# 1. Knowledge Models

ECON 7219 – Games With Incomplete Information

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# Selling Farmland

A Rice Farmer considers selling his/her land to a Tea Farmer.

**Annual average yield:**

- Tea: 5.35m NTD/km$^2$.
- Rice: 4.2m NTD/km$^2$.

**Incomplete information:**

- Suppose high/low-quality soil yields 50% above/below average.
- The Rice Farmer knows the quality, but the Tea Farmer does not.
- Can trade between the two farmers occur? If so, at which price?

Source: https://www.visualcapitalist.com/the-worlds-most-valuable-cash-crop/

# Modeling This Interaction

**Uncertainty:**

- We must describe the farmers' information about the soil quality.
- In absence of information, the Tea Farmer will form beliefs about the soil's quality, which we need to model.

**Game theory:**

- Each farmer's utility of holding the land depends on the soil quality and the price at which trade occurs.
- We need a model for the strategic aspect of the trade.
- Players choose action that maximizes their expected utility, given their beliefs about the state and the action of opponent.
- This requires us to describe the players' higher-order beliefs.

# Information and Knowledge

# Modeling Uncertainty

*"While in theory randomness is an intrinsic property, in practice, randomness is incomplete information."*

*– Nassim Nicholas Taleb*

**States of nature:**

- One or several players have incomplete information about a payoff-relevant variable $\theta$, called the state of nature.
- The set $\Theta$ is the set of all possible values that $\theta$ can take.
- We denote by $\vartheta$ a specific (deterministic) element in $\Theta$.

**Example:**

- Suppose the quality of soil $\theta$ can take the values $\Theta = \{L, M, H\}$.
- The Tea Farmer knows $\Theta$ and, hence, each element $\vartheta$ in $\Theta$.
- However, he/she does not know the random variables $\theta$.

# Information Sets

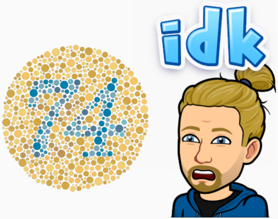

**Information sets:**

- A player lacks of information about the state of nature if there are $\vartheta$ and $\vartheta'$, for which he/she cannot distinguish whether $\theta = \vartheta$ or $\theta = \vartheta'$.
- We group states that player $i$ cannot distinguish into information sets $\tau_i$.
- Idea: player $i$ cannot distinguish any states in $\tau_i$.

**Example:**

- Tea Farmer does not know the quality of the soil.
- Tea Farmer has only one information set $\tau_T = \{L, M, H\}$.
- Rice Farmer knows $\theta$ from experience.
- Rice Farmer has information sets $\tau_R^L = \{L\}$, $\tau_R^M = \{M\}$, and $\tau_R^H = \{H\}$.

# Color Blindness

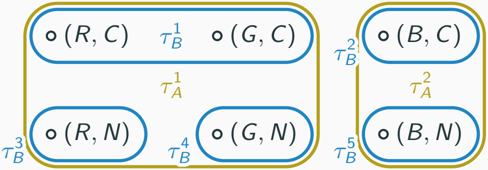

Alan and Ben are looking at a hat, wondering what color it is.

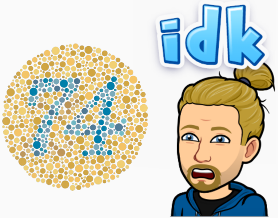
- The hat can have one of three colors $\vartheta \in \{R, G, B\}$.
- Alan is color blind and cannot distinguish $R$ and $G$.
- Ben is not color blind and can distinguish all three colors.

Very intuitive, but it does not allow us to model higher-order knowledge:

Does Ben know whether Alan knows the color of the hat?

# Color Blindness




**Extend the model:**

- Add 2nd dimension to the state: Ben is (C)olor blind, (N)ot color blind.
- Set of all states is $\{(R, C), (R, N), (G, C), (R, N), (B, C), (B, N)\}$.

**Can we describe:**

- Everyone knows Alan is color blind and cannot distinguish $R$ and $G$.
- Nobody except Ben knows whether Ben is color blind.

