Introduction

ECON 7219 – Games With Incomplete Information

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Decisions Under Uncertainty

> *"Our new Constitution is now established, and has an appearance that promises permanency; but in this world nothing can be said to be certain, except death and taxes."*
>
> *– Benjamin Franklin*

Should I Take This Class or Drop It?

Available actions:

- Stay in this class / drop it.

Impact on your life:

- Long-term impact: grade achieved, skills/knowledge gained.
- Short-term impact: enjoyment.
- Opportunity cost: time invested.

Incomplete information:

- Content: What is this class about? Will I learn anything useful?
- Conduct: Is this class going to be hard? Is it going to be fun?
- Prerequisites: What prior knowledge is required/helpful?

What Is This Class About?

Job Applications

Job application:

- What employers really want to know: how well can you do the job?
- Applicants are well informed, but also have an incentive to lie.
- In order to credibly distinguish themselves from others, applicants need to take into account what proof others can produce how easily.

Poker

To Call or Not to Call?

Ziigmund's decision:

- Available actions: call / fold / raise.
- Goal: maximize expected winnings in this hand.
- Expectation is subjective, given the players' beliefs, which depend on:
  - Hard information: distribution of cards, prior actions of opponent.
  - Strategic considerations: playing style of opponent.
  - Soft information: body language of opponent.

A call is profitable if

$$\mathbb{E}_Z[u_Z(\text{"Call"})] = 149.2 \cdot P_Z(\text{"better hand"}) - 44.3 \geq 0,$$

which is equivalent to $P_Z(\text{"better hand"}) \geq 29.69\%$.

How to Make it Look Like a Bluff?

Tom's decision:

- Available actions: how much to raise.
- Goal: maximize expected winnings in this hand.
- Expectation depends on higher-order beliefs:
    - Does Tom believe Ziigmund believes Tom's cards are bad? $\rightarrow$ big raise.
    - Does Tom believe Ziigmund aware of the above? $\rightarrow$ small raise.
- Tom uses his "perceived hand range" to get paid off.

Caveat:

- Actions in this hand affect Tom's image in other hands.
- Tom needs to maximize his strategy across hands.

Part 1: Modeling and Solving Games

Weeks 1–2: Modeling situations with incomplete information

- Knowledge and belief hierarchies.

- Common knowledge and the agreement theorem.

- Minimal and universal belief spaces.

- Some probability theory.

Weeks 3–4: Solving games with incomplete information

- Bayesian and perfect Bayesian equilibrium.

- Signaling games, cheap talk, and communication.

- There is significant overlap with Econ 7011.

Propaganda

Should you trust the newspaper?

- A biased owner has an incentive to skew their stories. If the articles contain too little reliable information, however, they will not be read.
- If the owner indeed wishes to push an agenda, how close to the truth do the articles have to be to maximize impact?
- Knowing the owner's agenda, can/should you trust such a newspaper?

Part 2.1: Designing Information Policies

Week 5: Information design

- Design of an information revelation policy in a situation where the designer has a preference over the players' chosen actions.
- To which extent can a rational audience be influenced?
- What outcomes can be implemented by controlling information alone?
- Which information policy maximizes the designer's objectives?

More applications:

- Can a school's grading policy improve the job prospect of its graduates?
- How should Google obscure traffic information to improve traffic flow?
- How do you design the optimal profile on a dating app?
- How should I advertise this class, given that I'm bound to be truthful?

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Incentivize Truthful Information Revelation
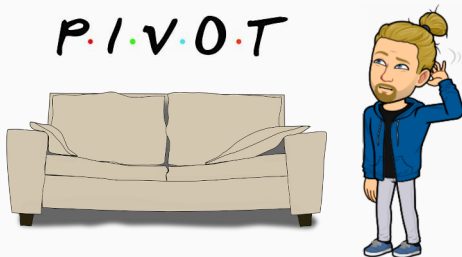
New appliance in a shared apartment:

- Should you and your roommate get that new couch, new X-box, etc?
- In theory it is easy. How much is it worth to each roommate?
- Problem: there is always this one roommate who reports a lower value because they want to pay as little as possible.

