Homework 1

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Collaborators statement: I have a study group with the following memebers:**b06201057 Yu-Chi Hsieh, r08323002 Ze-Wei Chen, b06202004 Han-Wen Chang.** We discuss the problems together but we always do the problem by ourselves first. So, if I specify I have collaborators, then I means I discuss with these group members.
This homework answers the problem set sequentially.

1. (a) **Collaborators: None.**
   Let $c = 0.1,\ n_0 = \frac{5}{2},\ cn^2 \leq 0.5n^2 - n\ \forall n \geq n_0$ holds. Thus, we prove that $0.5n^2 - n \in \Omega(n^2)$.

   (b) **Collaborators: Study Group Members.**
   If $f(n) \in \Omega(n^3)$, then $\exists\ c, n_0 > 0\ s.t.\ cn^3 \leq f(n)\ \forall n \geq n_0$. Let $c', n'_0 > 0$, $f(n) \geq cn^3 \geq cn'_0 \cdot n^2 \geq c'n^2,\ \forall n \geq n'_0$ holds. Thus, we can say that $\exists\ c', n'_0 > 0,\ s.t.\ f(n) \geq cn^2,\ \forall n \geq n'_0\ i.e.\ f(n) \in \omega(n^2)$.

2. **Collaborators: None.**

$$f_1 = (2n)!,\ f_2 = n^n,\ f_3 = n!,\ f_4 = 2^{2n},\ f_5 = (\log_2 n)!,$$

$$f_6 = n^3 + 5n^2,\ f_7 = 8^{\log_2 n},\ f_8 = \sqrt{n} + 3,\ f_9 = n^{0.01},\ f_{10} = \log_2 n,\ f_{11} = \ln n$$

   f_6, f_7 and f_{10}, f_{11} are pairs such that $f(n) \in \Theta(g(n))$.

3. By the Master Theorem, we can specify parameters in these two recurrences.

   (a) **Collaborators: None.**
   Let $a = 9,\ b = 3,\ f(n) = n^3,\ n^{log_b a} = n^2$. Since $f(n) = n^3 \in \Omega(n^2 \cdot n^\epsilon),\ \epsilon > 0$, which belongs to the third case in the Master Theorem. As a result, we can say that $T(n) \in \Theta(n^3)$.

   (b) **Collaborators: None.**
   Let $a = 9,\ b = 3,\ f(n) = n^2 + 20n\log n + 3,\ n^{log_b a} = n^2$, and $f(n) = n^2 + 20n\log n + 3$. Given $c_1 = 1,\ c_2 = 10,\ n_0 = 1,\ c_1 n^2 \leq f(n) \leq c_2 n^2$ when $n > n_0$, therefore we have $f(n) \in \Theta(n^2 \cdot (\log n)^0)$, which belongs to the second case in the Master Theorem. As a result, we can say that $T(n) \in \Theta(f(n)\log n) = \Theta(n^2 \log n)$.

4. (a) **Collaborators: Study Group Members.**
   Let $f_i(n) = i \cdot n\ i.e.\ f_1(n) = n,\ f_2(n) = 2n, \cdots, f_n(n) = n^2$, then $g(k) = \sum_{j=1}^{k} f_j(j) = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}\ i.e.\ g(n) \in n^3$, which implies the statement is false. Therefore, we can disprove the statement.