Theory SI Problem Solving

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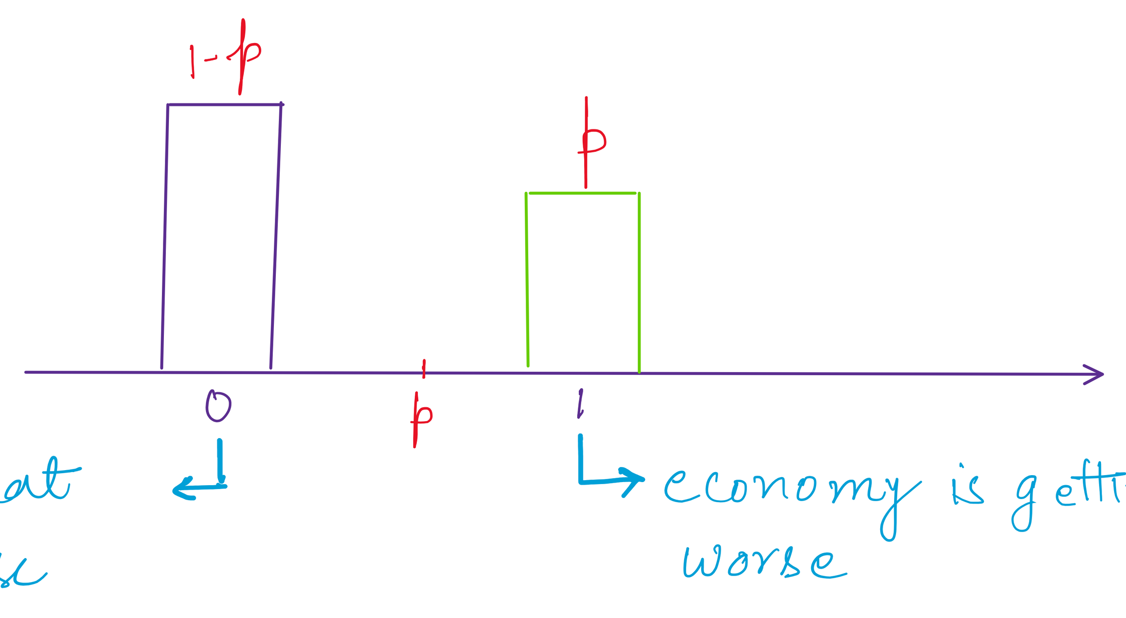
Since we have understood probability, random variable, central limit theorem, we can now solve the problem stated at the starting.

Problem: Assume that 40% population in India say that the economy is getting worse. If we take a sample of 250 people, how likely is that 50% or more of them will say they think the economy is getting worse. Calculate 99% confidence interval for proportion of the population who felt that the economy is getting worse.

Solution: We will not be able to survey the entire population to answer this question. But the entire population can be put in two buckets as follows



This is a binomial distribution.

Let we derive a sample of 250 people, in which 108 say the economy is getting worse.

We have 250 samples with in a sample

The sample mean $\bar{x} = \dfrac{1(108) + 0(142)}{250}$

$\bar{x} = \dfrac{108}{250} = 0.432$ (point estimate of the population who think that the economy is getting worse)

Sample variance $\sigma_1^2 = \dfrac{108(1-0.432)^2 + 142(0-0.432)^2}{250-1}$

$\sigma_1^2 = \dfrac{34.8434 + 26.5}{249} = \dfrac{61.344}{249} = 0.246$

$\sigma_1 = \sqrt{0.246} = 0.4963$

We want a 99% confidence interval.

Confidence interval:– It is a range of values that likely contains the true population parameter, with a specified level of confidence (like 95% or 99%)

Formula for a confidence interval (for population mean)

$$CI = \bar{x} \pm z\left(\frac{\sigma}{\sqrt{N}}\right)$$

where $\bar{x}$ = sample mean
z = z-score for the desired confidence level
σ = population standard deviation (or sample standard deviation if σ is unknown)
N = sample size

In our case, where $N = 1$ and σ is the standard deviation of the population.

Using CLT, we can say that $\bar{x}$ is comming from a Normal distribution with mean $\mu = \bar{x}$ and $\sigma_{\bar{x}} = \sigma/\sqrt{250}$

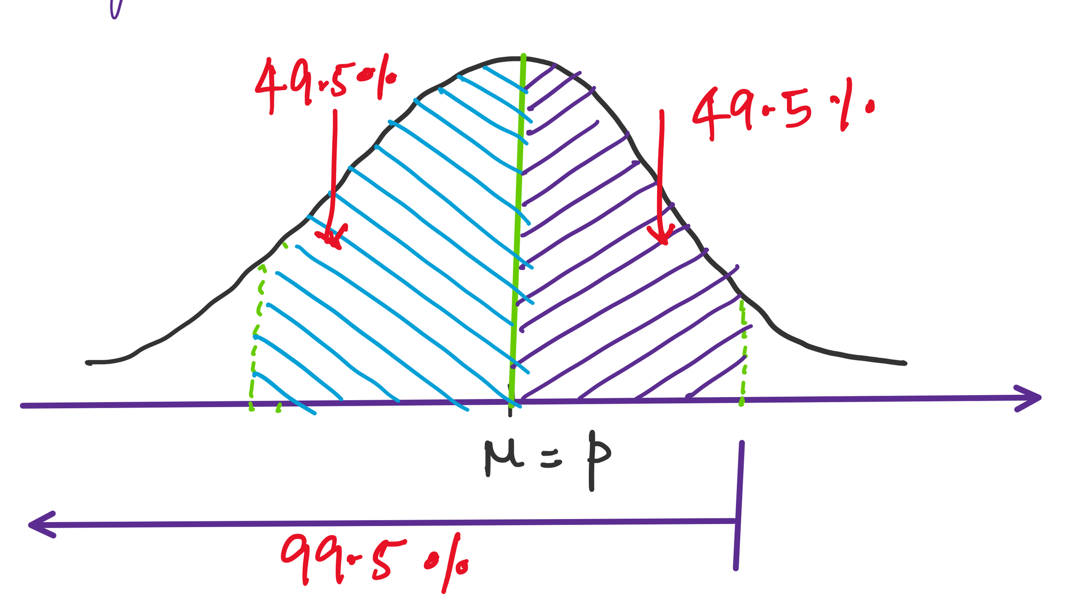
Recall that the variance of the sample mean

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} = \frac{\sigma^2}{250}$$

We do not know σ, so

$$\sigma_{\bar{x}} \approx \frac{\sigma_1}{\sqrt{250}} = \frac{0.4963}{\sqrt{250}} = 0.0314$$

For calculating the CI, we need to find z.



from the z-table, $0.5 + 0.495 = 0.995$ corresponds to $z = 2.58$. Thus, $2.58\,\sigma_{\bar{x}}$ away from the mean μ would give us the 99% confidence interval.

Thus the 99% CI $= \bar{x} \pm 2.58\,\sigma_{\bar{x}}$

$2.58\,\sigma_{\bar{x}} = 2.58 \times 0.0314 = 0.0810$

CI upper $0.432 + 0.08 = 0.513$
CI lower $0.432 - 0.08 = 0.352$

We are 99% confident that true population proportion is within the range 35.2% to 51.3%.
or
the true %age of the people who think that the economy is getting worse is in the range 35.2% to 51.3%. There is 99% chance of this.