$

$$V(2x + 3y) = 4V(x) + 9V(y) + 12Cov(x, y)$$

$$= 4 \times 0.63 + 9 \times 0.25 + 12 \times (-0.017) = 4.566$$

14. $f(x, y) = \begin{cases} k\,e^{-(x+y)} & 0 \le x \le \infty, 0 \le y \le \infty \\ 0 & \text{otherwise} \end{cases}$

Since, $\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} f(x, y)\,dy\right]dx = 1$

$\Rightarrow \int_0^{\infty}\left[\int_0^{\infty} k\,e^{-(x+y)}\,dy\right]dx = 1$

$\Rightarrow k\int_0^{\infty} e^{-x}\left[\int_0^{\infty} e^{-y}\,dy\right]dx = 1$

$\Rightarrow k\int_0^{\infty} e^{-x}.1\,dx = 1$

$\Rightarrow k.1 = 1$

$\Rightarrow k = 1$ $\qquad \therefore f(x, y) = e^{-(x+y)}$

$$f(x) = \int_{-\infty}^{+\infty} f(x, y)\,dy$$

$$= \int_0^{\infty} e^{-(x+y)}\,dy$$

$$= \int_0^{\infty} e^{-x} e^{-y}\,dy$$

$$= e^{-x}\int_0^{\infty} e^{-y}\,dy = e^{-x} \times 1 = e^{-x}$$

$$f(y) = \int_{-\infty}^{+\infty} f(x, y)\,dx$$

$$= \int_0^{\infty} e^{-(x+y)}\,dx$$

$$= e^{-y}\int_0^{\infty} e^{-x}\,dx = e^{-y}.1 = e^{-y}$$

$$f(x/y) = \frac{f(x, y)}{f(y)} = \frac{e^{-(x+y)}}{e^{-y}} = e^{-x}$$

$$f(y/x) = \frac{f(x, y)}{f(x)} = \frac{e^{-(x+y)}}{e^{-x}} = e^{-y}$$

Here, $f(x, y) = e^{-(x+y)}$

$f(x)\,f(y) = e^{-x} e^{-y} = e^{-(x+y)}$

$f(x, y) = f(x)\,f(y)$

Hence, x and y are independent

$$E(x) = \int_{-\infty}^{+\infty} x\,f(x)\,dx = \int_0^{\infty} x\,e^{-x}\,dx = 1$$

$$E(x^2) = \int_{-\infty}^{+\infty} x^2 f(x)\, dx = \int_0^{\infty} x^2 e^{-x}\, dx = 2$$

$$E(y) = \int_0^{\infty} y\, e^{-y}\, dy = 1$$

$$E(y^2) = \int_0^{\infty} y^2 e^{-y}\, dy = 2$$

$$E(xy) = \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} xy\, f(x, y)\, dy\right] dx$$

$$= \int_0^{\infty}\left[\int_{-\infty}^{+\infty} xy\, e^{-(x+y)}\, dy\right] dx$$

$$= \int_0^{\infty} x\, e^{-x}\left[\int_0^{\infty} y\, e^{-y}\, dy\right] dx$$

$$= \int_0^{\infty} x\, e^{-x} . 1\, dx$$

$$= 1$$

$$E(x^2 y^2) = \int_0^{\infty}\left[\int_0^{\infty} x^2 y^2 f(x, y)\, dy\right] dx$$

$$= \int_0^{\infty}\left[\int_0^{\infty} x^2 y^2 e^{-(x+y)}\, dy\right] dx$$

$$= \int_0^{\infty} x^2 e^{-x}\left[\int_0^{\infty} y^2 e^{-y}\, dy\right] dx$$

$$= \int_0^{\infty} x^2 e^{-x} . 2\, dx = 2\int_0^{\infty} e^{-x} x^2\, dx = 2\times 2 = 4$$

$$V(x) = E(x^2) - [E(x)]^2 = 2 - 1^2 = 1$$

$$V(y) = E(y^2) - [E(y)]^2 = 2 - 1^2 = 1$$

$$V(xy) = E(x^2y^2) - [E(xy)]^2 = 4 - 1^2 = 3$$

$$Cov(xy) = E(xy) - E(x)\, E(y) = 1 - 1 . 1 = 0$$

$$r(x, y) = \frac{Cov(x, y)}{\sqrt{V(x)}\,\sqrt{V(y)}} = \frac{0}{\sqrt{1}\sqrt{1}} = 0$$

$$E(3x - 2y) = 3\, E(x) - 2E(y) = 3 \times 1 - 2 \times 1 = 1$$

$$V(3x + 2y) = 9\, V(x) + 4\, V(y) + 12\, Cov(xy) = 9 \times 1 + 4\times 1 + 12 \times 0 = 13$$

15. $$f(x, y) = \begin{cases} k\,(x + 2)\, e^{-y} & 0 \le x \le 1, y \ge 0 \\ 0 & \text{elsewhere} \end{cases}$$

Since, $$\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} f(x, y)\, dy\right] dx = 1$$

$$\Rightarrow \int_0^{1}\left[\int_0^{\infty} k\,(x + 2)\, e^{-y}\, dy\right] dx = 1$$

$$\Rightarrow k\int_0^{1} (x + 2)\left[\int_0^{\infty} e^{-y}\, dy\right] dx = 1$$

$$\Rightarrow k\int_0^{1} (x + 2)\, 1\, dx = 1$$

$$\Rightarrow \quad k\left[\frac{x^2}{2} + 2x\right]_0^1 = 1$$

$$\Rightarrow \quad k\left(\frac{1}{2} + 2\right) = 1$$

$$\Rightarrow \quad k = \frac{2}{5} \qquad \therefore f(x, y) = \frac{2}{5}(x + 2)\, e^{-y}$$

$$f(x) = \int_{-\infty}^{+\infty} f(x, y)\, dy$$

$$= \int_0^{\infty} \frac{2}{5}(x + 2)\, e^{-y}\, dy$$

$$= \frac{2}{5}(x + 2) \int_0^{\infty} e^{-y}\, dy$$

$$= \frac{2}{5}(x + 2)$$

$$f(y) = \int_{-\infty}^{+\infty} f(x, y)\, dx$$

$$= \int_0^1 \frac{2}{5}(x + 2)\, e^{-y}\, dx$$

$$= \frac{2}{5} e^{-y}\left[\frac{x^2}{2} + 2x\right]_0^1$$

$$= \frac{2}{5} e^{-y}\left[\frac{1}{2} + 2\right] = \frac{2}{5} e^{-y} \times \frac{5}{2} = e^{-y}$$

$$f(x/y) = \frac{f(x, y)}{f(y)} = \frac{\frac{2}{5}(x + 2)\, e^{-y}}{e^{-y}}$$

$$f(y/x) = \frac{f(x, y)}{f(x)} = \frac{\frac{2}{5}(x + 2)\, e^{-y}}{\frac{2}{5}(x + 2)} = e^{-y}$$

Here, $f(x, y) = \frac{2}{5}(x + 2)\, e^{-y}$

$f(x)\, f(y) = \frac{2}{5}(x + 2)\, e^{-y}$

Hence, x and y are independent.

$$E(x) = \int_{-\infty}^{+\infty} x\, f(x)\, dx$$

$$= \int_0^1 x\, \frac{2}{5}(x + 2)\, dx$$

$$= \frac{2}{5}\left[\frac{x^3}{3} + \frac{2x^2}{2}\right]_0^1$$

$$= \frac{2}{5}\left[\frac{1}{3} + 1\right] = \frac{2}{5} \times \frac{4}{3} = \frac{8}{15}$$

$$E(x^2) = \int_{-\infty}^{+\infty} x^2 f(x)\, dx$$

$$= \int_0^1 x^2 \frac{2}{5}(x+2)\, dx$$

$$= \frac{2}{5}\int_0^1 (x^3 + 2x^2)\, dx$$

$$= \frac{2}{5}\left[\frac{x^4}{4} + \frac{2x^3}{3}\right]_0^1$$

$$= \frac{2}{5}\left[\frac{1}{4} + \frac{2}{3}\right] = \frac{2}{5} \times \frac{3+8}{12} = \frac{11}{30}$$

$$E(y) = \int_{-\infty}^{+\infty} y\, f(y)\, dy = \int_0^{\infty} y\, e^{-y}\, dy = 1$$

$$E(y^2) = \int_{-\infty}^{+\infty} y^2 f(y)\, dy = \int_0^{\infty} y^2 e^{-y}\, dy = 2$$

$$E(xy) = \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} xy\, f(x, y)\, dy\right] dx$$

$$= \frac{2}{5}\int_0^1 x\left[\int_0^{\infty} y\, e^{-y}\, dy\right] dx$$

$$= \frac{2}{5}\int_0^1 (x^2 + 2)\,.\,1\, dx$$

$$= \frac{2}{5}\left[\frac{x^3}{3} + \frac{2x^2}{2}\right]_0^1 = \frac{2}{5}\left(\frac{1}{3} + 1\right) = \frac{8}{15}$$

$$E(x^2y^2) = \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} x^2 y^2 f(x, y)\, dy\right] dx$$

$$= \int_0^1 x^2 (x+2)\left[\int_0^{\infty} y^2 e^{-y}\, dy\right] dx$$

$$= \frac{2}{5}\int_0^1 (x^3 + 2x^2)\, 2\, dx$$

$$= \frac{4}{5}\left[\frac{x^4}{4} + \frac{2x^3}{3}\right]_0^1$$

$$= \frac{4}{5}\left[\frac{1}{4} + \frac{2}{3}\right] = \frac{11}{15}$$

$$V(x) = E(x^2) - [E(x)]^2 = \frac{11}{30} - \left(\frac{8}{15}\right)^2 = 0.082$$

$$V(y) = E(y^2) - [E(y)]^2 = 2 - 1^2 = 1$$

$$V(xy) = E(x^2y^2) - E(xy)]^2 = \frac{11}{15} - \left(\frac{8}{15}\right)^2 = 0.449$$

$$Cov(xy) = E(xy) - E(x)\, E(y) = \frac{8}{15} - \frac{8}{15} \times 1 = 0$$

$$r(x, y) = \frac{\text{Cov}(x, y)}{\sqrt{V(x)}\sqrt{V(y)}} = \frac{0}{\sqrt{0.082}\sqrt{1}} = 0$$

16. $f(x, y) = \begin{cases} 2 & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$

$$f(x) = \int_x^1 f(x, y)\, dy$$

$$= \int_x^1 2\, dy$$

$$= 2\,[y]_\lambda^1 = 2\,[1 - \lambda]$$

$$f(y) = \int_0^y x\, f(x)\, dx$$

$$= \int_0^y 2\, dx = 2\,[x]_0^y = 2y$$

$$E(x) = \int_0^1 x\, f(x)\, dx$$

$$= \int_0^1 x\, 2\,(1 - x)\, dx = 2\int_0^1 (x - x^2)\, dx = 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{1}{3}$$

$$E(x^2) = \int_0^1 x^2\, f(x)\, dx$$

$$= \int_0^1 x^2\, 2\,(1 - x)\, dx = 2\int_0^1 (x^2 - x^3)\, dx = 2\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 2\left[\frac{1}{3} - \frac{1}{4}\right] = \frac{1}{6}$$

$$E(y) = \int_0^1 y\,(f(y))\, dy = \int_0^1 y\, 2y\, dy = 2\int_0^1 y^2\, dy = 2\left[\frac{y^3}{3}\right]_0^1 = \frac{2}{3}$$

$$E(y^2) = \int_0^1 y^2\, f(y)\, dy = \int_0^1 y^2\, 2y\, dy = 2\int_0^1 y^3\, dy = 2\left[\frac{y^4}{4}\right]_0^1 = \frac{2}{4} = \frac{1}{2}$$

$$E(xy) = \int_0^1 \left[\int_0^1 xy\, f(x, y)\, dy\right] dx$$

$$= \int_0^1 \left[\int_0^1 xy\,.\,z\, dy\right] dx$$

$$= \int_0^1 2x\left[\int_0^1 y\, dy\right] dx = \int_0^1 2x\left[\frac{y^2}{2}\right]_0^1 dx = \int_0^1 x\, dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$$

$$V(x) = E(x^2) - [E(x)]^2 = \frac{1}{6} - \left(\frac{1}{3}\right)^2 = \frac{1}{18}$$

$$V(y) = E(y^2) - [E(y)]^2 = \frac{1}{2} - \left(\frac{2}{3}\right)^2 = \frac{1}{18}$$

$$\text{Cov}(xy) = E(xy) - E(x)\,E(y) = \frac{1}{2} - \frac{1}{3} \times \frac{2}{3} = \frac{5}{18}$$

17. $f(x, y) = \dfrac{9(1+x+y)}{2(1+x)^4(1+y)^4}$ $\quad 0 < x < \infty$, $0 < y < \infty$

$$f(x) = \int_{-\infty}^{+\infty} f(x, y)\, dy = \frac{9}{2}\int_0^{\infty} \frac{1+x+y}{(1+x)^4(1+y)^4}\, dy$$

$$= \frac{9}{2}\int_0^{\infty} \frac{x+1+y}{(1+x)^4(1+y)^4}\, dy$$

$$= \frac{9}{2}\left[\int_0^{\infty} \frac{x}{(1+x)^4(1+y)^4}\, dy + \int_0^{\infty} \frac{1+y}{(1+y)^4(1+x)^4}\, dy\right]$$

$$= \frac{9}{2}\left[\frac{x}{(1+x)^4}\int_0^{\infty}(1+y)^{-4}\, dy + \frac{1}{(1+x)^4}\int_0^{\infty}(1+y)^{-3}\, dy\right]$$

$$= \frac{9}{2}\left[\frac{x}{3(1+x)^4} + \frac{1}{2(1+x)^4}\right]$$

$$= \frac{9}{2}\left[\frac{2x+3}{(1+x)^4}\right] = \frac{3}{4}\frac{(2x+3)}{(1+x)^4}$$

$$f(y) = \int_{-\infty}^{+\infty} f(x, y)\, dx = \frac{9}{2}\times\frac{2y+3}{6(1+y)^4} = \frac{3}{4}\times\frac{(2y+3)}{(1+y)^4}$$

$$f(y/x) = \frac{f(x/y)}{f(x)} = \frac{\dfrac{9(1+x+y)}{2(1+x)^4(1+y)^4}}{\dfrac{3}{4}\dfrac{2x+3}{(1+x)^4}} = \frac{6(1+x+y)}{(2x+3)(1-y)^4}$$

18. Comment on

Let $x \sim B(n, p)$

$E(x) = 7$

$V(x) = 11$

Now,

$E(x) = np = 7$ .... (i)

$V(x) = npq = 11$ .... (ii)

Substitute the value of np from (i) in (ii)

$7q = 11$

Or, $q = \dfrac{11}{7} = 1.57 > 1$, Which is impossible.

Hence, given information is incorrect.

