Overlapping Generations Model

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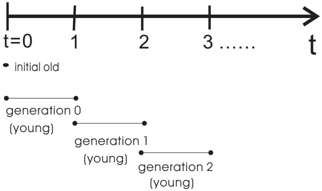
Introduction

- The Neo-classical growth model:
  - assumes there is a representative agent living for infinite periods;
  - usually applied to Real Business Cycles with productivity shocks;
  - abstracts from modeling life-cycle choices, for example,
    - when to retire?
    - how much to save for retirement?
    - how much for intergenerational transfer?
- We are studying age structure within a population. It matters that multiple generations living at the same periods of time.
- The overlapping generations (OLG) model is usually adopted in the demographic literature.

Basic Concepts

An agent lives for two periods, young adulthood and old adulthood (finite-lived).



- In each period of time, there exists two generations: young and old.
- During the young adulthood, each agent has one unit of labor.
- Agents retire when they enter the old adulthood. The labor supply is zero.

Household's Problem 1

An agent born at $t \geq 0$ solves the following maximization problem:

$$\max_{\{c_t^y, c_{t+1}^o, s_t\}} \{u(c_t^y) + \beta u(c_{t+1}^o)\}$$

∵不含 s_{t+1} ∴不需要 law of motion

subject to

$$\begin{aligned} c_t^y + s_t &= w_t; & (1) \\ c_{t+1}^o &= s_t R_{t+1}; & (2) \\ R_{t+1} &= 1 + r_{t+1} - \delta; & (3) \end{aligned}$$

此model的儲蓄是資本，∴還是會折舊

儲蓄率由下期決定，∵是下期才領出來

where equation (1) is the budget constraint for young adulthood and equation (2) is the budget constraint for old adulthood.

Household's Problem 2

- Substituting s_t in (2) to (1) to obtain the "life-time budget constraint":

$$c_t^y + \frac{c_{t+1}^o}{R_{t+1}} = w_t. \tag{4}$$

- The optimal allocation of an agent born at $t \geq 0$ is given by:

$$\frac{u'(c_t^y)}{u'(c_{t+1}^o)} = \beta R_{t+1}. \tag{5}$$

$\mathcal{L} = u(c_t^y) + \beta u(c_{t+1}^o) + \lambda [w_t - c_t^y - \frac{c_{t+1}^o}{R_{t+1}}]$

$u'(c_t^y) = \lambda \longrightarrow$ FOC to c_t^y

$\beta u'(c_{t+1}^o) = \lambda \cdot \frac{1}{R_{t+1}} \longrightarrow$ FOC to c_{t+1}^o

$\Rightarrow u'(c_t^y) = \beta u'(c_{t+1}^o) \cdot R_{t+1} \Rightarrow \frac{u'(c_t^y)}{u'(c_{t+1}^o)} = \beta R_{t+1}$

$u'(c_t^y) \rightarrow u_{c_t^y}(c_t^y)$

$u'(c_{t+1}^o) \rightarrow u_{c_{t+1}^o}(c_{t+1}^o)$

Household's Problem 3

The initial old solves the following problem:

$$\max_{c_0^o} u(c_0^o)$$

subject to

$$c_0^o = s_{-1} R_0. \tag{6}$$

The optimal decision is $c_0^o = s_{-1} R_0$.

Population

- At any period t, N_t young adults are born and the growth rate of generation is $\frac{N_{t+1}}{N_t} = 1 + n$.
- Total population at period t is $\overset{\text{young}}{N_t} + \overset{\text{old}}{N_{t-1}}$; total population at period $t+1$ is $N_{t+1} + N_t$.
- Total population growth rate is given by:

$$\frac{N_{t+1} + N_t}{N_t + N_{t-1}} = \frac{N_t(1+n) + N_{t-1}(1+n)}{N_t + N_{t-1}} = 1 + n,$$

which is equal to the growth rate of generation.

↳ 每期都只有一期出生，∴成长率相同

Production 1

The production side is perfectly competitive, using physical capital and labor as inputs. The production technology is $F(K, N)$ which satisfies the following properties:

1. $F(K, N)$ is constant return to scale (CRS). $\longrightarrow$ 两者都增加1倍，总产生加1倍
2. $F(0, N) = F(K, 0) = 0$. $\longrightarrow$ 缺一不可
3. $F(K, N)$ is increasing in K and N. $\longrightarrow$ 越多越好
4. $F(K, N)$ is strictly concave in K and N. $\longrightarrow$ 边际递减
5. $\lim_{K \to 0} F_K(K, N) = \lim_{N \to 0} F_N(K, N) = \infty$. $\longrightarrow$ 去除角解

Production 2

If $F(K,N)$ is CRS, $F_1(K,N)$ and $F_2(K,N)$ are homogenous of degree zero.

