Homework Assignment #3

Due Time/Date

2:10PM Tuesday, October 6, 2020. Late submission will be penalized by 20% for each working day overdue.

Note

Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in Yih-Kuen Tsay's mail box on the first floor of Management College Building 2. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. (3.4) Below is a theorem from Manber's book:

   2019Q1 For all constants $c > 0$ and $a > 1$, and for all monotonically increasing functions $f(n)$, we have $(f(n))^c = o(a^{f(n)})$.

   Prove, by using the above theorem, that for all constants $a, b > 0$, $(\log_2 n)^a = o(n^b)$.

2. (3.5) For each of the following pairs of functions, say whether $f(n) = O(g(n))$ and/or $f(n) = \Omega(g(n))$. Justify your answers.

   2019Q2

	$f(n)$	$g(n)$
(a)	$(\log n)^{\log n}$	$\frac{n}{\log n}$
(b)	$n^3 2^n$	3^n

3. Suppose $f(n)$ is a strictly increasing function, i.e., if $n_1 < n_2$, then $f(n_1) < f(n_2)$, and $f(n) = O(g(n))$. Is it true that $\log f(n) = O(\log g(n))$? Please justify your answer. What about $2^{f(n)} = O(2^{g(n)})$? What if $f(n)$ is constant?

4. (3.12) Solve the following recurrence relation:

   2019Q3

   $$\begin{cases} T(1) = 1 \\ T(n) = n + \sum_{i=1}^{n-1} T(i), \quad n \geq 2 \end{cases}$$

5. (3.18) Consider the recurrence relation

   2019Q4 $T(n) = 2T(n/2) + 1$, $T(2) = 1$.

   We try to prove that $T(n) = O(n)$ (we limit our attention to powers of 2). We guess that $T(n) \leq cn$ for some (as yet unknown) c, and substitute cn in the expression. We have to show that $cn \geq 2c(n/2) + 1$. But this is clearly not true. Find the correct solution of this recurrence (you can assume that n is a power of 2), and explain why this attempt failed.