(a)

$$\nabla_x x^T \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix} x = \begin{bmatrix} 10 & 2 \\ 2 & 10 \end{bmatrix} x$$

(b) $\nabla_x^2 f_0 = \begin{bmatrix} 10 & 2 \\ 2 & 10 \end{bmatrix}$

(c)

$$-\nabla_x f_0(x^{(0)}) = \begin{bmatrix} -34 \\ -26 \end{bmatrix}$$ 沒代 -1

(d)

$$\Delta x_{nt} = -\left(\nabla^2 f_0(x^{(0)})\right)^{-1} \nabla f_0(x^{(0)}) = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$ -1

2. 2/3 ~~Without~~

(a) $g = \begin{cases} p^* & a_i^T x = 0 \text{ where } p^* = \min\limits_x \left(\max\limits_{i=1,\dots,m} (a_i^T x + b_i) \right) \\ -\infty & \text{otherwise} \end{cases}$

(b)

$$g(\nu) = \inf_{x,y} \left\{ \max_{i=1,\dots,m} y_i + \nu^T (A^T x + b - y) \right\} \quad \text{where } A = [a_1\ a_2\ \cdots\ a_m]$$

$$\nabla_x g(\nu) = 0 \Rightarrow A\nu = 0$$

$$g(\nu) = \inf_y \left\{ \max_{i=1,\dots,m} y_i + \nu^T b - \nu^T y \right\}$$ 1/4

① If there exists one element of ν, $\nu_k < 0$, we can choose $y_k \to -\infty$ which makes $g(\nu) \to -\infty$. Therefore $\nu_k \geq 0$.

② $\max\limits_{i=1,\dots,m} y_i - \nu^T y \geq \max\limits_{i=1,\dots,m} y_i - \nu^T 1 y_i$. If $\nu^T 1 < 1$, we can choose $y = k1$ and $k \to -\infty$ which makes $g(\nu) \to -\infty$. If $\nu^T 1 > 1$, we choose $k \to \infty$ which also makes $g(\nu) \to -\infty$, So $\nu^T 1 = 1$

$$\Rightarrow g(\nu) = \begin{cases} \nu^T b & A^T \nu = 0,\ \nu^T 1 = 1,\ \nu \succeq 0 \\ -\infty & \text{otherwise} \end{cases}$$

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(c)

maximize $\nu^T b$ 2

subject to

$A^T \nu = 0$ 1

$\nu^T \mathbf{1} = 1$ 1

$\nu \succeq 0$ 1

(d) Let $t = \max_{i=1,\dots,m} y_i$

$$\Rightarrow \boxed{\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & \\ & A^T x + b \preceq t\mathbf{1} \end{array}}$$

where $A = [a_1 \dots a_m]$

(e) $g(\lambda) = \inf_{t,x} \left(t + \lambda^T(A^T x + b - t\mathbf{1})\right)$

$= \inf_{t} \left(t + \lambda^T b - \lambda^T \mathbf{1}\, t \right)$

$= \lambda^T b \quad , \quad \lambda^T \mathbf{1} - 1 = 0 \;;\; \lambda \succeq 0 \;;\; A\lambda = 0$

ans (e)

maximize $\lambda^T b$ 4

subject to

$A\lambda = 0$ 2

$\lambda^T \mathbf{1} - 1 = 0$ 2

$\lambda \succeq 0$ 2

(f) We substitute "t" in problem (d) with s.

$\Rightarrow$ minimize $ts - \sum_{i=1}^{m} \log(s - a_i^T x - b_i)$

(g) KKT conditions: ① $\nabla_x f_0 = 0 \quad \Rightarrow \sum_{i=1}^{m} \frac{a_i}{s - a_i^T x - b_i} = 0$

2 2

② $\nabla_s f_0 = 0 \quad \Rightarrow t - \sum_{i=1}^{m} \frac{1}{s - a_i^T x - b_i} = 0$

2 2

2

3.

(a)

$$\phi(x) = -\sum_{i=1}^{m} \log(b_i - a_i^T x)$$

$$(tf_0 + \phi)(x) = t\left(\frac{1}{2}x^T x + c^T x\right) - \sum_{i=1}^{m} \log(b_i - a_i^T x)$$

(b) $\Delta x_{nt} = -(\nabla_x^2 h)^{-1} \nabla_x h$ where $h(x) = (tf_0 + \phi)(x)$

3 3

$$\nabla_x h(x) = t(x + c) + \sum_{i=1}^{m} \frac{a_i}{b_i - a_i^T x}$$

$$\nabla_x^2 h(x) = tI + \sum_{i=1}^{m} \frac{a_i a_i^T}{(b_i - a_i^T x)^2}$$

3 3 3

$$\Rightarrow \Delta x_{nt} = -\left(tI + \sum_{i=1}^{m} \frac{a_i a_i^T}{b_i - a_i^T x} \right)^{-1} \left(t(x+c) + \sum \frac{a_i}{b_i - a_i^T x} \right)$$

$\nabla_x h(x)$ $\nabla_x^2 h(x)$ 3

4.

(a) Yes.

I. $\psi(x)$ is concave.

II. Sublevel set is closed

III. $\nabla^2 \psi$ exists

IV. $\mathrm{dom}(K) = R_{++}^3 = \mathrm{int}(K)$ 2

V. $\nabla^2 \psi = \begin{bmatrix} -x_1^{-2} & & \\ & -x_2^{-2} & \\ & & -x_3^{-2} \end{bmatrix} \prec 0$

VI. $\psi(sx) = \psi(s) + 6\log(s)$ with $\theta = 6 > 0$

(b) No. 2

$\mathrm{dom}\,\psi = \{x \mid x_1 + x_2 + x_3 > 0\} \neq \mathrm{int}\{K\}$

(c) No 2

$\nabla_x^2 \psi(x) = \begin{bmatrix} -x_1^{-2} & 0 \\ 0 & x_2^{-2} \end{bmatrix}$ is not negative-definite.