Macroeconomic Theory: Solow Model

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Solow Model

1. K

meaning

- Solow model demonstrates how **capital accumulation** and
  2. A **technological progress** generates economic growth
- 3. Solowe model captures the characteristics of economic growth in reality nicely → why some countries cannot growth, some grow fast?

Good

- Households decisions in terms of consumption and savings are very simple, and this makes Solow model a very simple framework

Bad

- Solow model fails to capture some characteristics of business cycles, and this can be complemented by the Ramsey growth model

Solow Model

PS: 目前的部分假設沒有市場 (中央集權國家)

只有一個家戶，自產自消

∴ 這個假設目前不太重要

⇐ why不直接假設只有一個?

∵這樣那個就有能力操控價格

Not 只有一個家戶

只是全部看作一個整體 (一種衡量方法)

- Time is discrete and goes to infinite: $t = 0, 1, 2, 3, \ldots$
- There are households with **mass one** in the economy

input

- A household owns one unit of labor in each period → Assume $l_t = 1,\ \forall t$
- A household owns capital k_t

output

- A household has a technology for output production:

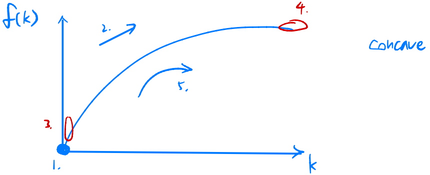
$$y_t = F(k_t, 1) \equiv f(k_t) \tag{1}$$

這裡的 $y_t = Y_t$

i.e. 人均產出 = 總產出

∵ $l_t = 1\ \forall t$

- 1. $f(0) = 0$, 2. $f'(k) > 0$, 3. $\lim_{k\to 0} f'(k) = \infty$, 4. $\lim_{k\to\infty} f'(k) = 0$, 5. $f''(k) < 0$



Households' Behavior

- **Key assumption**: households save a fixed fraction s of its income/output and consume a fraction $(1 - s)$
- c_t : consumption
- x_t : investment

$y \begin{cases} c : 1-s \\ x : s \end{cases}$

$$c_t + x_t = y_t \tag{2}$$

$$x_t = sy_t \tag{3}$$

$$c_t = (1 - s)y_t \tag{4}$$

- Investment becomes capital next period

$$k_{t+1} = (1 - \delta)k_t + x_t \tag{5}$$

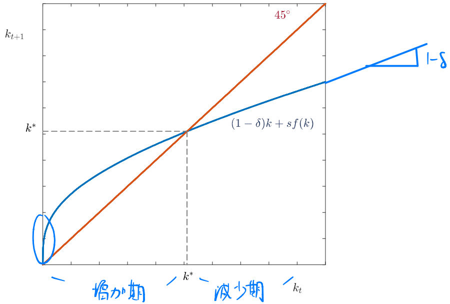
where δ denotes the depreciation rate of capital

- The initial capital k_0 is taken as given

Households' Behavior

- Combine (1), (3), (5), we obtain the following law of motion of capital

$$k_{t+1} = \underbrace{(1-\delta)k_t}_{\text{上期剩的}} + \underbrace{sf(k_t)}_{\text{这期新增的}} \tag{6}$$

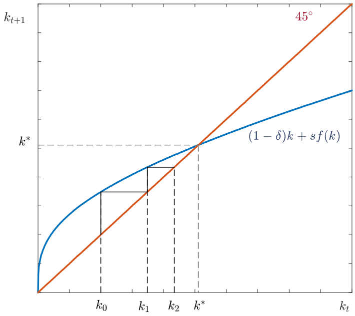


- The 45 degree line represents the function $k_{t+1} = k_t$
- Given k_t, if the law of motion (6) is above the 45 degree line, then $k_{t+1} > k_t$
- If the law of motion (6) is below the 45 degree line, then $k_{t+1} < k_t$