# Aumann Model of Incomplete Information

## Definition 1.1

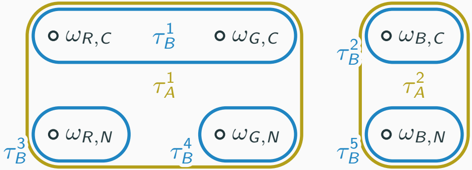
Let $\Theta$ be a set of finitely many states of nature. An Aumann model of incomplete information over $\Theta$ consists of:

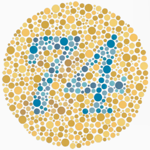
1. A finite set of players $\mathcal{I} = \{1, \ldots, n\}$.
2. A finite set $\Omega$ of possible states of the world $\omega$.
3. A partition $\mathcal{T}_i = \left\{\tau_i^1, \ldots, \tau_i^{m_i}\right\}$ of $\Omega$ for each player $i \in \mathcal{I}$.
4. A function $\theta : \Omega \to \Theta$, indicating the state of nature $\theta(\omega)$ in each $\omega$.

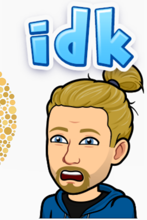
**Remarks:**

- We are interested in the players' knowledge of the states of nature.
- States of the world are needed to describe higher-order knowledge.
- An element $\tau_i \in \mathcal{T}_i$ is an information set of player $i$.
- Player $i$ cannot distinguish $\omega$ and $\omega'$ in the same information set $\tau_i$.

# Color Blindness






**Determining the state of nature:**

- Suppose the players place a bet on the color of the hat.
- Then the state of nature $\theta$ is the color of the hat. Color blindness is relevant only insofar as it affects the players' knowledge of $\theta$.

**Apply the formalism:**

- We have $\Theta = \{R, G, B\}$ and $\Omega = \{\omega_{R,C}, \omega_{R,N}, \omega_{G,C}, \omega_{G,N}, \omega_{B,C}, \omega_{B,N}\}$.
- The state of nature is the map $\theta(\omega_{col,blind}) = col$.

# Knowledge

## Definition 1.2

For any state $\omega \in \Omega$, denote by $T_i(\omega)$ the partition element of $\mathcal{T}_i$ that contains $\omega$, i.e., $T_i(\omega)$ is the set of all states that $i$ deems possible in $\omega$.

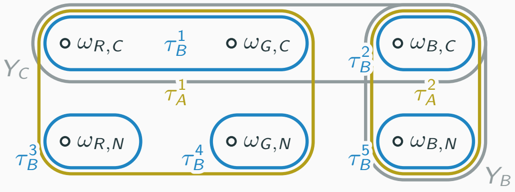
1. An event $Y$ is a subset of $\Omega$.
2. An event $Y$ obtains (is true) in state $\omega$ if $\omega \in Y$.
3. Player $i$ knows event $Y$ in state $\omega$ if $T_i(\omega) \subseteq Y$.
4. Define player $i$'s knowledge operator $K_i : 2^\Omega \to 2^\Omega$ as

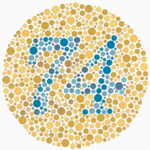
$$K_i(Y) := \{\omega \in \Omega \mid T_i(\omega) \subseteq Y\}.$$

**Remarks:**

- $2^\Omega$ is the set of all subsets of $\Omega$, i.e., the set of all events.
- $K_i Y = K_i(Y)$ is the event that player $i$ knows $Y$.

# Color Blindness



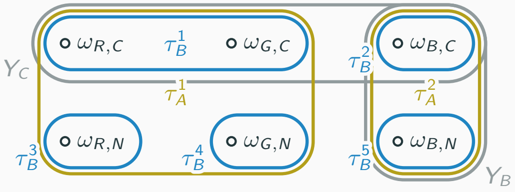

## Can define several events:

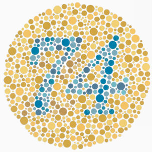
- "The hat is blue" is $Y_B = \{\omega \in \Omega \mid \theta(\omega) = B\} = \{\omega_{B,C}, \omega_{B,N}\}$.
- "Ben is color blind" $Y_C = \{\omega_{R,C}, \omega_{G,C}, \omega_{B,C}\}$.

