

Operations Research, Spring 2020 (108-2)

Case Assignment 3

Group 16

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1 Compact Mathematical Formulation

Let I be the set of CSRs, S be the set of shifts, e_{jp} be lack amounts for p th period and j th day, D_{jp} be the demand for p th period and j th day, x_{ijk} be a binary variable to decide CSR i 's assignment, and w_{kp} be the shifting arrangement for p th period and k th shift. We use the sequence of CSRs to represent in the mathematical formulation instead of CSRs' id number. Also, we use the number on the data sheet to represent the date here. For example, if CSR i is assigned to shift k in March third, it denotes $x_{i,3,k}$. Note that

$$w_{kp} = \begin{cases} 1 & \text{if the CSR is on the shift} \\ 0 & \text{if the CSR is not on the shift} \end{cases} \quad (1)$$

$$x_{ijk} = \begin{cases} 1 & \text{if CSR } i \text{ is assigned to shift } k \text{ in day } j \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

The full formulation is in the next page.

$$\begin{aligned}
& \min \quad \sum_{j=1}^{31} \sum_{p=1}^{24} e_{jp} \\
& \text{s.t.} \quad e_{jp} \geq D_{jp} - \sum_{i=1}^{40} x_{ijk} \cdot w_{kp} \quad \forall p = 1, \dots, 24, \quad \forall j = 1, \dots, 31, \quad j, p \in \mathbb{Z}_+ \\
& \quad \sum_{k=0}^{13} x_{ijk} = 1 \quad \forall i = 1, \dots, 40, \quad \forall j = 1, \dots, 31, \quad i, j \in \mathbb{Z}_+ \\
& \quad \sum_{j=1}^{31} x_{ij0} = 8 \quad \forall i = 1, \dots, 40, \quad j \in \mathbb{Z}_+; \quad x_{27,10,5} = 1, x_{19,17,6} = 1, x_{12,27,13} = 1 \\
& \quad \sum_{j=1}^3 x_{2,j,0} = 3, x_{37,14,0} = 1, x_{18,20,0} = 1, x_{20,15,0} = 1, x_{1,3,0} = 1, \sum_{j=19}^{20} x_{19,j,0} = 2, x_{4,31,0} = 1 \\
& \quad \sum_{j=n}^{j+6} \sum_{k=11}^{13} x_{ijk} \leq 1 \quad \forall n = 1, \dots, 25, \quad \forall i = 1, \dots, 40, \quad n, i \in \mathbb{Z}_+ \\
& \quad \sum_{j=n}^{j+6} \sum_{k=7}^{10} x_{ijk} \leq 2 \quad \forall n = 1, \dots, 25, \quad \forall i = 1, \dots, 40, \quad n, i \in \mathbb{Z}_+ \\
& \quad \sum_{j=n}^{j+6} x_{ij0} \geq 1 \quad \forall n = 1, \dots, 25, \quad \forall i = 1, \dots, 40, \quad n, i \in \mathbb{Z}_+ \\
& \quad \sum_{k=11}^{13} x_{ijk} \geq 1 \quad \forall i \in \{3, 4, 5, 10, 13, 17, 19, 20, 22, 31\}, \quad \forall j \in \{1, 15, 29\} \\
& \quad \sum_{k=7}^{10} x_{i,10,k} \geq 1 \quad \forall i \in \{3, 4, 5, 10, 13, 17, 19, 20, 22, 31\} \\
& \quad \sum_{k=11}^{13} x_{i,22,k} \geq 1 \quad \forall i \in \{13, 17, 23\} \\
& \quad \sum_{k=11}^{13} x_{i'jk} \geq 0.45 \sum_{k=11}^{13} x_{ijk} \quad \forall j \in \{1, 8, 15, 22, 29\}, \quad i' \in I - \{34, 39\} \\
& \quad \sum_k x_{i'jk} \geq 0.55 \sum_k x_{ijk} \quad \forall j \in \{3, 10, 17, 24, 31\}, \quad i' \in I - \{21, 33, 34, 39\}, \quad k \in S - \{0, 6, 13\} \\
& \quad \sum_{k=11}^{13} x_{i'jk} \geq 0.3 \sum_{k=11}^{13} x_{ijk} \quad \forall j \in \{4, 11, 18, 25\}, \quad i' \in I - \{34, 39\} \\
& \quad \sum_{k=11}^{13} x_{i'jk} \geq 0.3 \sum_{k=11}^{13} x_{ijk} \quad \forall j \in \{4, 11, 18, 25\}, \quad i' \in I - \{21, 33, 34, 39\} \\
& \quad \sum_{k=11}^{13} x_{i'jk} \geq 0.3 \sum_{k=11}^{13} x_{ijk} \quad \forall j \in \{5, 12, 19, 26\}, \quad i' \in I - \{21, 33, 34, 39\} \\
& \quad x_{ijk} \geq 0, \quad e_{jp} \geq 0 \quad \forall i = 1, \dots, 40, \quad \forall j = 1, \dots, 31, \quad \forall k = 0, \dots, 13, \quad \forall p = 1, \dots, 24, \quad i, j, k, p \in \mathbb{Z}_+
\end{aligned}$$

2 Computer Program

The result of computer program for our Integer Programming is attached as below. We will submit code files as well.