Part 2.2: Designing Games / Mechanisms

Weeks 6–8: Mechanism design

- Design of a game with players that hold private information.
- Can truthful information be elicited efficiently?
- Is there a mechanism, in which players cannot benefit from strategic consideration about information reported by others?
- Is there a mechanism, in which players know they cannot benefit from strategic consideration about information reported by others?

More applications:

- What is the revenue-maximizing auction format?
- Is there an efficient voting mechanism?

There is some overlap with Econ 8008.

Which Bubble Tea Shop Is Better?

Lining up for bubble tea:

- Do those people line up because they know this shop is the best?
- In that case, it is in my best interest to line up for the shop as well.
- Or did they make the same considerations as I did and simply line up, expecting others in the line must have information?

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Part 3: Learning over Time

Week 10: Social learning

- Will society eventually learn which store is the better store?
- Under what conditions can herding on the "wrong store" be avoided?
- Is social learning robust to societies with inhomogeneous preferences?

Weeks 11–12: Reputations

- Can a strategic store owner build a reputation for quality tea?
- Is the benefit of the reputation worth the cost of building it?
- When will a store owner have an incentive to deplete his/her reputation?

Week 13 (?): Strategic experimentation

- When should I explore new opportunities vs. sticking to old ones?
- Can I incentivize others to experiment?

Syllabus

Purpose of This Class

Types of students:

1. Students from technical backgrounds who are interested in learning more about game theory.

2. Economics students who wish to explore whether they might be interested in doing research in economic theory.

3. Economics graduate students who do research in economic theory.

Objectives:

- For any given situation, everybody should be able to recognize what kind of incomplete information is present.

- Everybody will learn the tools/skills to solve basic problems.

- For the third type of students, sometimes I need to go beyond what is exam relevant. I will highlight those slides in a separate color theme.

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Course Grade

Grade distribution:

- 10% Participation.
- 20% Assignments.
- 30% Midterm.
- 40% Student presentations in groups of two.

Participation:

- Answer the weekly questions on Poll Everywhere.
- Attend other student presentations.

Student presentations:

- Present an influential and/or recent paper to one of the topics.
- Write a short 2–4 page LaTeX summary of the paper and relate its contribution to the existing literature.

Poll Everywhere

Free polling software:

- Available in Google play / App store.
- Also available on the web at
  https://pollev.com/bbernard442.
- Please register using your student ID.
- Answers do not have to be correct.

I would like to know:

- Why do you take this class?

Scan me

Books

Game Theory by M. Maschler, E. Solan, and S. Zamir, 2013. W1–2.

- Modern book on game theory with an eclectic selection of topics.
- Contains many many practice problems in each chapter.
- Presentation is not always the most intuitive.

An Introduction to the Theory of Mechanism Design by T. Börgers, 2015. W6–8.

- Well written introduction to mechanism design.
- The theory is completely developed using examples.

Repeated Games and Reputations by G. Mailath and L. Samuelson, 2006. W11–12.

- Very thorough, but still surprisingly easy to read.
- Covers mostly repeated games with complete information.

Game Theory – An Introduction by S. Tadelis, 2013. W3–4.

- Extremely well explained. Interesting examples. Actually fun to read.
- More of an advanced undergraduate textbook.

Preliminary Knowledge

Tools we use: game theory, probability theory, calculus.

Required:

1. I can take the derivatives of $f(x) = 2x^2$ and $g(x) = \frac{1}{x}$.
2. I can integrate $f(x) = 2x^2$ and $g(x) = e^x$.

Scan me

Helpful to have seen it:

3. I can compute a mixed-strategy Nash equilibrium in a 2×2 game.
4. I can find a subgame perfect equilibrium in a 2-period game.
5. I can compute the expected value $\mathbb{E}[X]$ of a random variable X with a simple density function $f(x)$.

Not relevant for the class, just for me to know:

6. I can write down the definition of a martingale.