# Mathematical Model for Engineering Drawing (COP 290 )

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**Abstract**

We hope to develop a mathematical model for engineering drawing. We present the various theorems and lemmas regarding the same.

## 1. Orthographic Projection of 3D objects

Here we present the projection, scaling and rotation matrices needed while projection a 3D object onto a 2D surface.
**Input Specifications:**A list of 3D points. (Each representing intersection of two lines in 3D space).
**Output Specifications:** A list of 2D points. (Each representing intersection of two lines in 2D space).

### *1.1. Projection Matrices*

To construct the top view of an object, we must project a 3D point $(x_i, y_i, z_i)$ onto a 2D point $(x_i, y_i)$ on the x-y plane. We define the following function :-

$$f : R^3 \rightarrow R^3 \ni f(x_i, y_i, z_i) = (x_i, y_i, 0)$$

We construct the following matrix:

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We define the vector representing the 3D point:

$$X = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

Then we observe that:

$$PX = \begin{bmatrix} x_i \\ y_i \\ 0 \end{bmatrix}$$

Thus PX represents a point on the x-y plane. Thus, P defines the required function, P is called a projection matrix.

*1.2. Translation Matrices*

If we need to project a 3D point onto the planes of a translated origin, we need a translation matrix. We define the following function :-

$$f : R^3 \to R^3 \ni f(x_i, y_i, z_i) = (x_i + x_0, y_i + y_0, z_0)$$

where

$$(x_0, y_0, z_0)$$

represents the transformed origin. We construct the following matrix:

$$T = \begin{bmatrix} 1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & y_0 \\ 0 & 0 & 0 & z_0 \end{bmatrix}$$

We define the vector representing the 3D point:

$$X = \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}$$

Then we observe that:

$$TX = \begin{bmatrix} x_i + x_0 \\ y_i + y_0 \\ z_0 \end{bmatrix}$$

Thus TX represents a point on the x-y plane for the transformed origin. Thus T defines the required function, T is called a translation matrix.

*1.3. Scaling Matrices*

If we need to scale a 3D point, we need a scaling matrix. We define the following function :-

$$f : R^3 \to R^3 \ni f(x_i, y_i, z_i) = (x_i s_x, y_i s_y, z_i s_z)$$

where

$$(s_x, s_y, s_z)$$

represents the scaling factors. We construct the following matrix:

$$S = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}$$

We define the vector representing the 3D point:

$$X = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

Then we observe that:

$$SX = \begin{bmatrix} x_i s_x \\ y_i s_y \\ z_i s_z \end{bmatrix}$$

S defines the required function, S is called a scaling matrix.