## Possible states in $\omega_{B,C}$:

- Alan knows that the true state is in $T_A(\omega_{B,C}) = \{\omega_{B,C}, \omega_{B,N}\}$.
- Ben knows that the true state is in $T_B(\omega_{B,C}) = \{\omega_{B,C}\}$.
- Ben knows the state since $T_B(\{\omega_{B,C}\}) \subseteq \{\omega_{B,C}\}$.

# Color Blindness




**Who knows events $Y_B$ and $Y_C$ in state $\omega_{B,C}$?**

- Note: both events obtain (are true) in $\omega_{B,C}$ since $\omega_{B,C} \in Y_B, Y_C$.
- Since $T_B(\omega_{B,C}) \subseteq Y_B, Y_C$, Ben knows $Y_B$ and $Y_C$.
- The states, in which Ben knows $Y_B$ is $K_B(Y_B) = \{\omega_{B,N}, \omega_{B,C}\}$.
- Since $T_A(\omega_{B,C}) = \{\omega_{B,N}, \omega_{B,C}\} \subseteq Y_B$, Alan knows $Y_B$ as well.
- Alan does not know whether Ben is color blind since $T_A(\omega_{B,C}) \nsubseteq Y_C$.
- Still, Alan knows Ben knows $Y_B$ because $T_A(\omega_{B,C}) \subseteq K_B(Y_B)$.

# Uncertainty as Random Variables

**Modeling uncertainty:**

- The state of the world $\omega$ is an elementary outcome of the model's entire uncertainty, both payoff-relevant and irrelevant.
- In this finite model, any function $X$ of $\omega$ is a random variable.

**Examples:**

- The state of nature $\theta : \Omega \to \Theta$ is a random variable, whose outcome is simply not observed by (some of) the players.
- Similarly, $T_i(\omega)$ is a random variable because it is a priori unknown.
- An event $Y$ is not random, but it is random whether the event obtains.

**Notation:**

- We use "smaller" letters $\vartheta, \tau, \omega$ to represent deterministic elements and "larger" letters $\Theta, \mathcal{T}, \Omega$ to denote sets of elements.

# Characterization of Knowledge Operators

**Proposition 1.3**

*An operator $K_i : 2^\Omega \to 2^\Omega$ is a knowledge operator if and only if it satisfies the following properties:*

(K1) *Axiom of awareness:* $K_i\Omega = \Omega$,

(K2) *Axiom of knowledge:* $K_iY \subseteq Y$ *for any event* $Y$,

(K3) *Distribution axiom:* $K_i(X \cap Y) = K_iX \cap K_iY$ *for any events* $X, Y$,

(K4) *Axiom of introspection:* $K_i\big(K_iY\big) = K_iY$ *for any event* $Y$,

(K5) *Axiom of negative introspection:* $(K_iY)^c = K_i\big((K_iY)^c\big)$ *for any* $Y$.

**Notes:**

- (K2) implies that you cannot know something that is not true.
- (K3) is sometimes stated in a different, but equivalent form.

# Equivalent Representation of Knowledge Operator

**Lemma 1.4**

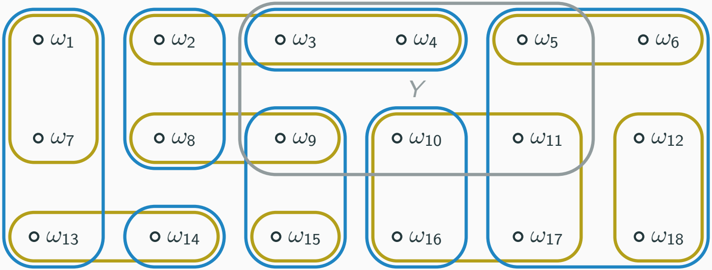
*Fix an information partition $\mathcal{T}_i$ of $\Omega$. For any event $Y$, the event $K_i(Y)$ is the union of all of $i$'s information sets that are fully contained in $Y$:*

