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Assignment-2

(1) $T(n) = \begin{cases} T(n-1) + 1 & n > 0 \\ 1 & n = 0 \end{cases}$

Master Theorm can't be applied because b=1 but in master's theorm b should be greater than 1

$$\begin{array}{ll} T(n) & 1 \\ & | \\ T(n-1) & 1 \\ & | \\ T(n-2) & 1 \\ & \vdots \\ T(0) & 1 \end{array}$$

$$\Rightarrow \sum_{i=0}^{n} 1 = n+1$$

$T(n) = n+1$

$T(n) = \Theta(n)$

(2) $T(n) = \begin{cases} T(n-1) + n & n > 0 \\ 1 & n = 0 \end{cases}$

Master Theorm cant be applied because b=1.

Tree

$$\begin{array}{ll} T(n) & n \\ & | \\ T(n-1) & n-1 \\ & | \\ T(n-2) & n-2 \\ \vdots & \vdots \\ T(0) & n-n \end{array}$$

$T(n) = n + n-1 + n-2 + n-3 \dots n-n$

$T(n) = n^2 + n - \frac{(n^2+1)}{2} = \frac{n^2+n}{2}$

$T(n) = \Theta(n^2)$

(3) $T(n) = \begin{cases} T(n-1) + \log(n) & n>0 \\ 1 & n=0 \end{cases}$

Master's Theorm can't be applied because it is not of form $aT(\frac{n}{b}) + f(n)$ also $b=1$. here.

Tree

$T(n) = \log(n)$

$T(n-1) \quad \log(n-1)$

$T(n-2) \quad \log(n-2)$

⋮

$T(0) \quad \cancel{\log(0)}\ 1$

$$T(n) = \sum_{i=0}^{n} \log(n-i)$$

$$\Rightarrow \sum_{i=0}^{n} \log(n-i) \le n \log n$$

$$T(n) = O(n \log n)$$

(4) $T(n) = \begin{cases} 2T(n-1)+1 & n>0 \\ 1 & n=0 \end{cases}$

No master's theorm can't be applied becau it is not in the form $aT(\frac{n}{b}) + f(n)$ also $b=1$ here.

<u>Tree</u>

$T(n) \quad 1$

$T(n+1) \quad 1 \quad 1$

$T(n-2 \quad 1 \quad 1 \quad 1 \quad 1$

⋮

$T(0) \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$

$$T(n) = 1 + 2^1 + 2^2 + 2^3 \dots 2^n$$

$$= 1 + \frac{(2^n - 1)\,1}{1}$$

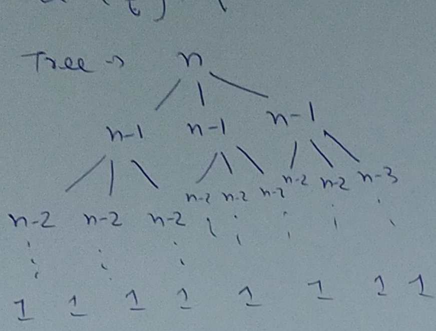
$$= 2^n$$

$$T(n) = \Theta(2^n)$$

(5) $T(n) = \begin{cases} 3T(n-1) + n & n > 0 \\ 1 & n = 0 \end{cases}$

No Master Theorm cause $3T(n-1) + n$ is not of form $aT(\frac{n}{b}) + f(n)$ where $a \geq 1$, $b > 0$.

Tree ->



$$T(n) = \sum_{i=0}^{n} 3^i (n-i)$$

$$n \cdot 3^n \geq \sum_{i=0}^{n} 3^i (n-i)$$

$$T(n) = n \cdot 3^n$$

(6) $T(n) = \begin{cases} T(n/2) + 1 & n > 1 \\ 1 & n = 1 \end{cases}$

$$T(n) = O(\log_2 n)$$

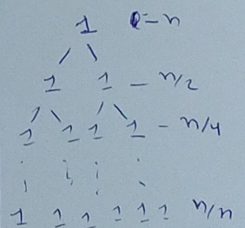
Master's Theorm can be applied.

$a = 1 \quad b = 2$

$n^{\log_b a} = n^{\log_2 1} = n^0 = 1$

$f(n) = 1 = n^{\log_b a} = 1$

So $T(n) = (\log n)$



$$T(n) = \sum_{i=0}^{\log n} 1$$

$$T(n) = (\log n)$$

7. $T(n) = \begin{cases} 1 & n=1 \\ T(n/2)+n & n>1 \end{cases}$

Applying Master's Theorm

$a=1 \quad b=2$

$n^{\log_b a} = n^{\log_2 1} = n^0 = 1$

$f(n) = n > 1$

But $f(n) = n = n^{\log_2 1 + 1} = n^1 \quad \boxed{\epsilon = 1}$

Also $a f(\frac{n}{b}) \leq c f(n)$

$a=1 \quad b=2$

$f(\frac{n}{2}) = \frac{n}{2} \leq \frac{n}{2} \quad \boxed{c = 1/2}$

Thus ~~$T(n) = \Omega(n^{\log_b a}$~~

$f(n) = \Omega(n^{\log_b a + \epsilon})$

$= \Omega(n^{\log_2 1 + 1}) \quad \boxed{\epsilon = 1}$

Thus $T(n) = \Theta(n)$

Tree

n

$\frac{n}{2}$

$\frac{n}{4}$

$\vdots$

1

$\Rightarrow T(n) = \frac{n}{2^0} + \frac{n}{2^1} + \frac{n}{2^2} + \frac{n}{2^3} + \cdots 1$

