Macroeconomic Theory - Competitive Equilibrium

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Outline

之前学的

- So far, we have studied the centralized economy (planner's problem.)
- However, there is no market, there is no price, there is no interaction between agents in the economy
- The social planner's economy is like a home production economy

Now!

- In competitive equilibrium, firms produce, households work and invest capital
  2 agents!
- Households take salary, interest return, and dividend payment, and the income is used to purchase goods from firms
- We study the decentralized, market economy in this lecture

Sequential Trading Equilibrium

Environment

- There are mass one households and mass one firms
- A household holds capital and make capital accumulation decisions
- Each household also own one unit of labor force in each period: $n = 1$
- A household supplies labor and rents capital
- Firms are capital and labor demanders

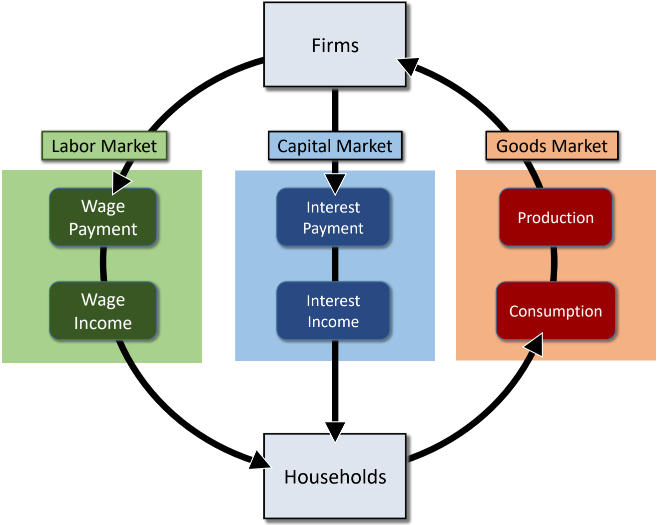
Environment

- Firms produce output following the production function $F(k, n)$
- Household owns the share of firms
- In the planner's economy, we consider that households work for their own firms

$$f(k) = F(k, 1)$$

↳ 之前其实我们是 labor 全投入

Economic Models



Firms

- Households are identical. Firms are also identical.
- It is sufficient to analyze the representative household and the representative firm
- We assume that the firm's goal is to maximize profit. It is given by,

$$\max_{k_t, n_t} F(k_t, n_t) - \underline{r_t k_t} - \underline{w_t n_t}$$

Cost of the input

First order conditions:

$$\begin{aligned} r_t &= F_k(k_t^*, n_t^*) \\ w_t &= F_n(k_t^*, n_t^*) \end{aligned}$$

- The function $F(k, n)$ is homogeneous of degree one (constant returns to scale).

$$F(ak, an) = aF(k, n) \tag{1}$$

for all k, n

Firms

透过对a微分求极值

? 不懂这个a在干嘛?

ok

这就是为了把F_k, F_n
两个串在一起，就先用a
造出连接两者的式

- Take derivative w.r.t. a on both side of (1):

$$F_k(ak, an)k + F_n(ak, an)n = F(k_t, n_t)$$

Let $a = 1$:

$$F_k(k, n)k + F_n(k, n)n = F(k_t, n_t)$$

- The firms' profit:

$$\begin{aligned}
\pi &= F(k_t^*, n_t^*) - r_t k_t^* - w_t n_t^* \\
&= F_k(\cancel{k_t^*}, n_t^*)k_t^* + \cancel{F_n(k_t^*, n_t^*)n_t^*} \\
&\quad - r_t \cancel{k_t^*} - \cancel{w_t n_t^*} \\
&= 0
\end{aligned}$$

的结果，会互消

公司不能获利

本来要把公司获利拿给家户
(∵ 持有股份)

- Therefore, we will neglect to include dividend payment in the consumer's budget constraint

但既然一定是0，
就不用特地加一个内生变数

Households

- Households own labor and capital:

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

假设 $n=1$，全投入工作

$$\text{subject to} \quad \begin{aligned} c_t + x_t &= w_t + r_t k_t \\ k_{t+1} &= (1-\delta)k_t + x_t \\ k_{t+1} &\geq 0 \end{aligned}$$

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{subject to} \quad \begin{aligned} c_t + k_{t+1} - (1-\delta)k_t &= w_t + r_t k_t \\ k_{t+1} &\geq 0 \end{aligned}$$

