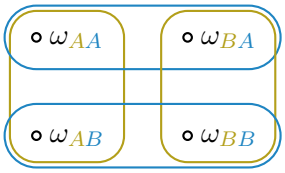
ECON 7219, Semester 110.1, Assignment 1

Use the formal model of knowledge to answer below questions and provide a justification for all your answers. Please hand in the assignment by Friday Oct 15, 23:59 online or in person.

1. A misunderstanding occurs between two people if the Listener believes something different has been expressed than what the Speaker intended to express. We can model a misunderstanding in an Aumann model of incomplete information with four states as follows



where ω_{xy} means statement x was intended by the Speaker and statement y was understood by the Listener. This blog post states that there are two main types of misunderstandings:

(i) The Speaker is not aware that they are being misunderstood.

(ii) The Listener is unaware that they are not understanding what was intended.

   (a) Express the two types of misunderstandings using quantifiers in our formal model. Are statements (i) and (ii) correct in the above Aumann model?

   (b) Do you agree that the two types of misunderstandings are fundamentally different?

2. Prove or disprove: the minimal belief space for any belief hierarchy β consists of only one common knowledge component. *Hint: this is easier than it sounds.*

3. Darren writes numbers 5, 3, and 2 on Anya's, Berndette's, and Cheryl's forehead and announces: "each of you has a distinct positive number on your forehead such that two of the numbers add up to the third." The players take turns in alphabetic order and announce whether they know the number on their forehead until everybody knows their number.

   (a) Describe an Aumann model of incomplete information that captures the above interaction after Darren's but before anybody else's announcements.

   (b) Show how the players' update their information set with each announcement. Who last announces that she knows her number?

   (c) Give a set of numbers, for which the above process terminates in round 4.

   (d) Can the players arrive at the same conclusion without knowing each number is distinct?

4. Before the semester starts, Aaron, Blake, and Cédric disagree on which day of the week their favorite class is scheduled. Aaron believes it is scheduled on Monday, but that Blake and Cédric both believe it is scheduled on Tuesday and that this is common belief among the three. Blake believes it is scheduled Monday with 40% probability, that Aaron and Cédric both know on which day it is scheduled and that they believe the correct day is common belief. Cédric believes it is scheduled on Monday with 75% probability, that Aaron and Blake both believe Monday and Tuesday are equally likely and that whichever day it is, they both believe everybody else knows the correct day of the week and that this is common belief.

   (a) Find the minimal belief space that contains the above belief hierarchy.

   (b) Find the event Y that Aaron and Blake are willing to place an even bet, in which Aaron wins if the class is on Monday and Blake wins if the class is on Tuesday.

   (c) How likely do Aaron and Blake believe this bet to be?

   (d) Does your answer in (c) contradict Aumann's agreement theorem? Explain.