Calibration Analysis: Integrating Theory with Data[1]

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[1] Some materials in this slide are borrowed from Prof. Ping Wang's lecture notes.

Why Calibration?

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- To conduct more serious policy experiments than simple numerical analysis before a policy is implemented into the real economy

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e.g. FOC, BC, . . .

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e.g. β

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- Use model equations and stable relations that we observe in data to pin down the values of parameters in the model
- Stable relations could be:
  - First moment: time-series average; cross-sectional average; ratio e.g. $\overline{GDP}$
  - Second moment: growth rate; variance

An Example

Consider a social planner's **stationary** problem (the model with exogenous population growth and labor-augmenting technological progress):

$$\max_{\{\hat{c}_t, \hat{k}_{t+1}, h_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta\eta)^t [\log \hat{c}_t - Bh_t]$$

subject to:

$$\hat{c}_t + \eta\gamma\hat{k}_{t+1} = \hat{k}_t^{\theta} h_t^{1-\theta} + (1-\delta)\hat{k}_t$$

Steady-state Equations

The three equations that characterizes the steady state are given by:

$$\begin{aligned}\frac{(1-\theta)\bar{k}^{\theta}\bar{h}^{-\theta}}{\bar{c}} &= B;\\ \gamma &= \beta(\theta\bar{k}^{\theta-1}\bar{h}^{1-\theta}+1-\delta);\\ \bar{c}+(\eta\gamma+\delta-1)\bar{k} &= \bar{k}^{\theta}\bar{h}^{1-\theta}.\end{aligned}$$

In the steady state, we have 3 endogenous variables $\bar{c}$, $\bar{k}$, and $\bar{h}$. Once those parameters are determined (calibrated), we can solve for the 3 endogenous variables.

Parameters in the Model

- We have six parameters to be calibrated:
  - Preference: β, η, and B → weight of utility from working
  - Technology: γ, θ, and δ

growth rate　income share of capital ⇒ 這些都算得出來

- Some of them are observable (η, γ, θ), but others are lack of measurement in the real world (β, B, δ). Thus, we do calibration to determine these parameters.

↳ 無法觀察，∴只能用假設

- Serious Calibration: $\sharp$ of parameters equals $\sharp$ of data moments.

if <, then over identified

p.s. δ 不是 accounting 的那種折舊

Target Economy

- Target economy: calibrate the model to data from the US after the Korean War
- Six stable relations for the US economy (annual basis, directly taken from data):
  - $\gamma = 1.014$: average growth rate of per capita output (recall $g = \gamma$)
  - $\eta = 1.015$: average population growth rate
  - $\theta = 0.4$: capital income share $\frac{r\bar{k}}{\bar{y}}$
  - $\bar{h} = 0.31$: average fraction of time that people spend on working
  - $\frac{\bar{k}}{\bar{y}} = 3.5$: capital-output ratio
  - $\frac{\bar{c}}{\bar{y}} = 0.75$: consumption-output ratio

data moments

从文献得到

Parameters, Data, and Equations

- What we have at this stage:

$u(\ ) = \Sigma$

s.t. $c + k = Y$

$\pi = Y - wn - rk$

$$\begin{cases} k^f = k \\ c + i = Y \end{cases}$$

Parameters, Data, and Equations

- What we have at this stage:
  - 6 unknown parameters: β, η, B, γ, θ, and δ
  - 6 stable relations: γ, η, θ, $\bar{h}$, $\frac{\bar{k}}{\bar{y}}$, and $\frac{\bar{c}}{\bar{y}}$
  - 3 model equations at steady state $\longrightarrow$ P. 9

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- Thus, γ, η, and θ are directly determined by data.
- Next, solve β, B, and δ using the rest 3 stable relations and 3 model equations.

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- We obtain $B = 2.581$, $\beta = 0.946$, and $\delta = 0.042$.
- Together with $\eta = 1.015$, $\gamma = 1.014$, and $\theta = 0.4$, **the steady-state calibrated economy** is given by:

$$
\begin{aligned}
\bar{k} &= [\frac{\beta\theta}{\gamma - \beta(1-\delta)}]^{\frac{1}{1-\theta}} \bar{h} = 2.509; \\
\bar{y} &= [\frac{\beta\theta}{\gamma - \beta(1-\delta)}]^{\frac{\theta}{1-\theta}} \bar{h} = 0.715; \\
\bar{c} &= [\frac{\beta\theta}{\gamma - \beta(1-\delta)}]^{\frac{\theta}{1-\theta}} (\frac{1-\theta}{B}) = 0.537; \\
\bar{I} &= \bar{y}(1 - \frac{\bar{c}}{\bar{y}}) = 0.179.
\end{aligned}
$$

值不重要
真正重要的是
ratio!!

Research Steps

- Find a research question
- Construct a theoretical model
- Make sure the model is stationary → by detrend
- Solve the maximization problems
- Collect data and calibrate the model
- Do impulse response or conduct policy experiments