Programming Languages: Functional Programming
4. Introduction to Haskell: A Quick Note on Type Classes

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Parametric Polymorphism in Haskell

The Type of *take*

- Recall the definition:

$$\begin{aligned} take\ 0\ xs &= [\,] \\ take\ (1+n)\ [\,] &= [\,] \\ take\ (1+n)\ (x : xs) &= x : take\ n\ xs \quad . \end{aligned}$$

- The first argument has to be of a numeric type (e.g. Int), since we pattern matched it against 0 and $1+$.
- The second argument must be a list, since we patten matched it against $[\,]$ and $(:)$.
- But the element of the list is not examined at all. It is merely copied to the output.

The Type of *take*

- The type of *take* can be
  - $Int \to List\,Int \to List\,Int$;
  - $Int \to List\,Char \to List\,Char$, etc.
- There is a *most general* type: $Int \to List\,a \to List\,a$.
  - The small letter means that a is a type variable. One can imagine that there is an implicit $\forall$ that quantifies all type varaibles: $\forall a. Int \to List\,a \to List\,a$.

The Identity Function

- For a more obvious example, consider the (simple but important) identity function

$$id\ x = x \quad .$$

- The argument is not touched at all.
- It may have type $Int \to Int$, $Char \to Char$, or even $(Int \to Int) \to (Int \to Int)$.
- The most general type is $a \to a$.

Filter

- Recall *filter*:

$$\begin{array}{lll} \mathit{filter} & :: & (a \to \mathit{Bool}) \to \mathit{List}\,a \to \mathit{List}\,a \\ \mathit{filter}\ p\ [\,] & = & [\,] \\ \mathit{filter}\ p\ (x : xs) & \mid p\ x & = \ x : \mathit{filter}\ p\ xs \\ & \mid \mathbf{otherwise} & = \ \mathit{filter}\ p\ xs \quad . \end{array}$$

- Still, in $\mathit{filter}\ p\ (x : xs)$ we merely passes x to p, without looking into x.
- Therefore *filter* works for any type a for which there exists functions of type $a \to \mathit{Bool}$ — which is true for all type a.

- For a counterexample, consider the following function:

$$
\begin{aligned}
&\mathit{lowers} && :: \mathit{List}\ \mathit{Char} \to \mathit{Int} \\
&\mathit{lowers}\ [\,] && = 0 \\
&\mathit{lowers}\ (x : \mathit{xs}) && = \textbf{if}\ \mathit{isLower}\ x \\
& && \quad\ \textbf{then}\ 1 + \mathit{lowers}\ \mathit{xs} \\
& && \quad\ \textbf{else}\ \mathit{lowers}\ \mathit{xs} \quad .
\end{aligned}
$$

- The function counts the number of lowercase characters in a string.
- It is equivalent to $\mathit{length} \cdot \mathit{filter}\ \mathit{isLower}$.
- x is passed to *isLower*, which forces x to be a *Char*.

Polymorphism

The term *polymorphism* comes in many forms, and refers to something slightly different in different programming languages. Here is a general definition:

Polymorphism: allowing a piece of code to have many types, such that it can be used in many occasions.

- Indeed, *take* can be applied to all types of lists. We do not need to define a separate version for $\mathit{List}\ \mathit{Int}$, $\mathit{List}\ (\mathit{Int} \to \mathit{Int})$.
- Meanwhile, $(+)$ can be applied to Int, Double...
- but these two cases of "polymorphism" are quite different in nature.

Parametric Polymorphism

- *Parametric* polymorphism, as we have seen just now, is common in many functional programming languages.
- When $take\ n :: List\,a \rightarrow List\,a$ is applied to an argument, say $[1, 2, 3]$, the type variable a is instantiated to the type of the argument (Int in this case).
  - The type variable a behaves like a parameter, thus the name.
  - Observe: the same piece of code (e.g. $take$, $filter$) works for all instantiations of a .
- Object-oriented languages often adopt another kind of polymorphism for operator overloading, called *ad-hoc* polymorphism, to be introduced later.