19. Find p if

$n = 6$, $9p(x = 4) = p(x = 2)$, $p = ?$

Here, $9p(x = 4) = p(x = 2)$

Or, $9c(6, 4)p^4q^2 = c(6, 2)p^2q^4$

Or, $9 \times 15p^4q^2 = 15p^2q^4$

Or, $9p^2 = q^2$

Or $9p^2 - q^2 = 0$

Or, $9p^2 - (1-p)^2 = 0$

Or, $9p^2 - (1 - 2p + p^2) = 0$

Or, $9p^2 - 1 + 2p - p^2 = 0$

Or, $8p^2 + 2p - 1 = 0$

Or, $8p^2 + 4p - 2p - 1 = 0$

Or, $4p(2p + 1) - 1(2p + 1) = 0$

Or, $(2p + 1)(4p - 1) = 0$

$\therefore$ $p = -\frac{1}{2}, \frac{1}{4}$

$p = -\frac{1}{2}$ is not possible $\therefore p = \frac{1}{4}$

20. In a binomial distribution

Let $x \sim B(n, p)$ where x = no of success

$n = 6$

$p(x = 3) = 0.2457$

$p(x = 4) = 0.0818$

Now,

$p(x = 3) = c(6, 3)\, p^3 q^3 = 0.2457$ .... (i)

$p(x = 4) = c(6, 4)\, p^4 q^2 = 0.0818$ .... (ii)

Divide (i) by (ii)

$$\frac{C(6, 3)\, p^3 q^3}{C(6, 4)\, p^4 q^2} = \frac{0.2457}{0.0818}$$

Or, $\frac{20\,q}{15\,p} = 3.003$

Or, $4q = 3p \times 3.003$

Or, $4(1 - p) = 9.009\,p$

Or, $4 - 4p = 9.009\,p$

Or, $4 = 13.009\,p$

Or, $p = \frac{4}{13.009}$

$\therefore$ $p = 0.307$

$q = 1 - p = 1 - 0.307 = 0.619$

Mean $= n\,p = 6 \times 0.307 = 1.842$

Variance $= n\,p\,q = 6 \times 0.307 \times 0.619 = 1.14$

21. The mean and variance

Let $x \sim B(n, p)$

$E(x) = np = 3$ (i)

$V(x) = npq = 2$ (ii)

Substitute value of np from (i) in (ii)

$3q = 2 \qquad$ Or, $\quad q = \frac{2}{3}$

$p = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}$

substitute the value of p in (i)

$n \times \frac{1}{3} = 3 \qquad$ Or, $\quad n = 9$

(i) $p(x \le 2) = p(x = 0) + p(x = 1) + p(x = 2)$

$= C(9, 0)\, p^0 q^9 + C(9, 1)\, p^1 q^8 + C(9, 2)\, p^2 q^7$

$= 1 \times \left(\frac{2}{3}\right)^9 + 9 \times \frac{1}{3} \times \left(\frac{2}{3}\right)^8 + 3\,C \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^7$

$= 0.026 + 0.117 + 0.234 = 0.377$

(ii) $p(x \ge 7) = p(x = 7) + p(x = 8) + p(x = 9)$

$= C(9, 7)\, p^7 q^2 + C(9, 8)\, p^8 q^1 + C(9, 9)\, p^9 q^0$

$= 36 \times \left(\frac{1}{3}\right)^7 \left(\frac{2}{3}\right)^2 + 9 \times \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right) + 1 \times \left(\frac{1}{3}\right)^9$

$= \left(\frac{1}{3}\right)^7 \left[16 + 2 + \frac{1}{9}\right] = \frac{18.111}{2187} = 0.0082$

22. 12% of the items

Let, defective items = x

Prob. of defective p = 12% = 0.12, q = 1 – p = 0.88

n = 20

$p(x = 5) \quad = c(20, 5)\, p^5 q^{15} = c(20, 5)\, (0.12)^5 (0.88)^{15} \; = 0.056$

23. The average no.

Let x = defective pieces

Prob. Of defective pieces (p) $= \frac{1}{10}$, $q = \frac{9}{10}$

n = 10

$p(x = 3) = c(10, 3)\, p^3 q^7 = c(10, 3) \left(\frac{1}{10}\right)^3 \left(\frac{1}{10}\right)^9 = 120 \times 0.001 \times 0.4782 = 0.057$

24. Let x = defective computers

P = 5% = 0.05, q = 0.95

n = 20,

$P(X = 3) = {}^{20}C_3\, (0.05)^3\, (0.95)^{17} = 1140 \times 0.0000522 = 0.059$

In a bombing

Prob. of hitting target p = 50% = 0.5

q = 1 – p = 0.5

No. of times hitting target = x

No. of trial n = ?

(i) $P(x \geq 1) \geq 99.9\%$

Or, $1 - P(x < 1) \geq 99.9\%$

Or, $1 - P(x = 0) \geq 0.999$

Or, $1 - C(n, 0)\ p^0 q^n \geq 0.999$

Or, $1 - (0.5)^n \geq 0.999$

Or, $1 - 0.999 \geq (0.5)^n$

Or, $0.001 \geq (0.5)^n$

By trial method n = 10

(ii) $P(x \geq 2) \geq 99.9\%$

Or, $1 - P(x < 2) \geq 99.9\%$

Or, $1 - [P(x = 0) + P(x = 1)] \geq 0.999$

Or, $1 - [C(n, 0)\ p^0 q^n + C(n - 1)\ p^n q^{n-1}] \geq 0.99$

Or, $1 - [q^n + n\ p\ q^{n-1}] \geq 0.999$

Or, $1 - [(0.5)^n + n\ (0.5)\ (0.5)^{n-1}] \geq 0.999$

Or, $1 - [(0.5)^n + n\ (0.5)^n] \geq 0.999$

Or, $1 - (0.5)^n\ [1 + n] \geq 0.999$

Or, $1 - 0.999 \geq (0.5)^n\ [1 + n]$

Or, $0.001 \geq (0.5)^n\ (1 + n)$

By trial method n = 14

26. Let x = No. of times success

p = probability of hitting target

$p = \frac{1}{3}, q = \frac{2}{3}$

$P(x \geq 1) > 90\%$

Or, $1 - P(X < 1) > 0.9$

Or, $0.1 > C(n, 0) \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^n$

Or, $0.1 > \left(\frac{2}{3}\right)^n$

By trial method n = 6

27. The probability that

Let, x = No. of students graduate

Prob. of students graduate p = 0.4, q = 1 – p = 1 – 0.4 = 0.6

(i) $P(x = 0) = C(n, x)\ p^x q^{n-x} = C(5, 0)\ (0.4)^0 (0.6)^{5-0} = 0.077$

(ii) $P(x = 1) = C(n, x)\ p^x q^{n-x} = C(5, 1)\ (0.4)^1 (0.6)^{5-1} = 0.25$

(iii) $P(x \geq 1) = 1 - P(x < 1) = 1 - P(x = 0) = 1 - 0.077 = 0.923$

(iv) $P(x = 5) = C(n, x)\ p^x q^{n-x} = C(5, 5)\ (0.4)^5 (0.6)^0 = 0.0102$

28. At a particular university

Let x = No. of students with draw without computing course

p = prob. of students withdraw without computing course

p = 20% = 0.2, q = 1 – p = 0.8, n = 18

i. $P(x = 0) = C(18, 0)\ (0.2)^0\ (0.8)^{18-0} = (0.8)^{18} = 0.01$

ii. $P(x \geq 1) = 1 - P(X < 1) = 1 - P(x = 0) = 1 - 0.01 = 0.99$

iii. $P(x \leq 2) = P(x = 0) + P(x = 1) + P(x = 2)$

$= C\ (18, 0)\ (0.2)^0\ (0.8)^{18} + C(18, 1)\ (0.2)^1\ (0.8)^{17} + C(18,2)\ (0.2)^2\ (0.8)^{16}$

$= (0.8)^{16}\{\ 0.64 + 2.88 + 6.12) = (0.8)^{16} \times 9.64 = 0.271$

29. Solution.

When no. of success = 2

No. of failure = 1

Total trial = 2 + 1 = 3

Probability of success (p) = $\frac{2}{3}$ $\therefore q = \frac{1}{3}$

Let x = no. of trial,

n = 6

$P(x \geq 4) = P(x = 4) + P(x = 5) + P(x = 6)$

30. The incidence of

Let x = No. of workers suffering from diseases

Probability of workers suffer from disease (p) = 20% = 0.2

$q = 1 - p = 0.8,\ n = 6,\ P(x \geq 4) = ?$

$P(x \geq 4) = P(x = 4) + P(x = 5) + P(x = 6)$

$= C(6, 4)\ (0.2)^4\ (0.8)^2 + C(6, 5)\ (0.2)^5\ (0.8)^1 + C(6, 6)\ (0.2)^6\ (0.8)^0$

$= 0.016$

31. From the past experience

Let, x = no. of telephone calls which are ordered

Prob. of ordered telephone calls (p) = 70% = 0.7

n = 8

(i) $P(x = 5) = C(n, x)\ p^x\ q^{n-x} = C(8, 5)\ (0.7)^5\ (0.3)^3 = 0.254$

(ii) $P(x \geq 6) = P(x = 6) + P(x = 7) + P(x = 8)$

$= C(8, 6)\ (0.7)^6\ (0.3)^2 + C(8, 7)\ (0.7)^7\ (0.3)^1 + C(8, 8)\ (0.7)^8$

$= (0.7)^6\ [28 \times 0.09 + 8 \times 0.21 + 0.49] = 0.551$

32. A discrete

Let $x \sim B(n, p)$

$E(x) = np = 6$ .... (i)

$V(x) = n\ pq = 2$ .... (ii)

Substitute value of np from (i) in (ii)

$6q = 2$

Or, $q = \frac{1}{3}$ $\therefore p = \frac{2}{3}$

Substitute p in (i)

$n \times \frac{2}{3} = 6$

or, $n = \frac{18}{2} = 9$

$P(5 < x < 7) = P(x = 6) = C(n, x)\, p^x q^{n-x}$

$$= C(9, 6)\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^{9-6} = 84 \times 0.087 \times 0.037 = 0.27$$

33. X = No. of program must be upgrade

Prob. of upgrading program (p) $= \frac{5}{12}$; $q = 1 - \frac{5}{12} = \frac{7}{12}$

$n = 4$

(i) $P(X \ge 2) = 1 - P(X < 2)$

$= 1 - [P(x = 0) + P(x = 1)]$

$$= 1 - \left[{}^4C_0\left(\frac{5}{12}\right)^0\left(\frac{7}{12}\right)^4 + {}^4C_1\left(\frac{5}{12}\right)^1\left(\frac{7}{12}\right)^3\right]$$

$$= 1 - \left[\left(\frac{7}{12}\right)^4 + 4\,\frac{5}{12} \times \left(\frac{7}{12}\right)^3\right]$$

$$= 1 - \left(\frac{7}{12}\right)^3 \times \frac{27}{12} = 1 - (0.583)^3 = 0.554$$

$E(x) = np = 4 \times 0.554 = 2.216$

34. Let X = No. of player buy advanced version of game

Prob. of player buy advanced version game (b) = 40% of 50% $= \frac{40}{100} \times \frac{50}{100} =$ 0

$q = 0.8,\ n = 12$

$E(x) = np = 12 \times 0.2 = 2.4 \approx 2$

$P(X \ge 2) = 1 - P(X < 2) = 1 - [P(x = 0) + P(X = 1)]$

$= 1 - [{}^{12}C_0(0.2)^0\,(0.8)^{12} + 12C_1(0.2)^1\,(0.8)^{11}$

$= 1 - (0.8)^{11} \times 3.2 = 0.725$

$P(X \le 3) = P(X = 0) + P(X = 11) + P(X = 2) + P(X = 3)$

$= {}^{12}C_0\,(0.2)^0\,(0.8)^{12} + {}^{12}C_1\,(0.2)^1\,(0.8)^{11} + {}^{12}C_2\,(0.2)^2\,(0.8)^{10} + {}^{12}C_3\,(0.2)^3\,(0.8)^9$

$= (0.8)^{12} + 12 \times 0.2 \times (0.8)^{11} + 66 \times 0.04 \times (0.8)^{10} + 220 \times 0.08 \times (0.8)^9$

$= 0.794$

35. An Electronic device

Let, x = no. of items

i. Average no. of items (λ) = 3

$$P(x = 0) = \frac{e^{-3}\,3^0}{0\,!} = 0.049$$

ii. Average no. of items per 3 minute (λ) = 3 × 3 = 9

$P(x \le 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4)$

$$= \frac{e^{-9}\,9^0}{0\,!} + \frac{e^{-9}\,9^1}{1\,!} + \frac{e^{-9}\,9^2}{2\,!} + \frac{e^{-9}\,9^3}{3\,!} + \frac{e^{-9}\,9^4}{4\,!}$$

$$= e^{-9}\left[1 + 9 + \frac{81}{2} + \frac{243}{6} + \frac{6561}{24}\right]$$

$$= e^{-9} \times 364.375 = 0.044$$

A car hire firm have

Average (λ) = 1.5

Let, x = No. of cars

$P(x = 0) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-1.5} (1.5)^0}{0!} = 0.223$

$P(x > 2) = 1 - P(x \leq 2)$

$= 1 - [P(x = 0) + P(x = 1) + P(x = 2)]$

$= 1 - \left[\frac{e^{-1.5} (1.5)^0}{0!} + \frac{e^{-1.5} (1.5)^1}{1!} + \frac{e^{-1.5} (1.5)^2}{2!}\right]$

$= 1 - [0.223 + 0.3495 + 0.2508] = 1 - 0.823 = 0.17$

37. An automatic machine

Let, x = no. of defectives

Prob. of defective (p) = $\frac{1}{400}$

n = 100

Average defective (λ) = n p = $100 \times \frac{1}{400} = 0.778$

i. $P(x = 0) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-0.25} (0.25)^0}{0!} = 0.778$

ii. $P(x \geq 1) = 1 - P(x < 1) = 1 - P(x = 0) = 1 - 0.778 = 0.221$

iii. $P(x < 2) = P(x \leq 1) = P(x = 0) + P(x = 1)$

$= 0.778 + \frac{e^{-0.25} (0.25)^1}{1!} = 0.778 + 0.194 = 0.972$

38. The chance of

Let x = No. of traffic accidents

Probability of traffic accident in a street (p) = 0.0005

No. of street (n) = 1000

Now,

$\lambda = n p = 1000 \times 0.0005 = 0.5$

i. $P(x = 0) = \frac{e^{-0.5} (0.5)^0}{0!} = 0.606$

Consider in a year

No. of days with no accidents = $365 \times P(x = 0)$

$= 365 \times 0.606 = 221.38 = 221$

iii. $P(x > 3) = 1 - P(x \leq 3)$

$= 1 - [P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)]$

$= 1 - \{\left[\frac{e^{-0.5} (0.5)^0}{0!} + \frac{e^{-0.5} (0.5)^1}{1!} + \frac{e^{-0.5} (0.5)^2}{2!}\right] + e^{-0.5}(0.5)^3/3!\}$

$= 1 - e^{-0.5}\{1 + 0.5 + 0.125 + 0.0208\}) = 1 - 1.645\, e^{-0.5} = 0.003$

No. of days with more then three accidents = 365 × P(x > 3)

= 365 x 0.003 = 1.14 = 1

39. Let λ = No. of telephone calls

$\lambda = 3$ per minute

$$P(X = 0) = \frac{e^{-3}\,3^0}{0!} = 0.049$$

$\lambda = 3 \times 3 = 9$ per three minute

$$P(X \le 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$$

$$= \frac{e^{-9}\,9^0}{0!} + \frac{e^{-9}\,9^1}{1!} + \frac{e^{-9}\,9^2}{2!} + \frac{e^{-9}\,9^3}{3!} + \frac{e^{-9}\,9^4}{4!}$$

$$= 0.945$$

40. The average

Let x = No. of network error

Average no. network error (λ) = 2.4

i. $P(x = 0) = \frac{e^{-2.4}\,(2.4)^0}{0\,!} = 0.09$

ii. $P(x \ge 1) = 1 - P(x < 1) = 1 - P(x = 0) = 0.91$

iii. $P(x = 1) = \frac{e^{-2.4}\,(2.4)^1}{1\,!} = 0.21$

41. **Solution.**

Let x = No. of messages arises

$\lambda = 9$ per hour

(a) $P(x \ge 3) = 1 - P(X < 3) = 1 - \left[\frac{e^{-9}\,9^0}{0!} + \frac{e^{-9}\,9^1}{1!} + \frac{e^{-9}\,9^2}{2!}\right]$

$$= 0.997$$

(b) $\lambda = 9 \times 2 = 18$ per hour

$$P(x = 5) = \frac{e^{-18}\,18^5}{5!} = 0.066$$

42. A hospital has

x = no. of emergency calls

Average no. of emergency calls (λ) = 4 per 10 min.

i. $\lambda = \frac{4}{10} = 0.4$ per min.

$$P(x \le 2) = P(x = 0) + P(x = 1) + P(x = 2)$$

$$= \frac{e^{-0.4}\,(0.4)^0}{0\,!} + \frac{e^{-0.4}\,(0.4)^1}{1\,!} + \frac{e^{-0.4}\,(0.4)^2}{2\,!} = 0.67 + 0.268 + 0.053 = 0.99$$

ii. $\lambda = 4$ calls per 10 min.

$$P(x = 3) = \frac{e^{-\lambda}\,\lambda^x}{x!} = \frac{e^{-4}\,4^3}{3\,!} = 0.195$$

