1. Probability that Head appears at k^{th} toss,

$$P(X=k) = p\cdot(1-p)^{k-1}$$

k	$P(X=k)$
1	p
2	$p\cdot(1-p)$
3	$p(1-p)^2$
|	|
k	$p(1-p)^{k-1}$

For Any Integer α,

$$P(X>\alpha) = P(\alpha+1) + P(\alpha+2) + \ldots$$
$$= p(1-p)^{\cancel{(\alpha+1)-1}} + p(1-p)^{(\alpha+2)-1} + \ldots$$
$$= p(1-p)^{\alpha} + p(1-p)^{\alpha+1} + p(1-p)^{\alpha+2} \ldots$$

$$= p(1-p)^{\alpha} = \left[1 + (1-p) + (1-p)^2 + \ldots\right]$$

[Infinite GP]

$$= p(1-p)^{\alpha-1}\ \frac{1}{\cancel{(1-p)}}$$

$$P(X>\alpha) = p(1-p)^{\alpha-1}$$

~~$P(X>m+n \mid X>n) = P(X>m+n) \cap P(X>n)$~~

~~$= p(1$~~

$$P(X>m+n \mid X>n) = \frac{P(X>m+n \cap X>n)}{P(X>m+n)} = P(X>m)$$

$= RHS$

2. The probability distribution of a Poisson random variable X with parameter λ,

$$P(X) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad \begin{array}{l} \lambda - \text{parameter.} \\ x - \text{no. of occurrence} \end{array}$$

Now,

$$\text{Mean}(x) = \sum_{x=0}^{\infty} x \left(\frac{e^{-\lambda}\lambda^{x}}{x!}\right)$$

$$= \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^{x}}{(x-1)!}$$

$$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda}\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \text{——}\right]$$

$$= \lambda \cancel{e^{-\lambda}}\,(\cancel{e^{\lambda}}) \quad \left\{\sum \frac{\lambda^{x}}{x!} = e^{\lambda}\right\}$$

$$= \lambda$$

Hence, Mean$(X) = \lambda$. $= E(X)$

$$\text{Var}(x) = E(X^2) - (E(X))^2$$

$$= E(X^2 - X + X) - (E(X))^2$$

$$= E(X(X-1) + X) - (E(X))^2$$

$$= E(X(X-1)) + E(X) - (E(X))^2$$

$$= E(X(X-1)) + \lambda - \lambda^2$$

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$$E(X(X-1)) = \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$= \sum_{x=2}^{\infty} x(x-1) \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$= \sum_{x=2}^{\infty} \frac{e^{-\lambda} \lambda^{x}}{(x-2)!} = e^{-\lambda} \lambda^{2} \sum \frac{\lambda^{x-2}}{(x-2)!}$$

$$= \cancel{e^{-\lambda}} \lambda^{2} \cancel{e^{(\lambda)}} = \lambda^{2}$$

$$Var(x) = \cancel{\lambda^2} + \lambda - \cancel{\lambda^2} = \lambda.$$

$$\boxed{Mean(x) = Var(x) = \lambda}$$

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3. Mean - μ ; Var - σ^2 ; X - normal random variable

$$X \sim N(\mu, \sigma^2).$$

<u>To Prove</u> $X = \sigma Z + \mu$; $Z \sim N(0,1)$

<u>Proof</u> ~~Let~~ ~~X=~~

If $X = \sigma Z + \mu$

$$Z = \frac{X-\mu}{\sigma}$$

If we proof $Z \sim N(0,1)$ then we can conclude the proof.

$$Z = \frac{1}{\sigma} X - \frac{\mu}{\sigma}$$

Let $a = \frac{1}{\sigma}$, $b = -\frac{\mu}{\sigma}$

$$Z = aX + b$$

$$\sim N(a\mu + b, a^2\sigma^2)$$ [Linear transform of a normal is another normal]

$$\sim N\left(\frac{\mu}{\sigma} - \frac{\mu}{\sigma}, \frac{\sigma^2}{\sigma^2}\right)$$

$$\sim N(0,1)$$

Hence Proved

<u>Note</u>

If $E(X) = \mu$ $Var(X) = \sigma^2$

$E(aX+b) = a\mu + b$ $Var(aX+b) = a^2\sigma^2$

$$\boxed{aX + b \sim N(a\mu + b, a^2\sigma^2)}$$

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