Who's Who in the Network. Wanted: Key Player

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June 3, 2022

Motivation

- As a decision-maker or policymaker, we may want to find the most influential player in the network to break or strengthen such effect.
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Literature Review

Outline

Nash-Bonacich Equilibrium

Theorem 1

Let $\mu_1(\boldsymbol{G})$ be the largest eigenvalue of $\boldsymbol{G}$, the matrix $\beta[\boldsymbol{I} - \lambda^*\boldsymbol{G}]$ is well-defined and nonnegative if and only if $\beta > \lambda\mu_1(\boldsymbol{G})$, thus the unique interior Nash equilibrium is given by $\boldsymbol{x}^*(\boldsymbol{\Sigma}) = \frac{\alpha}{\beta+\gamma b(\mathcal{G},\lambda^*)} b(\mathcal{G}, \lambda^*)$.

- Given the unique Nash equilibrium $\boldsymbol{x}^*(\boldsymbol{\Sigma}) = \frac{\alpha}{\beta+\gamma b(\mathcal{G},\lambda^*)} b(\mathcal{G}, \lambda^*)$, we want to analyze how three different effects influence the equilibrium.
  - There exists no equilibrium if the matrix of cross-effects $\boldsymbol{\Sigma}$ reduces to $\lambda \boldsymbol{G}$.
  - There is a unique equilirium if $\boldsymbol{\Sigma}$ reduces to $-\beta \boldsymbol{I} - \gamma \boldsymbol{U}$.

Model

Proposition 1

Let $\mu_1(\boldsymbol{G})$ be the largest eigenvalue of $\boldsymbol{G}$, [1] *the matrix $\beta[\boldsymbol{I} - \lambda^*\boldsymbol{G}]$ is well-defined and nonnegative if and only if $\beta > \lambda\mu_1(\boldsymbol{G})$, thus the unique interior Nash equilibrium is given by $\boldsymbol{x}^*(\boldsymbol{\Sigma}) = \frac{\alpha}{\beta+\gamma b(\mathcal{G},\lambda^*)} b(\mathcal{G}, \lambda^*)$.*

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[1] $\mu_1(\boldsymbol{G})$ is well-define and larger than 0 since all eigenvalues of a symmetric matrix $\boldsymbol{G}$ are real, and the diagnal of $\boldsymbol{G}$ is zero.