The Dynamics

- The law of motion for capital

$$k_{t+1} = (1-\delta)k_t + sf(k_t)$$

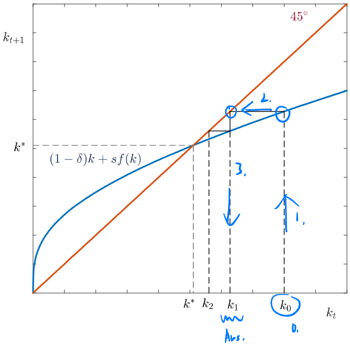


- If $0 < k_0 < k^*$, captial increases and converges to k^*

The Dynamics

- The law of motion for capital

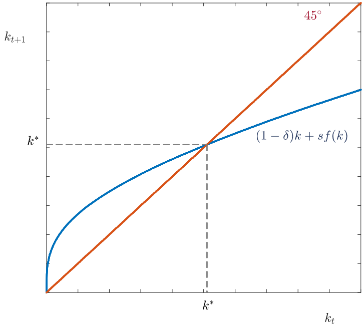
$$k_{t+1} = (1-\delta)k_t + sf(k_t)$$



- If $k_0 > k^*$, captial decreases and converges to k^*

Steady State

$$k_{t+1} = (1-\delta)k_t + sf(k_t)$$



和45°線交點 (∵各期都不再變)

- Steady state: we say that an economy is in a steady state if the variables are unchanging in time
- The steady state values of capital are 0 and k^*

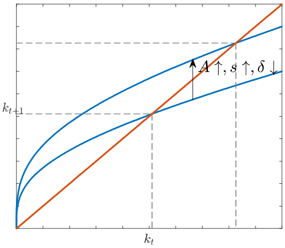
一變就往k^*跑: unstable　　stable：即使k小變動,也會回k^*

Steady state

- How changes in parameters influence the steady state capital k^*

$$k_{t+1} = (1-\delta)k_t + sf(k_t)$$

$f(x) = \underbrace{A}_{\text{TFP}} k^{\alpha}$



	A	δ	s
k^*	+	-	+
y^*	+	-	+

Steady State - Consumption

$$c^* = (1-s)y = (1-s)f(k^*)$$

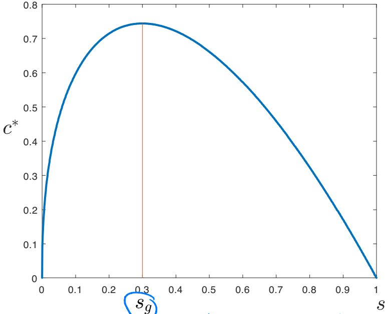
	A	δ	s
k^*	+	-	+
y^*	+	-	+
c^*	+	-	?

- An increase in saving ($s \uparrow$) has two effects in steady state:
  1. Greater steady state capital and output: $k^* \uparrow \Rightarrow Af(k^*) \uparrow$
  2. Smaller consumption rate: $(1-s) \downarrow$

The Paradox of Thrift 節儉

$$c^* = (1 - s)Af(k^*)$$

s 同時影響這兩者



golden rule saving rate

- $s \to 0 : k^* \to 0 \Rightarrow c^* \to 0$
- $s \to 1 : (1 - s) \to 0 \Rightarrow c^* \to 0$

Golden Rule

- **Golden Rule savings rate** is the rate of savings s that maximizes steady state level of consumption c^*

$$
\begin{aligned}
c^* &= (1-s)f(k^*) \\
\Rightarrow \quad c^* &= \underbrace{f(k^*)}_{y^*} - \underbrace{sf(k^*)}_{x^*} \qquad \text{(GC)}
\end{aligned}
$$

- Steady state capital is a function of the saving rate

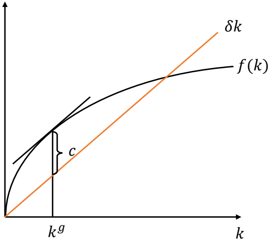
$$
\begin{aligned}
k^* \quad &= \quad (1-\delta)k^* + sf(k^*) \qquad \text{(GK)} \\
\Rightarrow \quad \underbrace{\delta k^*}_{k\text{减少量}} &= \underbrace{sf(k^*)}_{k\text{增加量}}
\end{aligned}
$$

- In the steady state, the investment is just enought to compensate the depreciation of the capital
- We can also consider saving rate as a function of capital