偏导

CRS
$\Updownarrow$

Proof :

$F(K,N)$ is homogenous of degree 1, thus $F(\lambda K, \lambda N) = \lambda F(K,N)$.
Differentiate with respect to K to get:

$$\begin{aligned} \lambda F_1(\lambda K, \lambda N) &= \lambda F_1(K,N), \\ F_1(\lambda K, \lambda N) &= F_1(K,N). \end{aligned}$$

Thus, $F_1(K,N)$ is homogenous of degree 0. Similarly, we can show that F_2 is homogenous of degree 0.

Production 3

- Profit maximization leads to the following conditions:

$$\left.\begin{aligned} r &= F_1(K,N), \\ w &= F_2(K,N). \end{aligned}\right\}$$

∵完全競爭的 setting
∴自然會如此

- per capita term: $\frac{F(K,N)}{N} \overset{\text{CRS}}{=} F(k,1) \equiv f(k)$.
  - $r = F_1(K,N) \overset{\text{homogeneous of degree zero.}}{=} F_1(k,1) = f'(k)$, $K \cdot r + N \cdot w = 1$
  - Because $F(K,N)$ is CRS, $\underline{KF_1(K,N) + NF_2(K,N) = F(K,N)}$,
  - Divided by N to obtain $kF_1(K,N) + F_2(K,N) = F(k,1)$ and $kf^{'}(k) + w = f(k)$,
  - Thus, $\underline{w = f(k) - kf^{'}(k)}$.

↳ P15 會用到，精神是全換成 k 的函數後，比較好求出 policy func.

Market Clearing Conditions 1

這裡其實假設
儲蓄是以資本的方式存

- **Asset market:**
  - Capital tomorrow = Total savings today, $\underset{\text{demand}}{K_{t+1}} = \underset{\text{supply}}{N_t s_t}.$
  - Divided by N_t:

$$
\begin{aligned}
\frac{N_{t+1}}{N_t}\frac{K_{t+1}}{N_{t+1}} &= s_t \\
(1+n)k_{t+1} &= s_t \\
k_{t+1} &= \frac{s_t}{1+n}
\end{aligned}
$$

save 和使用capital的人不同

上一期人　　下一期人

∴必須考慮成長率，供需才會均衡

- **Labor market:**

$$N_t = N_t.$$

Market Clearing Conditions 2

- **Goods market:**
  - Resource constraint: $c_t^y N_t + c_t^0 N_{t-1} + I_t = F(K_t, N_t)$.
  - Divided by N_t:
  $$c_t^y + \frac{c_t^0}{1+n} + i_t = f(k_t).$$
- **Law of motion of capital:**
  - $K_{t+1} = I_t + (1-\delta)K_t$.
  - Divided by N_t:
  $$(1+n)k_{t+1} = i_t + (1-\delta)k_t.$$
- Substitute for i_t to obtain:

$$c_t^y + \frac{c_t^o}{1+n} + (1+n)k_{t+1} = f(k_t) + (1-\delta)k_t.$$

Competitive Equilibrium

A competitive equilibrium is allocations $\{c_t^y, c_t^o, s_t, k_t\}_{t=0}^{\infty}$, and allocation for the initial old, c_{-1}^o, prices $\{r_t, w_t\}_{t=0}^{\infty}$ such that

1. Households born at time $t \geq 0$ solves the maximization problem, and the initial old solves his maximization problem.
2. The firm maximizes its profits.
3. All markets clear.

An Example 1

Suppose the utility function is given by $u(c) = \log(c)$ and the production technology is $F(K, N) = AK^{\alpha}N^{1-\alpha}$, where $0 < \alpha < 1$.

- Production function (per capita): $f(k_t) = Ak_t^{\alpha}$.
- Euler equation: $\frac{c^o_{t+1}}{c^y_t} = \beta R_{t+1}$.
- Budget constraints: $c^y_t = w_t - s_t$ and $c^o_{t+1} = s_t R_{t+1}$.

Substituting budget constraints into Euler equation to obtain:

$s_t = \frac{\beta}{1+\beta} w_t$.

