Admissibility of Recursive Structures in the UNNS Substrate

UNNS Research Collective

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Abstract

We introduce admissibility axioms governing which recursive structures may be legitimately subjected to analysis within the UNNS (Unbounded Nested Number Sequences) framework. These axioms do not determine observability, persistence, or physical interpretation. Instead, they delimit the class of structures for which such questions are well-posed. Admissibility is formulated independently of any *specific* computational implementation. These axioms are designed to prevent circular validation and post hoc parameter tuning, ensuring that both positive and negative analytical outcomes carry structural significance.

1 Motivation

UNNS analyses recursive structures through operators that test stability, closure, and observability. Without a prior restriction on admissible inputs, any subsequent success risks being explained away as selection bias, tuning, or circular validation.

This paper fixes a methodological prerequisite: admissibility must precede observability. Failure of admissibility is a meaningful structural outcome, not an error.

2 Candidate Structure Space

Let $\mathcal{S}$ denote the space of candidate recursive structures, arising from symbolic rules, numerical recursions, hybrid constructions, or stochastic generators. We define a distinguished subset $\mathcal{S}_{\text{adm}} \subset \mathcal{S}$ of admissible structures by axioms A1–A7.

3 Admissibility Axioms

3.1 Axiom A1 (Finite Generability)

A structure $s \in \mathcal{S}$ is admissible only if it is generated by a finite, explicit rule set. There exists a finite description $\mathcal{R}$ such that

$$s = G(\mathcal{R}).$$

Structures requiring infinite rule definitions, oracle access, or non-constructive specification are inadmissible.

3.2 Axiom A2 (Bounded Initial Data)

An admissible structure must be initialized from bounded data. There exists $B < \infty$ such that

$$\|s_0\| \leq B.$$

Structures whose initialization requires divergent amplitudes, undefined values, or infinite precision are inadmissible.

3.3 Axiom A3 (Well-Posed Evolution or Terminal Law) *(revised)*

An admissible structure must admit one of the following:

- **(Extension)** For all finite depths $n \in \mathbb{N}$, the update

$$s_{n+1} = F(s_n)$$

  is well-defined; *or*

- **(Terminal law)** there exists a finite depth N at which the evolution reaches a terminal event, and this terminal event is itself well-defined by the rule set (e.g. absorbing state, controlled collapse, or explicit singular termination rule).

Thus, finite-time breakdown is not automatically inadmissible; it is inadmissible only when the breakdown is ill-defined, non-reproducible, or requires extrinsic choice to resolve.

3.4 Axiom A4 (Spectral Measurability) *(revised)*

An admissible structure must admit a well-defined spectral *measure*. Let μ_s be a finite Borel measure on an appropriate spectral domain (e.g. frequency or wavenumber space) associated with s. Admissibility requires that such a μ_s exists and is finite on bounded sets.

This axiom does not require an absolutely continuous spectrum or a smooth density $P_s(k)$. Pure point, absolutely continuous, and singular continuous components are all admissible, provided the spectral object is well-defined.

3.5 Axiom A5 (Operator Neutrality)

Admissibility is independent of all downstream analytical operators. No admissibility criterion may depend on closure success, observability outcome, invariant preservation, or persistence.

Formally, for any downstream operator $\mathcal{O}$ acting on s,

$$\mathcal{O}(s) \text{ is not evaluated in determining admissibility.}$$

3.6 Axiom A6 (Parameter Independence / Anti-Circularity) *(revised)*

Let $G(\mathcal{R}, s_0, \theta)$ be a generator with parameters θ. Admissibility forbids *outcome-targeted tuning* of θ to force a specific downstream analytical conclusion.

A6(a) Prohibited tuning. A parameter choice θ is inadmissible if it is selected by optimizing an explicit objective that encodes a downstream success condition (e.g. "choose θ to maximize closure score" or "choose θ to pass the observability gate").

A6(b) Permitted discovery. Parameter *discovery* is admissible when θ is selected to satisfy a criterion that is:

- *defined independently* of the downstream property later reported as a primary result, and
- *documented prior* to evaluating the downstream property.

In particular, searching for θ that stabilizes an evolution, minimizes numerical instability, or achieves a pre-registered equilibrium criterion is permitted, provided the subsequent claims concern a different property not used in the search objective.

Illustrative protocol. A parameter $\theta^\star$ may be discovered by searching for numerical equilibration or stability under iteration. If, after $\theta^\star$ is fixed and documented, a structurally unrelated property (e.g. scale invariance or spectral regularity) is evaluated and reported as a primary result, the procedure satisfies A6(b)–(d). The admissibility of such discovery does not depend on the nature of the reported property, but on the documented independence of the search criterion.

A6(c) Multi-stage protocol rule. If a parameter $\theta^\star$ is discovered in stage A using criterion C_A, then using $\theta^\star$ in stage B to test criterion C_B is admissible only if C_B is not logically equivalent to (or a re-encoding of) C_A.

A6(d) Documentation requirement. Any admissible parameter discovery must record: (i) the search objective, (ii) the search domain, (iii) stopping rules, (iv) the independence statement specifying which later claims were *not* targeted. (v) the timestamp and version identifier of the search procedure relative to subsequent analysis.

