# Kalman Filter Implementation Report

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## 1 Equations of Motion Used

The Equations of Motion Used Are:

1. $s_t = s_o + v_{t-1}T + \frac{1}{2}aT^2$
2. $v_t = v_{t-1} + aT$

## 2 State Vector

This is the state vector $\vec{X}$

$$\vec{X} = \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \\ a_z \end{bmatrix}$$

$a_z$ is always going to be 9.81, it will never change but i needed g in the equation so that i can write the $A$ matrix.

Transition state:

$$X_{t+1} = A \times X_t$$

There is no noise as we are not taking any sensor measurements, so I have not put the white noise

## 3 Equations for prediction step

The equations governing the model are:

$$x^{t+1} = x^t + v^{xt} \times T$$

$$y^{t+1} = y^t + v^{yt} \times T$$

$$z^{t+1} = z^t + v^{zt} \times T - \frac{1}{2} g T^2$$

$$v_x^{t+1} = v_x^t$$

$$v_y^{t+1} = v_y^t$$

$$v_z^{t+1} = v_x^t - gT$$

So The State Transition Equation Looks like this

$$\begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \\ a_z \end{bmatrix}_{t+1} = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & T & -\frac{1}{2}T^2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -T \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \\ a_z \end{bmatrix}_t$$

Which can be rewritten as:

$$X_{t+1} = A \times X_t$$

## 4 Equation for the update state

The assumption is that the camera will give us noisy $X$ , $Y$ and $Z$ coordinates. In real life we will have to adjust vibrations in the camera with it's inbuilt IMU, but for simplicity that is not taken into account

so for the equation

$$Y_t = CX_t + V_t$$

which is used in the update state,

$Y = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

So The Equation is,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \\ a_z \end{bmatrix} + V$$

The random vector $V$, during simulation is calculated so that it has a specefic covariance matrix and mean using the cholesky distribution methood.

The function calculating the sensor data in the simulation takes the projectile sate as input, adds a noise to it and takes the first 3 elements and returns them as a vector

The Covariance matrix of V is defined in the Kalman Filter Class