This is the third homework assignment. The problems are to be presented on exercise session on October 28, 2021. Students should tick in TUWEL problems they have solved and are prepared to present their detailed solutions. The problems should be ticked by **20:00 on October 27, 2021**.

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(1) **Profit function**
Based on its analysis of the future demand for its products, the financial department at a certain corporation has determined that there is a 0.17 probability that the company will lose 1.2 million dollars during the next year, a 0.21 probability that it will lose 0.7 million dollars, a 0.37 probability that it will make a profit of 0.9 million dollars, and a 0.25 probability that it will make a profit of 2.3 million dollars.

(a) Let X be a random variable that denotes the profit (in million dollars) earned by this corporation during the next year. Write the probability distribution of X.

(b) Compute the expectation and standard deviation of the probability distribution of X.

(c) Give a brief interpretation of the value of the expectation.

(d) Compute $P(|X| \leq 1)$ and $F_X(1.5)$, where $F_X(x)$ is the cumulative distribution function (cdf) of X.

(2) **A left-turn lane problem**
A civil engineer is studying a left-turn lane that is long enough to hold six cars. Let X be the number of cars in the lane at the end of a randomly chosen red light. The engineer believes that the probability that $X = x$ is proportional to $(x+1)(7-x)$.

(a) Find the probability mass function (pmf) of X.

(b) Compute the probability that X will be at least 4.

(c) Calulate the expectation and standard deviation of X.

Note: `R` might be useful.

(3) **Density and distribution function**
Let X a continuous random variable with the probability density function (pdf)

$$f_X(x) = \begin{cases} \frac{x^2}{a}, & x \in (0,3) \\ 0, & \text{otherwise} \end{cases},$$

where a ia a nonzero constant.

(a) Determine the value of a.

(b) Find the cumulative distribution function of X.

(c) Compute the expectation of $Z = 2X^3 + 5$.

(4) **River floods**
A certain river floods every year. Suppose that the low-water mark is set at 1 and the high-water mark Y has cumulative distribution function

$$F_Y(y) = P(Y \leq y) = \begin{cases} 1 - \frac{1}{y^2}, & 1 \leq y < \infty \\ 0, & y < 1 \end{cases}.$$

(a) Verify that $F_Y(y)$ is a cumulative distribution function.

(b) Compute $P(-1 \leq Y \leq \frac{1}{2})$.

(c) If the low-water mark is reset at 0 and we use a unit of measurement that is $\frac{1}{10}$ of that given previously, the high-water mark becomes $Z = 10(Y-1)$. Find the cdf $F_Z(z)$ and the probability density function (pdf) $f_Z(z)$.

(5) **Continuous random variable**
Let X be a random variable whose cumulative distribution function (cdf) is of the form

$$F(x) = \frac{e^x}{1+e^x}, \quad x \in \mathbb{R}.$$

(a) Determine the associated probability density function (pdf) $f(x)$.

(b) Use `R` function `plot()` to sketch the cdf F and pdf f.

(c) Find an expression for the p-quantile x_p and then determine the three quartiles (25%, 50% and 75% quantiles) of the distribution.

*H*int: Recall, x_p is the p-quantile if it holds $F(x_p) = p$.

(6) `R` **-functions**

(a) Using `R` define a vector x that contains the values of the column `Height` from the dataset `trees`. Round the values of x to the tenth place and then output the number of each type. The commands `round()` and `table()` should be used.

Note that `trees` is the `R` Dataset Package containing Diameter, Height and Volume for 31 Black Cherry trees.

(b) Write a function in `R` that outputs the first n Fibonacci numbers. Calculate the sum of the first 15 and the first 25 Fibonacci numbers.

(c) Consider the curve given by the parametrization

$$t \mapsto \begin{pmatrix} \cos 2t \\ \sin 3t \end{pmatrix} \quad \text{for} \quad 0 \leq t < 2\pi\,.$$

Set the working directory by using `R` function `setdw()`. Use `plot()` to plot the curve and `dev.print()` to save the result as a pdf file.