4. Perfect Bayesian Equilibrium

ECON 7219 – Games With Incomplete Information

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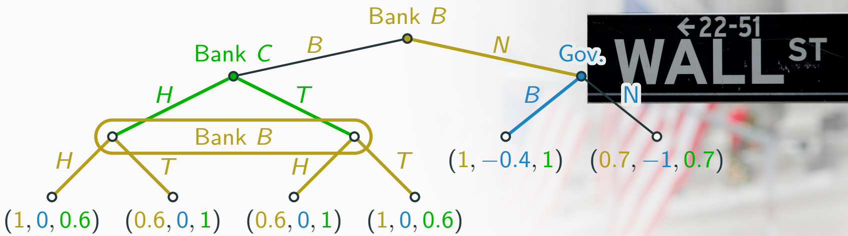
Perfect Bayesian Equilibrium

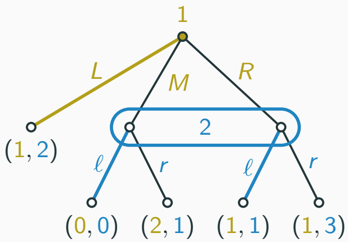
Bail-in or Bailout?



Default resolution:

- The default of bank A causes losses both to its creditors Bank B and Bank C as well as the economy.
- Bank B and Bank C have an incentive to bail-in Bank A.
- The Government has an incentive to bail out Bank A.
- What does subgame perfection predict?

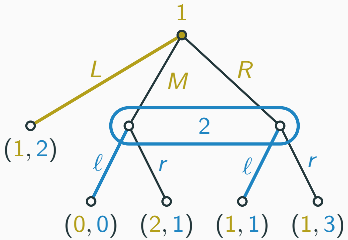
Seemingly Dominated Strategies



Two subgame-perfect equilibria:

- Given that Player 2 gets to act, it is strictly dominant to choose r.
- Nevertheless, (L, ℓ) is a subgame-perfect equilibrium.
- By definition, a subgame starts at a singleton information set, hence the only subgame is the entire game itself.
- As a consequence, all Nash equilibria are trivially subgame perfect.

Refining Subgame Perfection



Basic idea:

- If a player has beliefs μ_h over an information set h, he/she can aggregate expected payoffs from subgames $\mathcal{G}(x)$ that start at $x \in h$.
- We want players to act optimally, given their beliefs.
- Moreover, beliefs should be derived from Bayes' rule.

Beliefs on the Equilibrium Path

Beliefs over continuation games:

- For any information set h that is reached with positive probability under σ, Bayesian updating induces beliefs over h given by

$$\mu_h(x) := P_\sigma(\{x\} \mid h) = \frac{P_\sigma(\{x\})}{P_\sigma(h)}.$$

- Moreover, any node x' that can be reached from h has conditional probability of being reached, given that h is reached, of

$$P_\sigma(\{x'\} \mid h) = \frac{P_\sigma(\{x'\})}{P_\sigma(h)}.$$

- This allows us to aggregate expected payoffs on h by maximizing the conditional expected value $\mathbb{E}_\sigma[u_i(Z) \mid h]$.
- However, we want strategies to be optimal also off the path.

Beliefs off the Equilibrium Path

Beliefs over continuation games:

- Suppose a player has some beliefs μ_h over information set h.
- Given beliefs μ_h, any node x that is reachable from $x_0 \in h$ via sequence $(x_0, x_1, \dots, x_k)$ with $x = x_k$ is reached with probability

$$P_{\mu_h,\sigma}(\{x\}) = \mu_h(\{x_0\}) \prod_{j=0}^{k-1} \sigma_{i(x_j)}(h(x_j); a_{x_{j+1}})$$

  when continuation profile σ is played, where $h(x_j)$ denotes the information set at x_j and $a_{x_{j+1}}$ denotes the taken at x_j that leads to x_{j+1}.

Remark:

- Probability $P_{\mu_h,\sigma}(\{x\})$ multiplies $\mu(\{x_0\})$ with the probability of all edges in the tree between x_0 and x.
- On the path, $P_{\mu_h,\sigma}$ coincides with $P_\sigma(\,\cdot \mid h\,)$ if $\mu_h = P_\sigma(\,\cdot \mid h\,)$.

Sequential Rationality

Definition 4.1

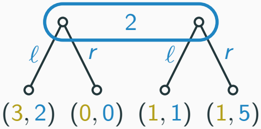
1. A belief system $\mu = (\mu_h)_{h \in \mathcal{H}}$ assigns beliefs to each information set.
2. A strategy profile σ is sequentially rational, given belief system μ, if for every player i, every information set $h \in \mathcal{H}_i$, and every deviation s_i,

$$\mathbb{E}_{\mu_h, \sigma}[u_i(Z)] \geq \mathbb{E}_{\mu_h, s_i, \sigma_{-i}}[u_i(Z)].$$

3. A pair (μ, σ) is called an assessment.

Sequential rationality:

- Allows us to "cut through" information sets.
- Allows us to analyze improper subgames.



Perfect Bayesian Equilibrium

Definition 4.2

An assessment (μ, σ) is a perfect Bayesian equilibrium for prior P if

1. σ is sequentially rational given μ,
2. μ is derived from P via Bayes rule "wherever possible."
3. μ can be anything where Bayes rule "is impossible."

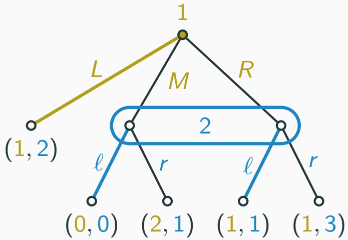
Wherever possible means:

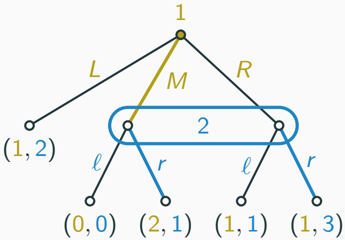
- On the equilibrium path, we must have $\mu_h = P_\sigma(\cdot \mid h)$.
- At any singleton information set $h = \{x\}$, we must have $\mu_h = \delta_x$.[1]
- For any proper subgame $\mathcal{G}(x)$, we must have $\mu_h = P_{\delta_x,\sigma_x}(\cdot \mid h)$ on the path of the continuation equilibrium $\sigma|_x$.

Implication: Any perfect Bayesian equilibrium is subgame perfect.

[1] δ_x is called the Dirac measure and places probability 1 on x.

Seemingly Dominated Strategies

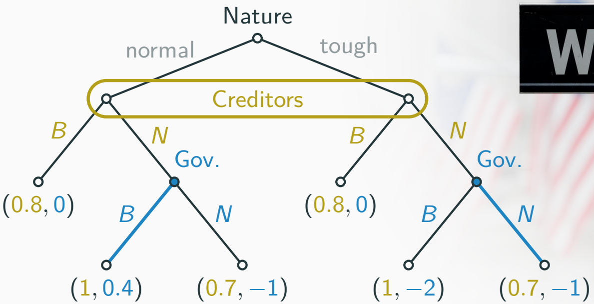


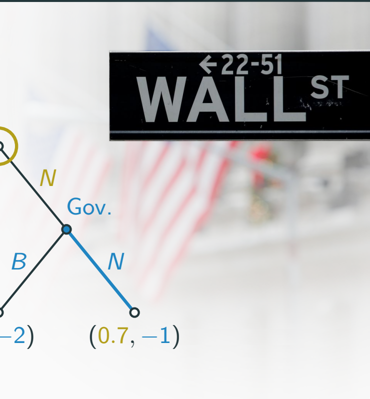


Proper refinement off the path:

- (L, ℓ) is not a perfect Bayesian equilibrium.
- Regardless of Player 2's belief at his/her information set, it cannot be sequentially rational to play a dominated strategy.
- (M, r) is a perfect Bayesian equilibrium.