$\beta \uparrow \Rightarrow$ more patient $\Rightarrow$ saving $\uparrow$

An Example 2

Asset market clearing condition: $k_{t+1} = \frac{s_t}{1+n}$.
Substituting $s_t = \frac{\beta}{1+\beta} w_t$ and $w = f(k) - kf'(k)$, we have:

$$
\begin{aligned}
k_{t+1} &= \frac{\beta}{(1+\beta)(1+n)} [f(k_t) - f'(k_t)k_t] \\
&= \frac{\beta}{(1+\beta)(1+n)} (Ak_t^{\alpha} - A\alpha k_t^{\alpha}) \\
&= \frac{A\beta(1-\alpha)}{(1+\beta)(1+n)} k_t^{\alpha}. \longrightarrow \text{optimal policy func.}
\end{aligned}
$$

At steady state: $k_{ss} = [\frac{A\beta(1-\alpha)}{(1+\beta)(1+n)}]^{\frac{1}{1-\alpha}}$, which is negatively related to population growth rate n.

$n \uparrow \Rightarrow$ 人均资本能拿到的和减少 $\Rightarrow k_{ss} \downarrow$

An Example 3

choose to

MPK ↑ ⇒ old generation save more ⇒ young produce more

⇒ more income to save

⇒ k_t ↑

↓

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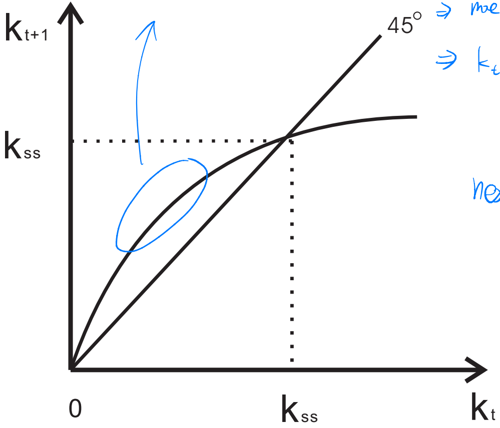

An Example 4

In an OLG model, suppose at period t the economy starts with capital-labor ratio $k_t < k_{ss}$

$\Rightarrow$ marginal product of capital is large

$\Rightarrow$ generation t saves more $\because MPK = r$ & "$r \uparrow \Rightarrow$ save more"

$\Rightarrow$ the wage of generation $t+1$ $\uparrow$ and they save more $\because$ save more $\Rightarrow K \uparrow \Rightarrow F \uparrow \Rightarrow MPL \uparrow \Rightarrow w_{t+1} \uparrow$

$\Rightarrow$ the wage of generation $t+2$ $\uparrow$ and they save more ... $\because$同上, $w_{t+2} \uparrow$

$\Rightarrow$ capital-worker ratio $\uparrow$ $\frac{K}{N} = k$

$\Rightarrow$ due to diminishing return to k, it converges to k_{ss}.

An Example 5

Solving for other variables:

$$
\begin{aligned}
w_t &= A(1-\alpha)k_t^{\alpha}; \\
r_t &= \alpha A k_t^{\alpha-1}; \\
R_t &= \alpha A k_t^{\alpha-1} + 1 - \delta; \\
s_t &= \frac{A\beta(1-\alpha)}{1+\beta}k_t^{\alpha}; \\
c_t^y &= \frac{A(1-\alpha)k_t^{\alpha}}{1+\beta}; \\
c_{t+1}^o &= \frac{A\beta(1-\alpha)}{1+\beta}k_t^{\alpha}(A\alpha k_{t+1}^{\alpha-1} + 1 - \delta);
\end{aligned}
$$

where $k_{t+1} = \frac{A\beta(1-\alpha)}{(1+\beta)(1+n)}k_t^{\alpha}$.

Dynamic Inefficiency 1

$$c_t^y + \frac{c_t^o}{1+n} + (1+n)k_{t+1} = f(k_t) + (1-\delta)k_t$$

At SS, $c_{ss}^y + \frac{c_{ss}^o}{1+n} = f(k_{ss}) - (n+\delta)k_{ss}$ FOC to k_{ss}

$$f'(k_{ss}) - (n+\delta) = 0$$

$$\Rightarrow f'(k_{ss}) = n+\delta.$$

- **Golden rule of savings**: the saving rate that maximizes steady state per capita consumption. In an OLG model, the Golden rule of savings implies $f'(k_{ss}) = n + \delta$.
- **Definition**: an economy is dynamically efficient if no one can be made better off without making someone else worse off.
- **Proposition**: An OLG economy is dynamically inefficient if and only if $f'(k_{ss}) < n + \delta$.