43. If A random variable

$x \sim P(\lambda)$

Here, $P(x = 1) = P(x = 2)$

Or, $\frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$

Or, $\lambda = \frac{\lambda^2}{2}$

Or, $\lambda = 2$

$\therefore$ Mean $= E(x) = \lambda = 2$

Variance $= V(x) = \lambda = 2$

44. Calculate mean and

$x \sim P(x)$

$P(x = 4) = P(x = 5)$

Or, $\frac{e^{-\lambda}\lambda^4}{4!} = \frac{e^{-\lambda}\lambda^5}{5!}$

Or, $\lambda = 5$

$\therefore$ Mean $= E(x) = \lambda = 5$

Variance $= V(x) = \lambda = 5$

45. A manufacturer of pen drive

Let x = no. of defective

Probability of defective (p) = 3% = 0.03

No. of boxes (n) = 200

Average no. of defective (λ) = n p = 200 × 0.03 = 6

$P(x \le 2) = P(x = 0) + P(x = 1) + P(x = 2)$

$= \frac{e^{-6}6^0}{0!} + \frac{e^{-6}6^1}{1!} + \frac{e^{-6}6^2}{2!} = e^{-6}[1 + 6 + 18] = e^{-6} + 25 = 0.061$

46. Let λ = no. of field affected by virus

$n = 250$

$p = 0.032$

$\lambda = np = 250 \times 0.032 = 8$

$P(X > 5) = ?$

$P(X > 5) = 1 - P(X \le 5)$

$= 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)]$

$= 1 - \left[\frac{e^{-8}8^0}{0!} + \frac{e^{-8}8^1}{1!} + \frac{e^{-8}8^2}{2!} + \frac{e^{-8}8^3}{3!} + \frac{e^{-8}8^4}{4!} + \frac{e^{-8}8^5}{5!}\right]$

$= 0.808$

Let x = No. of computer crashed

$p = \frac{1}{1000}$; $n = 5000$, $\lambda = r\,p = 5000 \times \frac{1}{1000} = 5$

(i) $P(X < 3)$

$= P(X = 0) + P(X = 1) + P(X = 2)$

$= \frac{e^{-5} 5^0}{0!} + \frac{e^{-5} 5^1}{1!} + \frac{e^{-5} 5^2}{2!} = 0.124$

$P(X = 10) = \frac{e^{-5} 5^{10}}{10!} = 0.018$

47. A manufacturer produces IC chips

Let x = No. of IC chips defective

Probability of IC chips is defective (p) = 1% = 0.01

No. of boxes (n) = 100

$\lambda = n\,p = 100 \times 0.01 = 1$

$P(x = 0) = \frac{e^{-1} 1^0}{0\,!} = 0.36$

48. The probability of

Let x = No. of error in transmission of a bit

Probability of error in transmission of a bit (p) = 0.001

No. of bit (n) = 1000

$\lambda = n\,p = 1000 \times 0.001 = 1$

$P(x > 3) = 1 - P(x \le 3)$

$= 1 - [P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)]$

$= 1 - \left[\frac{e^{-1} 1^0}{0\,!} + \frac{e^{-1} 1^1}{1\,!} + \frac{e^{-1} 1^2}{2\,!} + \frac{e^{-1} 1^3}{3\,!}\right]$

$= 1 - e^{-1}\left[1 + 1 + \frac{1}{2} + \frac{1}{6}\right] = 1 - 0.367 \times 2.66 = 0.021$

49. X = No. of electronic message; $\lambda = 9$

$P(X \ge 3) = 1 - P(X < 5)$

$= 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$

$= 1 - \left[\frac{e^{-9} 9^0}{0!} + \frac{e^{-9} 9^1}{1!} + \frac{e^{-9} 9^2}{2!}\right]$

$= 1 - e^{-9} [1 + 9 + 9]$

$= 0.997$

$P(X = 5) = \frac{e^{-9} 9^5}{5!} = 0.066$

50. Prob. that computer crash (p) $= \frac{1}{1000}$

$n = 5000;\ \lambda = n\,p = 5000 \times \frac{1}{1000} = 5$

a. $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = \left[\frac{e^{-5} 5^0}{0!} + \frac{e^{-5} 5^1}{1!} + \frac{e^{-5} 5^2}{2!}\right] = 0.124$

b. $P(\lambda = 10) = \frac{e^{-5} 5^{10}}{10\,!} = 0.018$

Suppose that the

Let, x = no. of calls between 10 a. m. and 11 a. m.

y = no. of calls between 11 a. m. and 12 noon.

$x \sim P(\lambda);\ \lambda = 2$

$y \sim P(\lambda);\ \lambda = 4$

$z = x + 4$ = no. of calls between 10 a. m. and 12 noon

$z \sim P(\lambda) \qquad \lambda = 2 + 4 = 6$

$P(z \geq 5) = 1 - P(z < 5)$

$= 1 - [P(z = 0) + P(z = 1) + P(z = 2) + P(z = 3) + P(z = 4)]$

$$= 1 - \left[\frac{e^{-6}\,6^0}{0\,!} + \frac{e^{-6}\,6^1}{1\,!} + \frac{e^{-6}\,6^2}{2\,!} + \frac{e^{-6}\,6^3}{3\,!} + \frac{e^{-6}\,6^4}{4\,!}\right]$$

$= 1 - [0.0024 + 0.0148 + 0.0446 + 0.0892 + 0.133]$

$= 1 - 0.2848 = 0.7151$

52. A source of

Mean no. of bacteria per c.c. $(\lambda) = 3$

No. of text tube of capacity 1 c. c. $(n) = 10$

Here, $\lambda = n\,p$

Or, $p = \frac{\lambda}{n} = \frac{3}{10},\ q = 1 - p = \frac{7}{10}$

Let x = No. of bacteria

i. $P(x \geq 1) = 1 - P(x < 1) = 1 - P(x = 0)$

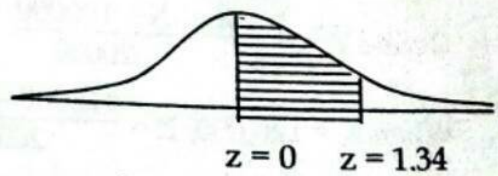
$= 1 - C(10, 0)\,(0.3)^0\,(0.7)^{10-0} = 1 - 0.7^{10} = 0.971$

ii. $P(x = 7) = C(10, 7)\,(0.3)^7\,(0.7)^{10-7} = 120 \times (0.3)^7 \times (0.7)^3 = 0.009$

53. If Z is standard

$Z \sim N(0, 1)$

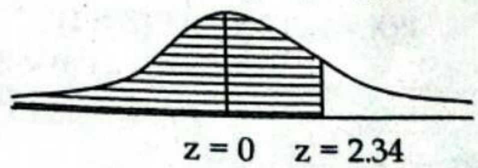
$P(0 < Z < 1.34) = 0.4099$



$P(Z < 1.34) = 0.5 + P(0 < 2 < 1.34)$

$= 0.5 + 0.4099$

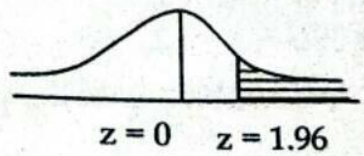
$= 0.9099$



$P(Z > 1.96) = 0.5 - P(0 < Z < 1.96)$

$= 0.5 - 0.4750$

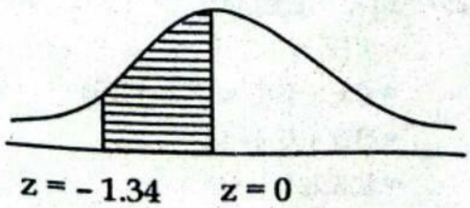
$= 0.025$



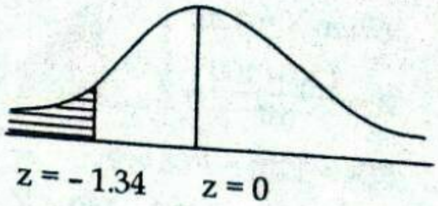
$P(1.34 < Z < 0)$

$= P(0 < Z < 1.34)$

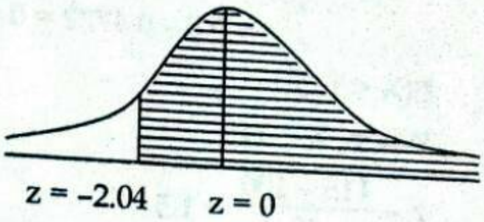
$= 0.4099$



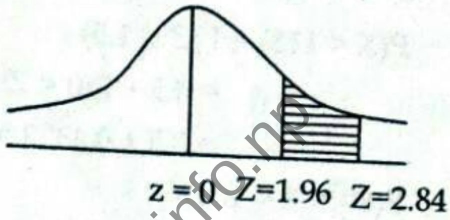
e. $P(Z < -1.34)$
$= 0.5 - P(-1.34 < Z < 0)$
$= 0.5 - P(0 < Z < 1.34)$
$= 0.5 - 0.4099 = 0.0901$



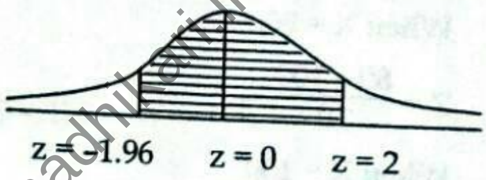
f. $P(Z > -2.04)$
$= P(-2.04 < Z < 0) + 0.5$
$= P(0 < Z < 2.04) + 0.5$
$= 0.4793 + 0.5 = 0.9793$



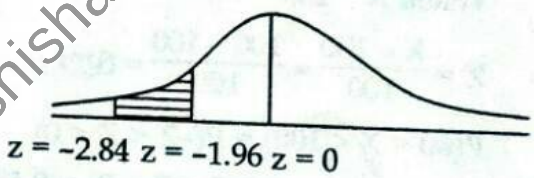
g. $P(1.96 < Z < 2.84)$
$= P(0 < Z < 2.84) - P(0 < Z < 1.96)$
$= 0.4977 - 0.4750$
$= 0.0227$



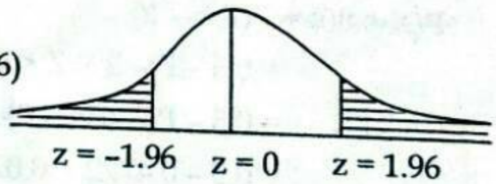
h. $P(-1.96 < Z < 2)$
$= P(-1.96 < Z < 0) + P(0 < Z < 2)$
$= P(0 < Z < 1.96) + P(0 < Z < 2)$
$= 0.4750 + 0.4772$
$= 0.9522$



i. $P(-2.48 < Z < -1.96)$
$= P(-2.48 < Z < 0) - P(-1.96 < Z < 0)$
$= P(0 < Z < 2.84) - P(0 < Z < 1.96)$
$= 0.4977 - 0.4750$
$= 0.0227$

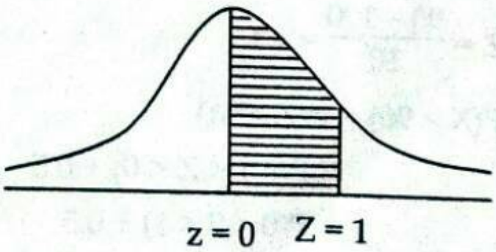


j. $P(Z < -1.96 \text{ or } Z > 1.96)$
$= 0.5 - P(-1.96 < Z < 0) + 0.5 - P(0 < Z < 1.96)$
$= 1 - 2P(0 < Z < 1.96)$
$= 1 - 2 \times 0.4750$
$= 1 - 0.95 = 0.05$



56. Suppose X follows the
$\mu = 100, \sigma = 10$
$X \sim N(\mu, \sigma^2)$

a. $P(100 < X < 110) = ?$



Define $Z = \dfrac{X - \mu}{\sigma} = \dfrac{X - 100}{10}$

When $X = 100$, $Z = \dfrac{100 - 100}{10} = 0$

When $X = 110$, $Z = \dfrac{110 - 100}{10} = 1$

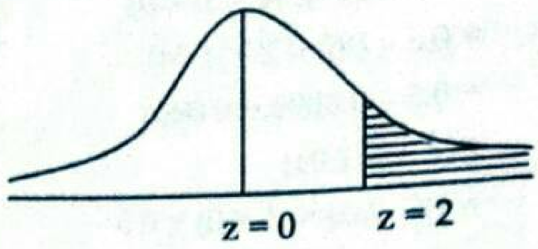
$P(100 < X < 110) = P(0 < Z < 1)$
$= 0.3413$

b. $P(X > 120)$

When $X = 120$,

$$Z = \frac{120 - 100}{10} = 2$$

$$P(X > 120) = P(Z > 2)$$
$$= 0.5 - P(0 < Z < 2)$$
$$= 0.5 - 0.4772 = 0.0228$$

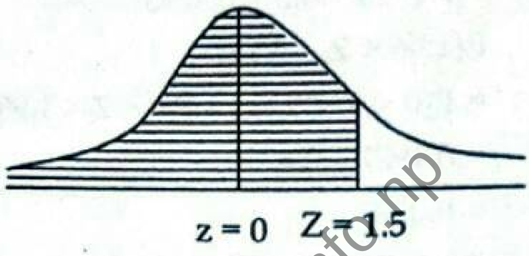


c. $P(X < 115)$

When, $X = 115$

$$Z = \frac{115 - 100}{10} = 1.5$$

$$P(X < 115) = P(Z < 1.5)$$
$$= 0.5 + P(0 < Z < 1.5)$$
$$= 0.5 + 0.4332 = 0.8332$$



d. $P(80 < X < 100)$

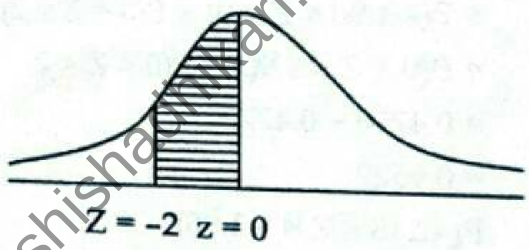
When $X = 80$,

$$Z = \frac{80 - 100}{10} = 2$$

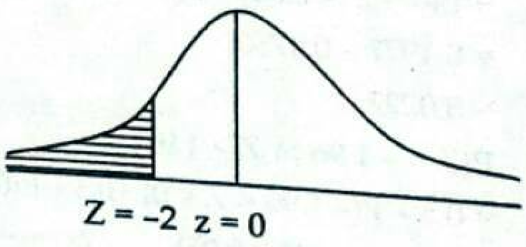
When $X = 100$

$$Z = \frac{X - 100}{100} = \frac{100 - 100}{100} = 0$$

$$P(80 < X < 100) = P(-Z < Z < 0)$$
$$= P(0 < Z < 2) = 0.4772$$



e. $P(X < 80) = P(Z < -2)$
$$= 0.5 - P(-2 < Z < 0)$$
$$= 0.5 - P(0 < Z < 2)$$
$$= 0.5 - 0.4772 = 0.028$$

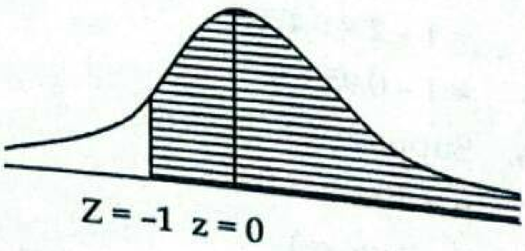


f. $P(X > 90)$

When $X = 90$

$$Z = \frac{90 - 100}{10} = -1$$

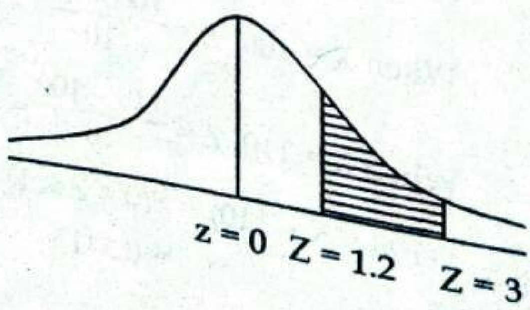
$$P(X > 90) = P(Z > -1)$$
$$= P(-1 < Z < 0) + 0.5$$
$$= P(0 < 2 < 1) + 0.5$$
$$= 0.3413 + 0.5 = 0.8413$$



g. $P(112 < X < 130)$

When $X = 112$, $Z = \frac{112 - 100}{10} = 1.2$

When $X = 130$, $Z = \frac{130 - 100}{10} = 3.0$

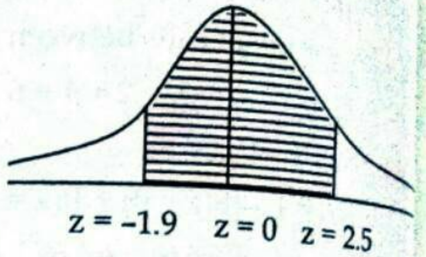


$P(112 < X < 130) = P(1.2 < Z < 3)$
$= P(0 < Z < 3) - P(0 < Z < 1.2)$
$= 0.49865 - 0.3849$
$= 0.11375$

h. $P(85 < X < 125)$

When $X = 85, Z = \dfrac{85 - 100}{10} = -1.5$

When $X = 125, Z = \dfrac{125 - 100}{10} = 2.5$

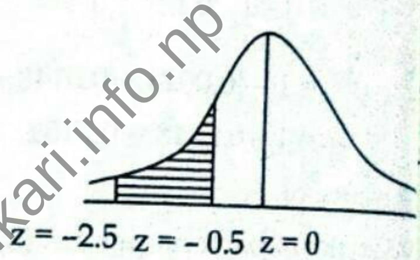


$P(85 < X < 125) = P(-1.5 < Z < 2.5)$
$= P(-1.5 < Z < 0) + P(0 < Z < 2.5)$
$= 0.4332 + 0.4938 = 0.9270$

i. $P(75 < X < 95)$

When $X = 75, Z = \dfrac{75 - 100}{10} = -2.5$

When $X = 95, Z = \dfrac{95 - 100}{10} = -0.5$



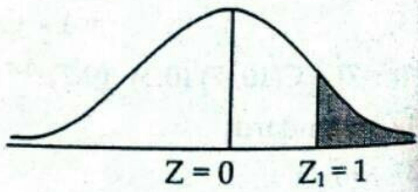
$P(75 < X < 95) = P(-2.5 < Z < -0.5)$
$= P(-2.5 < Z < 0) - P(-0 < Z < 0)$
$= P(0 < Z < 2.5) - P(0 < Z < 0.5)$
$= 0.4938 - 0.1915 = 0.3023$