Two Types of Government



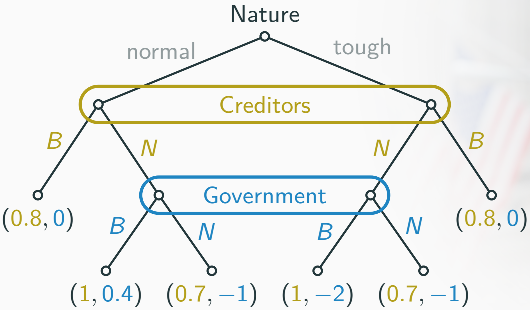


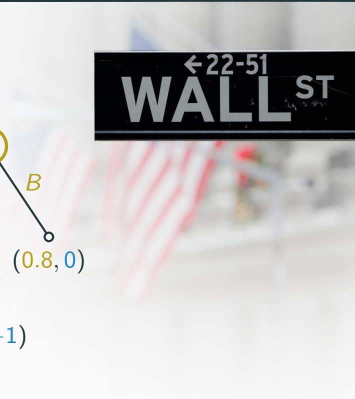
Tough Government:

- Instead of being the welfare-maximizing normal type, the Government may be tough on banks because elections are coming up.
- What is the perfect Bayesian Nash equilibrium for prior $P(\text{tough}) = \mu_0$?

Sequential Equilibrium

Unknown Type of Government





Unknown voter preferences:

- The government is not aware whether voters prefer a Government that is tough on banks or not.
- What are the perfect Bayesian Nash equilibrium with $P(\text{tough}) = 0.3$?

Forgetful Government

In equilibrium (B, N):

- The Government's information set lies off the equilibrium path of any proper subgame, hence beliefs are unrestricted in a PBE.
- The choice N is sequentially rational for strong beliefs that voters prefer a tough Government.
- Perfect Bayesian equilibrium allows Government to "forget" its prior over types simply because the Creditors have acted unexpectedly.

This issue is resolved by:

- Sequential equilibria in general extensive-form games.
- Perfect extended Bayesian equilibria in multi-stage games.

Consistent Belief Systems

Definition 4.3

A belief system $\mu = (\mu_h)_{h \in \mathcal{H}}$ is consistent with strategy profile σ if there exists a sequence of completely mixed strategies $(\sigma_k)_{k \geq 0}$ such that

$$\lim_{k \to \infty} \sigma_k = \sigma, \qquad \mu_h = \lim_{k \to \infty} P_{\sigma_k}(\cdot\,|h).$$

Consistency imposes:

- Along the approximating sequence, $P_{\sigma_k}(\cdot\,|h)$ is well defined for any h.
- As a consequence, $\mu = \lim_{k \to \infty} P_{\sigma_k}$ is a belief system and beliefs in μ are given by Bayesian updating wherever possible.
- If players are called upon to act unexpectedly, players have an explanation how they ended up there (via "trembles" σ_k).

Sequential Equilibrium

Definition 4.4

An assessment (σ, μ) is a sequential equilibrium if

1. σ is sequentially rational, given μ,
2. μ is consistent with σ.

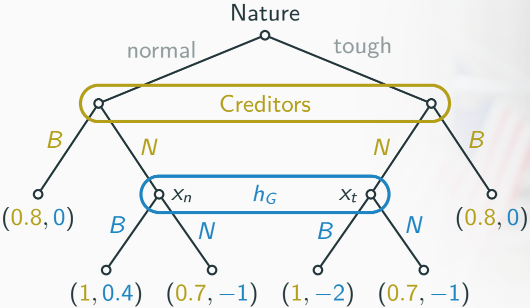
Refinements:

- Every sequential equilibrium is a perfect Bayesian equilibrium and hence also a subgame-perfect equilibrium.

Advantages and disadvantages:

- Restricts off-path beliefs to consistent beliefs.
- Verifying that an assessment is a sequential equilibrium can be tedious.

Unknown Type of Government



Unknown voter preferences:

- Any completely mixed approximating sequence σ^k must place positive probability on N, hence $P_{\sigma^k}(\{x_t\} \mid h_G) = 0.3$.
- The unique beliefs in the limit as $k \to \infty$ is $\mu_{h_G}(\{x_t\}) = 0.3$.
- In the unique sequential equilibrium, (N, B) is played.

Summary

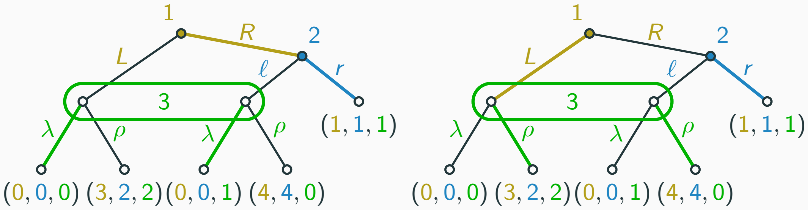
Solution concepts:

Nash $\supsetneq$ Subgame perfect $\supsetneq$ Perfect Bayesian $\supsetneq$ Sequential

Refinements:

- Subgame perfection eliminates non-credible threats in proper subgames.
- PBE: sequential rationality eliminates non-credible threats after "improper subgames," i.e., starting at non-singleton information sets.
- Sequential equilibria: consistency ensures that belies are consistent with beliefs held earlier in the game.

Check Your Understanding



True or false:

1. Subgame-perfect equilibria are sequentially rational at singleton information sets.
2. Sequential equilibria and perfect Bayesian equilibria both eliminate non-credible threats.
3. We cannot refine sequential equilibria any further.

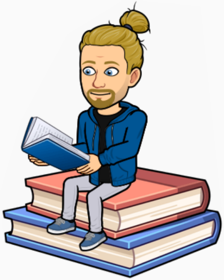
Scan me

Short-answer question:

4. Which of the two subgame-perfect equilibria are sequential equilibria?

Literature

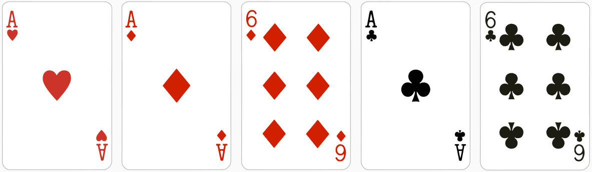
- D. Fudenberg and J. Tirole: Game Theory, Chapter 8, MIT Press, 1991
- M. Maschler, E. Solan, and S. Zamir: Game Theory, Chapter 7.4, Cambridge University Press, 2013
- S. Tadelis: Game Theory: An Introduction, Chapter 15, Princeton University Press, 2013
- D.M. Kreps and R. Wilson: Sequential Equilibria, Econometrica, **50** (1982), 863–894
- D. Fudenberg and J. Tirole: Perfect Bayesian Equilibrium and Sequential Equilibrium, Journal of Economic Theory, **53** (1991), 236–260



Signaling Games

Good Fold? Bad Fold?

Bluffing on the River



Model the situation:

- John Juanda holds 6♡ 6♠.
- He loses to A♠, but wins against anything else.
- What are the types? actions? strategies?
- What is Juanda's best reply here?
- What are the perfect Bayesian equilibria of the last round?