代表没達到 golden rule.

Pf: ($\Rightarrow$) P20 ~ P25

($\Leftarrow$) P26

Dynamic Inefficiency 2

可以减少某一人储蓄给其他人，
同时让所有人更爽

- Want to show: $f'(k_{ss}) < n + \delta \Rightarrow$ dynamic inefficiency
- Steps:
  1. show that there is an overaccumulation of capital by the young at steady state
  2. show that making the young saving less and transferring some of their consumption to the old can improve everyone's utility
  3. The Pareto improvement implies that the original steady state is inefficient

Dynamic Inefficiency 3

Step 1 for Proof:

The market clearing condition for the goods market is given by

$$c_t^y + \frac{c_t^o}{1+n} + (1+n)k_{t+1} = f(k_t) + (1-\delta)k_t.$$

At steady state, the aggregate consumption per worker is

$$c_{ss}^y + \frac{c_{ss}^o}{1+n} = f(k_{ss}) - (n+\delta)k_{ss}.$$

Maximize the steady state consumption per worker (FOC with respect to k_{ss}) to obtain the Golden rule of saving $f'(k_{ss}) = n + \delta$.

Dynamic Inefficiency 4

MPK is smaller ⇒ k_{ss} is larger ⇒ overaccumulation

Now we have $f'(k_{ss}) < n + \delta$, which implies overaccumulation of capital. Because,

- $f'(k_{ss})$ is a decreasing function of k_{ss}.
  修正
- $k_{ss} = \frac{s_{ss}}{1+n}$, thus, $s_{ss} \downarrow \to k_{ss} \downarrow \to f(k_{ss}) \downarrow$, $(n+\delta)k_{ss} \downarrow$ and $f'(k_{ss}) < n + \delta$. $\Longrightarrow \left(c^y_{ss} + \frac{c^o_{ss}}{1+n} \right) \uparrow$
- Therefore, $f(k_{ss}) - (n+\delta)k_{ss} \uparrow$ and aggregate consumption per worker $\uparrow$.

Dynamic Inefficiency 5

- Suppose we pick any steady state level of capital per worker k_{ss}^{PI} such that $k_{ss}^{PI} < k_{ss}$ and $f^{'}(k_{ss}) < f^{'}(k_{ss}^{PI}) < n + \delta$.
- The above choice implies that $\Rightarrow f(k_{ss}^{PI}) - (n+\delta) k_{ss}^{PI} > f(k_{ss}) - (n+\delta) k_{ss}$

$$
\begin{aligned}
\Rightarrow \quad c_{ss}^{y,PI} + \frac{c_{ss}^{o,PI}}{1+n} &> c_{ss}^{y} + \frac{c_{ss}^{o}}{1+n}; \quad \text{lifetime consumption} \uparrow \\
s_{ss}^{PI} = (1+n)k_{ss}^{PI} &< s_{ss} = (1+n)k_{ss}; \\
\Rightarrow \quad \left\{ w_{ss}^{PI} = A(1-\alpha){k_{ss}^{PI}}^{\alpha} \right. &< w_{ss} = A(1-\alpha)k_{ss}^{\alpha};
\end{aligned}
$$

- $k_{ss} \downarrow \rightarrow s_{ss} \downarrow$ proportionally , but $w_{ss} \downarrow$ less than proportionately.

  ↳ ∵有α在次方

  $\uparrow \quad \Leftarrow \quad \downarrow$少 $\quad \downarrow$多
- Since $c_{ss}^{y} = w_{ss} - s_{ss}$, $c_{ss}^{y,PI} > c_{ss}^{y}$.
- $s_{ss} \downarrow \Rightarrow c_{ss}^{o} \downarrow$. $\left(\because \underset{\downarrow}{C_{ss}^{o}} \Leftarrow = \underset{\downarrow}{S_{ss}} R_{ss}\right)$
- We have $c_{ss}^{y} \uparrow$, $c_{ss}^{o} \downarrow$, and $(c_{ss}^{y} + \frac{c_{ss}^{o}}{1+n}) \uparrow$. Thus, c_{ss}^{y} increases more than $\frac{c_{ss}^{o}}{1+n} \downarrow$. i.e. $\Delta C_{ss}^{y} > \Delta \left(\frac{C_{ss}^{o}}{1+n}\right)$

Dynamic Inefficiency 6

- Now we know that lower savings can increase young consumption, decrease old consumption, and $\Delta c_{ss}^{y} > \Delta \frac{c_{ss}^{o}}{1+n}$.
- This provide a room for pareto improvement.

