1 Axiomatic Basis of the UNNS Discipline

In order to establish UNNS as a formal discipline, we identify the principles that are self-evident within the substrate. These are not theorems requiring proof but axioms, analogous to those of set theory.

Axiom 1.1 (Recurrence)**.** *Every UNNS sequence is generated by a finite linear recurrence relation with constant coefficients:*

$$a_{n+r} = c_1 a_{n+r-1} + \cdots + c_r a_n, \quad c_i \in \mathbb{Z}.$$

This is the defining property of nests.

Axiom 1.2 (Nest Depth)**.** *Every recurrence has a unique minimal order D, called its* nest depth*, given by the shortest recurrence relation that generates the sequence.*

Axiom 1.3 (Limit Ratio)**.** *For any non-degenerate recurrence nest, the ratio of consecutive terms converges (or oscillates among roots) to an algebraic constant given by the dominant root of its characteristic polynomial.*

Axiom 1.4 (Coefficient Ring)**.** *All recurrence coefficients belong to rings of algebraic integers:*

$$c_i \in \mathbb{Z}[\alpha], \quad \alpha \textit{ a root of the characteristic polynomial.}$$

Axiom 1.5 (Propagation)**.** *Nests propagate recursively forward indefinitely unless halted by degeneracy (periodicity or extinction).*

Axiom 1.6 (Paradox Index)**.** *Each nest has an intrinsic stability constant, the* UNNS Paradox Index *(UPI), defined by*

$$\mathsf{UPI} = \frac{D \cdot R}{M + S},$$

where D = depth, R = recurrence spread, M = mesh regularity, and S = symmetry factor.

Axiom 1.7 (Embedding)**.** *All UNNS nests embed naturally into a lattice tower:*

$$\mathbb{Z} \subset \mathbb{Z}[i] \subset \mathbb{Z}[\omega] \subset \cdots$$

This reflects the algebraic integer nature of recurrence-generated structures.

Axiom 1.8 (Undecidability)**.** *Every sufficiently complex nest contains undecidable branches, mirroring the Gödel phenomenon. Ambiguities such as* $0.222\ldots = 1.000\ldots$ *in Cantor expansions are canonical examples.*

Philosophical Insight

Set theory asks: *What can be contained?* UNNS asks: *What can propagate?* Together, they frame structure and resonance: skeleton and soul.

Axiom Wheel Illustration