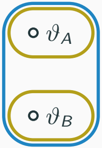
Bluffing on the River: Types and Actions

	C	F
B	$p+b, -b$	$p, 0$
C	$p, 0$	$p, 0$

ϑ_A

	C	F
B	$-b, p+b$	$p, 0$
C	$0, p$	$p, 0$

ϑ_B



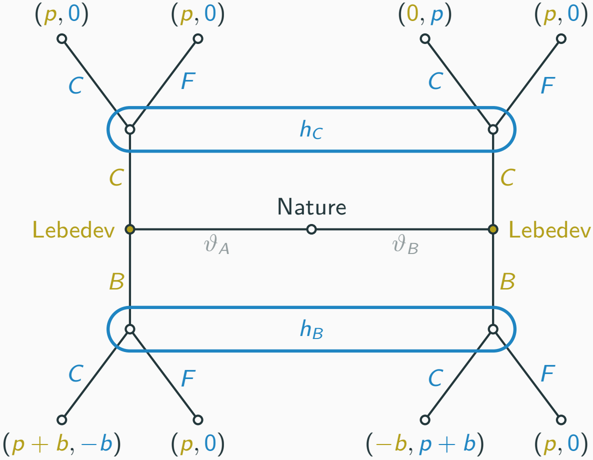
Types:

- Juanda needs to distinguish only two of Lebedev types: $\theta_L \in \{\vartheta_A, \vartheta_B\}$, where type ϑ_A holds A♠ and type ϑ_B does not (= the bluffing type).
- Suppose that Juanda has only one type, i.e., it is common knowledge that his hand is better than the board, but not the A♠.

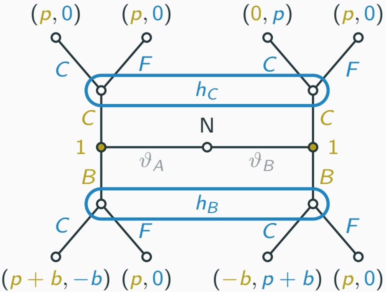
Actions:

- Let p denote the size of the pot going into the river.
- Lebedev can either (B)et a fixed amount b or (C)heck (= bet 0).
- Suppose that Juanda can only (C)all or (F)old (no raise).

Bluffing on the River: Extensive-Form Game



Bluffing on the River: Information Setting



Information setting:

- Different from a Bayesian game, Juanda observes Lebedev's action.
- As a result, Juanda has one information set for each history of play, each containing two nodes, corresponding to the two types.
- Because Juanda updates their beliefs with the new information, Lebedev's types can signal their strength through their action.

Signaling Games and Screening Games

Signaling game:

- A two-stage game, in which a player with private information moves first, followed by a player without private information.
- Since a player's strategy depends on their type, the first player's action sends a signal about their type to the second player.

Screening game:

- A two-stage game, in which the informed player moves last.
- The uninformed player, who moves first, can "screen" the types of the second player because they react differently.
- We will look at screening games next week.

Independent types:

- If there is one-sided information, types are trivially independent.

Signaling Game: Types and Strategies

Types:

- Denote again by Θ the set of payoff-relevant states or states of nature.
- Player 1's type T_1 is distributed according to a prior P on Ω that captures how T_1 is correlated to the state of nature.
- In many examples (like the poker example) we have $\mathcal{T}_1 = \Theta = \Omega$.
- Player 2's type is common knowledge.

Strategies:

- A strategy of player 1 is a map $\sigma_1 : \mathcal{T}_1 \to \Delta(\mathcal{A}_1)$.
- Player 2 observes A_1, but does not observe $\mathcal{T}_1$, hence his/her information sets (or histories) are $\mathcal{H}_2 = \mathcal{A}_1$.
- Player 2's strategy is a map $\sigma_2 : \mathcal{H}_2 \to \Delta(\mathcal{A}_2)$.

Signaling Game: Beliefs

Updating beliefs via Bayes' rule:

- Given $A_1 = a_1$, player 2 updates his/her beliefs about each $\vartheta \in \Theta$ to

$$\mu(\vartheta \mid a_1) = \frac{P_\sigma(A_1 = a_1 \mid \theta = \vartheta)P(\theta = \vartheta)}{P_\sigma(A_1 = a_1)}$$

$$= \frac{\sum_{\tau_1 \in \mathcal{T}_1} P_\sigma(A_1 = a_1 \mid T_1 = \tau_1)P(T_1 = \tau_1, \theta = \vartheta)}{\sum_{\tau_1 \in \mathcal{T}_1} P_\sigma(A_1 = a_1 \mid T_1 = \tau_1)P(T_1 = \tau_1)}$$

$$= \frac{\sum_{\tau_1 \in \mathcal{T}_1} \sigma_1(\tau_1; a_1)P(T_1 = \tau_1, \theta = \vartheta)}{\sum_{\tau_1 \in \mathcal{T}_1} \sigma_1(\tau_1; a_1)P(T_1 = \tau_1)}.$$

- If $\mathcal{T}_1 = \Theta$ with prior $\mu_0 \in \Delta(\Theta)$, then updating simplifies to

$$\mu(\vartheta \mid a_1) = \frac{\sigma_1(\vartheta; a_1)\mu_0(\vartheta)}{\sum_{\vartheta' \in \Theta} \sigma_1(\vartheta'; a_1)\mu_0(\vartheta')}.$$

Pooling and Separating Equilibria

Definition 4.5

1. A pooling equilibrium is a perfect Bayesian equilibrium, in which every type of the informed player chooses the same pure action.
2. A (fully) separating equilibrium is a perfect Bayesian equilibrium, in which no two types choose mixed actions with overlapping support.

Remark:

- Separating equilibria are fully revealing: the uninformed player will know exactly which type he/she is facing.
- Pooling equilibria reveal no information to the uninformed player.
- Because only one action is chosen on the path, we always have to specify off-path beliefs for pooling equilibria.
- Mixed-strategy PBE are also called semi-separating equilibria.

Bluffing on the River: Strategies and Beliefs

Parametrize strategies:

- Lebedev's strategy specifies $\sigma_L(\vartheta_A)$ and $\sigma_L(\vartheta_B)$ and Juanda's strategy specifies $\sigma_J(h_B)$ and $\sigma_J(h_C)$. We parametrize them by

$$\alpha = \sigma_L(\vartheta_A; B), \quad \beta = \sigma_L(\vartheta_B; B), \quad \gamma = \sigma_J(h_B; C), \quad \delta = \sigma_J(h_C; C).$$

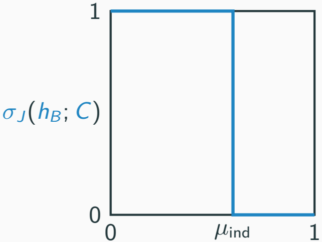
Parametrize beliefs:

- Suppose $\mu_0 \in (0,1)$ indicates Juanda's prior that Lebedev is of type ϑ_A.
- Under strategy profile $\sigma = (\sigma_L, \sigma_J)$, those beliefs are updated to

$$\mu(h_B) = \frac{P_\sigma(h_B \,|\, \vartheta_A) P_\sigma(\vartheta_A)}{P_\sigma(h_B \,|\, \vartheta_A) P_\sigma(\vartheta_A) + P_\sigma(h_B \,|\, \vartheta_B) P_\sigma(\vartheta_B)} = \frac{\alpha\mu_0}{\alpha\mu_0 + \beta(1-\mu_0)},$$

$$\mu(h_C) = \frac{(1-\alpha)\mu_0}{(1-\alpha)\mu_0 + (1-\beta)(1-\mu_0)}.$$

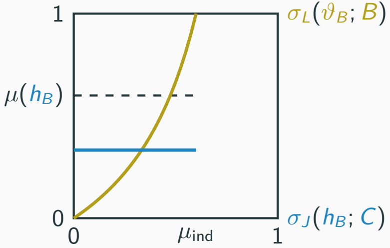
Bluffing on the River: Posterior Beliefs



Posterior beliefs:

- After observing B, the posterior beliefs $\mu(h_B)$ increase if $\alpha > \beta$, they decrease if $\alpha < \beta$, and they are μ_0 if $\alpha = \beta$.
- There exists a cutoff $\mu_{\text{ind}} = \frac{p+b}{p+2b}$ of posterior beliefs, for which Juanda is indifferent between calling a bet of size b and folding.
- If $\mu(h_B) < \mu_{\text{ind}}$, the unique best response is to call.
- If $\mu(h_B) > \mu_{\text{ind}}$, the unique best response is to fold.