Dynamic Inefficiency 7

Step 2 for Proof: Key: Gov try to use T to transform the wealth.

- Suppose c^y_{ss} increases by ε_1, $\frac{c^o_{ss}}{1+n}$ decreases by ε_2 and $\varepsilon_1 > \varepsilon_2$. Thus, we have $\overline{c}^y_{ss} = c^y_{ss} + \varepsilon_1$ and $\overline{c}^o_{ss} = c^o_{ss} - (1+n)\varepsilon_2$.
- Denote τ to be a lump-sum tax on the young's consumption. The new consumption for the young becomes $c^y_{ss} + \varepsilon_1 - \tau$.
- Total tax is equally distributed to the old. Each old gets $(1+n)\tau$. The new consumption for the old becomes $c^o_{ss} - (1+n)\varepsilon_2 + (1+n)\tau$.
- Then, for any $\varepsilon_2 < \tau < \varepsilon_1$, we have pareto improvement because

$\because \tau < \varepsilon_1$

$$
\begin{aligned}
c^{y,PI}_{ss} &= c^y_{ss} + \varepsilon_1 - \tau > c^y_{ss}, \\
c^{o,PI}_{ss} &= c^o_{ss} - (1+n)\varepsilon_2 + (1+n)\tau > c^o_{ss}.
\end{aligned}
$$

$\rightarrow$ young better off

$\rightarrow$ old better off.

$\because \varepsilon_2 < \tau$

So, $f'(k_t) < n+\delta \Rightarrow$ inefficient.

Dynamic Inefficiency 8

1. Goal

2. 要讓 f↑

$$\left(c^y_{ss} + \frac{c^o_{ss}}{1+n} \right)\uparrow = f(k_{ss}) - (n+\delta)k_{ss}$$

$k_{ss}\uparrow$ with $f'(k_{ss}) > \delta$ ①

3. 唯一讓 f 增加的方法是 $k_{ss}\uparrow$

② $s_{ss}\uparrow \Rightarrow \begin{cases} c^y_{ss}\downarrow = w_{ss} - s_{ss} \\ c^o_{ss}\uparrow = s_{ss}k_{ss} \end{cases}$

4. 但這會導致 $c^y_{ss}\downarrow$

- Now want to show dynamic inefficiency $\Rightarrow f'(k_{ss}) < n + \delta$. It is equivalent to show that $f'(k_{ss}) > n + \delta \Rightarrow$ dynamic efficiency.

5. 結論: 增加整體消費 by 增加 一定會犧牲某人

- Suppose not, someone can be better off without making anyone else worse off

  $\Rightarrow$ aggregate consumption per worker can be increased

  $\Rightarrow$ because $f'(k_{ss}) > n + \delta$, the only way is $k_{ss}\uparrow$. But $k_{ss}\uparrow \rightarrow s_{ss}\uparrow \rightarrow c^y_{ss}\downarrow$ and $c^o_{ss}\uparrow$

  $\Rightarrow$ to make $c^y_{ss}\uparrow$, we need a transfer from the old

  $\Rightarrow$ but initial old would be worse off

  $\Rightarrow$ no Pareto improvement exists.

Dynamic Inefficiency 9

Summary

Without government power (taxing), the trade between the young and the old won't occur. This is because the young will transfer to the old only if the transfer from tomorrow's young is guaranteed.

- With tax, it works.
- Without government's promise, any smart young will prefer to not giving to the old and still receiving from tomorrow's young.

OLG with Exogenous Growth 1

Considering a labor-augmenting technological progress. All equilibrium conditions are the same in the general form:

1. $\frac{u^{'}(c_t^y)}{u^{'}(c_{t+1}^o)} = \beta R_{t+1}$.
2. $c_t^y + s_t = w_t$.
3. $c_{t+1}^o = s_t R_{t+1}$.
4. $r_t = f^{'}(k_t)$.
5. $w_t = f(k_t) - k_t f^{'}(k_t)$.
6. $R_t = r_t + 1 - \delta$.
7. $c_t^y + \frac{c_t^o}{1+n} + (1+n)k_{t+1} = f(k_t) + (1-\delta)k_t$.
8. $(1+n)k_{t+1} = s_t$.

