ECON 7219, Semester 110.1, Assignment 2

Please justify all your answers and hand in the assignment by Friday Nov 5, 23:59.

1. A defendant in court is either guilty or innocent, i.e., $\Theta = \{\vartheta_I, \vartheta_G\}$ with common prior $\mu_0 = P(\theta = \vartheta_G) \in (0,1)$. Two judges $i = 1, 2$ listen to the trial and form an opinion $T_i \in [0,1]$ of the defendant's guilt with conditional densities $f(\tau_i \,|\, \vartheta_G) = 2\tau_i$ and $f(\tau_i \,|\, \vartheta_I) = 2 - 2\tau_i$. A high value of T_i indicates that judge i believes it is likely that the defendant is guilty. Suppose that T_1 and T_2 are conditionally independent, given θ. Each judge votes whether to (A)cquit or (C)onvict the defendant and the defendant is convicted only if both judges vote to convict. Each judge receives a payoff of 1 for convicting a guilty defendant and for acquitting an innocent defendant and he/she receives a payoff of 0 otherwise.
   (a) Find the unique symmetric Bayesian Nash equilibrium in cutoff strategies.
   (b) Suppose that $\mu_0 = 0.2$. With what probability is the defendant convicted in equilibrium? Provide an intuitive reason why this probability is different from μ_0.

2. In a Cournot duopoly, two firms $i = 1, 2$ each choose to produce a quantity $q_i \geq 0$, which affects the price of the product through the total supply. Consider a model with uncertainty about the demand θ, which can take one of two values $\Theta = \{\vartheta_L = 70, \vartheta_H = 90\}$, with a linear price function $p(\vartheta, q) = \vartheta - (q_1 + q_2)$. Suppose that Firm 1 has been in the business for a long time and it is common knowledge that Firm 1 knows the demand. Firm 2 started their business more recently and it is common knowledge that Firm 2 does not know θ and believes demand to be high or low with equal probability. Each firm i's ex-post utility is
$$u_i(\vartheta, q) = (p(\vartheta, q) - c)q_i,$$
where $c = 10$ is the unit cost of each firm.
   (a) Find all pure-strategy Bayesian Nash equilibria if the two firms act simultaneously.
   (b) Suppose now that Firm 1 chooses quantities first and that Firm 2 observes q_1.
      i. Find all separating PBE in pure strategies.
      ii. Find all pooling PBE in pure strategies.

      Hint: parametrize the firms' strategies and find conditions, under which neither on-path nor off-path deviations are profitable. For the latter, try to find those off-path beliefs, under which deviations are least attractive to Firm 1.
   (c) Compare the equilibria in (a) and (b.ii). How and why are the similar/dissimilar?
   (d) Does Firm 2 prefer a separating equilibrium in a sequential game over a Bayesian Nash equilibrium in the simultaneous-move game? Explain intuitively why this is or is not so.

3. The Government can choose to (I)nvest or (N)ot invest into its cyberdefense. A group of Hackers observe the Government's decision and choose to (A)ttack or (N)ot attack. The prior is μ_0 that the government is the competent type ϑ_C. The attack is successful either if the government is the incompetent type ϑ_I or if it does not invest. Utilities are as follows:

	A	N
I	$-1, -1$	$-1, 0$
N	$-3, 3$	$0, 0$

ϑ_C

	A	N
I	$-4, 2$	$-1, 0$
N	$-3, 3$	$0, 0$

ϑ_I

   (a) Find all perfect Bayesian equilibria.
   (b) Is this a signaling or a cheap-talk game? Explain.