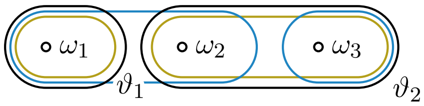
ECON 7011, Semester 110.1, Assignment 5

Please hand in your solutions via NTU Cool before 11:59pm on Tuesday, December 21

1. Consider the following 2-player Bayesian game, in which the players share a common prior $P(\{\omega_3\}) = \frac{1}{5}$ and $P(\{\omega_1\}) = P(\{\omega_2\}) = \frac{2}{5}$.



	L	R
T	$2, 2$	$-1, 3$
D	$3, -1$	$0, 0$

ϑ_1

	L	R
T	$0, 3$	$2, 0$
D	$3, 1$	$0, 2$

ϑ_2

   (a) Find the family of posterior beliefs for both players.

   (b) Find all Bayesian Nash equilibria of the game.

   (c) What do you predict will happen if the true state is ω_2?

2. Two political candidates compete for the presidency, whose value θ is known to the Incumbent, but not to the Challenger. The Challenger only knows that θ is either 1 or 2 with equal probability. The outcome of the election is independent of the value of the presidency, but it depends on the chosen campaign budgets b_I and b_C. The probability that the Incumbent wins the presidency is $\frac{b_I}{b_I+b_C}$. Interim expected utilities, are, therefore,

$$u_I(b, \vartheta) = \frac{b_I}{b_I + b_C}\vartheta - b_I, \qquad u_C(b, \vartheta) = \frac{b_C}{b_I + b_C}\vartheta - b_C.$$

   Suppose that a minimum budget of $\frac{1}{10}$ is required to participate in the election. Find the pure-strategy Bayesian Nash equilibria of this game.

3. Consider a first-price auction among two bidders, in which the two bidders' valuations are uniformly distributed on $\{\vartheta_L, \vartheta_H\}$. If the bids are a tie, the item is awarded to both players with equal probability, that is, ex-post payoffs for bids $b = (b_1, b_2)$ are given by

$$u_i(b, \vartheta_i) = \begin{cases} \vartheta_i - b_i & \text{if } b_i > b_{-i}, \\ \frac{1}{2}(\vartheta_i - b_i) & \text{if } b_i = b_{-i}, \\ 0 & \text{if } b_i < b_{-i}. \end{cases}$$

   Find the unique symmetric Bayesian Nash equilibrium with the following steps:

   (a) Show that in any equilibrium σ, we must have $\sigma_i(\vartheta_L) = \vartheta_L$.

   (b) Show that no pure-strategy Bayesian Nash equilibrium exists.

   Let $\sigma_i(\vartheta_H)$ be parametrized by distribution function F. One can show that F admits a density function f on an interval with end points $\underline{b} < \bar{b}$ (you do not have to show this).

   (c) Show that, in equilibrium, the lower end of the support of F is ϑ_L.

   (d) Show that ϑ_L itself is not in the support of F.

   Together, these steps show that $\operatorname{supp} F = (\vartheta_L, \bar{b}]$ *Note: $\bar{b}$ can be included by continuity.*

   (e) Use the indifference principle to find $\bar{b}$ and F.

   (f) Verify that the derived strategy profile is a Bayesian Nash equilibrium.

   Hint: You can solve parts (b)–(e) even if you did not solve earlier parts. You may want to start with the simpler parts (b), (e), and (f). These may then give you an idea how to solve (c) and (d). To prevent a circular argument, your answers to (c) and (d) should not depend on (e)–(f) and your answer to (a) should not depend on (b)–(f).