57. **Solution.**

Let x = monthly production of computer parts $x \sim N(\mu, \sigma^2)$
$\mu = 100{,}000$
$\sigma = 20{,}000$

(i) $P(x > 120{,}000)$

Define $Z = \dfrac{X - \mu}{\sigma} = \dfrac{X - 100000}{20000}$



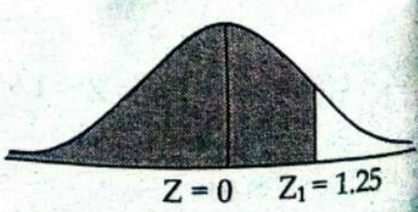
When $X = 120{,}000, Z = \dfrac{120000 - 100000}{20000} = 1$

$\therefore$ $P(X > 120{,}000) = P(Z > 1)$
$= 0.5 - P(0 < Z < 1)$
$= 0.5 - 0.3413 = 0.1587$

(ii) $P(x < 125000)$

When $X = 125{,}000$

$Z = \dfrac{125000 - 100000}{20000} = 1.25$

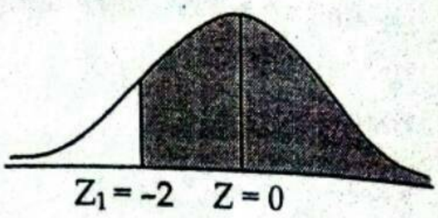


$\therefore$ $P(X < 125000)$
$= P(Z < 1.25)$
$= 0.5 + P(0 < Z < 1.25)$
$= 0.5 + 0.3944$
$= 0.8944$

) $P(x > 60{,}000)$

When $X = 60{,}000$

$$Z = \frac{60000 - 100000}{20000} = -2$$



$P(X > 60{,}000)$

$= P(Z > -2)$

$= P(-2 < Z < 0) + 0.5$

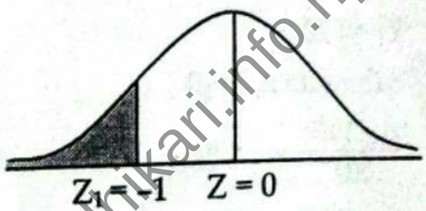
$= 0.4772 + 0.5$

$= 0.9772$

(iv) $P(X < 80{,}000)$

When $X = 80{,}000$

$$Z = \frac{80000 - 100000}{20000} = -1$$



$P(X < 80{,}000)$

$= P(Z < -1)$

$= 0.5 - P(-1 < Z < 0)$

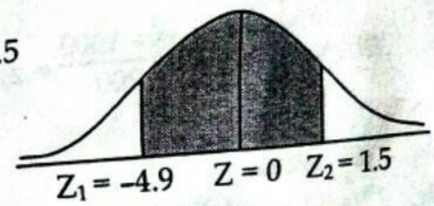
$= 0.5 - P(0 < Z < 1)$

$= 0.5 - 0.3413 = 0.1587$

(v) $P(1050 < X < 130000)$

When $X = 1050$, $Z_1 = \dfrac{1050 - 100000}{20000} = -4.9$

When $X = 130000$, $Z_2 = \dfrac{130000 - 100000}{20000} = 1.5$



$P(1050 < X < 130000)$

$= P(-4.9 < Z < 1.5) + P(0 < Z < 1.5)$

$= P(0 < Z < 4.9) + P(0 < Z < 1.5)$

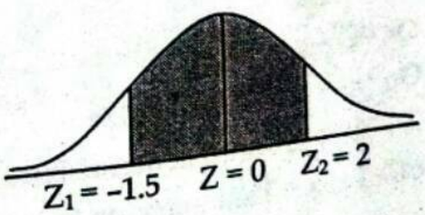
$= 0.5 + 0.4332$

$= 0.9332$

(vi) $P(70{,}000 < X < 140{,}000)$

When $X = 70{,}000$; $Z_1 = \dfrac{70000 - 100000}{20000} = -1.5$

When $X = 140{,}000$; $Z_2 = \dfrac{140000 - 100000}{20000} = 2$



$P(70{,}000 < X < 140{,}000)$

$= P(-1.5 < Z < 2)$

$= P(-1.5 M < Z < 0) + P(0 < Z < 2)$

$= P(0 < Z < 1.5) + P(0 < Z < 2)$

$= 0.4332 + 0.4772$

$= 0.9104$

(vii) $P(75{,}000 < X < 95{,}000)$

When $X = 75{,}000$; $Z_1 = \dfrac{75000 - 100000}{20000} = -1.25$

When $X = 55{,}000$; $Z_2 = \dfrac{95000 - 100000}{20000} = -0.25$

$P(75{,}000 < X < 95{,}000)$

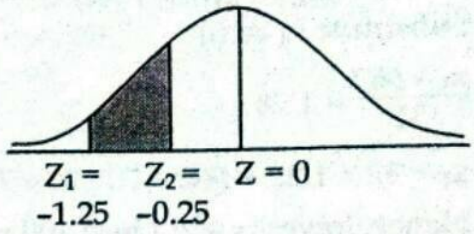
$= P(-1.25 < Z < -0.25)$

$= P(-1.25 < Z < 0) - P(-0.25 < Z < 0)$

$= P(0 < Z < 1.25) - P(0 < Z < 0.25)$

$= 0.3944 - 0.0987$

$= 0.2957$



58. The mean yield

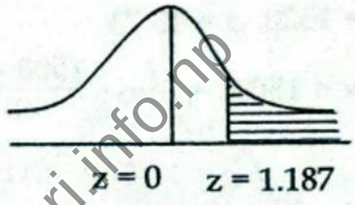
Let, x = yield in kilos

$x \sim N(\mu, \sigma^2)$, $\mu = 662$, $\sigma = 32$, $N = 1000$

i. $P(x > 700)$

Define, $Z = \dfrac{x - \mu}{\sigma} = \dfrac{x - 662}{32}$

When, $x = 700$, $z = \dfrac{700 - 662}{32} = 1.87$



$P(x > 700) = P(z > 1.187)$

$= 0.5 - P(0 < z < 1.187)$

$= 0.5 - 0.381 = 0.119$

$\therefore$ No. of plot $= N\,P(x > 700) = 1000 \times 0.119 = 119$

ii. $P(x < 650)$

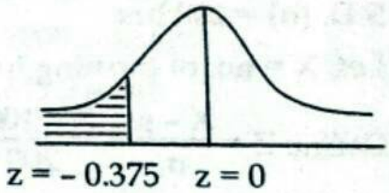
When, $x = 650$, $z = \dfrac{x - 662}{32} = \dfrac{650 - 662}{32} = -0.375$

$P(x < 650) = P(z < -0.375)$

$= 0.5 - P(-0.375 < z < 0.375)$

$= 0.5 - 0.144$

$= 0.356$



$\therefore$ No. of puts $= N\,P(x < 650) = 1000 \times 0.356 = 356$

ii. Let, lowest yield $= x_1$

$P(x \geq x_1) = \dfrac{100}{1000}$

Or, $P(x \geq x_1) = 0.1$

When $x = x_1$

$z = \dfrac{x_1 - 662}{32} = z_1$ (say) .... (i)

Then,

$P(x \geq x_1) = 0.1$

$\Rightarrow P(z \geq z_1) = 0.1$

$\Rightarrow 0.5 - P(0 \leq z \leq z_1) = 0.1$

$\Rightarrow 0.5 - 0.1 = P(0 \leq z \leq z_1)$

$\Rightarrow P(0 \leq z \leq z_1) = 0.4$

Then $z_1 = 1.28$

Substitute z_1 in (i)

$$\frac{x_1 - 662}{32} = 1.28$$

Or, $x_1 = 32 \times 1.28 + 662 = 702.96 \approx 703$

Hence, lowest yield f best 100 plots is 703 kilos.

59. Let x = time taken to download, n = 95

For one file

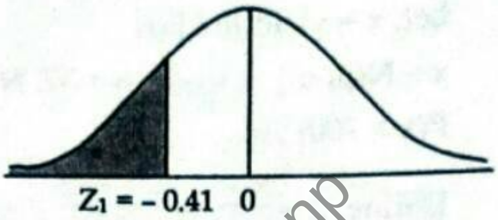
$\mu = 16, \sigma = 5, \sigma^2 = 25$

For 95 file

$\mu = 16 \times 95 = 1520$

$\sigma^2 = 25 \times 95 = 2375 \qquad \therefore \sigma = 48.73$

$\mu = 1520, \sigma = 48.73$



$$P(x < 1500) = P\left(z \le \frac{1500 - 1520}{48.73}\right)$$
$$= P(Z < -0.41)$$
$$= 6.5 - P(-0.41 < z < 0)$$
$$= 0.5 - 0.1591$$
$$= 0.3409$$

60. The local authorities

No. of lamps (N) = 1000

Average life of lamp (μ) = 1000 hrs

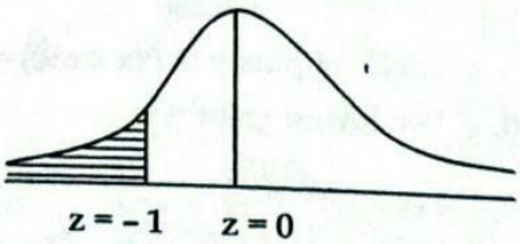
S.D. (σ) = 200 hrs

Let, X = no. of burning hrs of lamp

Define $Z = \frac{X - \mu}{\sigma} = \frac{X - 1000}{200}$

i. $P(X < 800)$

When, $X = 800$, $Z = \frac{800 - 1000}{200} = -1$



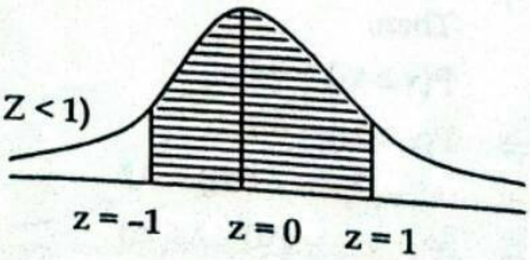
$$P(X < 800) = P(Z < -1)$$
$$= 0.5 - P(-1 < Z < 0)$$
$$= 0.5 - 0.3413 = 0.1587$$

No. of bulb burn fail in 800 hrs = N P(X < 800) = $1000 \times 0.1587 = 158.7 \approx 159$

ii. $P(800 < X < 1200)$

When $X = 1200$, $Z = \frac{1200 - 1000}{200} = 1$

$$P(800 < X < 1200) = P(-1 < Z < 1)$$
$$= P(-1 < Z < 0) + P(0 < Z < 1)$$
$$= 2\,P(0 < Z < 1)$$
$$= 2 \times 0.3413$$
$$= 0.6826$$



No. of bulb burn between 800 and 1200 hrs

$= N\,P(800 < X < 1200) = 1000 \times 0.6826 = 682.6 \approx 683$

Let, after $x = x_1$ hrs 10% lamp fail

$P(x < x_1) = 10\%$

When, $x = x_1$

$Z = \dfrac{x_1 - 1000}{200} = -Z_1$ (say) .... (i)

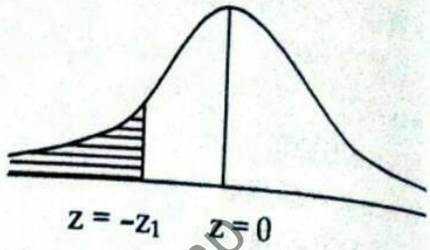
$P(x < x_1) = 0.1$

Or, $P(z < -z_1) = 0.1$

Or, $0.5 - P(-z_1 < z < 0) = 0.1$

Or, $0.5 - 0.1 = P(0 < z < z_1)$

Or, $P(0 < Z < Z_1) = 0.4$



$Z_1 = 1.28$

Substitute z_1 in (i)

$\dfrac{x_1 - 1000}{200} = -1.28$

Or, $x_1 - 1000 = -256$

Or, $x_1 = 1000 - 256 = 744$

∴ 10% lamps fails on burning 744 hrs

Let after $x = x_2$ hrs. 10% lamp continue burning

$P(x > x_2) = 10\%$

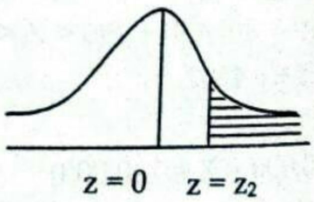
When $x = x_2$, $z = \dfrac{x_2 - 1000}{200} = z_2$ (say) .... (ii)

$P(x > x_2) = 0.1$

Or, $P(z > z_2) = 0.1$

Or, $0.5 - 0.1 = P(0 < z < z_2)$

Or, $P(0 < z < z_2) = 0.4$



$Z_2 = 1.28$

Substitute z_2 in (ii)

$\dfrac{x_2 - 1000}{200} = 1.28$

Or, $x_2 - 1000 = 256$

Or, $x_2 = 1000 + 256 = 1256$

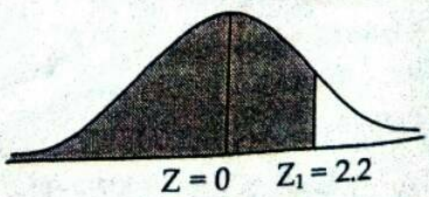
∴ 10% of lamps continue burning after 1256 hrs.