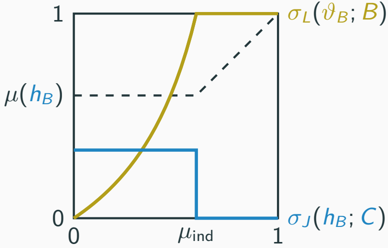
Bluffing on the River: Undominated PBE



Optimistic prior: $\mu_0 < \mu_{\text{ind}}$

- If type ϑ_B never bluffs, Juanda will never call type ϑ_A because he would be certain that he is facing type ϑ_A after seeing a bet.
- To keep calls in Juanda's best responses, type ϑ_B bluffs with a probability that makes Juanda indifferent between calling and folding.
- In equilibrium, Juanda must call a bet with a probability that makes the bluffing type ϑ_B indifferent between betting and checking.

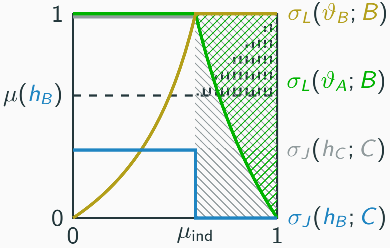
Bluffing on the River: Undominated PBE



Pessimistic prior: $\mu_0 > \mu_{\text{ind}}$

- If Lebedev wishes to keep calls in Juanda's range, he must decrease Juanda's posterior by not betting with certainty when he is of type ϑ_A.
- However, if Juanda calls with positive probability, type ϑ_A must bet.
- Thus, there is no equilibrium that keeps calls in Juanda's range.
- Since type ϑ_A cannot elicit a call, the only thing Lebedev can do is to protect the pot as type ϑ_B by betting with certainty.

Bluffing on the River: Dominated PBE



Pessimistic prior: $\mu_0 > \mu_{\text{ind}}$

- Since type ϑ_A cannot elicit a call, he is indifferent between betting and checking as long as folding remains a best response for Juanda.
- This requires ϑ_A to bet with a probability so that $\mu(h_B) \geq \mu_{\text{ind}}$.
- Since type ϑ_B bets with certainty, posterior beliefs $\mu(h_C)$ after seeing C assign probability 1 to ϑ_A, hence Juanda is willing to fold.

Bluffing on the River: Comparing PBE

Dominated PBE:

- Require players to be very certain of opponent's beliefs and strategies.
- To not bluff with certainty as type ϑ_B when $\mu_0 > \mu_{\text{ind}}$, Lebedev must know Juanda will fold with certainty because Juanda knows Lebedev checks the ace sufficiently frequently to not make a call profitable.

Undominated PBE:

- Are much more robust to misspecification of players beliefs or changes in higher-order knowledge about players' strategies.
- Does sequential equilibrium rule out the dominated PBE?

Bluffing on the River: Sequential Equilibria

Off-path beliefs:

- Off-path beliefs arise when an action is observed that no type is supposed to take under the conjectured strategy profile.
- Here, this happens if $\mu_0 \geq \mu_{\text{ind}}$ and $\sigma_L(\vartheta_B; B) = \sigma_L(\vartheta_A; B) = 1$.
- Response $\sigma_J(h_C; C) < 1$ is supported by $\mu(h_C) = 1$.

Approximation:

- Juanda must believe that type ϑ_A is infinitely more likely to deviate.
- Approximate σ_L by $\sigma_L^k(\vartheta_A; B) = 1 - \frac{1}{k}$ and $\sigma_L^k(\vartheta_B; B) = 1 - \frac{1}{k^2}$.
- The induced posterior after observing C is

$$\mu_k(h_C) = \frac{\frac{1}{k}\mu_0}{\frac{1}{k}\mu_0 + \frac{1}{k^2}(1-\mu_0)} = \frac{k\mu_0}{k\mu_0 + (1-\mu_0)} \xrightarrow{k\to\infty} 1.$$

Conclusion: All PBE in this example are also sequential equilibria.

Good Fold? Bad Fold?

What is the prior:

- It is a short-deck tournament, meaning there are no 2–5 in the deck.
- Juanda knows 7 out of 36 cards, 1 of the remaining 29 is the ace.
- There are five other players left in the tournament. If one of them had the ace, they would have stuck around, hence

$$\mu_0 = P(\theta_L = \vartheta_A \,|\, 7 \text{ cards}) = \frac{\binom{29}{9}}{\binom{29}{10}} = 50\%.$$

What have we observed:

- Juanda has folded, hence $\mu(h_B) \geq \mu_{\text{ind}} = \frac{p+b}{p+2b} = 84.3\%$.
- From $\mu(h_B) \geq \mu_{\text{ind}}$, we deduce that

$$\sigma_L(\vartheta_B; B) \leq \frac{\mu_0(1-\mu_{\text{ind}})}{\mu_{\text{ind}}(1-\mu_0)} = 18.6\%.$$

- Either Lebedev is known to be a very tight player or it was a bad fold.

Summary

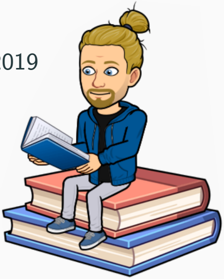
How to find PBE and SE:

1. Parametrize strategies efficiently ($=$ no strictly dominated strategies).
2. Write down expected utilities and Bayesian updating of beliefs.
3. Find a consistent set of parameters from the best-response functions:
   - Typically, we distinguish corner solutions (like $\alpha = \delta = 1$) and interior solutions (like $\alpha \in (0, 1)$) and verify consistency with the conditions imposed by Bayesian updating and the best response functions.
   - Signaling games: pooling and separating equilibria are corner solutions.
4. Specify required off-path beliefs if those occur in equilibrium.
5. For SE: verify which off-path beliefs are consistent.

Hint: Drawing a graph helps. If you realize you do not know an equilibrium for some prior μ_0, then you are missing equilibria.

Literature

- D. Fudenberg and J. Tirole: Game Theory, Chapter 8.2, MIT Press, 1991
- S. Tadelis: Game Theory: An Introduction, Chapter 16, Princeton University Press, 2013
- M. Acevedo: Modern Poker Theory, D&B Publishing, 2019
- I.-K. Cho and D.M. Kreps: Signaling Games and Stable Equilibria, Quarterly Journal of Economics, **102** (1987), 179–222



Signaling with a Continuum of Actions

Job-Market Signaling

Job application:

- Suppose a Worker's type is their productivity θ drawn from $\Theta \subseteq \mathbb{R}$.
- A Worker of type ϑ chooses an education level $e \geq 0$ at a cost $c(e, \vartheta)$.
- After observing e, the Firm offers a wage $w \geq 0$, leading to utilities

$$u_1(e, w, \vartheta) = w - c(e, \vartheta), \qquad u_2(e, w, \vartheta) = -(w - \vartheta)^2.$$

- Can high-type workers credibly distinguish themselves from lower-type workers by choosing a costly education?

Spence-Mirrlees Single-Crossing Property

Cost of education:

- We assume that education is costly on the margin and that it is less costly for more able workers, i.e.,

$$c_e(e,\vartheta) := \frac{\partial c(e,\vartheta)}{\partial e} > 0, \qquad c_{e\vartheta}(e,\vartheta) := \frac{\partial^2 c(e,\vartheta)}{\partial e\,\partial\vartheta} < 0.$$

- The latter condition is the Spence-Mirrlees single-crossing property, which ensures that iso-utility curves of different types intersect once.

