Probability-based Classification

Naïve Bayes Model

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Naïve Bayes Classification (1/15)

A very welcome classification method

Show you probabilities about classification predictions.



Naïve Bayes Classification (2/15)

Naïve Bayes classification is a probabilistic learning method.

In text classification, its goal is to find the "best" class for a document:

The class with the maximum a posteriori (MAP) probability:

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}} P(c \mid d)$$
 The probability of class c once we observe d
$$= \underset{c \in C}{\operatorname{argmax}} P(c)P(d \mid c)$$
 This is the classifier
$$\underset{c \in C}{\operatorname{argmax}} P(c)P(d \mid c)$$
 (classification model) of NB classification

• The distributions of P(c) and P(d|c) are model parameters that we learned (estimated) from training data.

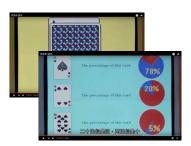
Naïve Bayes Classification (3/15)

Naïve Bayes classification is a way of **updating probabilities**.

- From P(c) to P(c|d)
- P(c) is called the prior probability of c.
 - You guess a class according to your prior knowledge.
- P(c|d) is called the posterior probability of c.
 - It updates (modifies) confidence that <u>c holds after we have seen d</u>.

Recall the card playing example:

- Before seeing the "tip", every card is possible (1/52).
- After seeing the "tip", the probability distribution is updated!!



Naïve Bayes Classification (4/15)

A further look at $\underset{c \in C}{\operatorname{argmax}} P(c) P(d \mid c)$

 $P(d | c) = P(t_1, t_2, ..., t_l | c)$

E.g., P('The', 'breakfast', 'is', 'terrible' | FOOD)

NB classification makes a conditional independence assumption:

Terms are independent of each other!!

$$P(d|c) = P(t_1, t_2, ..., t_l|c)$$

$$= P(t_1 | c) P(t_2 | c) ... P(t_l | c)$$

$$= \prod_{1 \le k \le l} P(t_k | c)$$

Without independence
assumption,
P(A B C) = P(A) * P(B|A) * P(C|A
B)
Very complex!!

E.g., P('The', 'breakfast', 'is', 'terrible' | FOOD)
 = P('The' | Food) P('breakfast' | Food) P('is' | Food) P('terrible' | Food)

Naïve Bayes Classification (5/15)

NB classification **ALSO** makes a **positional independence assumption**:

The probability of seeing a term is irrelevant to the term's position in a document!!

The two assumptions are too **naïve**....

- The probability of seeing 'Once' in the beginning of a sentence (document) is certainly higher than seeing it in other positions.
- The probabilities of seeing 'Hong' and 'Kong' may not be independent!!

We just make do with the assumptions. Without them, model parameters will be too many to implement the classification method.

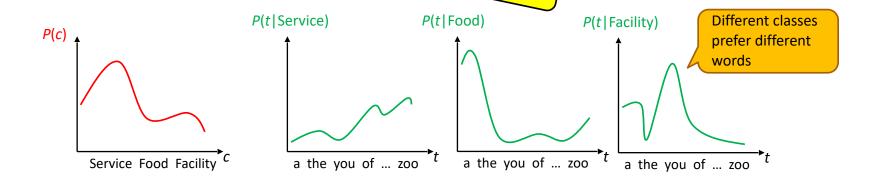
Naïve Bayes Classification (6/15)

The classification model now becomes:

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}} P(c) P(d \mid c) = \underset{c \in C}{\operatorname{argmax}} P(c) \prod_{1 \le k \le l} P(t_k \mid c)$$

To classify a document, we need to have the distributions of

P(c) and $P(t_k|c)$.



Naïve Bayes Classification (7/15)

The distributions are acquired from the training data *D*.

P(c) – the probability that a document belongs to category c (without seeing the document content).

$$P(c) = \frac{N_c}{N}$$
 The number of training documents in class c The number of training documents

Naïve Bayes Classification (8/15)

 $P(t_k|c)$ – the probability of seeing term t_k in a document about c.

$$P(tk \mid c) = \frac{T_{kc}}{\sum_{1 \leq m \leq M} T_{mc}} \frac{T_{mc}}{M_{-}} \text{ the number of occurrences of } t_k \text{ in all training documents about } c$$

Naïve Bayes Classification (9/15)

Example:

Training Phase

	review content	About "Food"?
Training data	i like the breakfast	Yes
	the breakfast is good	Yes
	that breakfast is terrible	Yes
	the location is perfect	No

- Vocabulary V = {i, like, the, breakfast, is, good, that, terrible, location, perfect},
 M = 10.
- P("Food") = 3/4 and P("not Food") = 1/4.
- P(breakfast | "Food") = 3/12, P(good | "Food") = 1/12, ...
- *P*(the | "not Food") = 1/4, ...