61. Let x = Time spent on training $X \sim N(\mu, \sigma^2)$

$\mu = 500, \sigma = 100$

$P(X < 420) = ?$

Let $Z = \dfrac{X - \mu}{\sigma} = \dfrac{X - 200}{100}$



When $X = 4200$, $Z = \dfrac{420 - 200}{100} = \dfrac{220}{100} = 2.2$

$P(X < 420) = P(Z < 2.2) = 0.5 + P(0 < Z < 2.2) = 0.5 + 0.4861 = 0.9861$

62. **Solution.**

Let x = life time of electronic component

$x \sim N(\mu, \sigma^2)$

$\mu = 5000, \sigma = 100$

$P(X < 5012) = ?$

$P(4000 < X < 6000) = ?$

$P(X > 7000) = ?$

Define $Z = \dfrac{X - \mu}{\sigma} = \dfrac{X - 5000}{100}$

When X = 5012

$Z = \dfrac{5012 - 5000}{100} = 0.12$

$P(X < 5012) = P(Z < 0.12) = 0.452$

When X = 4000

$Z = \dfrac{4000 - 5000}{100} = -10$

When X = 6000

$Z = \dfrac{6000 - 5000}{100} = 10$

$P(4000 < X < 6000) = P(-10 < Z < 10) = 0.998$

When X = 7000

$Z = \dfrac{7000 - 5000}{100} = 20$

$P(X > 7000) = P(Z > 20) \doteq 0$

63. Let, y = length of rod

$X \sim N(\mu, \sigma^2)$

$\mu = 50, \sigma = 0.06$

Prob. of rod accepted = $P(49.85 \le X \le 50.15)$

Let $Z = \dfrac{X - \mu}{\sigma} = \dfrac{X - 50}{0.06}$

When X = 49.85, $Z = \dfrac{49.85 - 50}{0.06} = -2.5$

When X = 50.15, $Z = \dfrac{50.15 - 50}{0.06} = 2.5$

$P(49.85 \le X \le 50.15$

$= P(-2.5 \le Z \le 2.5)$

$= P(-2.5 \le Z \le 0) + P(0 \le Z \le 2.5)$

$= 2P(0 \le Z \le 2.5)$

$= 2 \times 0.4938$

$= 0.9876$

Hence 98.76% rods are accepted.

Prob. of rod rejected = $P(X < 49.85) + P(x > 50.15)$

$= P(Z < -2.5) + P(Z > 2.5)$

$= 1 - 0.9876$

$= 0.0124$

Hence, 1.24% rods are rejected.

64. Let X = Size of battery

$X \sim N(\mu, \sigma^2)$

$\mu = 40, \sigma = 5$

$N = 1000$

$P(X > 35) = ?$

Let $Z = \frac{X - \mu}{\sigma} = \frac{X - 40}{5}$

When $X = 35$, $Z = \frac{35 - 40}{5} = -1$

$P(X > 35)$

$= P(Z > -1)$

$= P(-1 < Z < 0) + 0.5$

$= P(0 < Z < 1) + 0.5$

$= 0.3413 + 0.5$

$= 0.8413$

No. of cylinder need replacement after 35 days = $N\, P(X > 35)$

$= 1000 \times 0.8413$

$= 841.3 \approx 841$

65. Income of a computer operator

Let X = income

$\mu = 1520, \sigma = 160, X \sim N(\mu, \sigma^2)$

$N = 10{,}000$

% of poor $= \frac{2000}{10000} \times 100\% = 20\%$

% of rich $= \frac{1000}{10000} \times 100\% = 10\%$

Let highest income of poorest 2000 = x_1

Lowest income of richest 1000 = x_2

i. $P(X < x_1) = 20\%$

or, $P(X < x_1) = 0.2$

Define $Z = \frac{X - \mu}{\sigma} = \frac{X - 1520}{160}$

When, $X = x_1$, $Z = \frac{x_1 - 1520}{160} = -z_1$ (say) .... (i)

$P(X < x_1) = 0.2$

Or, $P(Z < -Z_1) = 0.2$

Or, $0.5 - P(-Z_1 < Z < 0) = 0.2$

Or, $0.5 - 0.2 = P(-Z_1 < Z < 0)$

Or, $P(0 < Z < Z_1) = 0.3$

$\therefore$ $Z_1 = 0.84$

Substitute Z_1 in (i)

$$\frac{x_1 - 1520}{160} = -0.84$$

Or, $x_1 = -134.4 + 1520 = 1385.6$

ii. $P(X > x_2) = 10\%$

Or, $P(X > x_2) = 0.1$

When $X = x_2$, $Z = \dfrac{x_2 - 1520}{160} = z_2$ (say) ....(ii)

$P(X > x_2) = 0.1$

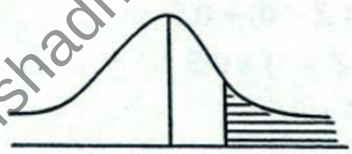
Or, $P(Z > Z_2) = 0.1$

Or, $0.5 - P(0 < Z < Z_2) = 0.1$

Or, $0.5 - 0.1 = P(0 < Z < Z_2)$

Or, $P(0 < Z < Z_2) = 0.4$

Or, $Z_2 = 1.28$



Substitute Z_2 in (ii)

$$\frac{x_2 - 1520}{160} = 1.28$$

$x_2 = 204.80 + 1520 = 1724.80$

66. Let life time of calculator = x

$X \sim N(\mu_1, \sigma^2)$, $\mu = 54$, $\sigma = 8$

Let warranty life time = x_1

$P(X \le X_1) = 22\% = 0.22$

Define $Z = \dfrac{X - \mu}{\sigma} = \dfrac{X - 54}{8}$

When $X = X_1$; $Z = \dfrac{X_1 - 54}{8} = -Z_1$ (say) .... (1)

$P(X \le X_1) = 0.22$

$\Rightarrow$ $P(Z \le -Z_1) = 0.22$

$\Rightarrow$ $0.5 - 0.22 = P(0 < Z < Z_1)$

$\Rightarrow$ $0.28 = P(0 < Z < Z_1)$

$\Rightarrow$ $Z_1 = 0.77$

bstitute Z_2 in (ii)

$$\frac{x_2 - 65}{\sqrt{40}} = -1.28$$

$$\frac{x_2 - 65}{6.324} = -1.28$$

$\Rightarrow$ $x_2 = 6.324 \times (-1.28) + 65 = 56.91$

Hence, highest marks of poorest 500 students is 56.91.

Let limit of marks of middle 80% students be x_3 and xy

$P(x_3 \le x \le x_4) = 80\%$

$\Rightarrow$ $P(X_3 \le X \le X_4) = 0.8$

When $X = X_3$;

$$Z = \frac{X_3 - 65}{6.324} = -Z_3 \text{ (Say) } \dots (2)$$

When $X = X_4$;

$$Z = \frac{X_4 - 64}{6.324} = Z_4 \text{ (Say) } \dots (3)$$

$P(X_3 \le X \le X_4) = 0.8$

$\Rightarrow$ $P(-2/3) \le Z \le Z_4) = 0.8$

$\Rightarrow$ $2P(0 \le Z \le Z_4) = 0.8$ (Since it is middle)

$\Rightarrow$ $P(0 \le Z \le Z_4) = 0.4$

$\Rightarrow$ $Z_4 = 1.28, Z_3 = 1.28$

Substitute values

$$\frac{X_3 - 65}{6.324} = -1.28$$

$\Rightarrow$ $X_3 = 56.91$

$$\frac{X_4 - 65}{6.324} = 1.28$$

$\Rightarrow$ $X_4 = 73.09$

68. Let x = marks secured

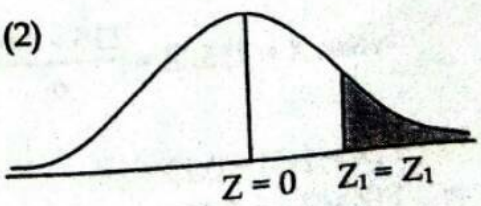
$P(X > 80) = 5\%$

$P(X < 30) = 10\%$

$$Z = \frac{X - \mu}{\sigma}$$

When $X = 80$, $Z = \dfrac{80 - \mu}{\sigma} = Z_1$ (Say) .... (1)

When $X = 30$, $Z = \dfrac{30 - \mu}{\sigma} = -Z_2$ (say) .... (2)



$P(X > 80) = 5\%$

Or $P(Z > Z_1) = 0.05$

Or $0.5 - P(0 < Z < Z_1) = 0.05$

Or $P(0 < Z < Z_1) = 0.45$

Or $Z_1 = 1.64$

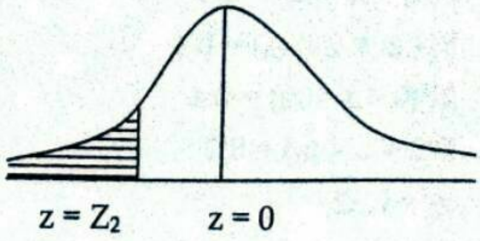
$P(X < 30) = 10\%$

Or $P(Z < -Z_2) = 0.1$

Or $0.5 - P(-Z_2 < Z < 0) = 0.1$

Or $0.4 = P(0 < Z < Z_2)$

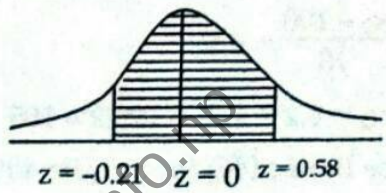
Or $Z_2 = 1.28$



Substitute Z_1 in equation (1) $\frac{80-\mu}{\sigma} = 1.64$ …. (3)

Substitute Z_2 in equation (2) $\frac{30-\mu}{\sigma} = -1.28$ …. (4)

Divide $\frac{80-\mu}{\sigma} = -1.3125$



Or $\mu = 49.02$

Substitute μ in (3); $\sigma = 18.88$

$P(45 < X < 60)$

When $X = 45$, $Z = \frac{45 - 49.02}{18.88} = -0.21$

When $X = 60$, $Z = \frac{60 - 49.02}{18.88} = 0.58$

$P(45 < X < 60) = P(-0.21 < Z < 0.58)$

$= P(-0.21 < Z < 0) + P(0 < Z < 0.58)$

$= 0.0832 + 0.219 = 0.3022 = 30.22\%$

69. The mean IQ

Let $X \sim N(\mu, \sigma^2)$ $\mu = 100, \sigma = 16$

X = IQ score of children

Define $Z = \frac{X-\mu}{\sigma} = \frac{X - 100}{16}$

i. $P(X < 80)$

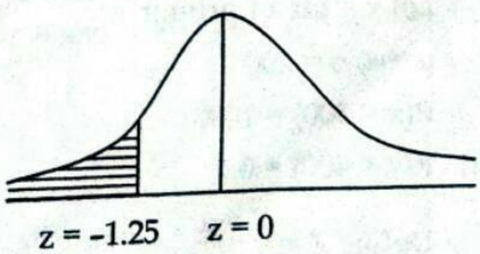
When $X = 80$, $Z = \frac{80 - 100}{10} = -1.25$

$P(X < 80) = P(Z < -1.25)$

$= 0.5 - P(-1.25 < Z < 0)$

$= 0.5 - P(0 < Z < 1.25)$

$= 0.5 - 0.3944 = 0.1056 = 10.56\%$



Let limit for middle 40% students be x_1 and x_2

$P(x_1 < x < x_2) = 40\%$

When, $x = x_1$, $Z = \frac{x_1 - 100}{16} = -Z_1$ ………. (i)

$x = x_2$, $Z = \frac{x_2 - 100}{16} = Z_1$ ………. (ii)

$P(x_1 < x < x_2) = 0.4$

Or, $P(-z_1 < z < z_1) = 0.4$

Or, $2P(0 < z < z_1) = 0.4$

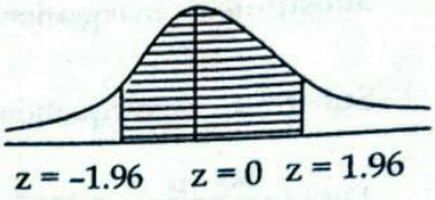
Or, $P(0 < z < z_1) = 0.2$

$\therefore$ $z_1 = 0.52$

Substitute z_1 in (i) and (ii)

$\frac{x_1 - 100}{10} = -0.52$

Or, $x_1 = -5.2 + 100 = 94.8 \approx 95$



$\frac{x_2 - 100}{10} = 0.52$

Or, $x_2 = 5.2 + 100 = 105.2 \approx 105$

Hence limit of IQ for middle 40% is 95 to 105

IQ range $= \mu \pm 1.96 \times 16$

$= 100 \pm 31.36$

$= (68.64, 131.36)$

$P(68.64 < X < 131.36)$

When $x = 68.64$, $Z = \frac{68.64 - 100}{16} = -1.96$

$x = 131.36$, $Z = \frac{131.36 + 100}{16} = 1.96$

$P(68.84 < X < 131.36) = P(-1.96 < Z < 1.96)$

$= P(-1.96 < Z < 0) + P(0 < Z < 1.96)$

$= 2P(0 < Z < 1.96)$

$= 2 \times 0.475 = 0.95$

$\therefore$ 95 children had IQ with rang $\mu \pm 1.96\sigma$

70. The life of

Let x = life of printer

$\mu = 4, \sigma = 200$

$P(x < 400) = 10\%$

Or, $P(x < 400) = 0.1$

Define $Z = \frac{X - \mu}{\sigma}$

When $X = 400$, $Z = \frac{400 - \mu}{200} = -z_1$ (say) .... (i)

$P(X < 400) = 0.1$

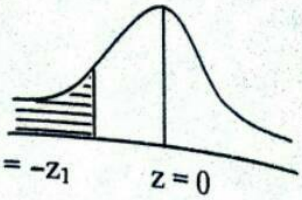
$\Rightarrow$ $P(Z < -Z_1) = 0.1$

$\Rightarrow$ $0.5 - P(-Z_1 < Z < 0) = 0.1$

$\Rightarrow$ $0.5 - 0.1 = P(-Z_1 < Z < 0)$

$\Rightarrow$ $P(0 < Z < Z_1) = 0.4$

$\therefore Z_1 = 1.28$



Substitute Z_1 in (i)

$$\frac{400 - \mu}{200} = -1.28$$

Or, $400 - \mu = -256$

Or, $400 + 256 = \mu$

$\therefore$ $\mu = 656$

Hence mean life of bulb is 656 hrs.

71. Solution.

Let Daily sale of book = X

$X \sim N(\mu, \sigma z)$

$\sigma = 5000; \mu = ?$

$P(X > 40{,}000) = 90\%$

$P(X > 40{,}000) = 0.9$

Let $Z = \dfrac{X - \mu}{\sigma}$

When X= 40,000; $Z = \dfrac{40000 - \mu}{5000} = -Z_1$ (say) .... (1)

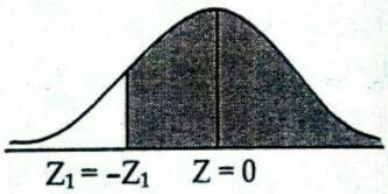
$P(X > 40{,}000) = 0.9$

$\Rightarrow$ $P(Z > -Z_1) = 0.9$

$\Rightarrow$ $0.5 + P(-Z_1 < Z < 0) = 0.9$

$\Rightarrow$ $P(0 < Z < Z_1) = 0.4$

$\Rightarrow$ $Z_1 = 1.28$



Substitute Z_1 in equation (1)

$$\frac{40000 - \mu}{5000} = -1.28$$

$\Rightarrow$ $40{,}000 - \mu = -6400$

$\Rightarrow$ $40{,}000 + 6400 = \mu$

$\mu = 46400$

72. Sacks of grain

Let, x = weight of bag

$\mu = 114$, $X \sim N(\mu, \sigma^2)$, $\sigma = ?$

Here $P(X > 115) = 15\%$

Or, $P(X > 115) = 0.15$

Define $Z = \dfrac{X - \mu}{\sigma}$

When X = 115, $Z = \dfrac{115 - 114}{\sigma} = \dfrac{1}{\sigma} = z_1$ (say) .... (i)

$P(X > 115) = 0.15$

$\Rightarrow$ $P(Z > Z_1) = 0.15$

$\Rightarrow$ $0.5 - P(0 < Z < Z_1) = 0.15$

$\Rightarrow$ $0.5 - 0.15 = P(0 < Z < Z_1)$

$\Rightarrow$ $P(0 < Z < Z_1) = 0.35$

$\Rightarrow$ $Z_1 = 1.04$

Substitute Z_1 in (i)

$\frac{1}{\sigma} = 1.04$

Or, $\sigma = \frac{1}{1.04} = 0.96$

Hence, S.D. = 0.96.

