Quiz 1 answer

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Problem 1

(a)	T	T	F	F
(b)	T	F	F	F
(c)	T	T	T	T
(d)	T	T	F	F
(e)	F	F	F	F
(f)	T	F	F	F
(g)	T	F	F	T
(h)	F	F	F	T
(i)	T	F	F	F
(j)	T	F	F	F
(k)	T	F	F	F

Problem 2

(a)	T	T	F
(b)	T	T	F
(c)	F	T	T
(d)	T	T	F
(e)	F	F	F
(f)	F	T	F

Problem 3

(a)	F	F	F	F	F
(b)	T	F	F	T/F	T
(c)	T	F	T	T	T
(d)	T	F	F	T	T

Problem 4

(a) Let

$$M = \sup_{c \in \mathcal{E}} (c^T x)$$

Then the original problem can be written as

$$\text{minimize} \quad M$$
$$\text{subject to} \quad \begin{cases} Ax \preceq b \\ c_1^T x \leq M \\ \vdots \\ c_K^T x \leq M \end{cases}$$

where x is a slack variable. It is not difficult to see the form is a LP, if an extended matrix is defined. Namely, for block matrix

$$\begin{bmatrix} A \\ c_1^T \\ \vdots \\ c_K^T \end{bmatrix} x \preceq \begin{bmatrix} b \\ M \\ \vdots \\ M \end{bmatrix}$$

(b) Similarly, let M be defined the same way.

$$M = \sup_{c \in \mathcal{E}} (c^T x)$$

Notice that, if we let $u = c - c_0$, we arrive at constraint $\|u\|_2 \leq \gamma$,

$$\begin{aligned} M &= \sup_{c \in \mathcal{E}} (c^T x) \\ &= \sup_{c \in \mathcal{E}} (c_0 + u)^T x \\ &= c_0^T x + \sup_{\|u\|_2 \leq \gamma} u^T x \\ &= c_0^T x + \gamma \|x\|_2 \end{aligned}$$

Then the original problem can be written as

$$\text{minimize} \quad M$$
$$\text{subject to} \quad \begin{cases} \|x\|_2 \leq \dfrac{M}{\gamma} - \dfrac{c_0^T x}{\gamma} \\ \|0x + 0\|_2 \leq -A_{1,1} x_1 - \cdots - A_{1,n} x_n + b_i \\ \vdots \leq \vdots \\ \|0x + 0\|_2 \leq -A_{n,1} x_1 - \cdots - A_{n,n} x_n + b_i \end{cases}$$

where x is a slack variable. It is not difficult to see the form is a SOCP.