# APPLICATION OF ARTIFICIAL NEURAL NETWORK FOR PREDICTING THE DYNAMIC PERFORMANCE OF FREE PISTON STIRLING ENGINE

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#### INTRODUCTION

A multilayer feedforward neural network is built to predict the dynamic performance of a  $\beta$  type free piston stirling engine(FPSE).

Compared with the kinematic Stirling engines, FPSEs have outstanding potential advantages, such as high efficiency, few moving parts, long life because of no lubrication and non-contact operation, and the possibility of generating power in a wide range of source temperature, which make it more appropriate for special applications. Stirling engines can be classified into three major types namely  $\alpha$ ,  $\beta$  and  $\gamma$ .  $\beta$  type FPSE is more compact and has higher power density features when compared to other types.

The data for the artificial neural network model was taken from the journal "Application of artificial neural network for predicting the dynamic performance of a free piston Stirling engine" published by "Wenlian Ye", "Xiaojun Wang" and "Yingwen Liu" in 'www.elsevier.com'.

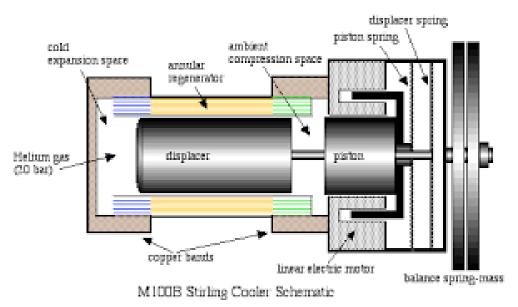


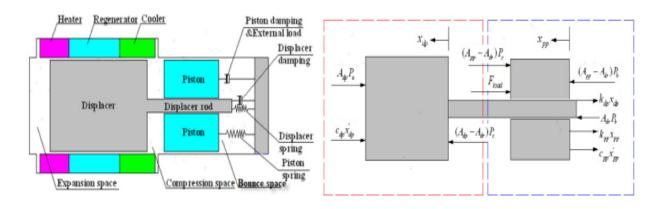
Fig: FPSE

In this study, we are taking six input parameters and three output parameters. The input parameters are spring stiffness values of diplacer and power piston, damping

coefficient of displacer and load, mass values of displacer and power piston. The output parameters are operating frequency, amplitude ratio and phase angle. 55 datasets were obtained from the above mentioned journal.

The results obtained from the ANN model was compared with the target values mentioned.

#### MATHEMATICAL MODEL



(a) Schematic of FPSE

(b) Free-body diagram of two pistons

The dynamic equations of displacer(DP) and power piston(PP) are expressed by Newton's second law.

$$m_{pp}x_{pp}^{"} + c_{pp}x_{pp}^{'} + k_{pp}x_{pp}^{} + F_{load}^{} = (A_{pp} - A_{dr})(p_c - p_b)$$

$$m_{\text{dp}} x^{"}_{\text{dp}} + c_{\text{dp}} x^{'}_{\text{dp}} + k_{\text{dp}} x_{\text{dp}} = -A_{\text{dp}} (p_{\text{e}} - p_{\text{c}}) - A_{\text{dr}} (p_{\text{c}} - p_{\text{b}})$$

 $m_{\mbox{\scriptsize dp}}$  - Mass value of displacer

 $m_{pp}$  - Mass value of power piston

x<sub>dp</sub> - displacement of displacer

 $x_{pp}$  - displacement of power piston

A<sub>dp</sub> - Area of displacer

 $A_{pp}$  - Area of power piston

 $p_{\text{e}}$  ,  $p_{\text{c}}$  and  $p_{\text{b}}$  are the pressures of working fluid inside the expansion, compression and bounded space.

F<sub>load</sub> - External load

Average pressure of Helium can be obtained by

$$p = m_g R / ((V_r / T_r) + (V_k / T_k) + (V_h / T_h) + (V_e / T_e) + (V_c / T_c))$$

Temperature in the regenerator is  $T_r = (T_h - T_k)/ln(T_h/T_k)$ 

 $T_h$  - Temperature of heater

T<sub>k</sub> - Temperature of cooler

The dynamic equations of FPSE are

$$x''_{pp} = D_{11}x'_{pp} + D_{12}x'_{pp} + K_{11}x_{pp} + K_{12}x_{dp}$$

$$\vec{x''_{dp}} = \vec{D_{21}} \vec{x_{pp}} + \vec{D_{22}} \vec{x_{dp}} + \vec{K_{21}} \vec{x_{pp}} + \vec{K_{22}} \vec{x_{dp}}$$

Operating frequency, 
$$f = \frac{1}{2\pi} \sqrt{\frac{D21K12 + D12K21 - D22K11 - D11K22}{D22 + D11}}$$

Phase angle, 
$$\alpha = arctan[\frac{\omega(D11 - D12(K11 + \omega^2))}{-(K12K11 + \omega^2(K12 + D12D11))}]$$

Piston amplitude ratio,  $\gamma = X_{dp}/X_{pp}$ 

$$=\frac{(\omega^{2}+K^{2}11)^{2}+\omega^{2}D^{2}11}{\sqrt{(K12(\omega^{2}+K11)+\omega^{2}D11D12)^{2}+\omega^{2}(D12(\omega^{2}+K11)-D11K12)^{2}}}$$

#### **METHODOLOGY**

Artificial Neural Network is used to train the network and predict the results of test data. Here sigmoid transfer function is used to get the output.

A total of 55 data set is used for the study. Among these, 45 are training data sets and 10 are testing data sets.

