ECON 7219, Semester 110.1, Assignment 3

Please justify all your answers and hand in the assignment by Thursday Nov 18, 23:59.

1. Consider a third-price auction with n buyers, in which the highest bidder wins the auction and pays the third-highest bid. Suppose that the buyers' valuations are independently drawn from a uniform distribution on $[0, \bar{\vartheta}]$.
   (a) Find the unique pure-strategy BNE in increasing symmetric strategies.
   Hint: use the revenue equivalence result, the fact that the joint density of the highest and second-highest realization $\theta^{(1)}$ and $\theta^{(2)}$ of a standard-uniform sample of size m is
   $$f_{\theta^{(1)},\theta^{(2)}}(x, y) = m(m-1)y^{m-2}1_{\{y \leq x\}},$$
   and that if $f = g$ for two differentiable functions f and g, then also $f' = g'$. Here, $1_{\{y \leq x\}}$ is the indicator function *that is equal to* 1 *if $y \leq x$ and equal to* 0 *otherwise. The last hint means you will not have to do any integration to find the strategy.*
   (b) How do bidders adjust their valuations in their equilibrium bid? Explain.
   (c) What is the ex-ante expected revenue of the seller in this auction?
   (d) Can a third-price auction be implemented with a dominant-strategy mechanism?

2. Mark and Lisa are planning a vacation and decide to go either to New Zealand, Switzerland, or Tanzania. Mark's preferences are commonly known to be $v_M(N) = 1$, $v_M(S) = 3$, and $v_M(T) = -1$. Mark believes Lisa's preferences to be either ϑ_L^a or ϑ_L^b, defined by
   $$v_L(N, \vartheta_L^a) = 2, \qquad v_L(S, \vartheta_L^a) = 1, \qquad v_L(T, \vartheta_L^a) = 6,$$
   $$v_L(N, \vartheta_L^b) = 4, \qquad v_L(S, \vartheta_L^b) = 3, \qquad v_L(T, \vartheta_L^b) = 2,$$
   with equal likelihood. Suppose that the outside options are $IR_M = 0$ and $IR_L = 4$.
   (a) Find the IR-VCG mechanism and show that it runs an expected surplus.
   (b) Suppose that Mark and Lisa decide to use the surplus to pay for a dinner on their vacation, from which they benefit equally. Mention two issues that may arise with this attempt at balancing the budget and illustrate them numerically in the example.

3. In a double auction, n potential buyers and sellers of a good each submit their bids and asks, respectively, to a market institution that chooses a market price p, at which the market clears. Suppose that buyers' and seller's types are distributed independently on $[\underline{\vartheta}_B, \bar{\vartheta}_B]$ and $[\underline{\vartheta}_S, \bar{\vartheta}_S]$, respectively, with some strictly positive density functions. For a fixed price p and a realization of valuations, one can construct a supply curve $S(p)$ and a demand curve $D(p)$ by ordering the valuations of sellers increasingly $\vartheta_S^1 \leq \vartheta_S^2 \leq \ldots \leq \vartheta_S^n$ and valuations of buyers decreasingly $\vartheta_B^1 \geq \vartheta_B^2 \geq \ldots \geq \vartheta_B^n$ and setting $S(p) = \max\{i \mid \vartheta_S^i \geq p\}$ and $D(p) = \max\{i \mid \vartheta_B^i \geq p\}$.
   (a) What are indirect and direct mechanisms in this setting?
   (b) Let $k(\vartheta) = \max\{i \mid \vartheta_B^i \geq \vartheta_S^i\}$. Show that the social state $q(\vartheta)$ defined as trade occurring between seller i and buyer i for $i \leq k(\vartheta)$ is ex-post efficient.
   (c) Find the IR-VCG mechanism and show that it is not ex-post budget balanced if the valuations of buyers and sellers have the same support, i.e., if $[\underline{\vartheta}_B, \bar{\vartheta}_B] = [\underline{\vartheta}_S, \bar{\vartheta}_S]$.
   (d) Suppose we are willing to give up dominant-strategy implementability. Does there exist an individually rational, incentive compatible, and ex-post efficient mechanism?