OLG with Exogenous Growth 2

Functional form:

$$
\begin{aligned}
u(c) &= \log(c); \\
F(K_t, N_t) &= AK_t^{\alpha}[(1+g)^t N_t]^{1-\alpha}; \\
f(k_t) &= A(1+g)^{(1-\alpha)t} k_t^{\alpha};
\end{aligned}
$$

where g is a labor-augmenting technological progress.

OLG with Exogenous Growth 3

The equilibrium conditions become:

1' $\frac{c^o_{t+1}}{c^y_t} = \beta R_{t+1}$.

2' $c^y_t + s_t = w_t$.

3' $c^o_{t+1} = s_t R_{t+1}$.

4' $r_t = A\alpha(1+g)^{(1-\alpha)t} k_t^{\alpha-1}$.

5' $w_t = (1-\alpha)A(1+g)^{(1-\alpha)t} k_t^{\alpha}$.

6' $R_t = r_t + 1 - \delta$.

7' $c^y_t + \frac{c^o_t}{1+n} + (1+n)k_{t+1} = A(1+g)^{(1-\alpha)t} k_t^{\alpha} + (1-\delta)k_t$.

8' $(1+n)k_{t+1} = s_t$.

OLG with Exogenous Growth 4

The detrended equilibrium conditions:

1" $\frac{(1+g)\widehat{c}^o_{t+1}}{\widehat{c}^y_t} = \beta \widehat{R}_{t+1}$.

2" $\widehat{c}^y_t + \widehat{s}_t = \widehat{w}_t$.

3" $(1+g)\widehat{c}^o_{t+1} = \widehat{s}_t \widehat{R}_{t+1}$.

4" $\widehat{r}_t = A\alpha \widehat{k}_t^{\alpha-1}$.

5" $\widehat{w}_t = (1-\alpha)A\widehat{k}_t^{\alpha}$.

6" $\widehat{R}_t = \widehat{r}_t + 1 - \delta$.

7" $\widehat{c}^y_t + \frac{\widehat{c}^o_t}{1+n} + (1+n)(1+g)\widehat{k}_{t+1} = A\widehat{k}_t^{\alpha} + (1-\delta)\widehat{k}_t$.

8" $(1+n)(1+g)\widehat{k}_{t+1} = \widehat{s}_t$.

OLG with Exogenous Growth 5

Solving for $\widehat{k}_{ss}$:

- Combine 1"2"3" to obtain $\widehat{s}_t = \frac{\beta}{1+\beta}\widehat{w}_t$.
- Combine with 8" to obtain $\widehat{k}_{t+1} = \frac{\beta}{(1+\beta)(1+n)(1+g)}\widehat{w}_t$.
- Substitute for $\widehat{w}_t$:

$$\widehat{k}_{t+1} = \frac{(1-\alpha)A\beta}{(1+\beta)(1+n)(1+g)}\widehat{k}_t^{\alpha}.$$

- At steady state,

$$\widehat{k}_{ss} = \left[\frac{(1-\alpha)A\beta}{(1+\beta)(1+n)(1+g)}\right]^{\frac{1}{1-\alpha}}.$$

OLG with Exogenous Growth 6

多這2項

- The goods market clearing condition:

$$\widehat{c}^y_{ss} + \frac{\widehat{c}^o_{ss}}{1+n} = A\widehat{k}^\alpha_{ss} - (\delta + g + n + ng)\widehat{k}_{ss}.$$

- Dynamic inefficiency iff $f'(\widehat{k}_{ss}) < \delta + g + n + ng$.

Is the US Dynamical Inefficiency? 1

To answer the question, we calibrate the model to the US postwar data:

- One model period: 35 years
- Population growth (quarterly): 0.35%
- Growth rate of real output per worker (quarterly): 0.46%
- Capital income share: 0.37
- Capital-output ratio (quarterly): 12.56
- Depreciation rate (yearly): 8%

Is the US Dynamical Inefficiency? 2

The condition for dynamically inefficiency is:
$\frac{\alpha(1+\beta)(1+n)(1+g)}{(1-\alpha)\beta} < \delta + g + n + ng$.

- $1 + n = (1 + 0.35\%)^{4\times 35} = 1.63$
- $1 + g = (1 + 0.46\%)^{4\times 35} = 1.90$
- $\alpha = 0.37$
- $\delta = 1 - (1 - 8\%)^{35} = 0.946$
- $\frac{k}{y} = \frac{12.56}{4} = 3.14$

These imply that $1.4859 < \beta$. But, $0 < \beta < 1$. Thus, we conclude that the US is dynamically efficient.