Each data set consists of 6 input parameters and 3 output parameters. The input parameters considered are masses of the displacer and power piston, the spring stiffness of displacer and power piston and the damping coefficients of load and displacer. The output parameters considered are operating frequency, phase angle and amplitude.

The obtained results are compared with the target output provided in the dataset. Mean square error is calculated for the training and test data set separately and analysis is done using the mean square error.

The final output of the neural network and error are stored to an output file. Plot is created between mean square error and number of iterations.

The optimal number of hidden neurons was found by trial and error method.

The test data set predicted output was in agreement with the output data from the data set.

Table 1 - Variation of MSE with learning rate and number of hidden neurons

LEARNING RATE	NUMBER OF HIDDEN NEURONS	MSE(TRAINING DATA SET)	MSE(TEST DATA SET)
0.1	5	0.058374	0.076240
0.1	8	0.062978	0.052583
0.1	10	0.056002	0.088070
0.1	15	0.324432	0.525123
0.2	5	0.061529	0.036163
0.2	8	0.062890	0.036664
0.2	10	0.061052	0.043050
0.2	15	0.056159	0.046507
0.3	5	0.053362	0.070224
0.3	8	0.053094	0.075521
0.3	10	0.058607	0.040084
0.3	15	0.048828	0.050204
0.4	5	0.051846	0.065715
0.4	8	0.054216	0.050938
0.4	10	0.034542	0.025272
0.4	15	0.038461	0.058398
0.5	5	0.057995	0.042604
0.5	8	0.041542	0.057966
0.5	10	0.041460	0.062749
0.5	15	0.038270	0.022668

5	0.048458	0.047923
8	0.038388	0.039328
10	0.040868	0.060027
15	0.042873	0.055961
5	0.047208	0.051707
8	0.024364	0.014204
10	0.016844	0.026167
15	0.025831	0.031492
5	0.038164	0.034772
8	0.034255	0.033737
10	0.045030	0.053987
15	0.028244	0.030953
5	0.034619	0.025680
8	0.018069	0.026829
10	0.023939	0.024836
15	0.031458	0.025863
	8   10   15   5   8   10   15   5   8   10   15   5   8   10   15   5   8   10	8       0.038388         10       0.040868         15       0.042873         5       0.047208         8       0.024364         10       0.016844         15       0.025831         5       0.038164         8       0.034255         10       0.045030         15       0.028244         5       0.034619         8       0.018069         10       0.023939

The least mean square error is obtained in the training data set for a learning rate of 0.7 and number of hidden neurons as 10. Here the total number of iterations are 10000.

Table 2 - Variation of MSE with momentum coefficient

MOMENTUM COEFFICIENT	MSE(TRAINING DATA SET)	MSE(TEST DATA SET)
0.1	0.038185	0.047886
0.2	0.031936	0.039648
0.3	0.024351	0.030201
0.4	0.031394	0.043333
0.5	0.026624	0.029700
0.6	0.021615	0.019112
0.7	0.044112	0.029966
0.8	0.027322	0.047839
0.9	0.031731	0.026584

The optimum momentum coefficient from the above data set is obtained as 0.6.

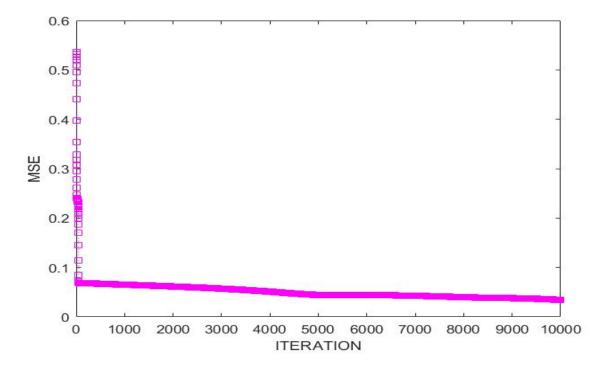


Fig: MSE vs Iteration

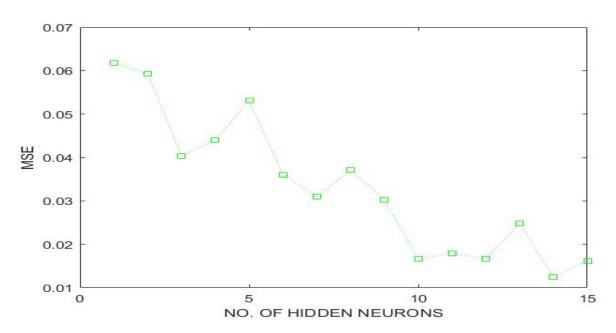


Fig: MSE vs No. of hidden neurons

#### **RESULT**

The mean square error for the training and test data set are obtained and the optimal number of hidden neurons, learning rate and momentum coefficient are found out.

Number of input parameters = 6

Number of output parameters = 3

Number of hidden neurons = 10

Number of training pattern = 45

Number of testing pattern = 10

Learning rate = 0.7

Momentum coefficient = 0.6

Total number of iterations = 10000

MSE of training data set = 0.016844

MSE of testing data set = 0.026167

### **CONCLUSION**

- 1. Test set error value was found to be close to training set error value.
- 2. The results vary from time to time due to random initialization of weights and randomizing of input data sets.
- 3. The optimal number of hidden neurons, learning rate and momentum coefficient are selected so that the MSE is minimum.
- 4. The momentum term has increased the convergence rate and more number of training patterns improves the accuracy.

## **REFERENCE**

- 1. Application of artificial neural network for predicting the dynamic performance of a free piston stirling engine by Wenlian Ye, Xiaojun Wang and Yingwen Liu.
- 2. Beale W. Free-piston Stirling engines-Some model tests and simulations. 1969. SAE Technical Paper 690230.