Algorithms Final Exam

June 20, 2019, 09:10 - 12:10

Please answer the following 7 questions on the answer sheets provided. Be sure to write your name and student ID on all answer sheets you use. You may bring one A4-sized, hand-written, double-sided "cheat sheet". No other books, notes, or calculators may be used during the exam.

If you want to use any result or theorem that has been taught in class (including homeworks), you may do so but you must state the result or theorem clearly before using it. You may assume that all arithmetic operations take $O(1)$ time.

Problem 1 (15%)

Given an undirected graph $G = (V, E)$. Three markers are initially placed at vertices s_1, s_2 and s_3 respectively. Each round, you may pick a pair of adjacent vertices u, v and either move all markers at vertex u to vertex v or move exactly one marker at vertex u to vertex v. Your goal is to move the three markers to vertices t_1, t_2 and t_3 respectively. Design a polynomial-time algorithm which finds the minimum number of rounds. Briefly justify the correctness and analyze the running time.

Problem 2 (15%)

There are n different airports $A_1, A_2, \ldots, A_n$. There are two types of flights from airport A_i to A_j: a regular flight which cost r_{ij} and a discounted flight with cost d_{ij}. When booking a trip with multiple flights, any two discounted flights must have at least one regular flight in between. Given all airports and all costs, design an algorithm to find the cheapest trip to fly from A_1 to A_n. You must design an efficient algorithm in order to get the full credit, but any polynomial time algorithm gives partial credit. Briefly justify the correctness and analyze the running time.

Problem 3 (15%)

n different houses are located along a road. The goal of this problem to build some fire stations to protect these houses. Each fire station must be built near one of the houses. Building a fire station near house i costs c_i and can cover houses $a_i, a_i+1, a_i+2, \ldots, b_i$. (You may assume that $a_i \leq i \leq b_i$.) Design an $O(n \log n)$-time algorithm to find the set of fire stations which covers all houses with minimum cost. Briefly justify the correctness and analyze the running time.

Problem 4 (15%)

For each of the following two statements, you must either prove it or find a counter example:

1. When all edge costs are distinct, the cheapest edge must be part of the minimum spanning tree.
2. When all edge costs are distinct, the most expensive edge must not be part of the minimum spanning tree.

For problems 5 and 6, you must either

1. design an algorithm with running time polynomial in n, briefly justify the correctness,

   or

2. prove that the problem is NP-complete.

You may use the fact that SAT, 3-SAT, Vertex Cover, Set Cover, Independent Set, Hamiltonian Path and Hamiltonian Cycle problems are all NP-complete. Also, you do not need to prove that problems 5, 6 are in NP.

For problems 5 and 6, a conference program committee has n members and need to review m papers. The i-th member of the committee can only review papers in a subset S_i due to conflict of interest. Given the list of all members, all papers and all conflict of interest S_i:

Problem 5 (15%)

Determine whether it is possible to select k members in the committee such that each paper has at least three selected members who can review it.

Problem 6 (15%)

Determine whether it is possible to assign each paper to exactly three committee members such that each committee member receives at least ten papers and there is no conflict of interest in the assignment.

Problem 7 (15%)

Given a directed graph $G = (V, E)$, the goal is to find the maximum subset of edges E' such that the graph $G' = (V, E')$ is acyclic (= no cycles). Design an $O(E + V)$-time 2-approximation algorithm. More specifically, you algorithm must find a set of edges E'' such that the graph $G'' = (V, E'')$ is acyclic and E'' has at least half the edges of the optimal answer E'. Briefly justify the correctness and analyze the running time. (hint: Label the vertices from 1 to n arbitrarily.)

Administrative issues:

1. The exam score and adjustment will be announced on 6/27(Thu) the latest.
2. You can check exam scores on 6/26(Wed) 10-12am in MD718.