73. **Solution.**

Let marks secured = X

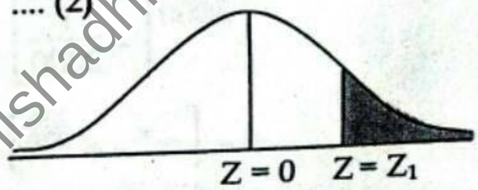
$X \sim N(\mu, \sigma^2)$

$P(X > 60) = 10\%$; $P(X < 40) = 30\%$

Define $Z = \frac{X - \mu}{\sigma}$

When $X = 60$, $Z = \frac{60 - \mu}{\sigma} = Z_1$ (say) .... (1)

When $X = 40$, $Z = \frac{40 - \mu}{\sigma} = -Z_2$ (say) .... (2)



$P(X > 60) = 10\%$

$\Rightarrow$ $P(Z > Z_1) = 0.1$

$\Rightarrow$ $0.5 - P(0 < Z < Z_1) = 0.1$

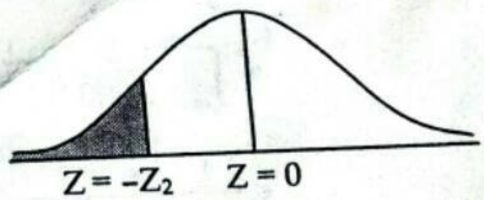
$\Rightarrow$ $0.5 - 0.1 = P(0 < Z < Z_1)$

$\Rightarrow$ $P(0 < Z < Z_1) = 0.4$

$\Rightarrow$ $Z_1 = 1.28$

Substitute Z_1 in equation (1)

$\frac{60 - \mu}{\sigma} = 1.28$ .... (3)



$P(X < 40) = 30\%$

$\Rightarrow$ $P(Z < -Z_2) = 0.3$

$\Rightarrow$ $0.5 - P(-Z_2 < Z < 0) = 0.3$

$\Rightarrow$ $0.5 - 0.3 = P(-Z_2 < Z < 0)$

$\Rightarrow$ $P(0 < Z < Z_2) = 0.2$

$\Rightarrow$ $Z_2 = 0.52$

Substitute Z_2 in equation (2)

$\frac{40 - \mu}{\sigma} = -0.52$ .... (4)

Divide (3) by (4)

$$\frac{\frac{60 - \mu}{\sigma}}{\frac{40 - \mu}{\sigma}} = \frac{1.28}{-0.52}$$

$\Rightarrow$ $\frac{60 - \mu}{40 - \mu} = -2.461$

$\Rightarrow$ $60 - \mu = -98.46 + 2.461\mu$

$\Rightarrow$ $60 + 98.46 = 2.461\mu + \mu$

$\Rightarrow \quad \mu = \frac{158.46}{3.461} = 45.78$

Substitute μ in equation (3)

$\frac{60 - 45.78}{\sigma} = 1.28$

$\Rightarrow \quad \sigma = \frac{14.22}{1.28} = 11.109$

74. Of a large group of

Let, $x \sim N(\mu, \sigma^2)$ where, x = height

$P(x < 60) = 0.05$

$P(60 < x < 65) = 0.4$

Define $z = \frac{X - \mu}{\sigma}$

When x = 60, $z = \frac{60 - \mu}{\sigma} = -z_1$ (say) .... (i)

When x = 65, $z = \frac{65 - \mu}{\sigma} = -z_2$ (say) .... (ii)

Now,

$P(x < 60) = 0.05$

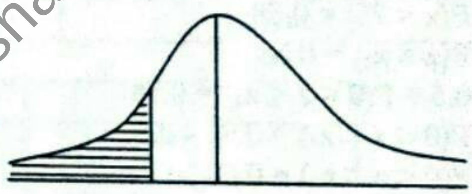
$\Rightarrow \quad P(z < -z_1) = 0.05$

$\Rightarrow \quad 0.5 - P(-z_1 < z < 0) = 0.05$

$\Rightarrow \quad 0.5 - 0.05 = P(-z_1 < z < 0)$

$\Rightarrow \quad P(0 < z < z_1) = 0.45$

$z_1 = 1.64$



Substitute z_1 in (i)

$\frac{60 - \mu}{\sigma} = -1.64$ .... (iii)

$P(60 < x < 65) = 0.4$

$\Rightarrow \quad P(-z_1 < z < -z_2) = 0.4$

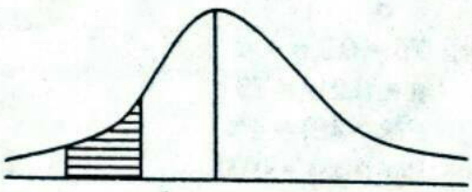
$\Rightarrow \quad P(-z_1 < z < 0) - P(-z_2 < z < 0) = 0.4$

$\Rightarrow \quad P(0 < z < z_1) - P(0 < z < z_2) = 0.4$

$\Rightarrow \quad 0.45 - 0.4 = P(0 < z < z_2)$

$\Rightarrow \quad P(0 < z < z_2) = 0.05$

$\therefore \quad z_2 = 0.13$



Substitute z_2 in (ii)

$\frac{65 - \mu}{\sigma} = -0.13$ .... (iv)

Divide (iii) by (iv)

$$\frac{\frac{60 - \mu}{\sigma}}{\frac{65 - \mu}{\sigma}} = \frac{-1.64}{-0.13}$$

$0.13(60 - \mu) = 1.64(65 - \mu)$

Or, $7.8 - 0.13\mu = 106.6 - 1.64\mu$

Or, $1.64\mu - 0.13\mu = 106.6 - 7.8$

Or, $1.51\mu = 98.8$

$\therefore \quad \mu = \dfrac{98.8}{1.51} = 65.43$

Substitute μ in (iv)

$\dfrac{65 - 65.43}{\sigma} = -0.13$

Or, $\sigma = \dfrac{-0.43}{-0.13} = 3.30$

Hence, mean = 65.43 and S.D. = 3.3

75. If the skulls

Let x = length breadth index

$X \sim N(\mu, \sigma^2)$

$P(x < 75) = 58\%$; $P(75 < x < 80) = 38\%$; $P(x > 80) = 4\%$

Define $z = \dfrac{X - \mu}{\sigma}$

When, $X = 75$, $z = \dfrac{75 - \mu}{\sigma} = z_1$ (say) .... (i)

When, $X = 80$, $z = \dfrac{80 - \mu}{\sigma} = z_2$ (say) .... (ii)

Now,

$P(x < 75) = 0.58$

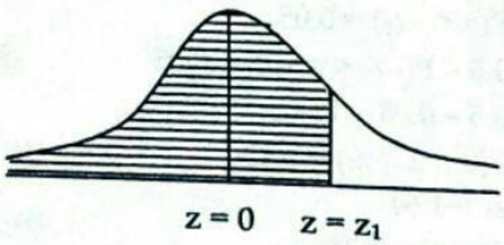
$\Rightarrow P(z < z_1) = 0.58$

$\Rightarrow 0.5 + P(0 < z < z_1) = 0.58$

$\Rightarrow P(0 < z < z_1) = 0.58 - 0.5$

$\Rightarrow P(0 < z < z_1) = 0.08$

$\therefore z_1 = 0.2$



Substitute z_1 in (i)

$\dfrac{75 - \mu}{\sigma} = 0.2$

Or, $75 = 0.2\sigma + \mu$

$\therefore \mu + 0.2\sigma = 75$ .... (iii)

$P(x > 80) = 4\%$

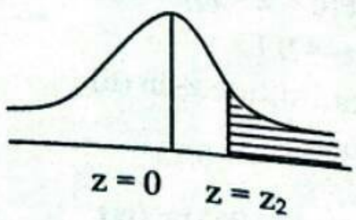
$\Rightarrow P(z > z_2) = 0.04$

$\Rightarrow 0.5 - P(0 < z < z_2) = 0.04$

$\Rightarrow 0.5 - 0.04 = P(0 < z < z_2)$

$\Rightarrow P(0 < z < z_2) = 0.46$

$Z_2 = 1.75$



Substitute z_2 in (ii)

$\dfrac{80 - \mu}{\sigma} = 1.75$

Or, $80 = 1.75\sigma + \mu$

$\therefore \mu + 1.75\sigma = 80$ .... (iv)

Solving (iii) and (iv) $\mu = 74.35$; $\sigma = 3.22$

76. Let waiting time = x hrs; $\theta = 5$

$P(x \geq \dfrac{15}{60}) = ?$

$$P(x \geq 0.25) = \int_{15}^{\infty} \theta e^{-\theta x} dx$$

$$= \int_{0.25}^{\infty} 5 e^{-5x} dx$$

$$= 5\left[\frac{e^{-5x}}{-5}\right]_{25}^{\infty} = -[e^{-5x}]_{0.25}^{\infty} = -[e^{-\infty} - e^{-1.25}] = e^{-1.25} = 0.28$$

$$E(x) = \frac{1}{\theta} = \frac{1}{5} \text{hr} = 12 \text{ min}$$

77. $y_\theta = 2000 \Rightarrow \theta = \frac{1}{2000}$

Let x = daily consumption of electricity

$$P(x \geq 1500) = \int_{1500}^{\infty} \theta e^{-\theta x} dx$$

$$= \int_{1500}^{\infty} \frac{1}{2000} e^{-\frac{1}{2000}x} dx$$

$$= \frac{1}{2000}\left[\frac{e^{-\frac{x}{2000}}}{-\frac{1}{2000}}\right]_{1500}^{\infty}$$

$$= -\left[e^{-\infty} - e^{-\frac{1500}{2000}}\right]$$

$$= e^{-0.75}$$

$$= 0.47$$

$$P(x \leq 2500) = \int_{\infty}^{2500} \theta e^{-\theta x} dx$$

$$= \int_{\infty}^{2500} \frac{1}{2000} e^{-\frac{1}{2000}x} dx$$

$$= \frac{1}{2000}\left[\frac{e^{-\frac{1}{2000}x}}{-\frac{1}{2000}}\right]_{0}^{2500}$$

$$= -\left[e^{-\frac{x}{2000}}\right]_{0}^{2500}$$

$$= -\left[e^{-\frac{2500}{2000}} - e^{-0}\right]$$

$$= -e^{-1.25} + 1$$

$$= 1 - e^{-1.25}$$

$$= 1 - 0.286$$

$$= 0.713$$

78. **Solution.**

Let x = No. of pages print by printer for min.

$\lambda = 15$

Here, $\mu = 15$, $\sigma = 15$

$P(x > 25.5) = ?$ $\quad$ x = 25.5 due to continuity correction for x > 25.5

$P(x < 10) = ?$

Let $z = \dfrac{x - \mu}{\sigma} = \dfrac{x - 15}{15}$

When $x = 25.5$

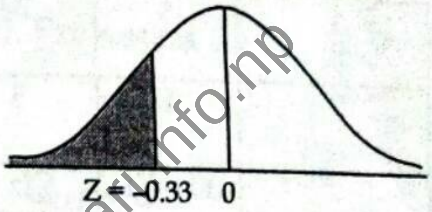
$z = \dfrac{25.5 - 15}{15} = \dfrac{10.5}{15} = 0.7$

$$\begin{aligned} P(x > 25.5) &= P(z > 0.7) \\ &= 0.5 - P(0 < z < 0.7) \\ &= 0.5 - 0.258 \\ &= 0.242 \end{aligned}$$

$x = 9.5$ due to continuity correlation for $x < 10$

When $x = 9.5$

$z = \dfrac{9.5}{15} = \dfrac{-5.5}{15} = -0.36$



$$\begin{aligned} P(x < 9.5) &= P(z < -0.36) \\ &= 0.5 - P(-0.36 < Z < 0) \\ &= 0.5 - P(0 < Z < 0.36) \\ &= 0.5 - 0.1406 \\ &= 0.3594 \end{aligned}$$

79. **Solution.**

$\alpha = 2$

$$\begin{aligned} P(x > 2) &= \int_2^\infty \frac{e^{-x} x^{2-1}}{\sqrt{2}} dx \\ &= \int_2^\infty \frac{e^{-x} x^{\alpha-1}}{\sqrt{\alpha}} dx \\ &= \int_2^\infty \frac{e^{-x} x^{2-1}}{\sqrt{2}} dx \\ &= \int_2^\infty x e^{-x} dx \\ &= \left[x \int e^{-x} dx - \int\left(\frac{dx}{dx} \int e^{-x} dx\right) dx\right]_2^\infty \\ &= [-x e^{-x} - e^{-x}]_2^\infty \\ &= -0 - 0 + 2e^{-2} + e^{-2} \\ &= 3e^{-2} = 3 \times 0.135 = 0.405 \end{aligned}$$

$$\begin{aligned} P(3 < x < 5) &= \int_3^5 \frac{e^{-x} x^{\alpha-1}}{\sqrt{\alpha}} dx \\ &= \int_3^5 x e^{-x} dx \\ &= [-x e^{-x} - e^{-x}]_3^5 \\ &= -5e^{-5} - e^{-5} + 3e^{-3} + e^{-3} \\ &= -6e^{-5} + 4e^{-3} \\ &= -0.040 + 0.199 \\ &= 0.159 \end{aligned}$$

Practical Questions

1. Fit the binomial

x	0	1	2	3	4	5	6	Total
f	7	6	19	35	23	7	1	$\sum f = 98$
fx	0	6	38	105	92	35	6	$\sum fx = 282$

$\bar{x} = \frac{\sum fx}{N} = \frac{282}{98} = 2.877$

Here, n = 6

$\bar{x} = np$

Or, $2.877 = 6p$

Or, $p = \frac{2.877}{6} = 0.479, q = 0.52$

x	$P(x) = C(n, x)\, p^x q^{n-x}$	Expected frequency = N P(x)
0	$C(6, 0)\,(0.479)^0\,(0.52)^6 = 0.019$	$1.86 \approx 2$
1	$C(6, 1)\,(0.479)^1\,(0.52)^5 = 0.1092$	$10.70 \approx 11$
2	$C(6, 2)\,(0.479)^2\,(0.52)^4 = 0.2516$	$24.65 \approx = 25$
3	$C(6, 3)\,(0.479)^3\,(0.52)^3 = 0.3090$	$30.28 \approx 30$
4	$C(6, 4)\,(0.479)^4\,(0.52)^2 = 0.2153$	$21.1 \approx 21$
5	$C(6, 5)\,(0.479)^5\,(0.52)^1 = 0.078$	$7.64 \approx 8$
6	$C(6, 6)\,(0.479)^6 = 0.012$	$1.176 \approx 1$

2. Five fair coin

No. of heads (x)	Frequency (f)	fx
0	2	0
1	10	10
2	24	48
3	38	114
4	18	72
5	8	40
	$N = \sum f = 100$	$\sum fx = 284$

$\bar{x} = \frac{\sum fx}{N} = \frac{284}{100} = 2.84$

n = 5

$\bar{x} = n\,p$

Or, $2.84 = 5p$

Or, $p = \frac{2.84}{5} = 0.568; q = 1 - p = 0.432$

$x \sim B(n, p)$

x	$P(x) = C(n, x)\, p^x q^{n-x}$	Expected frequency = N P(x)
0	$C(5, 0)\,(0.568)^0\,(0.432)^5 = 0.015$	$1.5 \approx 2$
1	$C(5, 1)\,(0.568)^1\,(0.432)^4 = 0.098$	$9.8 \approx 10$
2	$C(5, 2)\,(0.568)^2\,(0.432)^3 = 0.2601$	$26.01 \approx 26$
3	$C(5, 3)\,(0.568)^3\,(0.432)^2 = 0.341$	$34.10 \approx 34$
4	$C(5, 4)\,(0.568)^4\,(0.432)^1 = 0.224$	$22.4 \approx 22$
5	$C(5, 5)\,(0.568)^2\,(0.432)^3 = 0.059$	$5.9 \approx 6$

3. Solution.

192 Families

No. of girl child (x)	No. of families (f)	fx
0	77	0
1	90	90
2	20	40
3	5	15
	$N = \sum f = 192$	$\sum fx = 145$

$\bar{x} = \frac{\sum fx}{N}$

$= \frac{145}{192} = 0.755$

$n = 3$

(i) When both sexes are equally probable

$p = q = \frac{1}{2}$

$n = 3$

No. of girl child (x)	No. of families (f)	fx
0	77	0
1	90	90
2	20	40
3	5	15
	$N = \sum f = 192$	$\sum fx = 145$