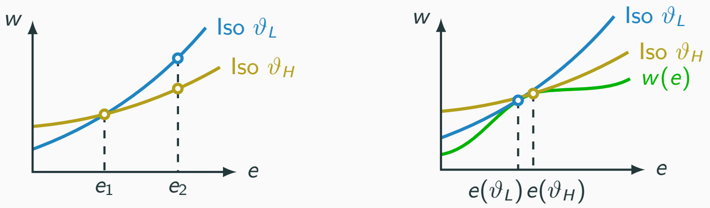
Iso-utility curves:

- Implicitly differentiating $u_1(e,w,\vartheta) = U$ yields

$$\left.\frac{\partial w}{\partial e}\right|_{u=U} = c_e(e,\vartheta) > 0, \qquad \left.\frac{\partial}{\partial\vartheta}\frac{\partial w}{\partial e}\right|_{u=U} = c_{e\vartheta}(e,\vartheta) < 0,$$

i.e., indifference curves in the (e,w)-space slope up.

Signaling Incentives



Single crossing:

- Suppose that indifference curves intersect at (e_1, w) for types ϑ_L, ϑ_H.
- Then iso-utility points (e_2, w_L) and (e_2, w_H) must satisfy

$$w_H - w_L = \int_{e_1}^{e_2} \int_{\vartheta_L}^{\vartheta_H} c_{e\vartheta}(x, y)\, \mathrm{d}y\, \mathrm{d}x < 0.$$

Signaling incentives:

- Since their indifference curves are flatter, higher-type workers choose a (weakly) higher education level for any wage function $w(e)$.

Job-Market Signaling: Two Types

Job application:

- Suppose there are two type of workers $\vartheta_L < \vartheta_H$ and that it is commonly known that a fraction $\mu_0 \in [0, 1]$ of Workers are type ϑ_H.
- For specificity, suppose that the cost of education is $c(e, \vartheta) = \frac{e}{\vartheta}$.
- What are the perfect Bayesian equilibria?
  - With a continuum of actions, there are potentially very many PBE.
  - Let us restrict attention to pooling and separating equilibria.

Competitive Labor Market

Firm's best response:

- Firm maximizes its conditional expected utility, given $A_1 = e$,

$$\mathbb{E}_\sigma[u_2(a,\theta)\,|\,e] = -\mathbb{E}_\sigma\left[(w-\theta)^2\,\middle|\,e\right]$$
$$= -\mu(e;\vartheta_H)(w-\vartheta_H)^2 - \mu(e;\vartheta_L)(w-\vartheta_L)^2.$$

- Since $\mathbb{E}_\sigma[u_2(a,\theta)\,|\,e]$ is differentiable and strictly concave in w, the maximum is attained where the derivative is 0.
- The derivative with respect to w is

$$\frac{\partial \mathbb{E}_\sigma[u_2(a,\theta)\,|\,e]}{\partial w} = -2\mu(e;\vartheta_H)(w-\vartheta_H) - 2\mu(e;\vartheta_L)(w-\vartheta_L) \overset{!}{=} 0.$$

- We deduce $w(e) = \mu(e;\vartheta_H)\vartheta_H + \mu(e;\vartheta_L)\vartheta_L = \mathbb{E}_\sigma[\theta\,|\,e]$, i.e., this utility function corresponds to a competitive labor market.

Pooling Equilibria

On the equilibrium path:

- By definition of a pooling equilibrium, $\sigma_1(\vartheta_L) = \sigma_1(\vartheta_H) = e_*$.
- After observing e_*, posterior is μ_0, hence $w(e_*) = \mu_0\vartheta_H + (1-\mu_0)\vartheta_L$.

Off the equilibrium path:

- Easiest way to prevent deviations: assign pessimistic beliefs $\mu(e; \vartheta_L) = 1$ after observing $e \neq e_*$.
- This results in a wage of $w(e) = \vartheta_L$ being offered.

Deviations:

- Incentives to deviate are higher for the low type.
- Low type has no profitable deviation if $\vartheta_L \leq \mu_0\vartheta_H + (1-\mu_0)\vartheta_L - \frac{e_*}{\vartheta_L}$.
- Solving for e_* yields $e_* \leq \mu_0\vartheta_L(\vartheta_H - \vartheta_L)$.

Separating Equilibria

No mixed actions:

- Since the low type will be identified, he/she gets ϑ_L regardless of education level. It is thus optimal to choose $\sigma_1(\vartheta_L) = 0$.
- If the high type mixes among education levels, the lowest of those already fully reveals his/her type by definition.
- The high type must choose a pure action, that is, $\sigma_1(\vartheta_H) = e_*$.

No deviations:

- High type does not want to deviate: $\vartheta_L \leq \vartheta_H - \frac{e_*}{\vartheta_H}$.
- Low types does not want to deviate: $\vartheta_L \geq \vartheta_H - \frac{e_*}{\vartheta_L}$.
- Together: $\vartheta_L(\vartheta_H - \vartheta_L) \leq e_* \leq \vartheta_H(\vartheta_H - \vartheta_L)$.

Pessimistic beliefs:

- Any such e_* can be supported given $\mu(\vartheta_L \mid e) = 1$ for $e \neq e_*$.

Least-Separating Equilibrium

Least-Separating equilibrium:

- It seems reasonable that type ϑ_H would select the cheapest education level $\underline{e} = \vartheta_L(\vartheta_H - \vartheta_L)$ that separate him/herself from type ϑ_L.
- However, if $e_* \neq \underline{e}$, then pessimistic off-path beliefs $\mu(\vartheta_L \mid e) = 1$ imply he/she will get the low wage for doing that.

Type ϑ_H could argue:

- It would not make sense for type ϑ_L to play $\underline{e}$ as he/she would pay for additional education and get the same wage.
- It can only make sense for type ϑ_H to choose $\underline{e}$.
- The pessimistic off-path beliefs $\mu(\vartheta_L \mid e) = 1$ are thus counter-intuitive.
- This is the idea behind the intuitive criterion, refining off-path beliefs.

Conditional Best-Response Correspondences

Definition 4.6

Player 2's conditional best-response correspondence to $a_1 \in \mathcal{A}_1$, given that play of a_1 convinces player 2 that player 1's type is in $\Theta' \subseteq \Theta$, is

$$\mathcal{B}_2(\Theta', a_1) = \bigcup_{\mu \in \Delta(\Theta')} \arg\max_{a_2 \in \mathcal{A}_2} \mathbb{E}_\mu[u_2(a_1, a_2, \theta)].$$

Interpretation:

- $\arg\max_{a_2 \in \mathcal{A}_2} \mathbb{E}_\mu[u_2(a_1, a_2, \theta)]$ is player 2's best-response correspondence if player 2 had specific beliefs μ over Θ.
- We want to say if player 2 revised his/her beliefs to *any* beliefs over types in Θ', player 1 would benefit.
- Therefore, we take the union over all such beliefs $\mu \in \Delta(\Theta')$.

Intuitive Criterion

Definition 4.7

Suppose Θ is finite. A perfect Bayesian equilibrium σ *fails* the intuitive criterion if there exists $\vartheta \in \Theta$, a set $\Theta' \subseteq \Theta \setminus \{\vartheta\}$, and $a_1 \in \mathcal{A}_1$ such that:

1. Type ϑ strictly benefits from playing a_1 if doing so separates him/herself from all the types in Θ', that is,

$$\min_{a_2 \in \mathcal{B}_2(\Theta \setminus \Theta', a_1)} u_1(a_1, a_2, \vartheta) > \mathbb{E}_{\vartheta,\sigma}[u_1(A, \theta)].$$

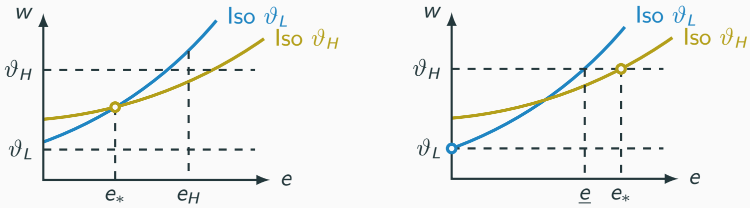
2. Any type $\vartheta' \in \Theta'$ strictly loses by playing a_1, that is,

$$\max_{a_2 \in \mathcal{B}_2(\Theta, a_1)} u_1(a_1, a_2, \vartheta') < \mathbb{E}_{\vartheta',\sigma}[u_1(A, \theta)].$$

Interpretation:

- The intuitive criterion refines off-path beliefs such that ϑ is separated from Θ' after play of a_1 without disadvantage to player 2 in equilibrium.