$= n\left(1 + \frac{1}{2^1} + \frac{1}{2^2} \cdots \frac{1}{2^k}\right) \quad \boxed{k = \log n}$

$= n\left(1 \cdot \frac{(1/2)^{k}-1}{\frac{1}{2}-1} + 1\right) = n\left(\left(\frac{1}{2}\right)^{k} - 1\right)$

$\Rightarrow T(n) = \Theta(n)$

(8) $T(n) = \begin{cases} 1 & n=1 \\ 2T(n/2)+n & n>1 \end{cases}$

By Master's theorm

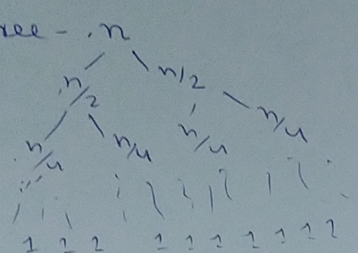
$a=2 \quad b=2$

$n^{\log_b a} = n^{\log_2 2} = n = f(n)$

$T(n) = \Theta(n\log n)$

$T(n) = O(n\log n)$

Tree -



$T(n) = \sum_{i=1}^{\log n+1} n$

$= (n\log n + n)$

$T(n) = O(n\log n)$

(9) $T(n) = \begin{cases} 8T(n/2)+n^2 & n>1 \\ 1 & n=1 \end{cases}$

By Master theorm

$a=8 \quad b=2$

$n^{\log_2 8} = n^3$

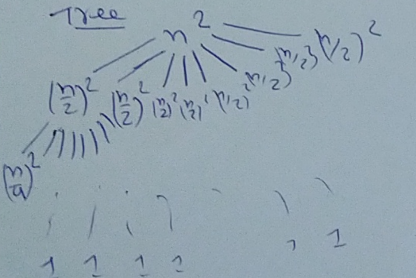
$f(n) = n^2 < n^3$

But $f(n) = n^2 = n^{3-1} = (n^{\log_2 8} - 1)$ $\quad \epsilon = 1$

$f(n) = O(n^{\log_b a - \epsilon})$ $\quad a=8, b=2, \epsilon=1$

$T(n) = \Theta(n^3)$

Tree



$\Rightarrow T(n) = \sum_{i=1}^{\log n} (8)^i \left(\frac{n}{2^i}\right)^2 = \sum_{i=0}^{\log n} 8^i \frac{n^2}{4^i}$

$= n^2 \sum_{i=0}^{\log n} 2^i = n^2(2^0 + 2^1 + 2^2 \cdots 2^{\log n})$

$= n^2\left(\frac{2^{\log n+1} - 1}{2-1}\right)$

$= 2n^3 - n^2$

$= O(n^3)$

(10) $T(n) = \begin{cases} 4T(n/2) + n^2 \log n^5 & n > 1 \\ 1 & n = 1 \end{cases}$

By Master's Theorm

$a = 4 \quad b = 2 \quad n^{\log_b a} = n^{\log_2 4} = n^2$

$f(n) = n^2 \log n^5$

Here master's theorm can't be applied because n^2 and $n^2 \log n^5$ are not polynomially same.

$n^2 \log n^5 \quad - 4^0$

$\left(\frac{n}{2}\right)^2 \log\left(\frac{n}{2}\right)^5 \quad - 4^1$

$\quad - 4^{\log_2 n}$

1

$$T(n) = \sum_{i=0}^{\log_2 n - 1} 4^i \left(\frac{n}{2^i}\right)^2 \log\left(\frac{n}{2^i}\right)^5 + 4^{\log_2 n}$$

$$= \sum_{i=0}^{\log_2 n - 1} 2^{2i} \frac{n^2}{2^{2i}} \log \frac{n^5}{2^{5i}} + 4^{\log_2 n}$$

$$= n^2 \sum_{i=0}^{\log_2 n - 1} \log \frac{n^5}{2^{5i}} + 4^{\log_2 n}$$

$$= n^2 \sum_{i=0}^{\log_2 n - 1} \log \frac{n^5}{2^{5i}} + n^2$$

$$\Rightarrow \sum_{i=0}^{\log n - 1} \log n^5 > \sum_{i=0}^{\log_2 n - 1} \log \frac{n^5}{2^{5i}}$$

$$\Rightarrow \log_2 n^5 \times \log_2 n = \log_2 n^c$$

$$= n^2 \log n^c + n^2$$

$$= O(n^2 \log^c n)$$

(12) $T(n) = \begin{cases} 4T(n/2) + n^3 \log n^2 & n > 1 \\ 1 & n = 1 \end{cases}$