$$K_i(Y) = \bigcup_{\tau_i \in \mathcal{T}_i : \tau_i \subseteq Y} \tau_i .$$

**Proof:**

- If $i$ knows $Y$ in state $\omega$, that is, $T_i(\omega) \subseteq Y$, then $i$ knows $Y$ in any state $\omega'$ with $T_i(\omega') = T_i(\omega)$ since then $T_i(\omega') \subseteq Y$.
- Thus, any information set with one state in $K_i(Y)$ must fully lie in $Y$.
- On the other hand, $K_i(Y)$ cannot contain any states of an information set $\tau_i$ that is not fully contained in $Y$ by definition of $K_i(Y)$.

# Example




**Example:**

- $K_2(Y) = \{\omega_3, \omega_4\}$: Player 2 knows event $Y$ in states $\omega_3$ and $\omega_4$.
- $K_1(Y) = \emptyset$: Player 1 cannot know event $Y$.
- Player 2 is aware that Player 1 does not know $Y$: $K_2(K_1^c(Y)) = \Omega$.

# Proof of Proposition 1.3

**Consequence of Lemma 1.4:**

- Lemma 1.4 immediately implies $K_i(Y) \subseteq Y$ (K2).
- If $Y$ is the union of some of $i$'s information sets, then $K_i(Y) = Y$.
- This readily implies $K_i(K_i Y) = K_i(Y)$ (K4) and $K_i(\Omega) = \Omega$ (K1).
- Since $\mathcal{T}_i$ is a partition of $\Omega$ and $K_i$ is a union of information sets, then so is $K_i^c(Y)$. In particular $K_i(K_i^c Y) = K_i^c(Y)$ (K5).

**Distribution axiom (K3):**

- $K_i(X)$ and $K_i(Y)$ are the union of information sets in $X$ and $Y$, resp.
- Thus, $K_i(X) \cap K_i(Y)$ is the union of information sets that lie in both.
- By Lemma 1.4, this set coincides with $K_i(X \cap Y)$.

See Bacharach (1985) for proof of converse.

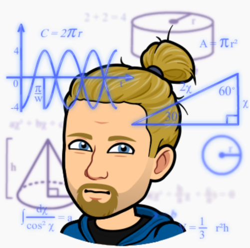
# How Strong Is This Concept of Knowledge?

**Corollary 1.5**

*If $X \subseteq Y$ and $\omega \in K_i(X)$, then $\omega \in K_i(Y)$.*

**Implications:**

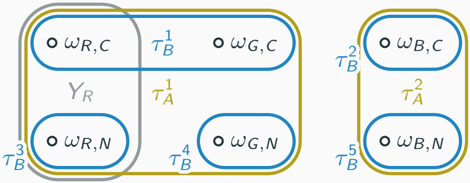
- If you know an event $X$, then you know everything that $X$ implies.
- If you know the 9 ZFC axioms, then for every mathematical statement you know whether it is true, false, or undecidable!




**Unbounded rationality:**

- In game theory and most of economics, we assume that players can make infinitely many logical deductions infinitesimally quickly.
- Behavioral game theory and bounded rationality relax this assumption.

# Check Your Understanding

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**True or false:**

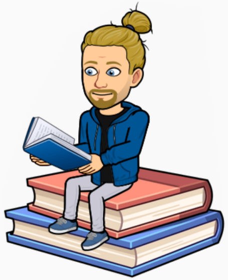
1. If $X \subseteq Y$ for two events $X$ and $Y$, then $Y$ is more informative than $X$.
2. For any player $i$, any state $\omega$ must lie in some information set $\tau_i \in \mathcal{T}_i$.

**Short-answer questions:**

3. What does Adam know about the color of the hat in state $\omega_{R,N}$?
4. What does Ben know about the color of the hat in state $\omega_{R,N}$?
5. Compute $K_A(K_B(Y_R))$. What does it mean?

# Literature


- D. Fudenberg and J. Tirole: Game Theory, Chapter 14.2, MIT Press, 1991
- M. Maschler, E. Solan, and S. Zamir: Game Theory, Chapter 9.1, Cambridge University Press, 2013
- R. Aumann: Agreeing to Disagree, Annals of Statistics, **4** (1976), 1236–1239.
- M. Bacharach: Some Extensions of a Claim of Aumann in an Axiomatic Model of Knowledge, Journal of Economic Theory, **37** (1985), 167–190.