Unintuitive Equilibria



Pooling equilibria:

- For each $e_* \leq (1-\mu_0)\vartheta_L(\vartheta_H - \vartheta_L)$, there exists a pooling equilibrium, in which both types choose e_*.
- Since type ϑ_H's iso-utility curves are flatter than type ϑ_L's, there is an education level e_H that only type ϑ_H is willing to choose.

Separating equilibria:

- All but the least separating fail the intuitive criterion because type ϑ_H could benefit by deviating to $\underline{e}$ without distorting type ϑ_L's incentives.

Intuitive Job-Market Signaling

Equilibrium selection:

- A high-productivity Worker can, in equilibrium, distinguish themselves from a low-productivity Worker by choosing education level $\underline{e}$.
- The intuitive criterion uniquely selects the least separating equilibrium among all pooling and separating equilibria.
- This is a frequent phenomenon in these signaling games.
- For other applications, we have the uniform divinity refinement.

Summary

Signaling games:

- Players reveal information indirectly by taking costly actions.
- In the "standard-signaling model," there is an order of types and actions so that higher types can take higher actions more cheaply/easily.
- These single-crossing property allow types to reveal themselves.

Separating equilibria:

- Separating equilibria are typically a focal point of the analysis because they reveal the most information.
- This is efficient for the receiver, but costly for the sender.
- If doing so is too costly, only pooling equilibria remain.

Check Your Understanding

True or false:

1. The description of every pooling equilibrium requires the specification of off-path beliefs.
2. During no stage of a poker game is it optimal to play a fully separating strategy.
3. A completely mixed Nash equilibrium (i.e., players mix at every node) is a sequential equilibrium.

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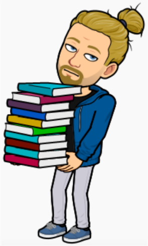
4. After analyzing pooling and separating equilibria, we can find the remaining PBE with the indifference principle.

Short-answer questions:

5. Given that an equilibrium requires players to correctly conjecture each other's strategies, is equilibrium analysis suitable to study poker?

Literature

- D. Fudenberg and J. Tirole: Game Theory, Chapter 8.2, MIT Press, 1991
- J. Levin: Dynamic Games with Incomplete Information, Lecture Notes, 2002
- S. Tadelis: Game Theory: An Introduction, Chapter 16, Princeton University Press, 2013
- M. Spence: Job Market Signaling, Quarterly Journal of Economics, **87** (1973), 355–374
- J.S. Banks and J. Sobel: Equilibrium Selection in Signaling Games, Econometrica, **55** (1987), 647–661
- I.-K. Cho and D.M. Kreps: Signaling Games and Stable Equilibria, Quarterly Journal of Economics, **102** (1987), 179–222



Cheap Talk

Job-Market Signaling

Separating equilibrium:

- High-type worker can successfully distinguish himself/herself from low-type workers because it is too costly for the low-type worker to pool.
- The education signal is credible because it is costly.

Costless signals:

- Suppose workers instead send a non-verifiable costless signal like "I am type H." Can meaningful information be conveyed in such a model?

Cheap-Talk Models

Setup:

- Player 1 has private information about his/her type $\theta \in \Theta$, which he/she can convey to Player 2 via a costless message $m \in \mathcal{M}$.
- Player 2 is initially uninformed with prior beliefs μ_0 on Θ.
- The payoffs of both players depend only on action a_2 taken by Player 2,

$$u_1(\vartheta, m, a_2) = u_1(\vartheta, a_2), \qquad u_2(\vartheta, m, a_2) = u_2(\vartheta, a_2).$$

- An important characteristic is that m is non-verifiable.

Examples:

- Any regular conversation between two individuals, asking somebody for a favor, persuading somebody without verifiable evidence, etc.
- A (possibly biased) expert providing advice to the government.

Cheap-Talk Job Market

	H	M	L
ϑ_H	$3, 1$	$2, 0$	$1, -2$
ϑ_L	$3, -2$	$2, 0$	$1, 1$

Cheap-talk job market:

- The Worker can send either the signal m_H "I am a high-ability worker" or the signal m_L "I am a low-ability worker."
- The Worker's payoff is independent of the signal and his/her type and it depends only on the wage $a_2 \in \{H, M, L\}$ chosen by the Firm.

Perfectly aligned preferences:

- Both types of workers prefer $H \succ M \succ L$.

Cheap-Talk Job-Market

Candidate separating equilibrium:

- If a separating equilibrium exists, we must have $\sigma_1(\vartheta_H) = m_1$ and $\sigma_1(\vartheta_L) = m_2$ for $m_1 \neq m_2$.
- Let us denote by $\mu(m)$ the Firm's posterior beliefs that $\theta = \vartheta_H$. Then,

$$\mu(m_1) = 1, \qquad \mu(m_2) = 0.$$

- A sequentially rational response is thus $\sigma_2(m_1) = H$ and $\sigma_2(m) = L$ for $m \neq m_1$, supported by off-path beliefs $\mu(m) = 0$ for $m \notin \{m_1, m_2\}$.

Incentives to deviate:

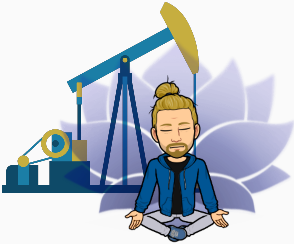
- The low type has an incentive to send signal m_1 instead since

$$u_1(m_1, \sigma_2(m_1), \vartheta_L) = 3 > 1 = u_1(m_2, \sigma_2(m_2), \vartheta_L).$$

- If incentives are perfectly aligned, there is no separating equilibrium.

Improving Company Image

	H	N
ϑ_O	$2, -2$	$0, 0$
ϑ_E	$-2, 2$	$0, 0$



Improving company image:

- In an attempt to improve company image, an Oil Company wants to hire an environmentally-friendly performer for a benefit gala.
- The Oil Company contacts a Performer and asks their type, which can be either ϑ_E "environmentally friendly" or ϑ_O "pro-oil ."

Misaligned preferences between players:

- The Performer accepts a job from an Oil Company only if they are ϑ_O.

Improving Company Image

Candidate separating equilibrium:

- Suppose σ with $\sigma_1(\vartheta_E) = m_1$ and $\sigma_2(\vartheta_O) = m_2$ for $m_1 \neq m_2$ is a PBE.
- Let us denote by $\mu(m)$ the Firm's posterior beliefs that $\theta = \vartheta_E$. Then,

$$\mu(m_1) = 1, \qquad \mu(m_2) = 0.$$

- In any sequentially rational response, $\sigma_2(m_1) = H$ and $\sigma_2(m_2) = N$.

Incentives to deviate:

- Both types have an incentive to change the message they send since

$$u_1(m_2, \sigma_2(m_2), \vartheta_E) = 2 > -2 = u_1(m_1, \sigma_2(m_1), \vartheta_E),$$
$$u_1(m_1, \sigma_2(m_1), \vartheta_O) = 2 > -2 = u_1(m_2, \sigma_2(m_2), \vartheta_O).$$

- If incentives between players are perfectly misaligned, there is no separating equilibrium because every type has an incentive to lie.

Coordination Game

	H	S
ϑ_H	5, 5	4, 4
ϑ_S	4, 4	5, 5

Coordination game:

- Suppose two friends want to meet up in New York. Each one of them could go see either Hamilton or Sleep No More.
- Player 1's type is his/her location and he/she can send a message, informing Player 2 of his/her location.
- It is clear that this game has a separating equilibrium, in which Player 1 truthfully reports his/her location.

