UKAEA – Jan 30-31 2024

ReMKiT1D Workshop January 2024
Complex Derivations in ReMKiT1D

This work was partly funded by the RCUK Energy Programme
[Grant number EP/W006839/1]

Introduction

2nd year PhD student at Imperial College London.

Improving sheath boundary conditions for scrape off layer modelling.

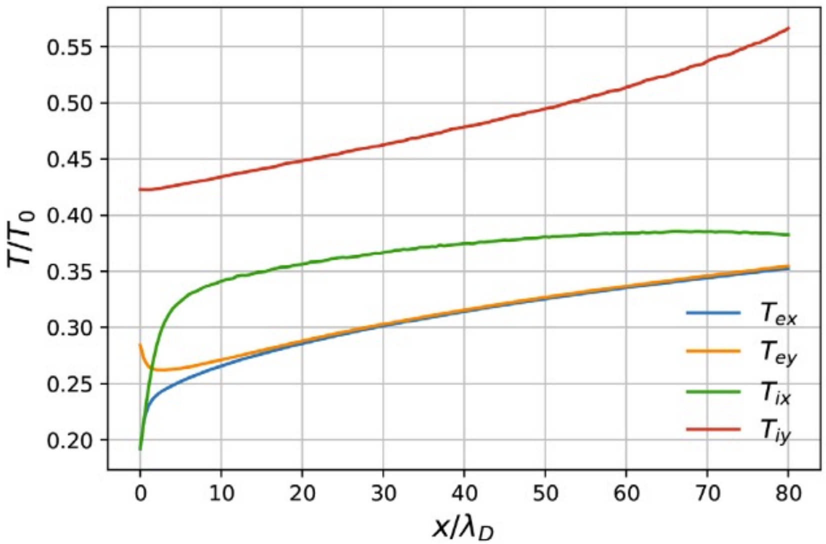
Active ReMKiT1D user, utilising ReMKiT1D's fluid modelling capabilities to build model testbench.

Presenting my own workflow using ReMKiT1D.

Background– Temperature Anisotropy in the SOL

High Magnetic field within the SOL causes anisotropy in parallel and perpendicular transport

When taking moments of the kinetic equation, distinguishing between parallel and perpendicular velocity components leads to two-temperature fluid model.



Li et al 2022

Trigger Warning,
Maths Ahead!

Anisotropic Fluid Model

UK Atomic Energy Authority

$$\boxed{\frac{\partial \rho^s}{\partial t} + \frac{\partial}{\partial x_\alpha}(\rho^s V_\alpha^s) = 0.} \tag{1}$$

$$\boxed{\rho^s \frac{\mathrm{d}^s V_\alpha^s}{\mathrm{d}t} + \frac{\partial}{\partial x_\beta} p_{\alpha\beta}{}^s - \rho^s\left[\frac{q^s}{m^s} E_\alpha + \omega_c{}^s[\mathbf{V}^s \times \mathbf{h}]_\alpha\right] = \sum_r I_\alpha{}^{sr}.} \tag{2}$$

$$\begin{aligned}\frac{\mathrm{d}^s p_\perp{}^s}{\mathrm{d}t} &+ \frac{\partial V_\alpha{}^s}{\partial x_\beta}[p_\perp{}^s \tau_{\alpha\beta} + 2p_\perp{}^s n_{\alpha\beta} + \pi_{\beta\gamma}{}^s n_{\alpha\gamma}] \\ &+ \frac{\partial}{\partial x_\alpha} S_\alpha{}^{s\perp} + S_\alpha{}^{s\perp} h_\alpha \frac{\partial}{\partial x_\beta} h_\beta + S_\alpha{}^{s\parallel} h_\beta \frac{\partial}{\partial x_\beta} h_\alpha + \tfrac{1}{2}\pi_{\alpha\beta}\frac{\mathrm{d}^s}{\mathrm{d}t}\tau_{\alpha\beta} = \tfrac{1}{2}\sum_r J_{\alpha\beta}{}^{sr} n_{\alpha\beta}\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\mathrm{d}^s p_\parallel{}^s}{\mathrm{d}t} &+ \frac{\partial V_\alpha{}^s}{\partial x_\beta}[p_\parallel{}^s n_{\alpha\beta} + 3p_\parallel{}^s \tau_{\alpha\beta} + 2\pi_{\beta\gamma}{}^s \tau_{\alpha\gamma}] \\ &+ \frac{\partial}{\partial x_\alpha} S_\alpha{}^{s\parallel} - 2S_\alpha{}^{s\parallel} h_\beta \frac{\partial}{\partial x_\beta} h_\alpha - 2S_\alpha{}^{s\perp} h_\alpha \frac{\partial}{\partial x_\beta} h_\beta - \pi_{\alpha\beta}\frac{\mathrm{d}^s}{\mathrm{d}t}\tau_{\alpha\beta} = \sum_r J_{\alpha\beta}{}^{sr}\tau_{\alpha\beta}.\end{aligned} \tag{4}$$