Naïve Bayes Classification (10/15)

Testing Phase

- New (testing) document: "good breakfast"
- The classifier (model):

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}} P(c) \prod_{1 \le k \le l} P(t_k \mid c)$$

- For class "Food": ¾ * 1/12 * 3/12 = 0.0156
- For class "not Food": ¼ * 0/4 * 0/4 = 0
- So... NB will label the document "Food" related.

Naïve Bayes Classification (11/15)

What about "perfect breakfast"?

• The classifier (model):

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}} P(c) \prod_{1 \le k \le l} P(t_k \mid c)$$

- For class "Food": $\frac{3}{4}$ * 0/12 * 3/12 = 0
- For class "not Food": ¼ * 1/4 * 0/4 = 0

Tie ... NB cannot classify the document 🕾

Worst of all ... the probabilities are all zero!!!

Naïve Bayes Classification (12/15)

 $P(t_k \mid c)$ is zero if a term t_k never occurs in the training documents about c.

• In the last example, it ruins the classification no matter how strong the evidence for the class "Food" from other terms.

The zero probability problem is unavoidable because of the text sparseness phenomenon (recall zipf's Law).

Don't give up ... here comes the solution – add-one smoothing.

Also called Laplace smoothing.

Naïve Bayes Classification (13/15)

The add-one smoothing:

$$P(tk \mid c) = \frac{T_{kc} + 1}{\sum_{1 \leq m \leq M} (T_{mc} + 1)} = \frac{T_{kc} + 1}{\sum_{1 \leq m \leq M} T_{mc} + M}$$
 Add a "pseudo" count (term occurrence) to every term in vocabulary

What about "perfect breakfast" now?

The classifier (model):

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}} P(c) \prod_{1 \le k \le l} P(t_k \mid c)$$



- For class "Food": $\frac{3}{4}$ * (0+1)/(12+10) * (3+1)/(12+10) = **0.006**
- For class "not Food": ¼ * (1+1)/(4+10) * (0+1)/(4+10) = 0.002

Naïve Bayes Classification (14/15)

Specifically, the classification model is the **multinomial** Naïve Bayes model:

$$\begin{aligned} c_{MAP} &= \underset{c \in C}{\operatorname{argmax}} P(c) \prod_{1 \leq k \leq l} P(t_k \, \Big| \, c) \\ &= \underset{c \in C}{\operatorname{argmax}} P(c) \prod_{1 \leq m \leq M} P(t_m \, \Big| \, c)^{tf_{m,d}} \end{aligned}$$



Multinomial distribution:

- Whenever an experiment is performed, one of the **disjoint** outcomes A_1 , A_2 , ..., A_M will occur.
- Let $P(A_i) = p_i$, $1 \le i \le M$, and $p_1 + p_2 + ... + p_M = 1$.
- If, in n independent performances of this experiment, Y_i , i = 1, 2, ..., M, denotes the number of times that A_i occurs.
- Then, $P(Y_1=y_1, Y_2=y_2, ..., Y_M=y_M)$, with $y_1+y_2+...+y_M=n$, is called multinomial distribution.

P(Y1=y1, Y2=y2, ..., YM=yM) =
$$\frac{n!}{y_1!y_2!...y_M!} \prod_{1 \le i \le M} p_i^{y_i}$$

Naïve Bayes Classification (15/15)

Because we are going to use SKLearn to build a NB model for text classification.

Before showing the code, the last equation implies <u>a term</u> <u>frequency vector</u> of each document!!

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}} P(c) \prod_{t_{c} \in \mathcal{N}} P(t_{m} | c)^{tf_{m,d}}$$

$$t_{1} \qquad t_{2} \qquad \dots \qquad t_{M-1} \qquad t_{M}$$

$$d = 0 \qquad 2 \qquad \dots \qquad 1 \qquad 0$$

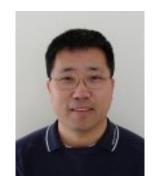
Coding Exercise (1/7)

SKLearn offers a set of APIs for the NB classification we just learned, and names it MultinomialNB.