Babbling Equilibrium

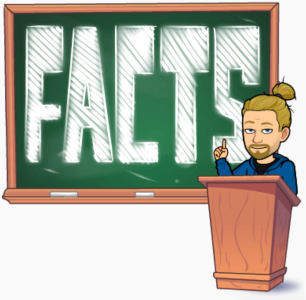
Definition 4.8

A babbling equilibrium is an perfect Bayesian equilibrium σ of a signaling game, in which $\sigma_1(\vartheta)$ is the uniform distribution over $\mathcal{M}$ and player 2 best responds to his/her prior beliefs.

Every cheap talk game has a babbling equilibrium:

- Since σ_1 has full support, there are no off-path messages.
- Given σ_1, player 2's posteriors coincide after any message.
- Since no deviation by player 1 affects player 2's posterior, it will not affect player 2's action. In particular, it is not profitable.

Soliciting Expert Opinions



Soliciting expert opinions:

- The Government solicits expert advice on an issue.
- The Government lacks the expertise to verify the Expert's message.
- While it may be costly to completely misrepresents his/her opinion, it may be costless to report the opinion with some personal bias.

Soliciting Expert Opinions

Setup:

- Suppose that the Government has a continuum of policies $\mathcal{A}_2 = [0, 1]$ available that it could implement.
- The Expert knows the correct policy, i.e., his/her type is $\theta \in \Theta = [0, 1]$.
- The Government's prior μ_0 is the uniform distribution over $[0, 1]$.
- The Government's and the Expert's utility functions are

$$u_1(a_2, \vartheta) = -(a_2 - \vartheta - c)^2, \qquad u_2(a_2, \vartheta) = -(a_2 - \vartheta)^2,$$

where $c > 0$ is the Expert's bias: he/she prefers a higher action.

Babbling equilibrium:

- The Expert recommends any policy with equal likelihood and the Government implements $a_2 = \frac{1}{2}$, which is sequentially rational, given μ_0.

Two-Message Equilibrium

Cutoff strategy:

- Suppose that a two-message equilibrium σ conveys information so that $\sigma_2(m_1) \neq \sigma_2(m_2)$. Without loss of generality $\sigma_2(m_1) < \sigma_2(m_2)$.
- The difference of payoffs for the Expert is

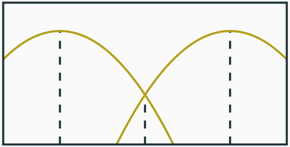
$$\Delta u_1(\vartheta) := (\sigma_2(m_1) - \vartheta - c)^2 - (\sigma_2(m_2) - \vartheta - c)^2.$$

- Since $\Delta u_1(\vartheta)$ is increasing, the Expert must use a cutoff strategy.
- Cutoff ϑ' is where the Expert is indifferent between $\sigma_2(m_1)$ and $\sigma_2(m_2)$.

Bayesian updating:

- After observing m_1, the Government has uniform beliefs on $[0, \vartheta')$.
- After observing m_2, the Government has uniform beliefs on $[\vartheta', 1]$.
- It follows that $\sigma_2(m_1) = \frac{\vartheta'}{2}$ and $\sigma_2(m_2) = \frac{\vartheta'+1}{2}$.

Two-Message Equilibrium



Solving for the cut-off:

- The indifference condition $u_1(\sigma_2(m_1), \vartheta') = u_1(\sigma_2(m_2), \vartheta')$ yields

$$\vartheta' + c - \frac{\vartheta}{2} = \frac{1 + \vartheta'}{2} - \vartheta' - c.$$

- This is equivalent to $\vartheta' = \frac{1}{2} - 2c$, which is positive if and only if $c < \frac{1}{4}$, that is, the Expert is not too biased.
- Note that the equilibrium is asymmetric. Since $\vartheta' < \frac{1}{2}$, message m_1 is more informative than message m_2.

Three-Message Equilibrium

Completely analogously:

- A three-message equilibrium exists if $c < \frac{1}{12}$ with cut-offs $\vartheta_1 = \frac{1}{3} - 4c$ and $\vartheta_2 = \frac{2}{3} - 4c$ such that the Government implements policy

$$\sigma_2(m_1) = \frac{\vartheta_1}{2}, \qquad \sigma_2(m_1) = \frac{\vartheta_1 + \vartheta_2}{2}, \qquad \sigma_2(m_1) = \frac{\vartheta_2 + 1}{2}.$$

- Types ϑ_i for $i = 1, 2$ are indifferent between sending m_i and m_{i+1}
- Observe that the bias must be even smaller to support this equilibrium.

Questions:

- Are all perfect Bayesian equilibria of this form?
- Under what conditions on the model are these the only PBE?

General Model

Setup:

- The informed player (sender)'s type θ is distributed on $\Theta = [0, 1]$ with density $f(\vartheta) > 0$ and sends a message m in $\mathcal{M} = [0, 1]$.
- Upon receiving the message, the uninformed player (receiver) takes an action $a(m) \in \mathcal{A} \in [0, 1]$.
- Utility functions $u_S(a, \vartheta)$ and $u_R(a, \vartheta)$ are both twice differentiable and strictly concave in a for each ϑ. Moreover, for $i = R, S$, we have

$$\frac{\partial^2 u_i(a, \vartheta)}{\partial a \, \partial \vartheta} > 0. \tag{1}$$

Interpretation:

- (1) implies that both sender and receiver prefer higher actions for higher types, that is, preferences are at least partially aligned.
- By concavity, both players have a unique preferred action for each ϑ.

Preferred Actions

Preferred actions:

- Let us denote by $a_S(\vartheta)$ and $a_R(\vartheta)$ the preferred actions of the sender and receiver, respectively, if the true state is ϑ.
- It will be convenient to denote by

$$a_R(\underline{\vartheta}, \bar{\vartheta}) = \max_{a \in \mathcal{A}} \int_{\underline{\vartheta}}^{\bar{\vartheta}} u_R(a, \vartheta) f(\vartheta) \, \mathrm{d}\vartheta$$

  the receiver's best response if he/she learns that $\theta \in [\underline{\vartheta}, \bar{\vartheta}]$.
- Since θ admits density f, a_R is continuous in both arguments.
- Moreover, because $\frac{\partial^2 u_R(a,\vartheta)}{\partial a \, \partial \vartheta} > 0$ and $f(\vartheta) > 0$, it follows that a_R is strictly increasing in both arguments.

Partition Equilibrium

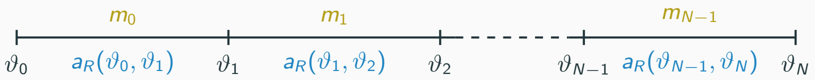
Lemma 4.9

Suppose $a_S(\vartheta) \neq a_R(\vartheta)$ for every $\vartheta \in \Theta$. Consider a partition of $\Theta = [0, 1]$ into intervals $[\vartheta_i, \vartheta_{i+1}]$ for $0 = \vartheta_0 < \vartheta_1 < \ldots < \vartheta_{N-1} < \vartheta_N = 1$. The strategy profile $\sigma = (\sigma_S, \sigma_R)$ with $\sigma_S(\vartheta) = m_i$ for $\vartheta \in [\vartheta_i, \vartheta_{i+1})$ and $\sigma_R(m_i) = a_R(\vartheta_i, \vartheta_{i+1})$ is a perfect Bayesian equilibrium if and only if

$$u_S(a_R(\vartheta_i, \vartheta_{i+1}), \vartheta_i) = u_S(a_R(\vartheta_{i-1}, \vartheta_i), \vartheta_i), \qquad \forall\ i = 1, \ldots, N-1. \quad (2)$$

Such an equilibrium is called a partition equilibrium of size N.