$$\begin{aligned}-\omega_c{}^s h_\delta(\varepsilon_{\alpha\gamma\delta}\pi_{\beta\gamma}{}^s + \varepsilon_{\beta\gamma\delta}\pi_{\alpha\gamma}{}^s) &+ p_\parallel{}^s\left(\frac{\partial V_\alpha{}^s}{\partial x_\delta}\tau_{\delta\beta} + \frac{\partial V_\beta{}^s}{\partial x_\delta}\tau_{\delta\alpha} - 2\frac{\partial V_\gamma{}^s}{\partial x_\delta}\tau_{\gamma\delta}\tau_{\alpha\beta}\right) \\ &+ p_\perp{}^s\left(\frac{\partial V_\alpha{}^s}{\partial x_\delta} n_{\delta\beta} + \frac{\partial V_\beta{}^s}{\partial x_\delta} n_{\delta\alpha} - \frac{\partial V_\gamma{}^s}{\partial x_\delta} n_{\gamma\delta} n_{\alpha\beta}\right) + (p_\parallel{}^s - p_\perp{}^s)\frac{\mathrm{d}^s}{\mathrm{d}t}\tau_{\alpha\beta} \\ &= \sum_r [J_{\alpha\beta}{}^{sr} - (\tfrac{1}{2}n_{\alpha\beta}n_{\gamma\delta} + \tau_{\alpha\beta}\tau_{\gamma\delta})J_{\gamma\delta}{}^{sr}].\end{aligned} \tag{5}$$

$$\begin{aligned}(p_\parallel{}^s\tau_{\beta\gamma} + p_\perp{}^s n_{\beta\gamma})\frac{\partial}{\partial x_\beta}\left[\frac{p_\perp{}^s}{\rho^s}(\tau_{\alpha\gamma} + 2n_{\alpha\gamma})\right] &- \omega_c{}^s[\mathbf{S}^{s\perp} \times \mathbf{h}]_\alpha \\ &- \tfrac{1}{2}(P_{\alpha\beta}{}^s P_{\gamma\delta}{}^s + P_{\alpha\gamma}{}^s P_{\beta\delta}{}^s + P_{\beta\gamma}{}^s P_{\alpha\delta}{}^s)\frac{\partial}{\partial x_\delta} n_{\beta\gamma} \\ &= \sum_r\left[\tfrac{1}{2}L_{\alpha\beta\gamma}^{sr} n_{\beta\gamma} - \frac{p_\perp{}^s}{\rho^s}(\tau_{\alpha\beta} + 2n_{\alpha\beta})I_\beta{}^{sr}\right].\end{aligned} \tag{6}$$

$$\begin{aligned}(p_\parallel{}^s\tau_{\beta\gamma} + p_\perp{}^s n_{\beta\gamma})\frac{\partial}{\partial x_\beta}\left[\frac{p_\parallel{}^s}{\rho^s}(3\tau_{\alpha\gamma} + n_{\alpha\gamma})\right] &- \omega_c{}^s[\mathbf{S}^{s\parallel} \times \mathbf{h}]_\alpha \\ &- (P_{\alpha\beta}{}^s P_{\gamma\delta}{}^s + P_{\alpha\gamma}{}^s P_{\beta\delta}{}^s + P_{\beta\gamma}{}^s P_{\alpha\delta}{}^s)\frac{\partial}{\partial x_\delta}\tau_{\beta\gamma} \\ &= \sum_r\left[L_{\alpha\beta\gamma}^{sr}\tau_{\beta\gamma} - \frac{p_\parallel{}^s}{\rho^s}(3\tau_{\alpha\beta} + n_{\alpha\beta})I_\beta{}^{sr}\right].\end{aligned} \tag{7}$$

