ECON 7011, Semester 110.1, Assignment 2

Please hand in your solutions via NTU Cool before 11:59pm on Tuesday, November 30

1. Consider the following two static games.

	L	C	R
T	$1,3$	$2,0$	$0,2$
D	$2,0$	$0,2$	$1,1$

	L	C	R
T	$2,2$	$0,4$	$2,1$
D	$2,1$	$1,0$	$3,2$

   (a) Find all Nash equilibria of both games.

   (b) Are you equally confident about the Nash predictions in the two games? Or are you more confident about the predictions in one of them? Explain.

2. Consider a Cournot competition among two firms $i = 1, 2$, in which Firm 1 can produce any quantity $q_1 \geq 0$, but is restricted to pure strategies, and Firm 2 can produce only quantities from the set $\{0, 20, 40\}$, but has the ability to mix. Suppose that the inverse demand function is $p(q) = 100 - q_1 - q_2$ and the unit cost is $c = 10$, which implies that firm i's utility is $u_i(q) = (p(q) - c)q_i$. Find all Nash equilibria of this game.

3. Dusty and Marcus compete in a snowboarding competition. Whoever finishes the more difficult routine without falling wins the competition. Suppose that shared victory is half as valuable as individual victory, and that if they both fall, some third competitor will win. We suppose that the probability of falling $P_i(d_i)$ is increasing in the difficulty d_i of the routine so that we can parametrize the game through the probability of falling p_i. Suppose that the two snowboarders are equally skilled so that $p_i > p_{-i}$ if and only if $d_i > d_{-i}$. Then i's expected utility is

$$u_i(p) = \begin{cases} 1 - p_i & \text{if } p_i > p_{-i}, \\ (1 - p_i)\frac{1}{2}(1 + p_{-i}) & \text{if } p_i = p_{-i}, \\ (1 - p_i)p_{-i} & \text{if } p_i < p_{-i}. \end{cases}$$

   (a) Find the pure-strategy best-response correspondences to pure strategies of the opponent.

   (b) Show that there is no pure-strategy Nash equilibrium.

   (c) Find a symmetric atomless mixed-strategy Nash equilibrium with support $[0, \bar{p}]$.

   (d) Explain why the guess in (c) is a good guess for a mixed-strategy Nash equilibrium.