ECON 7011, Semester 110.1, Practice Problems 7

You do no need to hand in the solution to these problems.

1. Many freemium games offer loot boxes in exchange for real money. Suppose that a game offers high-quality and medium-quality loot boxes, which are virtually costless to provide. The boxes are valued by the normal players ϑ_N and the so-called "whales" ϑ_W as follows:

	ϑ_W	ϑ_N
Good	10000	300
Medium	1030	30

   (a) For which ratio μ of whales is the normal type excluded from loot boxes?

   (b) Find the second-best contract as a function of μ.

2. Suppose the government outsources the provision of a public good to a private company, which may be one of two types $\vartheta_2 > \vartheta_1 > 0$. Type ϑ can produce an indivisible amount q of the public good at cost $c(\vartheta, q) = \vartheta q^2$, which the government values at $v(q) = q$. Suppose that a fraction μ_0 of firms are efficient and that utilities satisfy $u_1(p, q) = v(q) - p$ and $u_2(\vartheta, p, q) = p - c(\vartheta, q)$.

   (a) If the government knew the firms' types, what contract would it offer to each type?

   (b) What is the optimal contract when the government does not know the firms' types?

3. A TV network has two crews available that can produce one TV show each, subject to budget $b > 0$. Suppose that the audience's preferences are one-dimensional and characterized by $\theta \geq 0$ on the seriousness scale, where θ is exponentially distributed with mean $\frac{1}{\lambda}$. The production of a TV show with seriousness s and quality q incurs a cost $c(q, s) = qs$, that is, it is more costly to produce a good TV show when it is serious than when it is silly. Type ϑ's utility from watching show i is $q_i - (s_i - \vartheta)^2$ with outside option 0 by not watching either show. The TV Network's utility is $v(s, q, a_2) = r1_{\{a_2=\text{watching either show}\}} - c(q_1, s_1) - c(q_2, s_2)$.

   (a) Argue that in any optimal production, there exist $\underline{s}$, $\bar{s}$ such that show 1 captures viewers in the segment $[0, \underline{s}]$ and show 2 captures viewers in the segment $[\underline{s}, \bar{s}]$.

   (b) Assuming that b is low enough that the budget constraint binds, i.e., $s_1 q_1 + s_2 q_2 = b$ in the optimum, find the perfect Bayesian equilibria of this game.

   Hint: Use (a) to eliminate s_1 and s_2, then use the binding budget constraint to write q_2 as a function $q_2(q_1)$. The explicit expression of $q_2(q_1)$ is nasty, but we do not need it to find the PBE. Instead, we can directly maximize the objective function $V(q_1, q_2(q_1))$ with respect to q_1, where we find $q_2'(q_1)$ by implicitly differentiating the binding budget constraint.

   (c) Which show has the higher quality in equilibrium?

   (d) How does the ratio of the quality of the two shows change when the budget constraint is no longer assumed to be binding?