CSRs/days	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1	2	11	3	2	10	0	1	2	11	1	0	1	3	2	0	11	0	0	1	2	10	4	11	0	0	0	2	8	8	12	4
2	2	3	11	0	1	1	1	2	0	11	0	4	1	1	1	0	11	10	0	1	1	4	8	13	0	2	4	1	0	0	11
3	2	4	3	2	0	11	8	0	2	8	0	0	11	0	7	2	8	0	3	11	4	9	2	0	4	9	12	2	9	0	1
4	11	0	4	10	7	3	1	13	0	1	3	10	0	1	13	9	2	0	4	7	1	11	9	0	0	0	4	1	11	3	0
5	11	0	7	9	1	2	1	12	0	10	0	3	2	4	12	2	0	0	0	2	1	13	9	0	7	2	1	0	11	1	8
6	10	2	0	4	10	11	1	0	3	5	0	9	11	0	2	0	4	4	1	12	0	7	4	0	2	7	13	3	1	3	0
7	2	8	8	13	0	0	1	4	8	4	12	1	0	7	4	0	0	13	1	0	10	0	9	3	13	1	0	1	3	4	2
8	1	11	10	2	4	0	10	2	11	7	0	1	4	10	0	11	2	0	4	4	9	3	11	2	0	8	0	0	0	13	1
9	0	11	2	10	1	7	1	0	11	4	8	1	0	3	2	11	4	0	7	2	4	0	11	10	0	0	8	1	0	12	2
10	8	0	4	12	3	2	0	10	4	0	10	1	11	2	7	2	0	1	7	13	1	0	4	0	0	1	2	13	2	0	10
11	2	10	8	1	0	1	13	2	10	1	9	0	0	13	1	0	10	4	1	10	11	1	0	8	0	1	0	13	2	0	4
12	2	11	0	1	0	7	9	1	11	4	0	0	2	10	0	11	2	4	8	4	7	0	11	1	0	8	0	4	10	12	4
13	1	0	1	13	0	4	4	4	3	0	13	1	3	0	2	0	2	11	3	2	1	8	0	2	12	3	8	0	1	0	4
14	4	0	4	13	10	1	8	0	4	4	2	13	0	8	0	1	0	1	13	0	2	8	0	4	4	13	10	0	8	1	4
15	13	8	0	2	0	4	10	13	0	0	0	3	2	0	13	4	2	1	4	2	0	13	1	8	4	3	0	1	11	8	2
16	10	3	12	0	4	7	1	4	2	11	0	8	3	4	4	3	11	0	1	8	0	10	0	11	0	0	8	0	3	9	11
17	12	2	0	9	4	1	0	11	0	2	2	4	2	0	13	0	7	8	4	4	4	13	0	2	2	7	3	0	9	0	13
18	2	8	0	2	11	7	1	2	10	0	4	2	11	0	10	0	8	0	4	11	0	8	1	0	8	1	13	4	0	2	2
19	10	0	0	2	12	0	8	2	7	7	3	11	0	4	1	10	10	0	13	2	3	2	10	0	0	13	10	0	2	2	8
20	2	0	7	12	0	7	4	2	2	7	11	0	10	0	2	3	8	12	2	8	0	2	0	4	13	1	0	10	10	0	4
21	7	13	2	8	2	0	1	0	13	10	0	1	1	0	7	13	10	3	3	0	2	2	13	9	1	0	0	0	8	13	10
22	3	0	11	7	0	7	1	1	0	13	0	8	8	0	2	2	12	0	8	9	0	2	1	13	1	8	9	0	2	4	13
23	2	12	1	9	0	7	3	2	13	2	8	0	0	3	10	11	2	8	0	4	4	10	11	0	8	1	0	0	2	0	12
24	2	0	4	8	11	7	1	2	0	2	10	11	1	8	0	4	0	9	13	2	8	2	0	0	0	13	4	0	1	10	2
25	11	10	0	2	2	1	7	11	3	0	8	3	0	10	12	4	4	8	0	2	4	13	10	0	10	0	0	13	8	1	
26	7	0	4	13	7	3	4	2	0	0	11	7	0	2	4	10	0	13	7	2	0	2	7	4	13	7	0	0	2	8	2
27	9	11	4	2	0	3	8	2	11	1	0	1	0	2	2	13	9	0	4	4	0	10	13	0	9	0	1	2	7	13	0
28	11	0	1	7	2	7	1	13	0	0	7	1	0	0	11	4	8	7	4	3	0	13	0	7	9	0	1	2	11	4	2
29	2	0	13	9	3	7	1	2	0	13	0	1	0	7	9	0	13	0	2	2	3	2	0	13	1	0	8	2	8	2	13
30	2	7	0	2	13	7	1	1	8	0	0	0	13	0	10	3	7	2	4	13	0	2	3	0	1	10	13	10	3	4	0
31	11	0	9	1	0	7	3	11	1	3	0	1	0	0	12	1	8	1	10	2	0	13	1	4	3	10	0	8	12	0	2
32	2	11	10	0	3	4	7	2	12	8	0	1	1	9	2	11	8	0	4	2	2	2	12	0	8	0	0	0	10	13	0
33	11	10	0	2	0	4	9	11	10	0	4	1	1	2	11	10	0	0	4	2	8	13	0	2	1	0	0	10	11	7	2
34	0	0	0	11	1	4	3	3	0	0	1	13	2	3	4	4	0	4	12	2	7	3	0	2	1	11	10	1	0	1	8
35	10	2	11	0	1	3	4	10	0	13	1	0	10	1	0	7	11	0	4	10	1	3	8	11	0	4	8	0	0	4	11
36	0	2	10	0	13	3	1	2	1	10	0	11	2	8	2	2	0	1	7	1	13	3	2	0	2	0	9	0	12	1	0
37	0	2	2	8	9	13	1	0	1	0	2	0	11	1	0	8	2	0	0	12	7	4	1	9	2	0	1	13	7	3	4
38	8	4	10	0	13	4	2	0	2	0	4	2	0	12	3	1	6	2	0	0	13	2	2	7	1	4	0	11	0	8	10
39	8	0	2	11	2	7	1	8	0	1	13	0	8	2	0	3	2	13	0	4	3	7	0	7	13	0	0	4	7	4	3
40	0	1	12	7	1	10	1	0	0	13	8	2	7	1	1	0	11	10	0	8	1	4	1	11	3	0	0	7	0	4	11