Mass conservation

Momentum conservation

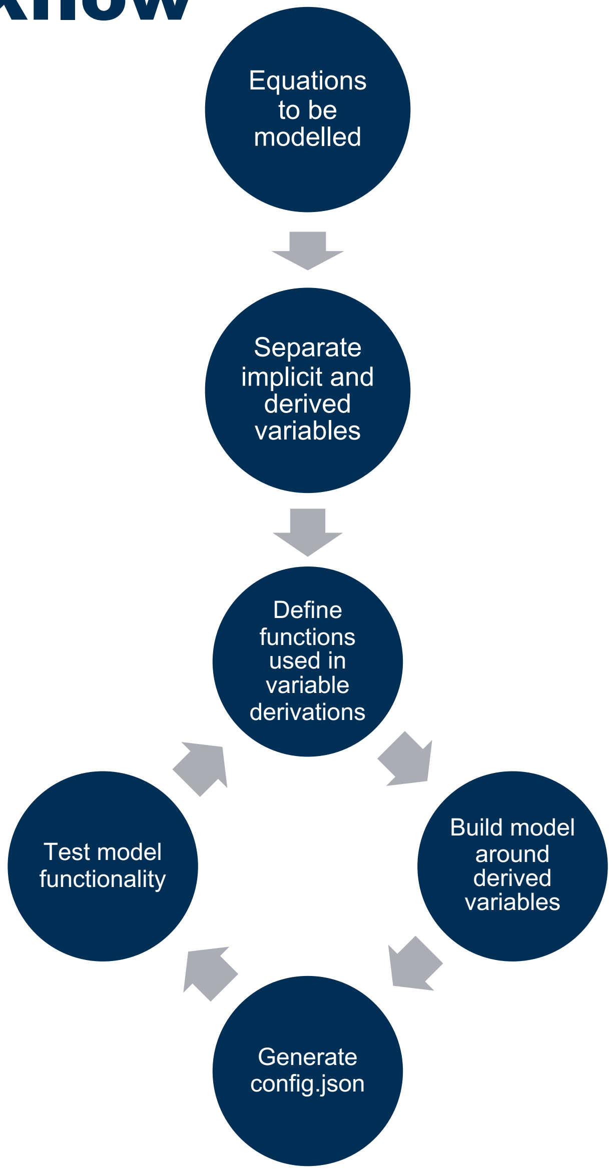
Perpendicular energy conservation

Parallel energy conservation

Heat flux

Chodura & Pohl 1971

ReMKiT1D Workflow



Collisional Energy Exchange and Temperature Isotropisation

Hands on session will explore a reduced set of equations for electrons:

$$\frac{\partial W_\perp}{\partial t} = 2nv[T_\perp(K_{002} - K_{200})]$$

$$\frac{\partial W_\parallel}{\partial t} = 4nv[T_\parallel(K_{200} - K_{002})]$$

$$\boxed{X = \frac{T_\perp}{T_\parallel} - 1}$$

Terms K_{200} and K_{002}:

$$K_{200} = \frac{1}{X}[-1 + (1 + X)\varphi(X)] \approx \frac{2}{3} - \frac{2}{15}X$$

$$K_{002} = \frac{2}{X}[1 - \varphi(X)] \approx \frac{2}{3} - \frac{2}{5}X$$

With the function $\varphi(X)$:

$$\varphi(X) = \left\{\begin{matrix} \dfrac{\tan^{-1}\left(\sqrt{X}\right)}{\sqrt{X}} & X > 0 \\ \dfrac{\log\dfrac{1+\sqrt{-X}}{1-\sqrt{-X}}}{2\sqrt{-X}} & X < 0 \end{matrix}\right\} \approx 1 - \frac{X}{3} + \frac{X^2}{5} + \cdots$$

Hands-on session