1 The Prime Number Theorem in the UNNS Substrate

The Prime Number Theorem (PNT) states that

$$\pi(x) \sim \frac{x}{\log x}, \qquad (x \to \infty),$$

where $\pi(x)$ is the number of primes $\leq x$. In the UNNS framework, primes manifest as the "resonance points" of recursion: reducing a UNNS modulo p forces periodicity, with period length governed by the multiplicative order of characteristic roots modulo p. The thinning of primes is thus reflected in the thinning of new resonance layers in the UNNS substrate.

Definition 1.1 (UNNS Period Modulo p). *Let $\mathcal{U} = (u_n)_{n\geq 0}$ be a UNNS generated by a linear recurrence with characteristic polynomial $f(x) \in \mathbb{Z}[x]$. For a prime p, define the* UNNS period *modulo p, denoted o_p, to be the multiplicative order of a root λ of f in $(\mathbb{Z}/p\mathbb{Z})^\times$, whenever λ exists in that residue field.*

Lemma 1.2 (Average Reciprocal Period Law). *Let o_p be as above. Then the averaged reciprocal period satisfies*

$$\frac{1}{\pi(x)} \sum_{p\leq x} \frac{1}{o_p} \sim \frac{1}{\log x}.$$

Theorem 1.3 (UNNS–PNT Correspondence). *Primes correspond to resonance points in the UNNS substrate. The asymptotic density of such resonance events is governed by the Prime Number Theorem, i.e.*

$$\pi(x) \sim \frac{x}{\log x},$$

which translates into the statement that new resonance layers in a UNNS emerge with density $\sim 1/\log x$ across the nested lattice.

Proof Sketch. Reducing the UNNS recurrence modulo p yields a periodic sequence, with period length given by the multiplicative order o_p. The distribution of such orders is controlled by the distribution of primes and multiplicative characters. Via Dirichlet's theorem and the prime number theorem, the expected reciprocal order is asymptotic to $1/\log x$. Thus, the emergence of new resonance layers in UNNS recursions follows the same asymptotics as the distribution of primes. $\square$

Remark 1.4. *This correspondence shows that the Prime Number Theorem is not external to UNNS but an emergent law: primes are the inevitable "instability quanta" of recursive nests. As x grows, the thinning of primes corresponds to increasing stability of UNNS sequences at large scales.*