# Common Knowledge

# Higher-Order Knowledge

# Knowledge Hierarchies

**Proposition 1.6**

*Let $\Theta$ be a set of states of nature. An Aumann model of incomplete information $(\mathcal{I}, \Omega, (\mathcal{T}_i)_{i\in\mathcal{I}}, \theta)$ over $\Theta$ uniquely defines a knowledge hierarchy $K_{i_1} K_{i_2} \dots K_{i_k}$ for any finite sequence of players $i_1, i_2, \dots, i_k$.*

**Proof:**

- For any event $Y$, the event $K_i Y$ is a well-defined subset of $\Omega$.
- By induction, $K_{i_1} K_{i_2} \dots K_{i_k} Y$ is well defined for any $i_1, i_2, \dots, i_k$.

**Application to Friends:**

- $Y = \{$Monica and Chandler are dating$\}$.
- Chandler, Monica, Rachel, and Phoebe could keep pranking each other until arbitrarily high order of knowledge about $Y$.
- If they talk about it in the same room, $Y$ becomes common knowledge.

# Common Knowledge

**Definition 1.7**

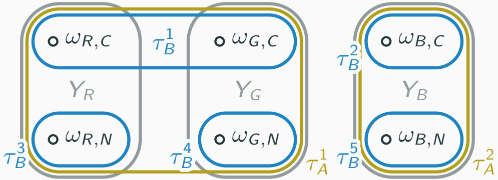
An event $Y \subseteq \Omega$ is common knowledge in state $\omega$ if for every finite sequence of players $i_1, \ldots, i_k$,

$$\omega \in K_{i_1} K_{i_2} \ldots K_{i_k} Y. \tag{1}$$

**Interpretation:**

- Crucially, equation (1) must hold for arbitrarily large $k$.
- Common knowledge = everybody knows $Y$, they all know that they know $Y$, they all know that they know $Y$, and so on.

# Color Blindness

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**Which events are common knowledge?**

- Since $K_A(Y_B) = K_B(Y_B) = Y_B$, it follows that $K_{i_1} K_{i_2} \ldots K_{i_k} Y_B = Y_B$.
- Therefore, $Y_B$ is common knowledge in $\omega_{B,N}$ and $\omega_{B,C}$.
- Similarly, $Y_R \cup Y_G$ is common knowledge in $\omega_{R,C}$, $\omega_{R,N}$, $\omega_{G,C}$, and $\omega_{G,N}$.
- Are there other events that are common knowledge?

# How to Find Events of Common Knowledge

**Definition 1.8**

An event $X$ is called self-evident if $K_i X = X$ for every player $i$.

**Lemma 1.9**

*An event $Y$ is common knowledge in $\omega$ if and only if there exists a self-evident event $X \subseteq Y$ with $\omega \in X$.*

**Showing equivalence between (A) and (B):**

- Show (A) implies (B) or, equivalently, not (B) implies not (A).
- Show (B) implies (A) or, equivalently, not (A) implies not (B).

We will show (B) implies (A) and not (B) implies not (A).

# Proof of Lemma 1.9

**Preparation:**

- Corollary 1.5 is equivalently stated as $K_i X \subseteq K_i Y$ for any $X \subseteq Y$.
- We say that $K_i$ is monotone for any player $i$.

**Suppose such self-evident X exists:**

- Monotonicity implies that for any sequence $i_1, i_2, \dots$

$$X = K_{i_1} X \subseteq K_{i_1} Y, \qquad X = K_{i_2} K_{i_1} X \subseteq K_{i_2} K_{i_1} Y, \qquad \dots$$

**Suppose no such self-evident event exists:**

- Then for every $X \subseteq Y$, there exists a player $i$ with $K_i X \subsetneq X$.
- There exists a sequence $i_1, i_2, \dots$ such that

$$Y \supsetneq K_{i_1} Y \supsetneq K_{i_2} K_{i_1} Y \supsetneq \dots$$

- Since $Y$ is finite, the sequence converges to the empty set.

