Problem Set 5

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Answer 1.

Suppose that $x(p,w)$ is the maximizer of preference relation on $B(p,w)$. Notice that for another w', we have $\frac{w'}{w}x(p,w) \succ \frac{w'}{w}y$ for all $y \in B(p,w)$. However, $\frac{w'}{w}y$ runs over $B(p,w')$, hence $\frac{w'}{w}x(p,w)$ is the maximizer of preference relation on $B(p,w')$.

Answer 2.

There are two cases, one is $x_1^* = 0$ remains not changed when w changes, the other is growing along with w grows.

Suppose that $x_1(p,w)$ is not zero and w grows to w'. Then $(x_1(p,w) + \frac{w'-w}{p_1}, x_2(p,w)) \succ (y_1(p,w) + \frac{w'-w}{p_1}, y_2(p,w))$ for all $y = (y_1, y_2) \in B(p,w)$. It remains to show that $(x_1(p,w) + \frac{w'-w}{p_1}, x_2(p,w)) \succ z$ when $z = (z_1, z_2)$ with $z_1 < \frac{w'-w}{p_1}$. However, since $(x_1(p,w) \neq 0$, if $z \succsim (x_1(p,w) + \frac{w'-w}{p_1}, x_2(p,w))$, then by strict convexity, we have

$$(\frac{x_1(p,w)}{2} + \frac{w'-w}{p_1}, x_2(p,w) + \frac{x_1(p,w)}{2}\frac{p_1}{p_2}),$$

which contradicts to our first part.

Answer 3.

a. When $p_1 = p_2$, $X(p,w) = \{(x_1,x_2) | x_1 + x_2 = \frac{w}{p}\}$. When $p_1 > p_2$, $X(p,w) = \{(0, \frac{w}{p_2})\}$. When $p_1 < p_2$, $X(p,w) = \{(\frac{w}{p_1}, 0)\}$.

b. Let $b^n \in X(p^n, w^n)$ and p^n, w^n, b^n are a convergent sequence with limit p, w, b respectively.

Not hard to see that $pb \leq w$. Suppose that $b \notin X(p,w)$, then there exists $z \in X(p,w)$. By continuity, there exists some $r > 0$ sucht that for all $z' \in B_r(z)$, $b' \in B_r(b)$, we have $z' \succ b'$.

However, for n large enough, we have $b^n \in B_r(b)$ and $B_r(z) \cap B(p^n, w^n) \neq \emptyset$. Choose such n and pick $z' \in B_r(z) \cap B(p^n, w^n)$, we have $z' \succ b^n$, which is impossible.

Thus, $b \in X(p,w)$.

Answer 4.

a. Yes. define $(1,k) \succ y$ if $y = (y_1, y_2)$ and $y_1 \neq 1$. Moreover, $(1,k_1) \succ (1,k_2)$ if $k_1 > k_2$. Easy to see that the preference is acyclical.

b. No, suppose it is and consider two cases. 1. $p_1 = 2, p_2 = 1$:

Not hard to calculate that $x_1 = \dfrac{p_1 w}{p_1^2 + p_2^2} = \dfrac{2}{5}w$ and $x_2 = \dfrac{1}{5}w$. Notice that $(\dfrac{1}{5}w, \dfrac{2}{5}w)$ is also a feasible choice, so $(\dfrac{1}{5}w, \dfrac{2}{5}w) \prec (\dfrac{2}{5}w, \dfrac{1}{5}w)$.

2. $p_1 = 1, p_2 = 2$:

Similarly, $(\dfrac{1}{5}w, \dfrac{2}{5}w) \succ (\dfrac{2}{5}w, \dfrac{1}{5}w)$.

The result of these two cases contradicts to each other. Done!