By MT

$a = 4 \quad b = 2 \qquad n^{\log_b a} = n^{\log_2 4} = n^2$ $\boxed{\text{case 3}}$

Here $n^3 \log n^2 = \Omega(\log_b^{a+\epsilon})$ $(\epsilon > 0)$

$af(n/b) \le c\, f(n)$

$a f(n/b) = 4 f\left(\frac{n}{2}\right) = 4 \times \frac{n^3}{8} \log \frac{n^2}{4} \le \underbrace{\frac{1}{2}}_{c} n^3 \log n^2$

$n^3 \log n^2$

$\left(\frac{n}{2}\right)^3 \log\left(\frac{n}{2}\right)^2$

$\left(\frac{n}{2^2}\right)^3 \log\left(\frac{n}{2^2}\right)^2$

$\vdots$

1

$$T(n) = \sum_{i=0}^{\log_2 n - 1} \left(\frac{n}{2^i}\right)^3 \log\left(\frac{n}{2^i}\right)^2 + n^2$$

$$= n^3 \sum_{i=0}^{\log_2 n - 1} \frac{1}{2^{3i}} \log \frac{n^2}{2^{2i}} + n^2$$

$$= n^3 \log^3 n + n^2$$

$$= O(n^3 \log^3 n)$$

$$T(n) = O(n^3 \log^3 n)$$

(15) $T(n) = T(9n/10) + n$

Applying MT $\Rightarrow a = 1 \quad b = 10/9$

$n^{\log_{10/9} 1} = n^0 = 1$

but $n^{\log_{10/9} 1 + 1} = n$

$f(n) = \Omega\left(n^{\log_b a + 1}\right)$

Als $af(n/b) \leq c f(n)$

$\frac{9n}{10} \leq \frac{9}{10} n \quad \boxed{c = \frac{9}{10}}$

$T(n) = \Theta(n)$

Tree

n

$\left(\frac{9}{10}\right)^1 n$

$\left(\frac{9}{10}\right)^2 n$

$\left(\frac{9}{10}\right)^3 n$

$\vdots$

1

$T(n) = \sum_{i=0}^{\infty} \left(\frac{9}{10}\right)^i n$ Taking ∞ steps

$= \cancel{n \frac{9}{10} \log_{10/9} n + 1}$

$= n \frac{\frac{9}{10}}{1 - \frac{9}{10}} = 9n$

$T(n) = O(n)$

(17) $T(n) = T(n-1) + \frac{1}{n}$

Master Theorm can't be applied because above is not of the form $aT\left(\frac{n}{b}\right) + f(n)$.

Tree

$1/n$

$1/n-1$

$\frac{1}{(n-2)}$

$\frac{1}{(n-3)}$

⋮

1

$$T(n) = \sum_{i=0}^{n-1} \frac{1}{(n-i)} + 1 = \int_1^n \frac{1}{x} + 1$$

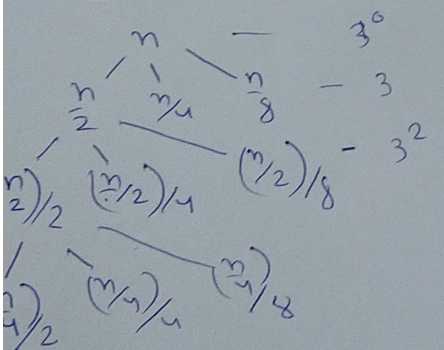
$$= \log_2 n + 1$$

$$T(n) = \Theta(\log_2 n)$$

(18) $T(n) = T(n/2) + T(n/4) + T(n/8) + n$

$T(n) = \Theta(n)$

No Master theorm



$$\Rightarrow T(n) = \sum_{i=0}^{\log n - 1} 3^i$$

(19) $T(n) = 4T(n/2) + n^2\sqrt{n}$

<u>MT</u>

$a=4 \quad b=2 \qquad n^{\log_b a} = n^2$

$n^2\sqrt{n} = \Omega(n^{2+\epsilon})$

$af(n/b) \le cf(n)$

$af(\frac{n}{b}) = 4\frac{n^2}{4}\sqrt{\frac{n}{2}} = \frac{1}{\sqrt{2}} n^2\sqrt{n}$

$\frac{1}{\sqrt{2}} \rightarrow c$

$n^2\sqrt{n}$

$(\frac{n}{2})^2\sqrt{\frac{n}{2}}$

$(\frac{n}{4})^2\sqrt{\frac{n}{4}}$

$$T(n) = \sum_{i=0}^{\log_2 n-1} \left(\frac{n}{2^i}\right)^{2} \sqrt{\frac{n}{2^i}} + n^2$$

$$= n^{5/2} \sum_{i=0}^{\log_2 n-1} \frac{1}{2^{5i}}$$

$$= n^2\sqrt{n} \cdot \frac{2^4}{31} + n^2$$

$$T(n) = O(n^2\sqrt{n})$$