Type Classes

Membership Test

- Given the definition below, $elem\ x\ xs$ yields $True$ iff. x occurs in xs.

$$
\begin{aligned}
&elem\ x\ [\,] && = False\\
&elem\ x\ (y : xs) \mid x == y && = True\\
&\qquad\qquad\quad\ \mid \textbf{otherwise} && = elem\ x\ xs\ .
\end{aligned}
$$

- It could have type $Int \to List\,Int \to Bool$, $Char \to List\,Char \to Bool$, etc.
- We do not want to define $elem$ once for each type, thus we wish that it has a polymorphic type, say $a \to List\,a \to Bool$.
- However, not all values can be tested for equality! The operator $(==)$ is defined for some types, but not all types. For example, we cannot in general decide whether two functions are equal.
- Thus $elem$ cannot have type, for example, $(Int \to Int) \to List\,(Int \to Int) \to Bool$.

THE *Eq* CLASS

- There is such a definition in the Standard Prelude:

$$\begin{array}{l}\mathbf{class}\ Eq\ a\ \mathbf{where}\\ \quad (==) :: a \to a \to Bool\quad .\end{array}$$

- which says that a type a is in the *type class* Eq if there is an operator $(==)$, of type $a \to a \to Bool$, defined.
- Int is in Eq since we can define $(==)$ for numbers. So is $Char$, although $(==)$ for $Char$ implements a different algorithm from that of Int.

Type of *elem*

- The most general type of *elem* is
  $Eq\ a \Rightarrow a \to List\,a \to Bool$,
  - which means that *elem* takes a value of type a and a list of type $List\,a$ and returns a $Bool$, provided that a is in Eq.
- The additional constraint arises from the fact that *elem* calls $(==)$.

- To use *elem* on concrete types, we have to teach Haskell how to check equality for each type. The following are defined somewhere in the Haskell Prelude:

$$\textbf{instance}\ Eq\ Int\ \textbf{where}$$
$$m == n = \ \ \{\text{- how to check equality for } Int \text{ -}\}$$

$$\textbf{instance}\ Eq\ Char\ \textbf{where}$$
$$m == n = \ \ \{\text{- how to check equality for } Char \text{ -}\}$$

- It is not possible to give a definition for, for example $Eq\ (a \to a)$. Thus *elem* cannot be applied to such types.

Instance Declaration

- When we define a new type, we might want to teach Haskell how to check equality:

$$\textbf{data}\ \mathit{Color}\ =\ \mathit{Red} \mid \mathit{Green} \mid \mathit{Blue} \ldots$$

-

$$\begin{aligned}&\textbf{instance}\ \mathit{Eq}\ \mathit{Color}\ \textbf{where}\\ &\quad \mathit{Red} \mathrel{==} \mathit{Red} \quad\ \ = \mathit{True}\\ &\quad \mathit{Red} \mathrel{==} \mathit{Green} = \mathit{False}\\ &\ldots\end{aligned}$$

Summary So Far...

- Class declaration:

  $$\textbf{class } Eq\ a \textbf{ where}$$
  $$\quad (==) :: a \to a \to Bool\ .$$

  - The *method* $(==)$ then has type $Eq\ a \Rightarrow a \to a \to Bool$.
- Instance declaration:

  $$\textbf{instance } Eq\ MyType \textbf{ where}$$
  $$\quad x == y = \ldots$$

  - $(==)$ above should have type $MyType \to MyType \to Bool$, but the type is not written.
- A function that calls a function with class constraint $Eq\ a$ (e.g. $(==)$) also has the constraint in its type:

  $$elem :: Eq\ a \Rightarrow a \to List\, a \to Bool$$
  $$elem = \ldots == \ldots$$
- $elem\ 2\ [1, 2, 3]$ is allowed because there is an instance declaration for $Eq\ Int$, while $elem\ id\ [id, (1+), (2+)]$ is not (unless you define and instance $Eq\ (Int \to Int)$).

- Note that $(==)$ for *Int* is a different program from that for *Char*.
- Type classes is thus a way to describe *operator loading* — using one name to refer to different piece of code.
- Such mechanisms are often called *ad-hoc* polymorphism.
- Compare with parametric polymorphism, where the same code, say, the same definition of *take*, works for all types.

Other Important Type Classes

- *Show*: things that can be printed (converted to string).
- *Read*: things that can be parsed from strings.
- *Num*: things that behave like numbers (with addition, multiplication, etc).
- *Integral*: things that behave like integers.
- *Monad*, *Functor*...hope we will be able to talk about them later!
- Use `:i` in `GHCi` to find out what methods and instances each class has!