Let's code it up!!

Data: Blog Author Gender Classification dataset

- Arjun Mukherjee and Bing Liu. "Improving Gender Classification of Blog Authors." Proceedings of Conference on Empirical Methods in Natural Language Processing (EMNLP-10).
- http://www.cs.uic.edu/~liub/FBS/blog-gender-dataset.rar



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Coding Exercise (2/7)

The format of the Excel file

- 3227 records. Each consists of text of a blog and a gender label.
 - Male: 1679 (52%), Female: 1548 (48%)
- I am a sucker for a Liberty print. I still have several items, including a paisley bag, which I bought from the store when Lused to live in London. So it's no surprise that Llike these pretty - in a old-fashioned yet contemporary 3219 I love the atmosphere the olympics are stirring up in my city. I love how I can get downtown after a 20 minute but F 3220 Monetary values change as consciousness changes. Prosperity Consciousness is Higher Consciousness becau M 3221And as the rain fell brilliantly against my window on a Saturday afternoon I noticed a pattern within the glitter o M 3222 That's the second book in Timothy Zahn's trilogy. I bet y'all thought that I'd forgotten about you concerning our fun M 3223 "weird"let's discuss what this word means to us. to me it's one of the most positive words i know. i think i generally F 3224 There are two types of fall guys; one who willingly accepts responsibility for something he didn't do, to cover for IM 3225 I like...flipping my blankets over to the cold side when I sleep, the smell of sun-dried laundry, back scratches, for F 3226 Alone for so long walking down the path of darkness with a lamp to guide my journey of life, the fates gust by the M 3227 It's been more than a month since I posted anything here. There is so much to do, plus my mood has been off. EF 3228 It was a scavenger style race with checkpoints throughout Boston and the surrounding neighborhoods. At each dM 3229 Finally! I got a full day's work done. Almost 4k, and if my hand hadn't started hurting I would have gotten even mcF 3230 At the height of laughter, the universe is flung into a kaleidoscope of new possibilities. ~Jean HoustonWhen peor M 3231 I like birds, especially woodpeckers and MOST especially the huge Campephilus woodpeckers. Four years ago M Oh friends, it's finally here! I thought the month between Christmas break and midwinter break wouldn't be too sld F

text Label: M/F

Coding Exercise (3/7)

Load data from the Excel file:

- texts: all text data
- labels: corresponding labels

Coding Exercise (4/7)

Convert the documents into frequency vectors

• Which are the input of the MultinomialNB classification.

```
In [7]: from sklearn.feature extraction.text import CountVectorizer
 In [8]: TF vectorizer = CountVectorizer()
In [9]: TF vectors = TF vectorizer.fit transform(texts)
In [10]: TF vectors.shape
Out[10]: (3227, 52456)
In [11]: x train = TF vectors[0:2500]
                                                Split the data into training/testing parts
        x test = TF vectors[2500:]
         y train = labels[0:2500]
         v test = labels[2500:]
                                                Training: The first 2500 instances
In [12]: x train.shape
                                                Testing: the remaining 727 instances
Out[12]: (2500, 52456)
In [13]: x test.shape
Out[13]: (727, 52456)
```

Coding Exercise (5/7)

Create a multinomial NB model

 The inputs of fit are the feature vectors and the corresponding labels

```
In [14]: from sklearn. naive_bayes import MultinomialNB
In [15]: model = MultinomialNB()
In [16]: model.fit(x_train,y_train)
Out[16]: MultinomialNB(alpha=1.0, class_prior=None, fit_prior=True)
```

alpha: the Laplace smoothing parameter

No given class priors, and learn prior probabilities

You can set the parameters when creating the model object

Coding Exercise (6/7)

Classify new data using predict

```
In [17]: predicted results = []
                                                                 Training ...
        expected results = []
                                                                 Testing ...
In [18]: expected results.extend(y test)
                                                             Everything done?
In [19]: predicted results.extend(model.predict(x test))
                                                             NO!! We need to
In [22]: print(predicted results)
                                                             know how good the
        model is
                                      'F', 'M', 'M', 'F',
                                        In [26]: model.predict proba(x test[2])
                                        Out[26]: array([[0.02662801, 0.97337199]])
```

Coding Exercise (7/7)

