ECON 7011, Semester 110.1, Practice Problems 7, Solutions

1. (a) Since there is no distortion at the top, the whales get the good loot boxes for 10000 NTD. The normal type is shut down if and only if the cost of information rent 1000μ exceeds the benefit $30(1-\mu)$ of selling to the normal type. This is equivalent to $\mu > \frac{3}{103}$.

   (b) If $\mu > \frac{3}{103}$, the company sells only the good loot boxes for 10000 NTD. If $\mu \leq \frac{3}{103}$, the company sells the medium loot boxes for 30 NTD and the good loot boxes for 9000 NTD.

2. (a) The first-best contract maximizes the joint surplus $u_1(p,q) + u_2(p,q,\vartheta) = q - \vartheta q^2$ at $q_\vartheta = \frac{1}{2\vartheta}$. Prices are given from the participation constraint, hence $p_\vartheta = \vartheta q_\vartheta^2 = \frac{1}{4\vartheta}$.

   (b) The setting is identical to the setting in class with reversed signs. Note that ϑ_1 is the more efficient type since the production cost is increasing in ϑ. This can also be seen from $c_{q\vartheta}(q,\vartheta) = 2q > 0$, which indicates that the lower numerical value corresponds to the higher type. It follows that the participation constraint for type ϑ_2 binds and the incentive constraint for type ϑ_1 binds. The high type produces the first-best amount $\hat{q}_1 = \frac{1}{2\vartheta_1}$ and the low type's production solves

$$1 = 2\hat{q}_2\vartheta_L - \frac{\mu}{1-\mu} 2\hat{q}_2(\vartheta_1 - \vartheta_2), \qquad \Rightarrow \qquad \hat{q}_2 = \frac{1-\mu}{2(\vartheta_2 - \mu\vartheta_1)} > 0.$$

   We find the prices from the binding (IR_2) and (IC_1) constraints

$$\hat{p}_2 = \vartheta_2 \hat{q}_2^2 = \frac{(1-\mu)^2\vartheta_2}{4(\vartheta_2 - \mu\vartheta_1)^2}, \qquad \hat{p}_1 = \frac{1}{4\vartheta_1} + \frac{(1-\mu)^2(\vartheta_2 - \vartheta_1)}{4(\vartheta_2 - \mu\vartheta_1)^2}.$$

3. (a) It follows from symmetry of the viewers' utility function that a show (s_i, q_i) attracts viewers in the segment $S_i = [s_i - \sqrt{q_i}, s_i + \sqrt{q_i}]$. Since the TV Network's utility does not increase if a viewer watches both shows, it is optimal that $S_1 \cap S_2$ has empty interior. Finally, the intervals are contiguous starting at 0 because the exponential density is decreasing and the cost of production is increasing with seriousness.

   (b) From (a), we conclude that $I_1 = [0, 2\sqrt{q_1}]$ and $I_2 = [2\sqrt{q_1}, 2\sqrt{q_1} + 2\sqrt{q_2}]$ as well as $s_1 = \sqrt{q_1}$ and $s_2 = 2\sqrt{q_1} + \sqrt{q_2}$. The TV network thus maximizes

$$V(q_1,q_2) = rP(\theta \leq 2\sqrt{q_1} + 2\sqrt{q_2}) - c(s_1,q_1) - c(s_2,q_2) \tag{1}$$

   subject to the budget constraint

$$\sqrt{q_1}^3 + 2\sqrt{q_1}q_2 + \sqrt{q_2}^3 \leq b.$$

   Let us simplify the notation by setting $x = \sqrt{q_1}$ and $y = \sqrt{q_2}$. The total production cost is $c(x,y) = x^3 + 2xy^2 + y^3$. Suppose first that the budget constraint binds. Then we can solve $c(x,y) = b$ for one of the variables, say $y(x)$, and write

$$V(x) = r - r\mathrm{e}^{-2\lambda(x+y(x))} - b.$$

   Since $c(x,y)$ is differentiable, so is $y(x)$. Therefore, $V(x)$ is maximized either at $x = 0$ or where the $V'(x) = 0$. We note that the budget constraint cannot bind if $x = 0$ is optimal, hence the maximizer (if the budget constraint binds) satisfies

$$V'(x) = 2r\lambda\mathrm{e}^{-2\lambda(x+y(x))}\Big(1 + y'(x)\Big) = 0.$$

   Implicitly differentiating $c(x,y) = b$ with respect to x yields

$$3x^2 + 2y^2 + 4xyy'(x) + 3y^2y'(x) = 0.$$

Therefore, $y'(x) + 1 = 0$ is equivalent to

$$-1 = y'(x) = -\frac{3x^2 + 2y^2}{4xy + 3y^2}. \tag{2}$$

This is a quadratic equation that we can solve for $y(x) = -2x \pm \sqrt{7}x$. Since we are interested in the positive solution, we deduce that $y(x) = kx$ for $k = \sqrt{7} - 2$. Substituting back into the binding budget constraint, we obtain

$$x = \left(\frac{b}{1 + 2k^2 + k^3}\right)^{\frac{1}{3}}, \qquad q_1 = x^2 = \left(\frac{b}{1 + 2k^2 + k^3}\right)^{\frac{2}{3}}, \qquad q_2 = k^2 q_1.$$

(c) The less serious show is the higher-quality show since $k < 1$.

(d) If the budget constraint does not bind, we simply maximize x and y independently. Taking the partial derivatives of $V(x, y) = r - r\mathrm{e}^{-2\lambda(x+y)} - c(x, y)$ with respect to both yields

$$\frac{\partial V(x, y)}{\partial x} = 2r\lambda\mathrm{e}^{-2\lambda(x+y)} - 3x^2 - 2y^2 = 0,$$
$$\frac{\partial V(x, y)}{\partial x} = 2r\lambda\mathrm{e}^{-2\lambda(x+y)} - 4xy - 3y^2 = 0.$$

Equating the two yields (2). We conclude that the ratio of the two shows' quality $k^2 = 11 - 2\sqrt{7} \simeq 0.8$ remains unchanged.