Figure 1: The result of AMPL

3 Summarization

As you can see in the results of computer programming, in order to satisfy the days-off constraint, we have no choice but to make people rest instead of working to reduce lack amounts. We observe that since CSRs only work for a few hour in the 5th and 6th shift, there are seldom allocation to these two shifts. Also, since there are more working

hours in the Afternoon and the Night categories, we mainly assign the shifts in these two categories. Senior limit actually has minus effect on scheduling since there are a few CSRs with the experience lower than 2 years.

In the end, the total lack amount returned by our computer program is 222 people.

4 Thoughts for the Case Assignment

1. Yu-Chieh Kuo

I found these three case assignments very interesting. These case assignments make us understand how to use Operation Research in the real world instead of only calculating on the textbook. After finishing these three assignments, I eventually know how the real world operates and how to deal with the real problem in math, which also reminds me the importance of math. After this class, I might go ahead to advanced optimization classes to realize how to solve the more difficult problems. One criticism is that the case assignment is a little bit difficult for OR amateurs, especially the demand to fill up the whole scheduling sheet. I think it will be better without that hard demand, but it doesn't change the fact that these assignments are all good training and testing for students.

2. Yu-Hsuan Chang

These three assignments are all good challenges for us, especially the first one since we didn't understand anything about operation research, we had to solve the problem by our tuition and experience. But after these three assignments, I understand how to utilize the operation research in the real world case. Writing computer program is my job in our team, although I want to invoke the gurobi to deal with the problems, I have no much time to learn it due to the numerous homework and exams this semester. One opinion is that I would like to have assignments for different topics instead of focusing on the single problem.

3. Oliver Dawson

I found the case assignments interesting mainly for the iterative process of looking at the same problem. Seeing the scale issues and finding the methods to solve them step for step was intriguing. As a point of criticism, I would have liked to have case assignments focused on different OR problems, as we only concentrated on shift scheduling. I

do understand however that to see most concepts in depth requires a certain degree of dedication to one optimization problem, yet I still feel like it would have been nice to see things like an in depth factory planning problem or the like.

4. Benjamin Pfundstein

The first case assignment was very hard. We had to approach a problem without any pre-knowledge. Accordingly there was no method or knowledge, we could apply and thus the learning effect was rather low. In the following assignments we could more and more apply methods and knowledges we gained within the course. All in all I learned a lot through the case assignments. However, a more detailed overview of methods we are supposed to use would be helpful and might increase the learning effect of those assignments.

5. Chieh-Wen Han

I think the case assignments were useful for seeing how these problems look like in a real business. It can be very complicated but also rewarding. I think the tasks were a bit too difficult sometimes as the descriptions were often very long and not always easy to find all the relevant information. However, it is a good exercise for students to learn the most important concepts.