Sklearn also provides a lot of APIs for performance measurement.

```
In [20]: from sklearn import metrics
In [21]: print(metrics.classification report(expected results, predicted results))
                                     recall f1-score
                        precision
                                                         support
                     F
                                       0.83
                             0.66
                                                  0.74
                                                             370
                     М
                             0.76
                                       0.56
                                                  0.65
                                                             357
                                                  0.70
             accuracy
                                                             727
                                       0.70
                                                  0.69
                                                             727
            macro avg
                             0.71
         weighted avg
                                       0.70
                             0.71
                                                  0.69
                                                             727
```

Bernoulli NB Text Classification Model (1/7)

Remember the multi-hot vector???

• A vector element $x_{t,d}$ is 1 if term t is present in the document, otherwise, it is 0.

	'the'	ʻa'	'cake'	'tea'		'dog'	'pay'	'bad'
doc_1	1	1	0	0	•••	1	1	1
doc ₂	1	1	1	1		1	1	0

If you represent documents as multi-hot vectors, you are using **Bernoulli** NB model!!

Bernoulli NB Text Classification Model (2/7)

Testing – the classification model:

$$= \underset{c \in C}{\operatorname{argmax}} \frac{P(c|d)}{P(d|c)}$$

$$= \underset{c \in C}{\operatorname{argmax}} \frac{P(c)P(d|c)}{P(d)}$$

$$= \underset{c \in C}{\operatorname{argmax}} P(c) P(d \mid c)$$

$$= \underset{c \in C}{\operatorname{argmax}} P(c) \prod_{1 \leq m \leq M}$$

 $P(x_{m,d} | c)$

Terms **not occuring** in document *d* also involve in classification decision!!

Training:

$$P(c) = \frac{N_c}{N}$$

N: the number of training documents N_c: the number of training documents in c

$$P(x_{m,d} = 1 \mid c) = \frac{N_{c,m}}{N_c}$$

$$P(x_{m,d} = 0 | c) = 1 - P(x_{m,d} = 1 | c)$$

N_c: the number of training documents in c

 $N_{c,m}$: the number of training documents in c containing term m

Bernoulli NB Text Classification Model (3/7)

Example:

Training Phase

	review content	About "Food"?
Training data	i like the breakfast	Yes
	the breakfast is good	Yes
	that breakfast is terrible	Yes
	the location is perfect	No

- Vocabulary V = {i, like, the, breakfast, is, good, that, terrible, location, perfect},
 M = 10.
- P("Food") = 3/4 and P("not Food") = 1/4.
- $P(i \mid \text{``Food''}) = (1+1)/(3+2), P(like \mid \text{``Food''}) = (1+1)/(3+2), ...$
- $P(i \mid \text{"not Food"}) = (0+1)/(1+2), P(like \mid \text{"not Food"}) = (0+1)/(1+2), ...$

Bernoulli NB Text Classification Model (4/7)

Testing Phase

- New (testing) document: "good breakfast"
- The classifier (model):

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}} P(c) \prod_{1 \le m \le M} P(x_{m,d} \mid c)$$

- For class "Food": ¾ * (1-2/5) * (1-2/5) * ... = 0.00318
- For class "not Food": ¼ * (1-1/3) * (1-1/3) * ... = 6.774E-05
- The Bernoulli NB thus will label the document "Food" related.

Bernoulli NB Text Classification Model (5/7)

Sklearn Example:

```
In [1]: import xlrd
In [2]: workbook = xlrd.open_wor
In [3]: booksheet = workbook.she
In [4]: texts = []
labels = []
In [5]: for i in range(booksheet labels.append(booksheet texts.append(booksheet))
```

```
In [6]: from sklearn.feature extraction.text import CountVectorizer
 In [7]: binary vectorizer = CountVectorizer(binary=True)
 In [8]: binary vectors = binary vectorizer.fit transform(texts)
         print(binary vectors.toarray())
         [[0 0 0 ... 0 0 0]
          [0 0 0 ... 0 0 0]
          [0 0 0 ... 0 0 0]
          [0\ 0\ 0\ \dots\ 0\ 0\ 0]
          [0 1 0 ... 0 0 0]
          [0 0 0 ... 0 0 0]]
 In [9]: binary vectors.shape
 Out[9]: (3227, 52456)
In [10]: x train = binary vectors[0:2500]
         x test = binary vectors[2500:]
         y train = labels[0:2500]
         v test = labels[2500:]
In [11]: x train.shape
Out[11]: (2500, 52456)
In [12]: x test.shape
Out[12]: (727, 52456)
```