Transversality Condition

- The finite horizon lagrangian:

$$L = \sum_{t=0}^{T} \beta^t \left\{ u(c_t) + \lambda_t \overbrace{\left\{ \begin{array}{c} [w_t + r_t k_t] \\ -\left[c_t + k_{t+1} - (1-\delta)k_t\right] \end{array} \right\}}^{\text{BC.}} + \mu_t k_{t+1} \right\}$$

cf. $f'(k_{t+1})+1-\delta$

$$\begin{aligned} c_t &: \quad u'(c_t) = \lambda_t \\ k_{t+1} &: \quad \lambda_t = \beta \lambda_{t+1} \underline{\left[r_{t+1} + (1-\delta)\right]} + \mu_t \\ k_{T+1} &: \quad \lambda_T = \mu_T \end{aligned}$$

- We must have $\mu_t = 0$ for $t < T$; 反證 because otherwise $\mu_t > 0$ implies that $k_{t+1} = 0$, and by the firm's problem, $r_{t+1} = f'(f_{t+1})$ will be infinite (not well-defined).

Transversality Condition

- complementary slackness

$$\begin{aligned} & \beta^T \mu_T k_{T+1} = 0 \\ \Rightarrow \quad & \beta^T \lambda_T k_{T+1} = 0 \\ \Rightarrow \quad & \beta^T u'(c_t) k_{T+1} = 0 \end{aligned}$$

Let $T \to \infty$

$$\text{TVC} : \lim_{T \to \infty} \beta^T u'(c_T) k_{T+1} = 0$$

Households

Definition

1

A sequential markets equilibrium is a sequence of prices $\{w_t, r_t\}_{t=0}^{\infty}$, 2 allocations for the representative household $\{c_t, k_{t+1}^s\}_{t=0}^{\infty}$, and 3 allocations for the representative firm $\{n_t^d, k_t^d\}_{t=0}^{\infty}$, such that

1. Given k_0 and $\{w_t, r_t\}_{t=0}^{\infty}$, allocations $\{c_t, \overset{\text{supply}}{k_{t+1}^s}\}_{t=0}^{\infty}$ solve the household maximization problem
2. Given $\{w_t, r_t\}_{t=0}^{\infty}$, allocations $\{n_t^d, \overset{\text{demand}}{k_t^d}\}_{t=0}^{\infty}$ solve the firm's maximization problem for $t \geq 0$
3. Markets clear: for all $t \geq 0$

n_t^s

$$
\begin{aligned}
n_t^d &= 1 && \rightarrow \text{Labor mkt} \\
k_t^d &= k_t^s && \rightarrow \text{Capital mkt} \\
F(k_t^d, n_t^d) &= c_t + \underbrace{k_{t+1}^s - (1-\delta)k_t^s}_{x_t} && \rightarrow \text{Good mkt}
\end{aligned}
$$

Market Clearings

- Recall that

$$f(k) = F(k, 1)$$

- Constant returns to scale implies that

$$F(k_t, 1) = F_k(k_t, 1)k_t + F_n(k_t, 1) \leftarrow 18$$

移项

- Recall that $f(k_t) = F(k_t, 1)$, then

FOC

$$r_t = F_k(k_t, 1) = f'(k_t)$$

$$\begin{aligned} w_t = F_n(k_t, \overset{1}{n_t}) &= \underline{F(k_t, 1)} - \underline{F_k(k_t, 1)}k_t \\ &= \underline{f(k_t)} - \underline{f'(k_t)}k_t \end{aligned}$$

- Substitute r_t and w_t into the household's budget constraint

$$c_t + k_{t+1} - (1 - \delta)k_t = \underbrace{w_t + r_t k_t}_{\text{把上面求出的 } r_t,\ w_t \text{ 代入}} = f(k_t)$$

即可得

Households

- By the first order condition of the household's problem

写错了

Euler: $$\beta^t u'(c_t) = \beta u'(c_{t+1}) \left[f'(k_{t+1}) + (1-\delta) \right]$$ ← P10 整理得到

- By market clearing condition and household's budget constraint

$$f(k_t) = k_{t+1} - (1-\delta)k_t + c_t$$

- By TVC

$$\lim_{T \to \infty} \beta^T u'(c_T) k_{T+1} = 0$$

- By the nonnegativity constraints

$$\begin{aligned} c_t &\geq 0 \\ k_{t+1} &\geq 0 \end{aligned}$$

- The equilibrium conditions are exactly the same as in the social planner's problem → 基本福利定理: CE = PO