# Common Knowledge Component

**Definition 1.10**

For any state $\omega \in \Omega$, the common knowledge component $C(\omega)$ in $\omega$ is the smallest self-evident event $X$ with $\omega \in X$.

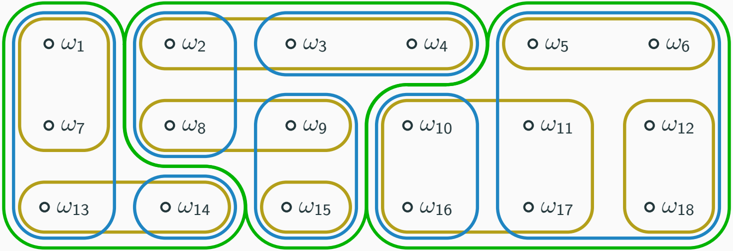
**Notes:**

- $C(\omega)$ is the most informative event that is common knowledge in $\omega$.
- An event $Y$ is common knowledge in $\omega$ if and only if $C(\omega) \subseteq Y$.

**Finding $C(\omega)$:**

1. Start with $C_0(\omega) = \{\omega\}$.
2. For $k \geq 1$, define $C_k(\omega)$ as the union over all information sets $\tau_i$ for all players $i$ with $\tau_i \cap C_{k-1}(\omega) \neq \emptyset$.
3. If $C_k(\omega) = C_{k-1}(\omega)$, then $C_k(\omega) = C(\omega)$.

# Finding Common Knowledge Component Graphically




**Common knowledge in $\omega_9$:**

- Start with $C_0(\omega_9) = \{\omega_9\}$.
- $C_1(\omega_9) = \{\omega_8, \omega_9, \omega_{15}\}$ contains all information sets, which contain $\omega_9$.
- We continue with $C_2(\omega_9) = \{\omega_2, \omega_8, \omega_9, \omega_{15}\}$ and

$$C(\omega_9) = C_3(\omega_9) = \{\omega_2, \omega_3, \omega_4, \omega_8, \omega_9, \omega_{15}\}.$$

Common knowledge components = connected components of the graph.

# Crossing Xinhai Road

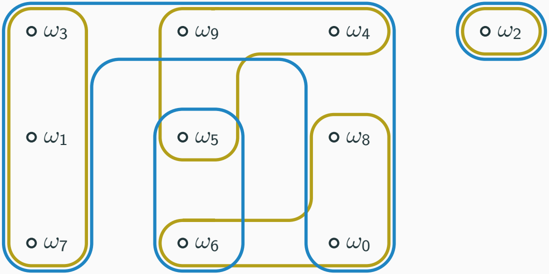

Layla and Rowan are waiting to cross Xinhai road. They stand at an angle, the countdown for the green light is partially obstructed:

- Layla sees only the left-most edge of the digits,
- Rowan sees only the right-most edge of the digits.

**Questions:**

- What are the information partitions of Layla and Rowan?
- Which digits do they know when they see it?
- What numbers are common knowledge?

# Crossing Xinhai Road




**Knowing digits:**

- Each player only knows the digit 2 when they see it.
- Digit 2 is the only number that is common knowledge.

# Decisions and Learning

# Logic Puzzles

Darren writes numbers 50, 30, and 20 on Anya's, Bernadette's, and Cheryl's forehead and announces: "each of you has a distinct positive number on your forehead such that two of the numbers add up to the third."

- Anya says: I don't know my number.
- Bernadette says: I don't know my number.
- Cheryl says: I don't know my number.
- Anya says: now I know my number.

**Communication:**

- The players reveal information with their announcements, leading the other players to refine their information sets.
- Similar puzzles include truth-telling puzzles, prisoners and hats, etc.

# Decision Rules

### Definition 1.11

Suppose player $i$ has a set of actions $\mathcal{A}_i$ available, with individual elements of $\mathcal{A}_i$ denoted by $a_i$. A decision rule by player $i$ is a map $\sigma_i : \mathcal{T}_i \to \mathcal{A}_i$.