Bernoulli NB Text Classification Model (6/7)

```
In [13]: from sklearn. naive bayes import BernoulliNB!
In [14]: model = BernoulliNB()
In [15]: model.fit(x train,y train)
Out[15]: BernoulliNB(alpha=1.0, binarize=0.0, class prior=None, fit prior=True)
In [16]: predicted results = []
       expected results = []
In [17]: expected results.extend(y test)
In [18]: predicted results.extend(model.predict(x test))
In [19]: print(predicted results)
```

Bernoulli NB Text Classification Model (7/7)

```
In [20]: from sklearn import metrics
In [21]: print(metrics.classification report(expected results, predicted results))
                                     recall f1-score
                        precision
                                                         support
                                       0.89
                             0.59
                                                  0.71
                                                             370
                             0.76
                                       0.36
                                                  0.49
                                                             357
                                                  0.63
                                                             727
             accuracy
                                                  0.60
                                                             727
            macro avq
                             0.67
                                       0.62
         weighted avg
                             0.67
                                       0.63
                                                  0.60
                                                             727
```

The result is much worse than that of the multinomial NB model ... WHY???

- The Bernoulli model uses binary occurrence information.
- Ignore the number of occurrences.
- Typically make many mistake when classifying long documents.

Properties of NB classification

The assumptions of NB classification are so naïve ... but the classifiers usually <u>have good classification performance</u>.

Also, both training and testing of NB classification are efficient.

The **efficiency** and **effectiveness** of NB classification are reasons why it is a popular text classification method.

Usually as a baseline in text classification research.

K-fold Cross Validation

```
In [6]: from sklearn.feature extraction.text import CountVectorizer
In [1]: import xlrd
In [2]: workbook = xlrd.open workbook('./blog-gender-dataset.xlsx'
                                                        In [7]: TF vectorizer = CountVectorizer()
In [3]: booksheet = workbook.sheet by name('data')
                                                        In [8]: TF vectors = TF vectorizer.fit transform(texts)
In [4]: texts = []
                                                        In [9]: TF vectors.shape
      labels = []
                                                        Out[9]: (3227, 52456)
In [5]: for i in range(booksheet.nrows):
         labels.append(booksheet.cell(i,1).value)
                                                       In [10]: x train = TF vectors[0:2500]
         texts.append(booksheet.cell(i,0).value)
                                                               x test = TF vectors[2500:]
                                                               v train = labels[0:2500]
                                                               y test = labels[2500:]
     In [11]: from sklearn.model selection import cross_val_score
                  from sklearn. naive bayes import MultinomialNB
                  model = MultinomialNB()
    In [12]: scores = cross_val_score(model, x_train, y_train, cv=10, scoring='f1_macro')
    In [13]:
                  scores.mean()
    Out[13]: 0.6709620232259874
```

ROC Curve (1/2)

```
In [13]: from sklearn.naive bayes import MultinomialNB
In [14]: model = MultinomialNB()
In [15]: model.fit(x train,y train)
Out[15]: MultinomialNB(alpha=1.0, class prior=None, fit prior=True)
In [16]: y_probs = model.predict_proba(x_test)[:,0]
                          predict proba: returns the classification
                             probabilities for each testing instance
In [17]: from sklearn.metrics import roc curve
          from sklearn.metrics import auc
In [18]: fpr, tpr, = roc curve(y test, y probs, pos lab
In [19]: auc score = auc(fpr, tpr)
```

ROC Curve (2/2)

0.0

0.2

0.4

False Positive Rate

0.6

```
In [20]: import matplotlib.pyplot as plt
In [21]: plt.figure(1)
          plt.plot([0,1], [0,1], "k--")
          plt.plot(fpr,tpr, label='test (AUC=%0.2f)' % auc score)
          plt.xlabel("False Positive Rate")
          plt.ylabel("True Positive Rate")
          plt.title("ROC Curve")
          plt.legend(loc='best')
          plt.show
Out[21]: <function matplotlib.pyplot.show(*args, **kw)>
                                 ROC Curve
             1.0
                    test (AUC=0.75)
             0.8
           True Positive Rate
             0.6
             0.2
             0.0
```

1.0

0.8