Theory SI Probability

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Probability:- with any random phenomenon, the probability of a particular outcome is the proportion of times that outcome would occur in long run of observations.

Example: A weather forecaster might say that probability of rain today is 0.70. This means that in a large number of days with atmospheric condition like those today, the proportion of days in which rain occurs is 0.70.

Sometimes probabilities are expressed as percentage, such as the weather forecaster might say that the probability of rain is 70%.

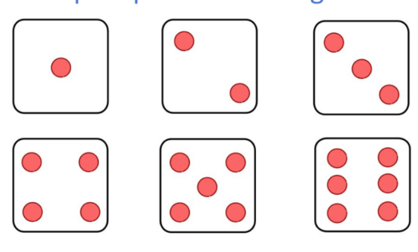
Probability Terminology:- Let us consider an example of rolling a die.

Sample space:- Sample space of an experiment consists of the set of all possible outcomes.

In the case of rolling a die, the sample space is

$S = \{1, 2, 3, 4, 5, 6\}$ — denoted by S.

Sample Space for Rolling a Die:



6 outcomes

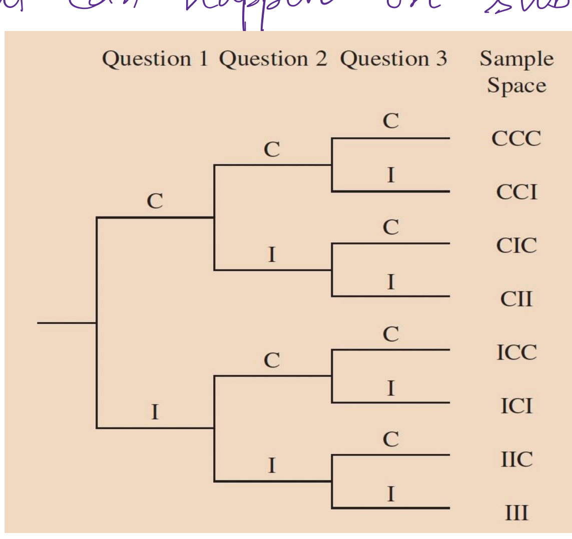
The integers 1 - - - 6 represent the number of dots on the six faces of the die.

Real-life example for sample space:- Let a statistics instructor gives a pop quiz with three multiple choice questions.

The student's answer is either correct (C) or incorrect (I). For example, if a student answered the first two questions correctly & the last question incorrectly, the student's outcome on the quiz can be symbolized by CCI.

What is the sample space of the pop quiz?

Solution:- One technique for listing the possible outcomes for finding the sample space is to draw a tree-diagram, with branches showing what can happen on subsequent trials.



From the diagram, a student's performance has eight possible outcomes. Thus, the sample space is

$\{CCC, CCI, CIC, CII, ICC, ICI, IIC, III\}$

Sample points:- The six possible outcomes of the rolling a die are the sample points of the experiment.

Events:- An event is a subset of 'S' and may consider any number of sample points.

For the pop quiz example, an event may be

A = student answer all three question correctly $= \{CCC\}$

B = student passes (at least two questions are correct)

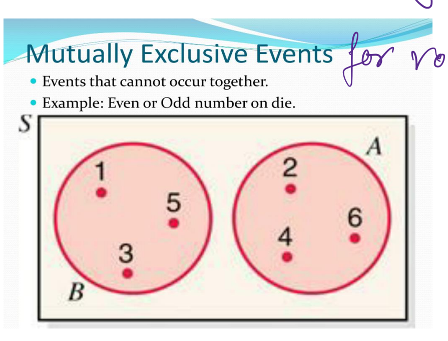
$= \{CCI, CIC, ICC, CCC\}$

Complement of an event:-

The compliment of an event A, denoted by $\bar{A}$, consists of all the sample points in 'S' that are not in A. If $A = \{2, 4\} \Rightarrow \bar{A} = \{1, 3, 5, 6\}$

Mutually Exclusive events:- Two events are said to be mutually exclusive if they have no sample points in common.

Example:



for rolling a die example.

The above two events are mutually exclusive or disjoint events.

Union of events:- The union (sum) of two events is an event that consists all the sample points in the two events. Let $A = \{1, 2\}$, $B = \{5, 6\}$

$$A \cup B = \{1, 2, 5, 6\}$$

The above implies $A \cup \bar{A} = S$ (Sample space)

Intersection of events:- Intersection two events is an event that consists of the points that are common to the two events.

If $A = \{2, 4\}$ and $B = \{1, 2, 3\} \Rightarrow A \cap B = \{2\}$

For mutually exclusive events (see fig)

$$A \cap B = \phi \text{ (Null event)}$$

Thus, $A \cap \bar{A} = \phi$

Probability rules:- Associated with each event A in S has probability of occurrence $P(A)$, which is defined as

$$\boxed{P(A) = \frac{\text{Number of points in } A}{\text{Number of points in } S}}$$

① The probability of an event A satisfies the condition

$$\boxed{P(A) \geq 0}$$

② The probability of the sample space 'S'

$$\boxed{P(S) = 1}$$

① and ② imply that $\boxed{0 \leq P(A) \leq 1}$

③ Let A_i for $i = 1, 2, \ldots n$ are mutually exclusive events in 'S', that is

$$A_i \cap A_j = \phi \quad , \quad i \neq j = 1, 2, \ldots n$$

Then, the probability of union of these mutually exclusive events satisfies the condition

$$\boxed{P\left(\bigcup_i A_i\right) = \sum_i P(A_i)}$$

Addition rule of Probability

If two events A and B are not disjoint or mutually exclusive

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example: Consider a family with two children. Let F and M denote child gender for female and male respectively

ⓐ find the sample space

ⓑ find the prob of the event A = first child is a girl

ⓒ find the prob of the event B = second child is a girl

ⓓ find the prob of the event $A \cup B$

Solution:- ⓐ Sample space $S = \{FF, FM, MF, MM\}$

ⓑ $A = \{FF, FM\} \Rightarrow P(A) = 2/4 = 0.5$

ⓒ $B = \{FF, MF\} \Rightarrow P(B) = 2/4 = 0.5$

ⓓ $A \cup B = \{FM, MF, FF\} \Rightarrow P(A \cup B) = 3/4 = 0.75$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$A \cap B = \{FF\} \Rightarrow P(A \cap B) = 1/4 = 0.25$

$P(A \cup B) = 0.5 + 0.5 - 0.25 = 0.75$