ECON 7219, Semester 110.1, Assignment 4

Please justify all your answers and hand in the assignment by Monday, Jan 3, 13:20.

1. Creating a Tinder profile is a problem of information design. Suppose that two people are either a good fit or a poor fit for each other, with prior 20% that they are a good fit. Each visitor of a Tinder profile receives a signal "good fit" or "poor fit." The visitor then chooses to swipe left or right, where swiping right signals a willingness to meet. A 2019 statistic reports that men swipe right 46% of the time and that women swipe right 14% of the time. Suppose that posteriors are distributed uniformly across the population so that this implies that men swipe right if their posterior is above 54% and women swipe right if their posterior is above 86%.
   (a) Romeo is using Tinder to meet women, Juliet is using Tinder to meet men. What conditional distribution of their signal should they aim for if their goal is to get as many right swipes as possible?
   (b) Suppose that a Bayesian rational Tinder user knows the distribution of right swipes, the ratio of good fits in the population, and that Juliet has crafted the optimal Tinder profile. What are the posteriors after observing the "poor fit" and the "good fit" signal?
   (c) Interpret the optimal Tinder profile at an intuitive level.
   (d) What percentage of right swipes can Romeo and Juliet secure with an optimal profile?
   (e) Under what additional assumption is the optimal Tinder profile you have derived consistent with our assumption that the distribution of posteriors is uniform across the population?
2. Consider the product-choice game with imperfect monitoring as on Slide 10 of the lecture slides. Suppose that the firm is patient, i.e., $\delta > \frac{1}{2(p-q)}$.
   (a) Characterize the set of all sequential equilibria in this setting.
   (b) Explain why the firm has to be more patient in order to build a reputation in the imperfect monitoring version of the game than in the perfect monitoring version.
3. Suppose that the mechanism designer sells to a single buyer with (conditional) distributions of ex-ante type $T \in [0,1]$ and ex-post type $\theta \in [0,1]$ given by

$$G(\tau) = \tau, \qquad F(\vartheta \mid \tau) = \vartheta + \tau\left(\left(\vartheta - \frac{1}{2}\right)^2 - \frac{1}{4}\right).$$

   Suppose that the seller places value $c \in [0,1]$ on the good.
   (a) Show that the assumptions of Proposition 15.10 are satisfied, that is,
      i. Show that $F(\vartheta \mid \tau)$ is decreasing in τ for any $\vartheta \in (0,1)$.
      ii. Show that the derivatives $\frac{\partial v(q,\vartheta)}{\partial \vartheta}$ and $\frac{\partial F(\vartheta \mid \tau)}{\partial \tau}$ are bounded in absolute value.
      iii. Show that $\psi(\tau,\vartheta) = \vartheta + \frac{1-G(\tau)}{g(\tau)} \frac{\partial F(\vartheta \mid \tau)/\partial \tau}{f(\vartheta \mid \tau)}$ is increasing in τ and ϑ.
   (b) Find the allocation $q(\tau,\vartheta)$ and the payment $\hat{p}(\tau)$ in the optimal selling mechanism. It is not necessary to find an explicit expression for $p_{\underline{\vartheta}}(\tau)$.
   (c) Show that the optimal selling mechanism is ex-post efficient if and only if $c \in \{0,1\}$.