Derived Instances

- The Haskell compiler may automatically construct some routine instance declarations, to save you some typing. E.g.

$$\textbf{data}\ \mathit{Colors} = \mathit{Red} \mid \mathit{Green} \mid \mathit{Blue}$$
$$\quad \textbf{deriving}\ (\mathit{Eq}, \mathit{Show}, \mathit{Read}) \quad .$$

- How do we check whether two lists are equal? We can do so if we know how to check whether their elements are equal.

$$
\begin{array}{l}
\textbf{instance}\ \mathit{Eq}\ a \Rightarrow \mathit{Eq}\ (\mathit{List}\ a)\ \textbf{where} \\
\quad [\,] == [\,] \qquad\qquad\qquad = \mathit{True} \\
\quad [\,] == (x : xs) \qquad\quad\;\; = \mathit{False} \\
\quad (x : xs) == [\,] \qquad\quad\;\; = \mathit{False} \\
\quad (x : xs) == (y : ys) = x == y \;\wedge\; xs == ys \;\; .
\end{array}
$$

- Note that in $x == y$, the $(==)$ refers to the method for type a, while the $(==)$ in $xs == ys$ is a recursive call.

Instance Inheritance

- Another example:

$$\textbf{instance } (Eq\ a, Eq\ b) \Rightarrow Eq\ (a, b) \textbf{ where}$$
$$(x_1, y_2) \mathrel{==} (x_2, y_2) \ = \ (x_1 \mathrel{==} x_2) \land (y_1 \mathrel{==} y_2) \ .$$

- All the three $(==)$ in the expression above refer to different methods!

The Type Class *Ord*

- Another type class *Ord* includes things are can be "ordered":

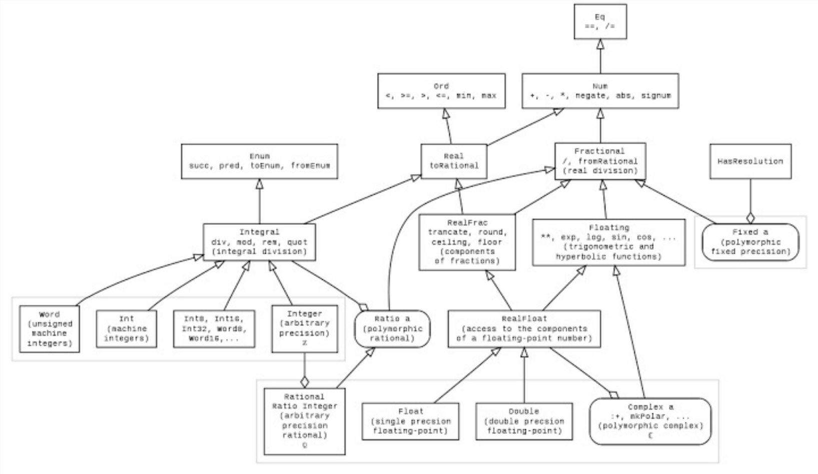
$$\begin{aligned}&\mathbf{class}\ Eq\ a \Rightarrow Ord\ a\ \mathbf{where}\\ &\quad (<) :: a \to a \to Bool\\ &\quad (\geq) :: a \to a \to Bool\\ &\quad (>) :: a \to a \to Bool\\ &\quad (\leq) :: a \to a \to Bool \quad \ldots\end{aligned}$$

- The declaration $Eq\ a \Rightarrow Ord\ a$ intends to mean that for a type a to be in class *Ord* it has to be in class *Eq*.
  - The methods $(<)$, $(\geq)$, etc, is allowed to use $(==)$.
  - Logically, it makes more sense to write $Eq\ a \Leftarrow Ord\ a$. But it's a historical mistake that has been made.
- The function *sort* that sorts a list might have type $Ord\ a \Rightarrow List\, a \to List\, a$.

Class Hierarchy

- Inheritance between *type classes* are not to be confused with inheritance between *types*.
- Through inheritance, type classes form a hierarchy.
- Types in the standard Haskell Prelude form a complex hierarchy.
- Other libraries may extend the existing hierarchy or build their own hierarchy.

Standard Haskell Numerical Classes



Notes

- The name “type class” is merely a mechanism for operator loading and shall not be confused with classes in object oriented languages.
- Type classes are an important feature of Haskell. Use of type classes has extended far beyond the inventors had imagined.
- We may use type classes in this course, but might not talk too much about their theoretical aspects, as they are orthogonal to the purpose of this course.