Golden Rule

- Rather than solving for the golden rule saving rate, we solve for the golden rule capital first
- By (GK), $s^* = \frac{\delta k^*}{f(k^*)}$, and substitute it into (GC), then

$$c^* = f(k^*) - \delta k^*$$



max c^*

FOC = 0

- The golden rule capital k^g satisfies $f'(k^g) = \delta$
- The golden rule saving rate s^g that solves 决定 k^g 后代入 s，就是 s^g

$$s^g = \frac{f(k^g)}{\delta k^g}$$ → 写反！

Dynamics - Productivity Shocks

- Consider an economy which is initially at **the steady state**
- Let $f(k) = Ak^{\alpha}$. At a certain point in time, the productivity A increases permanently A 永久增加
- What are the dynamic paths of variables such as capital, output, consumption?

Dynamics - Productivity Shocks

- Suppose that the productivity is equal to A for $t = -5, -4, -3, -2, -1$
- Initial capital stock is equal to the steady state capital stock $k = k^*$, where k^* satisfies $k^* = (1 - \delta)k^* + sAk^{*\alpha}$
- Suppose now the productivity increases to $A' > A$ permanently at $t = 0$

Dynamics - Productivity Shocks

- Suppose now the productivity increases to $A' > A$ permanently at $t = 0$

$1 = k^{-0.7}$

$k_{t+1} = (1-\delta)k_t + x_t$

$\delta k = s y$ $0.5 + sy$

$\cancel{0.5} \cdot k = \cancel{0.5} \cdot 1 \cdot k^{0.3}$

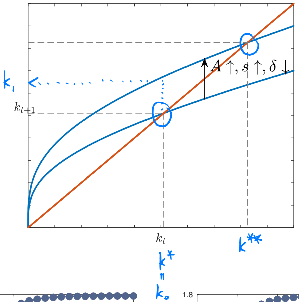
$k = k^{0.3}$

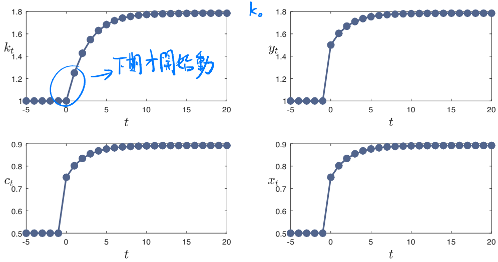
$k = 1$

$y = 1 \cdot 1^{0.7} = 1$

$c = 0.5$

$x = 0.5$



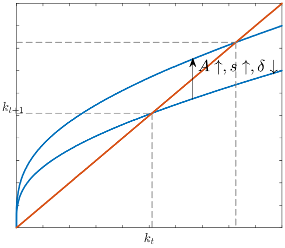


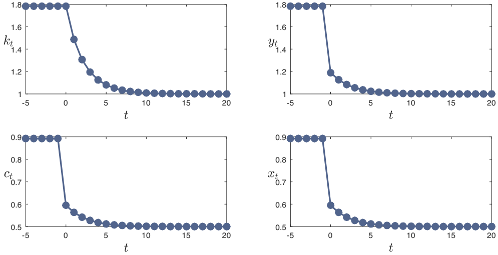
$A \to y \to c$
$\searrow x$

∴ y, c, x 當期就變

Dynamics - Productivity Shocks

- Suppose now the productivity decreases to $A' < A$ permanently at $t = 0$





Dynamics - Productivity Shocks

$sy = \delta k$ $\quad k_{t+1} = (1-\delta)k_t + x_t$

$0.5 \cdot$

Exercise 1. (20%) Consider the Solow growth model we discussed in class. Let the production be $f(k) = Ak^{\alpha}$, and $A = 1$, $\alpha = 0.3$, $\delta = 0.5$, $s = 0.5$. Suppose that the economy was at the steady state for $t = -5, -4, -3, -2, -1$. At $t = 0$, the TPF (A) increases to 1.5 permanently. Plot the dynamic path of capital, output, consumption, and investment from the old steady state to the new steady state (for $t = -5, ..., 0, 1, 2, \ldots, 20$)

Exercise 2. (20%) Now consider that at $t = 0$, the saving rate (s) decreases to 0.3 permanently. Plot the dynamic path of capital, output, consumption, and investment from the old steady state to the new steady state (for $t = -5, ..., 0, 1, 2, \ldots, 20$)