$\bar{x} = \frac{\sum fx}{N} = \frac{145}{192} = 0.755;$ $n = 3$

i. When both sexes are equally probable

$p = q = \frac{1}{2}$ $n = 3$

x	$P(x) = C(n, x)\, p^x\, q^{n-x}$	Expected frequency = N P(x)
0	$C(3, 0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 = 0.125$	24
1	$C(3, 1)\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 = 0.375$	72
2	$C(3, 2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = 0.375$	72
3	$C(3, 3)\left(\frac{1}{2}\right)^3 = 0.125$	24

ii. When probability vary

$\bar{x} = np$ Or, $0.755 = 3p$ Or, $p = \frac{0.755}{3} = 0.25$,

$q = 1 - p = 1 - 0.25 = 0.75$

x	$P(x) = C(n, x)\, p^x\, q^{n-x}$	Expected frequency = N P(x)
0	$C(3, 0)\,(0.25)^0\,(0.75)^3 = 0.421$	$80.83 \approx 81$
1	$C(3, 1)\,(0.25)^1\,(0.75)^2 = 0.421$	$80.83 \approx 81$
2	$C(3, 2)\,(0.25)^2\,(0.75)^3 = 0.1406$	$26.99 \approx 27$
3	$C(3, 3)\,(0.25)^3 = 0.0156$	3

4. Fit the Poisson

Mistaken per page (x)	No. of page (f)	fx	$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$	Expected frequency = N P(x)
0	142	0	$\frac{e^{-0.95} (0.25)^0}{0\,!} = 0.386$	154.4 ≈ 154
1	156	156	$\frac{e^{-0.95} (0.25)^1}{1\,!} = 0.367$	146.8 ≈ 147
2	69	138	$\frac{e^{-0.95} (0.25)^2}{2\,!} = 0.174$	69.6 ≈ 70
3	27	81	$\frac{e^{-0.95} (0.25)^3}{3\,!} = 0.055$	22
4	5	20	$\frac{e^{-0.95} (0.25)^4}{4\,!} = 0.013$	5.2 ≈ 5
5	1	5	$\frac{e^{-0.95} (0.25)^5}{5\,!} = 0.006$	2.57 ≈ 3
	$N = \sum f = 400$	$\sum fx = 380$		

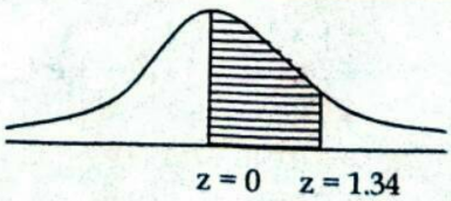
$$\lambda = \bar{x} = \frac{\sum fx}{N} = \frac{380}{400} = 0.95$$

5. Fit Poisson distribution

x	f	fx	$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$	Expected frequency = N P(x)
0	71	0	$\frac{e^{-1.74} (1.74)^0}{0\,!} = 0.175$	69.82 ≈ 70
1	112	112	$\frac{e^{-1.74} (1.74)}{1\,!} = 0.305$	121.695 ≈ 122
2	117	234	$\frac{e^{-1.74} (1.74)^2}{2\,!} = 0.265$	105.73 ≈ 106
3	57	161	$\frac{e^{-1.74} (1.74)^3}{3\,!} = 0.154$	61.446 ≈ 61
4	27	108	$\frac{e^{-1.74} (1.74)^4}{4\,!} = 0.067$	26.733 ≈ 27
5	11	55	$\frac{e^{-1.74} (1.74)^5}{5\,!} = 0.023$	9.177 ≈ 9
6	3	18	$\frac{e^{-1.74} (1.74)^6}{6\,!} = 0.0067$	2.67 ≈ 3
7	1	7	$\frac{e^{-1.74} (1.74)^7}{7\,!} = 0.0016$	0.67 ≈ 1
	$N = \sum f = 399$	$\sum fx = 695$		

$$\lambda = \bar{x}$$

$$= \frac{\sum fx}{N} = \frac{695}{399} = 1.74$$



1. If the covariance

$Cov.(x, y) = 36, \sigma_x^2 = 36, \sigma_y^2 = 100$

$$r = \frac{Cov\ (x, y)}{\sigma_x\ \sigma_y} = \frac{36}{6 \times 10} = 0.6$$

2. Find the r

$n = 10, \sigma_x = 2.05, \sigma_y = 2.06, \sum(x - \bar{x})\ (y - \bar{y}) = 40$

$$r = \frac{\frac{1}{n}\sum (x - \bar{x})\ (y - \bar{y})}{\sigma_x\ \sigma_y} = \frac{\frac{40}{10}}{2.05 \times 2.06} = 0.947$$

3. For 10 obs

$\sum x = 666, \sum y = 663, \sum x^2 = 44490, \sum y^2 = 44061, \sum xy = 44224, n = 10$

$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2}\sqrt{n \sum y^2 - (\sum y)^2}}$$

$$= \frac{10 \times 44224 - 666 \times 663}{\sqrt{10 \times 44490 - (666)^2}\sqrt{10 \times 44061 - (663)^2}} = 0.576$$

4. $$r = \frac{\frac{\Sigma xy}{n}}{\sqrt{\frac{\Sigma x^2}{n}}\sqrt{\frac{\Sigma y^2}{n}}}$$

$$0.8 = \frac{\frac{600}{n}}{\sqrt{\frac{9}{n}} \times 6}$$

or $0.8 = \frac{600}{n} \times \frac{\sqrt{n}}{6 \times 3}$

or $0.8 = \frac{33.33}{\sqrt{n}}$

or $n = 1735.83 \approx 1736$

5. **Solution**

x	y	x^2	y^2	xy
1	9	1	81	9
2	8	4	64	16
3	10	9	100	30
4	12	16	144	48
5	11	25	121	55
6	13	36	169	78
7	14	49	196	98
8	16	64	256	128
9	15	81	225	135
$\Sigma x = 45$	$\Sigma y = 108$	$\Sigma x^2 = 285$	$\Sigma y^2 = 1356$	$\Sigma xy = 597$

$$r = \frac{n\ \Sigma xy - \Sigma x\ \Sigma y}{\sqrt{n\ \Sigma x^2 - (\Sigma x)^2}\sqrt{n\ \Sigma y^2 - (\Sigma y)^2}} = \frac{9 \times 597 - 45 \times 108}{\sqrt{9 \times 285 - (45)^2}\sqrt{9 \times 1356 - (108)^2}} = 0.95$$

6. **Solution.**

size	No. of items	No. of defective items	% defective (y)	mid-size (x)	$u' = \frac{x-27.5}{5}$	$v' = \frac{y-50}{5}$	u'^2	v'^2	$u'v'$
15–20	200	150	75	17.5	–2	5	4	25	–10
20-25	270	162	60	22.5	–1	2	1	4	–2
25–30	340	170	50	27.5	0	0	0	0	0
30–35	360	180	50	32.5	1	0	1	0	0
35-40	400	180	45	37.5	2	–1	4	1	–2
40–45	300	120	40	42.5	3	–2	9	4	–6
					$\Sigma u' = 3$	$\Sigma v' = 4$	$\Sigma u'^2 = 19$	$\Sigma v'^2 = 34$	$\Sigma u'v' = -20$

$$r = \frac{n\,\Sigma u'v' - \Sigma u'\,\Sigma v'}{\sqrt{n\,\Sigma u'^2 - (\Sigma u')^2}\sqrt{n\,\Sigma v'^2 - (\Sigma v')^2}} = \frac{6 \times (-20) - 3 \times 4}{\sqrt{6 \times 19 - 3^2}\sqrt{6 \times 34 - 4^2}} = -0.939$$

7. **Solution.**

X	Y	xy	x^2	y^2
25	17	425	625	289
26	18	468	676	324
27	19	513	729	361
25	17	425	625	289
26	19	494	676	361
28	20	560	784	400
25	17	425	625	289
25	17	425	625	289
24	18	432	576	324
26	18	468	676	324
26	20	520	676	400
27	18	486	729	324
27	19	513	729	361
28	19	532	784	361
25	18	450	625	324
25	19	475	625	361
26	18	468	676	324
25	18	450	625	324
27	20	540	729	400
493	349	9069	12815	6429

Here n= 19, ΣX = 493, ΣY = 349, ΣXY = 9069 $\Sigma X^2 = 12815$, $\Sigma Y^2 = 6429$

Karl Pearson's Coefficient of correlation

$$r = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}} = \frac{19 \times 9069 - 493 \times 349}{\sqrt{13 \times 12815 - (493)^2}\sqrt{19 \times 6429 - (349)^2}}$$

$$= 0.654$$

$$P.E. = 0.6745 \times \frac{1 - r^2}{\sqrt{n}} = 0.6745 \times \frac{1 - (0.654)^2}{\sqrt{19}} = 0.088$$

$6 \times P.E. = 0.53$

Since $6 \times P.E. < r$. So the correlation coefficient is significant

8. When the coefficient

$\sum x = 127, \sum y = 100, \sum x^2 = 860, \sum y^2 = 549, \sum xy = 674, n = 20$

Wrong (x, y) = (10, 14) and (8, 6)

Correct (x, y) = (8, 12) and (6, 8)

Correct r = ?

Correct values

$\sum x = 127 - 10 - 8 + 8 + 6 = 123$

$\sum y = 100 - 14 - 6 + 12 + 18 = 100$

$\sum x^2 = 860 - 10^2 - 8^2 + 8^2 + 6^2 = 796$

$\sum y^2 = 549 - 14^2 - 6^2 + 12^2 + 8^2 = 525$

$\sum xy = 674 - 10 \times 14 - 8 \times 6 + 8 \times 12 + 6 \times 8 = 630$

$$r = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

$$= \frac{20 \times 630 - 123 \times 100}{\sqrt{20 \times 796 - (123)^2}\sqrt{20 \times 525 - (100)^2}} = \frac{300}{28.124 \times 22.36} = 0.47$$

9. A student

r = 0.795, n = 100

$$PE(r) = 0.6745 \times \frac{1 - r^2}{\sqrt{n}} = 0.6745\left\{\frac{1 - (0.795)^2}{\sqrt{100}}\right\} = 0.0248$$

Here, $r \neq PE(r)$

$6\ PE(r) = 6 \times 0.0248 = 0.1489$

Here, $r > 6\ PE(r)$. Hence, r is significant. So conclusion is correct.

10. n = 10, r = 0.81

$$PE(r) = 0.6745 \times \frac{1 - r^2}{\sqrt{n}} = 0.6745 \times \frac{1 - (0.81)^2}{\sqrt{10}} = 0.6745 \times 0.1087 = 0.0733$$

Here, $r = 0.81 \nless PE(r) = 0.0733$

$6PE(r) = 6 \times 0.0733 = 0.4398$

$r = 0.81 > 6\ PE(r) = 0.4398$

Hence, r is significant.

Limit of population correlation $\rho = r \pm PE(r)$

$= 0.81 \pm 0.0733$

Take –,	Take +
0.81 – 0.0733 = 0.7367	0.81 + 0.0733 = 0.8833

11. $r = \dfrac{N\Sigma fxy - \Sigma fx\,\Sigma gy}{\sqrt{N\Sigma fx^2 - (\Sigma fx)^2}\sqrt{N\Sigma fy^2 - (\Sigma fy)^2}}$

$= \dfrac{72 \times 172000 - 3560 \times 3260}{\sqrt{72 \times 196800 - (3560)^2}\sqrt{72 \times 168400 - (3260)^2}} = 0.52$

$PE(r) = 0.6745\dfrac{1 - r^2}{\sqrt{N}} = 0.6745 \times \dfrac{1 - (0.52)^2}{\sqrt{72}} = 0.057$

$r = 0.52 \nless PE(r) = 0.057$

$6PE(r) = 6 \times 0.057 = 0.342$

$r = 0.52 > 6\,PE(r) = 0.342$

Hence, r is significant.

12. The ranking of ten

A (R_1)	B (R_2)	$d = R_1 - R_2$	d^2
3	6	−3	9
5	4	1	1
8	9	−1	1
4	8	−4	16
7	1	6	36
10	2	8	64
2	3	−1	1
1	10	−9	81
6	5	1	1
9	7	2	4
		$\sum d = 0$	$\sum d^2 = 2.4$

$r_s = 1 - \dfrac{6\sum d^2}{n(n^2 - 1)} = 1 - \dfrac{6 \times 214}{10(10^2 - 1)} = 1 - \dfrac{1284}{990} = -0.296$

13. From following data

Variable x	Variable y	Rank of x (R_1)	Rank of y (R_2)	$d = R_1 - R_2$	d^2
10	100	1	1	0	0
15	120	3	5	−2	4
18	118	4	3	1	1
14	119	2	4	−2	4
30	130	6	6	0	0
27	105	5	2	3	9
				$\sum d = 0$	$\sum d^2 = 18$

$r_s = 1 - \dfrac{6\sum d^2}{n(n^2 - 1)}$

$= 1 - \dfrac{6 \times 18}{6(6^2 - 1)}$

$= 1 - \dfrac{108}{210} = 0.48$

14. Solution.

Age of husband (x)	Age of wife (y)	Rank of x (R_1)	Rank of y (R_2)	$d = R_1 - R_2$	d^2
23	21	1	1	0	0
27	22	2	2	0	0
28	23	3	3	0	0
29	24	4	4	0	0
30	25	5	5	0	0
31	26	6	6	0	0
33	28	7	7	0	0
35	29	8	8	0	0
				$\sum d = 0$	$\sum d^2 = 0$

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 0}{6(6^2-1)} = 1$$

15. Calculate Spearman's

x	y	Rank of x (R_1)	Rank of y (R_2)	$d = R_1 - R_2$	d^2
20	52	1	3	-2	4
25	50	3	1.5	1.5	2.25
60	55	7	4	3	9
45	50	5	1.5	3.5	12.25
80	60	10	5	5	25
25	70	3	7	– 4	16
55	72	6	8	– 2	4
65	78	8	9	– 1	1
25	80	3	10	– 7	49
75	63	9	6	3	9
				$\sum d = 0$	$\sum d^2 = 131.5$

Here, $m_1 = 3$, $m_2 = 2$

$$r_s = 1 - \frac{6\left\{\sum d^2 + \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12}\right\}}{n(n^2-1)}$$

$$= 1 - \frac{6\left[131.5 + \frac{3(3^2-1)}{12} + \frac{2(2^2-1)}{12}\right]}{10(10^2-1)} = 1 - \frac{6 \times 134}{990} = 0.187$$

16.

Applicant	Marks in Skill	Marks in Ability	R_x	R_y	$d = R_x - R_y$
A	38	48	2.5	4	-1.5
B	41	39	8	3	2
C	68	38	8	2	6
D	41	36	5	1	4
E	38	58	2.5	5	-2.5
F	55	61	7	6.5	0.5
G	85	72	10	8	2
H	81	83	9	10	-1
I	28	61	1	6.5	- 5.5
J	41	82	5	9	- 4
					$\Sigma d = 0$

$$R = 1 - \frac{6\left[\Sigma d^2 + \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12} + \frac{m_3(m_3^2-1)}{12}\right]}{n(n^2-1)}$$

$$= 1 - \frac{6\left[116 + \frac{2\times 3}{12} + \frac{3\times 8}{12} + \frac{2\times 3}{12}\right]}{10(100-1)}$$

$$= 1 - 0.72 = 0.278$$

17. Ten competitors in Nepal

1st Judge (R_1)	2nd Judge (R_2)	3rd Judge (R_3)	$d_{12} = R_1 - R_2$	d_{12}^2	$d_{13} = R_1 - R_3$	d_{13}^2	$d_{23} = R_2 - R_3$	d_{23}^2
1	4	6	–3	9	–5	25	–2	4
5	8	7	–3	9	–2	4	1	1
4	7	8	–3	9	–4	16	–1	1
8	6	1	2	4	7	49	5	25
9	5	5	4	16	4	16	0	0
6	9	10	–3	9	–4	16	–1	1
10	10	9	0	0	1	1	1	1
7	3	2	4	16	5	25	1	1
3	2	3	1	1	0	0	–1	1
2	1	4	1	1	–2	4	–3	9
			$\sum d_{12} = 0$	$\sum d_{12}^2 = 74$	$\sum d_{13} = 0$	$\sum d_{13}^2 = 156$	$\sum d_{23} = 0$	$\sum d_{23}^2 = 44$

$$r_{s(12)} = 1 - \frac{6\sum d_{12}^2}{n(n^2-1)} = 1 - \frac{6\times 74}{10(10^2-1)} = 0.55$$

$$r_{s(13)} = 1 - \frac{6\sum d_{13}^2}{n(n^2-1)} = 1 - \frac{6\times 156}{10(10^2-1)} = 0.054$$

$$r_{s(23)} = 1 - \frac{6\sum d_{23}^2}{n(n^2-1)} = 1 - \frac{6\times 44}{10(10^2-1)} = 0.73$$