Interpretation: Similar types pool together by sending the same message.



Proof of Sufficiency

Sequential rationality:

- Since message m_i is sent only by types in $[\vartheta_i, \vartheta_{i+1})$, Bayesian updating implies that the receiver's beliefs admit conditional density

$$f(\vartheta \mid m_i) = \frac{f(\vartheta)1_{\{\vartheta \in [\vartheta_i, \vartheta_{i+1})\}}}{P(\theta \in [\vartheta_i, \vartheta_{i+1}))}.$$

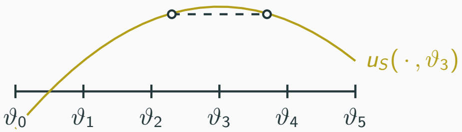
- Therefore, $\sigma(m_i) = a_R(\vartheta_i, \vartheta_{i+1})$ is sequentially rational.

Sender's incentives:

- Without loss of generality, we may assume $\mathcal{M} = \{m_0, \dots, m_{N-1}\}$.
- It is sufficient to show that $\vartheta \in [\vartheta_i, \vartheta_{i+1})$ satisfies

$$u_S(a_R(\vartheta_i, \vartheta_{i+1}), \vartheta) = \max_j u_S(a_R(\vartheta_j, \vartheta_{j+1}), \vartheta). \tag{3}$$

Proof of Sufficiency



Boundary types:

- Indifference for two consecutive m_i implies that u_R is maximized at m_i and m_{i+1} by convexity of $u_S(\cdot, \vartheta_i)$. In particular, (3) holds.

Interior types:

- Since $\frac{\partial^2 u_S(a,\vartheta)}{\partial a\, \partial \vartheta} > 0$, we obtain for any $0 \leq k \leq i$,

$$u_S(a_R(\vartheta_i, \vartheta_{i+1}), \vartheta) - u_S(a_R(\vartheta_k, \vartheta_{k+1}), \vartheta)$$
$$\geq u_S(a_R(\vartheta_i, \vartheta_{i+1}), \vartheta_i) - u_S(a_R(\vartheta_k, \vartheta_{k+1}), \vartheta_i) \geq 0.$$

- An analogous argument for $k \geq i$ shows that interior types satisfy (3).

Proof of Necessity

Necessity:

- Suppose that (2) is violated for some ϑ_i, that is,

$$u_S(a_R(\vartheta_i, \vartheta_{i+1}), \vartheta_i) > u_S(a_R(\vartheta_{i-1}, \vartheta_i), \vartheta_i).$$

- By continuity of u_S, there exists a type $\vartheta < \vartheta_i$ such that

$$u_S(a_R(\vartheta_i, \vartheta_{i+1}), \vartheta) > u_S(a_R(\vartheta_{i-1}, \vartheta_i), \vartheta).$$

- Type ϑ can thus benefit by sending signal m_i.

Partition Equilibria Are the Only PBE

Theorem 4.10

Suppose $a_S(\vartheta) \neq a_R(\vartheta)$ for every $\vartheta \in \Theta$. Then there exists N_* such that:

1. *For every $N \leq N_*$, there exists a partition equilibrium of size N.*
2. *All perfect Bayesian equilibria are realization equivalent to a partition equilibrium of size $N \leq N_*$.*

Remark:

- Perfect information transmission is impossible if communication is costless and non-verifiable.
- Expert opinions: the more bias the expert has, the smaller is N_*.
- Note the stark contrast to standard signaling games: even without refinement, there exist only finitely many PBE in cheap-talk games.

Proof of Theorem 4.10

Argument 1:

- Suppose that two actions $a_1 < a_2$ are played in a PBE.
- Thus, there are types ϑ_1, ϑ_2 such that ϑ_i weakly prefers a_i over a_{3-i}.
- By continuity, there exists $\vartheta' \in [\vartheta_1, \vartheta_2]$ such that $u_S(a_1, \vartheta') = u_S(a_2, \vartheta')$.
- Since $u_S(\,\cdot\,, \vartheta')$ is strictly convex, it follows that $a_1 < a_S(\vartheta') < a_2$.
- Moreover, because $\frac{\partial^2 u_S(a,\vartheta)}{\partial a\, \partial \vartheta} > 0$, we must have
  (i) a_1 is not induced by any type $\vartheta > \vartheta'$,
  (ii) a_2 is not induced by any type $\vartheta < \vartheta'$.
- Preferences over actions a_1 and a_2 invert at $\vartheta'(a_1, a_2)$.

Proof of Theorem 4.10

Finitely many actions in any PBE:

- Conditions (i) and (ii) imply via $\frac{\partial^2 u_R(a,\vartheta)}{\partial a\,\partial\vartheta} > 0$ that $a_1 \leq a_R(\vartheta') \leq a_2$.
- Since $a_S(\vartheta) \neq a_R(\vartheta)$ for any ϑ, continuity and compactness of Θ imply that there exists $\varepsilon > 0$ such that $\inf_{\vartheta\in\Theta}|a_S(\vartheta) - a_R(\vartheta)| \geq \varepsilon$.
- Together, these conditions imply that $a_2 - a_1 \geq |a_S(\vartheta') - a_R(\vartheta')| \geq \varepsilon$.
- Because $\frac{\partial^2 u_R(a,\vartheta)}{\partial a\,\partial\vartheta} > 0$, the least and the largest actions $\underline{a}$ and $\bar{a}$ in any perfect Bayesian equilibrium satisfy $a_R(0) \leq \underline{a} \leq \bar{a} \leq a_R(1)$.
- Because $a_R(1) - a_R(0) < \infty$, $a_2 - a_1 > \varepsilon$ implies that only finitely many actions are played in any perfect Bayesian equilibrium.

Proof of Theorem 4.10

Any PBE is a partition equilibrium:

- Order the actions $a_0 < a_1 < \ldots < a_{N-1} < a_N$ that are played in a perfect Bayesian equilibrium and apply Arugment 1 to $a_i < a_{i+1}$.

Existence of such an N_*:

- An upper bound on N_* is $(a_R(1) - a_R(0))/\varepsilon$.
- We will omit the proof that (2) has a solution for every $N \leq N_*$.
- See Crawford and Sobel (1982) and Kono and Kandori (2019).

Comment on the theorem's assumption:

- If $a_R(\vartheta) = a_S(\vartheta)$ for some ϑ, then there is no lower bound on $|a_2 - a_1|$.
- While partition equilibria still exist, they may no longer be finite.

Summary

Cheap-talk games:

- There is always a babbling equilibrium that conveys no information.
- Of more interest are separating equilibria, in which the informed player manages to get some information across despite non-verifiability.

Preference alignments:

- If preferences of types over actions are perfectly aligned, there is no separating equilibrium.
- If preferences between the two players are perfectly misaligned, there is no separating equilibrium.
- If preferences between players are partially aligned, we get partition equilibria, in which some noisy information is transmitted.
- Cheap talk works well in coordination games.

Check Your Understanding

True or false:

1. It is cheap talk if you discuss travel destinations with your friend and mention that Canada is too cold.
2. Conversation during a first date is cheap talk.
3. This class is cheap talk.
4. Academic reference letters are cheap talk.
5. A partition equilibrium of size 1 is a babbling equilibrium.
6. Without loss of generality, we can assume that the sender's message in a partition equilibrium is of the form "you should play action *a*."

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Literature

- J. Levin: Dynamic Games with Incomplete Information, Lecture Notes, 2002
- J. Sobel: Signaling Games, Lecture Notes, 2007
- V.P. Crawford and J. Sobel: Strategic Information Transmission, Econometrica, **50** (1982), 1431–1451
- H. Kono and M. Kandori: Corrigendum to Crawford and Sobel (1982) "Strategic Information Transmission," CIRJE Discussion Paper, 2019.