**Interpretation:**

- Player $i$'s decision cannot depend on information he/she does not have.
- In state $\omega$, player $i$ has information $T_i(\omega)$ and selects $\sigma_i(T_i(\omega))$.
- The outcome $\sigma_i \circ T_i$ of player $i$'s decision is also denoted by $A_i$.

**Example:**

- Ben leaves the room if he knows the color of the hat. This maps $\{\omega_{B,C}, \omega_{R,N}, \omega_{G,N}, \omega_{B,N}\}$ to "leaving" and $\{\omega_{R,C}, \omega_{G,C}\}$ to "staying".
- What does Alan learn by observing $\{A_B = \text{leave}\}$ and $\{A_B = \text{stay}\}$?

# Learning New Information

**Definition 1.12**

If player $i$ with information partition $\mathcal{T}_i$ learns information in $\mathcal{T}_*$, then his/her information partition with the new information is simply $\mathcal{T}_i \cap \mathcal{T}_*$ with information sets $\tau_i \cap \tau_*$ for any $\tau_i \in \mathcal{T}_i$ and $\tau_* \in \mathcal{T}_*$.

**Note:**

- Since player retains information in $\mathcal{T}_i$, information partition gets finer.

**Examples:**

- If a player observes event $Y$, then we have $\mathcal{T}_* = \{Y, Y^c\}$.
- If player $i$ shares his/her information in state $\omega$ with some player $i$, then we apply the above to $\mathcal{T}_* = \left\{T_j(\omega), T_j^c(\omega)\right\}$.
- If player $i$ shares his/her information in every state, then $\mathcal{T}_* = \mathcal{T}_j$.

# Crossing Xinhai Road

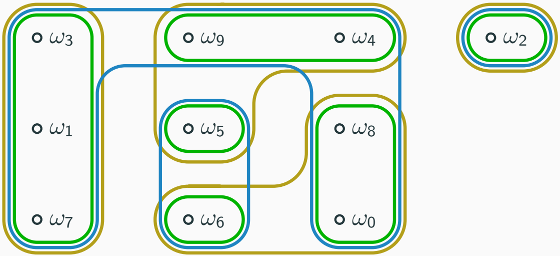

**Sharing information:**

- What numbers do Layla and Rowan know if they can communicate?
- Suppose Layla steps onto the road whenever the countdown could be zero and Rowan observes this. What is Rowan's information set now?

**Gathering information individually:**

- Suppose they observe the countdown for two consecutive seconds.
- What are Layla's and Rowan's information sets?
- What numbers are common knowledge now?

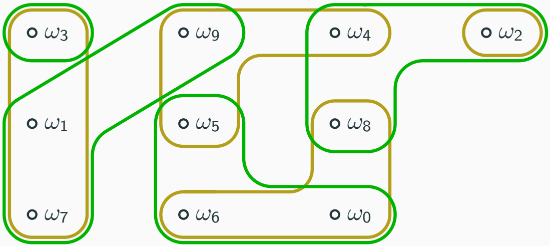
# Crossing Xinhai Road




**Learning:**

- If the players share their information in any state of the world, their joint information partition is $\mathcal{T}_L \cap \mathcal{T}_R$.
- If Rowan sees Layla step out on the road, he learns to distinguish $\{0, 6, 8\}$ and its complement $\{1, 2, 3, 4, 5, \}$.
- He learns the same if she does not step out on the road.

# Waiting For One Second




**Waiting for one second:**

- Initially, Layla's information sets are $\{0, 6, 8\}$, $\{1, 3, 7\}$, $\{2\}$, $\{4, 5, 9\}$.
- Layla learns to distinguish $\{1, 7, 9\}$, $\{2, 4, 8\}$, $\{3\}$, $\{5, 6, 0\}$.
- Consequently, $\mathcal{T}_L^{(1)} = \{\{0, 6\}, \{1, 7\}, \{2\}, \{3\}, \{4\}, \{5\}, \{8\}, \{9\}\}$.
- It is better to stand on the right to see the left-most digit.

