Algebra Poly Value

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Problem. *If* $P(x)$ *denotes a polynomial of degree* n *such that* $P(k) = \frac{k}{k+1}$ *for* $k = 0, 1, 2 \ldots, n$*, determine* $P(n+1)$*.*

Solution. *Consider the* $(n+1)$ *degree polynomial* $G(x) = (x+1)P(x) - x$*. Note that* $0, 1, 2 \ldots, n$ *are roots of* $G(x)$*, hence* $G(x) = C \prod_{i=0}^{i=n}(x-i)$ *where* C *is a constant. Now since* $(x+1)|G(x) + x$ *. This means -1 is a root to* $G(x) + x$*, so* $G(-1) - 1 = 0$*, i.e.* $G(-1) = 1$*, so* $C(-1)^{n+1}((n+1)!) = 1$*, hence* $C = \frac{(-1)^{n+1}}{(n+1)!}$ *. Hence* $P(n+1) = \frac{(G(n+1)+(n+1))}{n+2} = \frac{n+1+(-1)^{n+1}}{n+2}$

Exploration. *N/A*

Tags. *Polynomial , Algebra , Interpolation*