18. Solution.

Dell (R_1)	HP (R_2)	$d = R_1 - R_2$	d^2
5	10	–5	25
2	5	–3	9
9	1	8	64
8	3	5	25
1	8	–7	49
10	6	4	16
3	2	1	1
4	7	–3	9
6	9	–3	9
7	4	3	9
		$\Sigma d = 0$	$\Sigma d^2 = 216$

$$R = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6\times 216}{10\times 99} = -0.309$$

19. The coefficient of

$n = 10, r_s = 0.8$

Wrong d = 7 Correct d = 9

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

or, $0.8 = 1 - \frac{6\sum d^2}{10(10^2-1)}$

Or, $\frac{6\sum d^2}{990} = 1 - 0.8$ Or, $\sum d^2 = \frac{0.2 \times 990}{6} = 33$

Correct $\sum d^2 = 33 - 7 + 9 = 35$

Correct $r_s = 1 - \frac{6 \text{ Correct} \sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 35}{10(10^2-1)} = 0.78$

20. $R = 0.143, \Sigma d^2 = 48, n = ?$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

or $0.143 = \frac{6 \times 48}{n(n^2-1)}$

or $\frac{288}{n(n^2-1)} = 1 - 0.143$

or $n(n^2-1) = 336.05$

or By trial method n = 7

21. **Solution.**

$x = 22, \Sigma x = 950, \Sigma y = 26953, \Sigma y^2 = 35528893, \Sigma x^2 = 49250, \Sigma xy = 1263940$

For $y = a + bx$

$\Sigma y = na + b\,\Sigma x$

$26953 = 22a + 950\,b$ .... (1)

$\Sigma xy = a\,\Sigma x + b\,\Sigma x^2$

$1263940 = 950\,a + 49250\,b$ .... (2)

Solving (1) and (2)

$a = 699.957;\quad b = 12.162$

$y = a + bx = 699.957 + 12.162x$

$$TSS = \Sigma y^2 - n\,\overline{y} = 35528893 - 22\left(\frac{26953}{22}\right)^2 = 2507792.951$$

$SSE = \Sigma y^2 - a\,\Sigma y - \Sigma xy$

$= 35528893 - 699.957 \times 26953 - 12.162 \times 1263940$

$= 1290913.699$

$SSR = TSS - SSE$

$= 2507792.591 - 1290913.699$

$= 1216878.892$

$$R^2 = \frac{SSR}{TSS} = \frac{1216878.892}{2507792.591} = 0.4852$$

22. **Solution.** [Father = x, Son = y]

x	y	u = x – 66	v = y – 65	uv	u^2	v^2
67	68	1	3	3	1	9
63	66	–3	1	–3	9	1
66	65	0	0	0	0	0
71	70	5	5	25	25	25
69	67	3	2	6	9	4
65	67	–1	2	–2	1	4
62	64	–4	–1	4	16	1
70	71	4	6	24	16	36
61	62	– 5	–3	15	25	9
72	63	6	–2	–12	36	4
		$\Sigma u = 6$	$\Sigma v = 13$	$\Sigma uv = 60$	$\Sigma u^2 = 138$	$\Sigma v^2 = 93$

First find regression equation of v on u

$v = a + bu$

$\Sigma v = na + b\,\Sigma u$

$13 = 10a + 6b$ ..... (1)

$\Sigma uv = a\,\Sigma u + b\,\Sigma u^2$

$60 = 6a + 138\,b$

or $a = \dfrac{60 - 1386}{6}$

Substitute a in (1)

$13 = 10\,\dfrac{60 - 138b}{6} + 6b$

or $13 = \dfrac{600 - 1380b + 36}{6}$

or $78 = 600 - 1344\,b$

or $1344\,b = 600 - 78$

or $b = \dfrac{522}{1344} = 0.388$

Substitute b in (1)

$a = \dfrac{60 - 138 \times 0.388}{6} = \dfrac{6.401}{6} = 1.066$

$\therefore$ $v = a + bu$

or $v = 1.066 + 0.388\,u$

or $y - 65 = 1.066 + 0.388\,(x - 66)$

or $y = 1.066 + 65 + 0.388\,x - 25.608$

or $y = 40.45 + 0.388\,x$

When x = 70,

$y = 40.45 + 0.388 \times 70 = 67.61$

$$r = \frac{n\,\Sigma uv - \Sigma u\,\Sigma v}{\sqrt{n\,\Sigma u^2 - (\Sigma u)^2}\,\sqrt{n\,\Sigma v^2 - (\Sigma v)^2}} = \frac{10 \times 60 - 6 \times 13}{\sqrt{10 \times 138 - 6^2}\,\sqrt{10 \times 93 - 13^2}} = 0.51$$

Coefficient of determination, $R^2 = r^2 = (0.51)^2 = 0.266$

23. **Solution.**

Operator	x	y	xy	x^2
I	16	87	1372	256
II	12	88	1056	144
III	18	89	1602	324
IV	4	68	272	16
V	3	78	234	9
VI	10	80	800	100
VII	5	75	375	25
VIII	12	83	996	144
	$\Sigma x = 80$	$\Sigma y = 648$	$\Sigma xy = 6727$	$\Sigma x^2 = 1018$

Regression equation of y on x is

$y = a + bx$

Now, $\Sigma y = na + b\,\Sigma x$

or $648 = 8a + 80b$ .... (1)

$\Sigma xy = a\,\Sigma x + b\,\Sigma x^2$

or $6727 = 80a + 1018\,b$

or $a = \dfrac{6727 - 1018b}{80}$ .... (2)

Substitute 'a' in equation (1)

$$648 = 8 \times \frac{6727 - 1018\,b}{80} + 80b$$

or $648 = \dfrac{6727 - 1018 + 800\,b}{10}$

or $6480 = 6727 - 218\,b$

or $b = \dfrac{6727 - 6480}{218} = 1.133$

Substitute b in equation (2)

$$a = \frac{6727 - 1018 \times 1.133}{80} = 69.67$$

$\therefore$ $y = a + bx = 69.67 + 1.133x$

When $x = 8$,

$y = 69.67 + 1.133 \times 8 = 78.73$

24. **Solution.**

The city council

x (football game)	y (minor accidents)	xy	x^2	y^2
20	6	120	400	36
30	9	270	900	81
10	4	40	100	16
12	5	60	144	25
15	7	105	225	49
25	8	200	625	64
34	9	306	1156	81
$\sum x = 146$	$\sum y = 48$	$\sum xy = 1101$	$\sum x^2 = 3550$	$\sum y^2 = 352$

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} = \frac{7 \times 1101 - 146 \times 48}{7 \times 3550 - (146)^2} = \frac{699}{3534} = 0.197$$

$$\bar{x} = \frac{\sum x}{n} = \frac{146}{7} = 20.85$$

$$\bar{y} = \frac{\sum y}{n} = \frac{48}{7} = 6.85$$

Regression equation of y on x

Or, $y - \bar{y} = b_{yx}(x - \bar{x})$

Or, $y - 6.85 = 0.197(x - 20.85)$

Or, $y = 0.197x - 4.107 + 6.85$

$\therefore$ $y = 2.742 + 0.197x$ It is in the form $y = a + bx$

$y = 2.742 + 0.197 \times 30 = 8.65 \approx 9$

Hence no. of minor accidents is 9.

Now, $TSS = \sum y^2 - n\bar{y}^2 = 352 - 7 \times (6.85)^2 = 23.542$

$SSE = \sum y^2 - a\sum y - b\sum xy$

$= 352 - 2.752 \times 48 - 0.197 \times 1101 = 3.007$

$SSR = TSS - SSE = 23.542 - 3.007 = 20.535$

Coefficient of determinants $(R^2) = \frac{SSR}{TSS} = \frac{20.535}{23.542} = 0.87$

Standard error of estimate $= \sqrt{\frac{SSE}{n-k-1}} = \sqrt{\frac{3.007}{7-1-1}} = 0.77$

25. Solution.

x	y	xy	x^2	y^2
27	57	1539	729	3249
45	64	2880	2025	4096
41	80	3280	1681	6400
19	46	874	361	2116
35	62	2170	1225	3844
39	72	2868	1521	5184
19	52	988	361	2704
$\Sigma x = 225$	$\Sigma y = 433$	$\Sigma xy = 74539$	$\Sigma x^2 = 7903$	$\Sigma y^2 = 27593$

Regression equation of y on x

$y = a + bx$

Here, $\Sigma y = na + b\Sigma x$

$433 = 7a + 225b$ .... (1)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

$14539 = 225a + 7903b$

$a = \frac{14539 - 7903b}{225}$ .... (2)

Substitute a in equation (1)

$$433 = 7 \times \left(\frac{14539 - 7903b}{225}\right) + 225\,b$$

or $433 = \dfrac{101773 - 55321b + 550625\,b}{225}$

or $97425 = 101773 - 4696\,b$

or $b = \dfrac{4348}{4696} = 0.926$

Putting the value of b in equation (2)

$$a = \frac{14539 - 7903\,b}{225} = \frac{14539 - 7903 \times 0.926}{225} = 32.09$$

$y = a + bx = 32.09 + 0.926x$

where $x = 35$

$\therefore y = 64.5$

$R^2 = \dfrac{SSR}{SST}$

26. Solution.

x	y	x^2	y^2	xy
0	12	0	144	0
5	15	25	225	75
10	17	100	289	170
15	22	225	484	330
20	24	400	576	480
25	30	625	900	750
$\Sigma x = 75$	$\Sigma y = 120$	$\Sigma x^2 = 1375$	$\Sigma y^2 = 248$	$\Sigma xy = 1805$

To fit

$y = a + bx$

$\Sigma y = na + b\,\Sigma x$

$120 = 6a + 75\,b$ .... (1)

$\Sigma xy = a\,\Sigma x + b\,\Sigma x^2$

$1805 = 75a + 1375\,b$

$a = \dfrac{1805 - 1375\,b}{75}$ .... (2)

Substitute a in (1)

$$120 = 6\,\frac{1805 - 1375\,b}{75} + 75\,b$$

or $120 = \dfrac{10830 - 8250\,b + 5625\,b}{75}$

or $9000 = 10830 - 2625\,b$

or $2625\,b = 10830 - 9000$

or $b = 0.697$

Putting the value of b in equation (2)

$$a = \frac{1805 - 1375 \times 0.697}{75} = \frac{846.625}{75} = 11.29$$

$y = a + bx = 11.25 + 0.697x$

Now, coefficient of determination

$$R^2 = \frac{SSR}{TSS}$$

$$TSS = \Sigma y^2 - n\,\overline{y}^2$$

$$= 2618 - 6 \times \left(\frac{120}{6}\right)^2$$

$$= 218$$

$$SSE = \Sigma y^2 - a\,\Sigma y - b\,\Sigma xy$$

$$= 2618 - 11.29 \times 10 - 0.697 \times 1809$$

$$= 5.115$$

$$SSR = TSS - SSE = 218 - 5.115 = 212.88$$

$$R^2 = \frac{212.88}{218} = 0.97$$

27. Solution.

y	x	xy	x^2	y^2
3	50	150	2500	9
1	200	200	40000	1
4	70	280	4900	16
1	100	100	10000	1
2	90	180	8100	4
3	40	120	1600	9
$\Sigma y = 14$	$\Sigma x = 550$	$\Sigma xy = 1030$	$\Sigma x^2 = 67100$	$\Sigma y^2 = 40$

Now,

$\Sigma y = na + b\,\Sigma x$

$14 = 6a + 550b$ .... (1)

$\Sigma xy = a\,\Sigma x + b\,\Sigma x^2$

or $1030 = 550a + 67100\,b$ .... (2)

Solving equation (1) and (2)

$a = 3.725, \quad b = -0.01518$

$y = a + bx = 3.725 - 0.01518\,x$

When $x = 110$

$y = 3.725 - 0.01518 \times 110 = 2.055$

$$TSS = \Sigma y^2 - x\,\overline{y}^2 = 40 - 6 \times \left(\frac{14}{6}\right)^2 = 7.333$$

$$SSE = \Sigma y^2 - a\,\Sigma y - b\,\Sigma xy = 40 - 3.725 \times 14 - (-0.01518) \times 1030 = 3.487$$

$$SSR = TSS - SSE = 7.333 - 3.487 = 3.846$$

$$R^2 = \frac{SSR}{TSS} = \frac{3.846}{7.333} = 0.524$$

28. **Solution.**

Year	Adv. exp (x)	y	xy	x^2	y^2
1	10	20	200	100	400
2	12	30	360	144	900
3	14	37	518	196	1369
4	16	50	800	256	2500
5	18	56	1008	324	3136
6	20	78	1560	400	6084
7	22	85	1958	484	7921
8	24	100	2400	576	10000
9	26	120	3120	676	14400
10	28	110	3080	784	12100
	$\Sigma x = 190$	$\Sigma y = 690$	$\Sigma xy = 15004$	$\Sigma x^2 = 3940$	$\Sigma y^2 = 58810$

$$r = \frac{n\,\Sigma xy - \Sigma x\,\Sigma y}{\sqrt{n\,\Sigma x^2 - (\Sigma x)^2}\,\sqrt{n\,\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{10 \times 15004 - 190 \times 690}{\sqrt{10 \times 3940 - (190)^2}\,\sqrt{10 \times 58810 - (690)^2}} = 0.985$$

For $y = a + bx$

$\Sigma y = na + b\,\Sigma x$

$690 = 10a + 190\,b$ .... (1)

$\Sigma xy = a\,\Sigma x + b\,\Sigma x^2$

$15004 = 190\,a + 3940\,b$ .... (2)

Solving equation (1) and (2)

$a = -40.048$

$b = 5.739$

$\therefore$ $y = a + bx = -40.048 + 5.739\,x$

When $x = 27$,

$y = -40.048 + 5.739 \times 27 = 114.915$

29. **Solution.**

x	y	x^2	y^2	xy
250	83	62500	6889	20750
340	89	115600	7921	30260
320	88	102400	7744	28160
330	89	108900	7921	29370
346	92	119716	8464	31832
260	85	67600	7225	22100
280	84	78400	7056	23520
395	92	156025	8464	36340
380	93	144400	8649	35340
400	96	160000	9216	38400
$\Sigma x = 3301$	$\Sigma y = 891$	$\Sigma x^2 = 1115541$	$\Sigma y^2 = 79549$	$\Sigma xy = 296072$

$$r = \frac{n\,\Sigma xy - \Sigma x\,\Sigma y}{\sqrt{n\,\Sigma x^2 - (\Sigma x)^2}\sqrt{n\,\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{10 \times 296072 - 3301 \times 891}{\sqrt{10 \times 1115541 - (3301)^2}\sqrt{10 \times 79549 - (891)^2}}$$

$$= 0.957$$

Equation, $y = a + bx$

$\Sigma y = na + b\,\Sigma x$

or $891 = 10a + 3301\,b$ .... (1)

$\Sigma xy = a\,\Sigma x + b\,\Sigma x^2$

or $296072 = 3301\,a + 1115541\,b$ .... (2)

Solving (1) and (2)

$a = 64.1915;\quad b = 0.075$

$\therefore\ y = a + bx = 64.1915 + 0.075\,x$

30. Solution.

Data size (x)	Processed request (y)	x^2	y^2	xy
6	40	36	1600	240
7	55	49	3025	385
7	50	49	2500	350
8	41	64	1681	328
10	17	100	289	170
10	26	100	676	260
15	16	225	256	240
$\Sigma x = 63$	$\Sigma y = 245$	$\Sigma x^2 = 623$	$\Sigma y^2 = 10027$	$\Sigma xy = 1973$

To find $y = a + bx$

$\Sigma y = na + b\sum x$

$245 = 7a + 63b$ .... (1)

$\Sigma xy = a\sum x + b$

$1973 = 63a + 623b$ .... (2)

Solving (1) and (2) $a = 72.278$, $b = -4.142$

$y = a + bx = 72.278 - 4.142x$

$TSS = \Sigma y^2 - n\bar{y}^2 = 10027 - 7 \times \left(\frac{245}{7}\right)^2 =$

$SEE = \Sigma y^2 - a\,\Sigma y - b\sum xy = 10027 - 72.278 \times 245 - (-4.142) \times 1973) =$

$SSR = TSS - SSE =$

$R^2 = \frac{SSR}{TSS} = 0.662$

When $x = 12$, $y = 22.57$

THANK YOU