人

(a)

$$L(x, \nu) = \frac{1}{2} x^T P x + q^T x + r + \nu^T (Ax - b)$$

$$D_x L(x, \nu) = Px + q + A^T \nu = 0 \quad \Rightarrow \quad x = -P^{-1}(q + A^T \nu)$$

Dual function:

$$g(\nu) = -\frac{1}{2} \left\{ (q + A^T \nu)^T P^{-1} (q + A^T \nu) \right\} - \nu^T b + r$$

Dual problem:

$$\text{maximize} \quad -\frac{1}{2} \left\{ (q + A^T \nu)^T P^{-1} (q + A^T \nu) \right\} - \nu^T b + r.$$

KKT conditions

(1) $Ax^* - b = 0$

(2) $Px^* + q + A^T \nu^* = 0$

b) $L(x, \lambda) = c^T x + \lambda^T \left(\begin{bmatrix} A \\ -I \end{bmatrix} x - \begin{bmatrix} b \\ 0 \end{bmatrix} \right)$ $\quad D_x L(x, \lambda) = c + \begin{bmatrix} A^T & -I \end{bmatrix} \lambda = 0$

Dual function:

$$g(\lambda) = \begin{cases} -\lambda^T \begin{bmatrix} b \\ 0 \end{bmatrix}, & c + \begin{bmatrix} A^T & -I \end{bmatrix} \lambda = 0 \\ -\infty & \text{otherwise} \end{cases}$$

Dual problem:

$$\begin{aligned} \text{maximize} \quad & -\lambda^T \begin{bmatrix} b \\ 0 \end{bmatrix} \\ \text{subject to} \quad & \lambda \succeq 0 \\ & c + \begin{bmatrix} A^T & -I \end{bmatrix} \lambda = 0 \end{aligned}$$

KKT conditions:

(1) $\begin{bmatrix} A \\ -I \end{bmatrix} x^* \preceq \begin{bmatrix} b \\ 0 \end{bmatrix}$

(2) $\lambda^* \succeq 0$

(3) $\lambda^{*T} \left(\begin{bmatrix} A \\ -I \end{bmatrix} x^* - \begin{bmatrix} b \\ 0 \end{bmatrix} \right) = 0$

(4) $c + \begin{bmatrix} A^T & -I \end{bmatrix} \lambda^* = 0$

(c)

$$L(x,\nu) = x^T W x + \sum_{i=1}^{n} \nu_i (x_i^2 - 1)$$

$$= x^T W x + x^T \mathrm{diag}(\nu) x - \nu^T 1$$

$$D_x L(x,\nu) = 2(W + \mathrm{diag}(\nu))x = 0.$$

Dual function

$$L(\nu) = \begin{cases} -\nu^T 1, & W + \mathrm{diag}(\nu) \succeq 0 \\ -\infty, & \text{otherwise.} \end{cases}$$

Dual problem

maximize $-\nu^T 1$

subject to $W + \mathrm{diag}(\nu) \succeq 0$

KKT conditions

(1) $x_i^{*2} = 1$

(2) $(W + \mathrm{diag}(\nu^*)) x^* = 0.$

2.

(a) $$\nabla f(x) = \begin{bmatrix} x_1 \\ \gamma x_2 \end{bmatrix}$$

$$\nabla^2 f(x) = \begin{bmatrix} 1 & 0 \\ 0 & \gamma \end{bmatrix}$$

(b) $$\mathrm{cond}(\nabla^2 f(x)) = \begin{cases} \gamma, & \gamma > 1 \\ \gamma^{-1}, & \text{otherwise} \end{cases}$$

Iteration k :

(c) $$x^{(k)} + t\Delta x^{(k)} = \begin{bmatrix} (1-t)x_1^{(k)} \\ (1-\gamma t)x_2^{(k)} \end{bmatrix} = \begin{bmatrix} (1-t)\gamma\left(\frac{\gamma-1}{\gamma+1}\right)^k \\ (1-\gamma t)\left(-\frac{\gamma-1}{\gamma+1}\right)^k \end{bmatrix}$$

Proof by induction.

$$f(x^{(k)} + t\Delta x^{(k)}) = \frac{1}{2}\left\{\gamma^2(1-t)^2 + \gamma(1-\gamma t)^2\right\}\left(\frac{\gamma-1}{\gamma+1}\right)^{2k}$$

$$D_t f = 0 \Rightarrow -2\gamma^2(1-t) - 2\gamma^2(1-\gamma t) = 0 \Rightarrow t = \frac{2}{\gamma+1}$$

Iteration k+1 :

$$x^{(k+1)} = x^{(k)} + t\,\Delta x^{(k)} = \begin{bmatrix} \left(\frac{\gamma-1}{\gamma+1}\right)\gamma\left(\frac{\gamma-1}{\gamma+1}\right)^k \\ \left(\frac{-\gamma+1}{\gamma+1}\right)\left(-\frac{\gamma-1}{\gamma+1}\right)^k \end{bmatrix} = \begin{bmatrix} \gamma\left(\frac{\gamma-1}{\gamma+1}\right)^{k+1} \\ \left(-\frac{\gamma-1}{\gamma+1}\right)^{k+1} \end{bmatrix}$$

記分欄 轉頁從此開始寫起。

(d) $\Delta x_{nt} = -\begin{bmatrix} 1 & 0 \\ 0 & \gamma \end{bmatrix}^{-1} \begin{bmatrix} x_1 \\ \gamma x_2 \end{bmatrix}$

$= -\frac{1}{\gamma} \begin{bmatrix} \gamma x_1 \\ \gamma x_2 \end{bmatrix}$

$= -\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(e)

$x + \Delta x_{nt} = 0 < 10^{-6} \Rightarrow$ 1 iteration.

3.

(a) F

(b) T

(c) F

(d) T

(e) F

(f) T