3.7 Axiom A7 (Reproducibility)

An admissible structure must be reproducible. There exists a finite specification $(\mathcal{R}, s_0, \theta)$ such that repeated executions produce statistically equivalent structures:

$$G(\mathcal{R}, s_0, \theta) \sim G(\mathcal{R}, s_0, \theta).$$

Non-reproducibility under identical specification is inadmissible.

4 Admissibility vs. Observability (clarification)

Admissibility answers: *Is the object legitimate to analyze at all?* Observability answers: *Does the object survive a particular operator-defined test?*

Admissibility is a precondition; observability is a downstream verdict. An admissible structure may still fail stability, closure, or observability. Conversely, an inadmissible structure is not meaningfully classifiable as observable or non-observable, because the question is ill-posed.

5 Failure Modes and Informative Exclusion

Admissibility failure is informative because it locates the boundary of analyzability.

5.1 Binary vs. graded inadmissibility

Axioms can be violated in degrees (e.g. marginal reproducibility, borderline spectral measurability). While admissibility is stated as a binary predicate $s \in \mathcal{S}_{\mathrm{adm}}$, it is often useful to track a *failure profile* indicating which axioms fail and how.

In practice, structures may exhibit marginal compliance, where axiom satisfaction depends on numerical thresholds or tolerances (e.g. boundedness up to machine precision, reproducibility within statistical confidence intervals). Such borderline cases should be explicitly flagged in $\mathrm{Cert}(s)$, accompanied by sensitivity analyses demonstrating robustness under reasonable threshold variation. Structures whose admissibility status is unstable under small perturbations should be treated as provisionally inadmissible.

5.2 Interpretation of failures

- A1/A2 failures indicate non-constructive or physically ungrounded specification.
- A3 failures indicate ill-defined evolution (as opposed to well-defined terminal behavior).
- A4 failures indicate that no stable spectral object can be associated, blocking spectral questions rather than all questions.
- A5 failures indicate logical circularity (inadmissible by design).
- A6 failures indicate post hoc steering toward a desired conclusion.
- A7 failures indicate non-scientific irreproducibility.

6 Enforcement Protocol (theoretical)

The axioms are implementation-agnostic but not enforcement-free. A structure should be accompanied by an admissibility certificate consisting of:

$$\mathrm{Cert}(s) = (\mathcal{R}, s_0, \theta;\ \text{A1–A7 claims};\ \text{supporting evidence})$$

where the supporting evidence includes rule descriptions, bounds, terminal-law statements (if relevant), spectral-object definition, and parameter-independence documentation.

7 Definition of Admissibility

A structure s is admissible if and only if it satisfies Axioms A1–A7. Admissibility implies neither observability, stability, nor physical relevance. It guarantees only that s is a legitimate object of further analysis.

8 Relation to Existing Frameworks

The admissibility axioms introduced here align with, but are not reducible to, existing methodological constraints in related fields.

In computational complexity theory, admissibility is often enforced implicitly through resource bounds and input regularity assumptions. The UNNS axioms play a similar role but are expressed at the level of structural legitimacy rather than computational cost.

In algorithmic information theory, finite specification and bounded description length serve as criteria for meaningful objects of study. Axioms A1 and A2 parallel these ideas while remaining agnostic to particular coding schemes.

In the philosophy of science, demarcation criteria distinguish legitimate scientific inquiry from unfalsifiable or circular claims. Axioms A5 and A6 function as explicit demarcation rules, preventing circular validation and post hoc outcome justification within recursive analysis.

The UNNS admissibility framework may thus be viewed as a synthesis of these traditions, specialized to recursive and operator-driven structure analysis.

9 Conclusion

Admissibility is a necessary precondition for meaningful analysis in UNNS. By fixing admissibility prior to implementation or interpretation, the framework prevents circular validation and preserves the structural meaning of both positive and negative outcomes.

A Example Cases (illustrative)

Case E1: Finite-time singularity with terminal law

A recursion that reaches a blow-up threshold at step N but specifies an absorbing terminal state thereafter satisfies A3 (terminal law) and may remain admissible.

Case E2: Fractal structure with singular continuous spectrum

A structure whose spectrum is not described by a smooth density $P_s(k)$ may still be admissible if a well-defined spectral measure μ_s exists (A4).

Case E3: Outcome-targeted tuning (inadmissible)

Choosing θ by maximizing a downstream closure score violates A6(a).

Case E4: Discovery-then-test (admissible)

Selecting $\theta^\star$ via a pre-registered stability criterion and only then testing an independent spectral property is admissible under A6(b)–(d).

Case E5: Re-encoded criterion violation (inadmissible)

A parameter θ is selected by minimizing τ-field variance during evolution. A subsequent claim reports success based on the emergence of maximal phase coherence.

If, under the recursion rules, phase coherence is a monotone or functionally equivalent re-encoding of τ-field variance, then the downstream criterion C_B is not independent of the discovery criterion C_A. In this case, the procedure violates A6(c), even if the reported property was not explicitly named during the search.

This constitutes outcome-targeted tuning via re-encoding and is inadmissible.