# Releasing Prisoners

**Three prisoners are to be released:**

- Each of the three has a mark on their forehead, which others can see but they themselves cannot.
- A prisoner with a mark can simply walk out.
- A prisoner without a mark that attempts to flee will be stopped and sentenced to a life in jail.

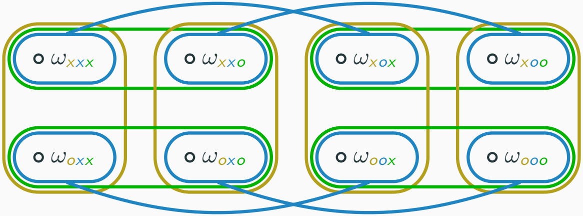
**Available information:**

- There are no reflective surfaces and communication is not allowed.
- All prisoners see each other once a day in the cafeteria for lunch.

**Questions:**

1. Will any prisoners leave? If so, after how many days?
2. If, additionally, the warden announces in the cafeteria that at least one prisoner is marked. Will anybody leave? If so, after how many days?

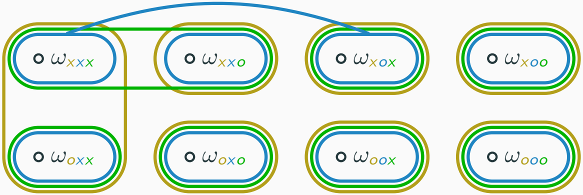
# Releasing Prisoners




**Reducing the scope of the problem:**

- Because of the strong punishment, prisoners will try to leave only if they know they carry the mark.
- By axiom of knowledge (K2), nobody without a mark will try to leave.
- It is enough to consider the three marked prisoners: there are 8 states.
- State $\omega_{xox}$ indicates that prisoners 1 and 3 are marked.

# Releasing Prisoners




**Learning by observing others:**

- Let $Y_i$ denote the event that $i$ is marked for freedom.
- Without the warden's announcement, we see that $K_i Y_i = \emptyset$.
- Warden's announcement contains the information $\{\omega_{ooo}, \omega_{ooo}^c\}$.
- The players know that they are marked in the events

$$K_1 Y_1 = \{\omega_{xoo}\}, \qquad K_2 Y_2 = \{\omega_{oxo}\}, \qquad K_3 Y_3 = \{\omega_{oox}\}.$$

- The players learn $\{K_1 Y_1, K_1^c Y_1\}$, $\{K_2 Y_2, K_2^c Y_2\}$, and $\{K_3 Y_3, K_3^c Y_3\}$.

# Summary

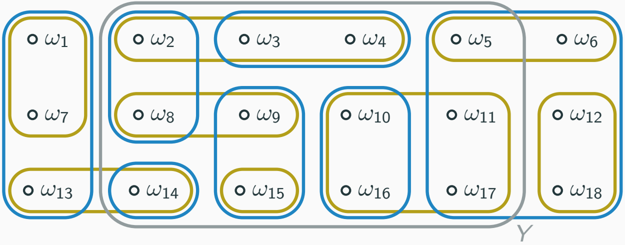
**Aumann model of incomplete information:**

- Formal model to express knowledge over a set of possible states.
- The concept of knowledge is understood in a very strong sense:
  - Players are unboundedly rational: if they know $Y$, they know everything that is deducible from $Y$.
  - This is in line with equilibrium concepts used in game theory.
- It is easy to verify graphically whether an event is common knowledge.

**Applications:**

- Solving games/riddles that involve higher-order knowledge.
- Game theory requires common knowledge of rationality.
- Especially useful if we enhance the model with beliefs over uncertainty.

# Check Your Understanding




Scan me

**True or false:**

1. Event $Y$ is common knowledge in state $\omega_9$.
2. Suppose two players share all the information they have. Then any known event is also commonly known.
3. Suppose prisoner $i$ leaves the prison in the setting of Slide 36. Does this carry the same information for other prisoners as if $i$ stays?

# Literature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- M. Maschler, E. Solan, and S. Zamir: Game Theory, Chapter 9.1, Cambridge University Press, 2013
- R. Aumann: Agreeing to Disagree, Annals of Statistics, **4** (1976), 1236–1239.
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