UNNS Inlaying: Recursive Self-Embedding in Number-Theoretic Substrates

UNNS Research Notes

September 22, 2025

Abstract

Unbounded Nested Number Sequences (UNNS) support recursive generation of algebraic, combinatorial, and topological structures. A central operation in this substrate is *inlaying*: the re-embedding of internal motifs within the substrate itself. This paper formalizes UNNS inlaying in detail, introducing precise definitions, mathematical properties, algorithmic procedures, and analogies to both biology (gene duplication) and topology (attaching maps). We show how inlaying can stabilize unstable regions, generate higher-order recurrences, and produce topological richness through recursive self-embedding.

Contents

1 Introduction

Recursion is the essence of UNNS, yet recursion without regulation leads to divergence. *Inlaying* provides a structural operation: taking motifs already present inside the substrate and embedding them elsewhere. Unlike inletting (external input) or repair (external correction), inlaying is an *internal self-coupling* operation.

This paper develops a formal framework for UNNS inlaying, including:

- precise definitions and examples,
- stability lemmas for contractive inlays,
- algorithmic procedures for motif selection, transformation, and placement,
- analogies to biology and topology,
- diagrams illustrating the inlaying process.

2 Preliminaries

Definition 2.1 (UNNS substrate)**.** *A* UNNS substrate $\mathcal{U}$ *consists of recursive structures: sequences generated by finite recurrences, or mesh labelings with edge/face coefficients, together with diagnostics such as local residues and growth rates.*

Definition 2.2 (Motif)**.** *A* motif P *is a finite subsequence or local mesh patch extracted from* $\mathcal{U}$*, together with its recurrence context (e.g. coefficients, phase, boundary conditions).*

3 Definition of UNNS Inlaying

Definition 3.1 (UNNS Inlaying)**.** *A* UNNS inlaying *is an operator*

$$\mathcal{I}_{P\to L} : \mathcal{U} \longrightarrow \mathcal{U},$$

which takes a motif $P \subset \mathcal{U}$*, applies a (possibly trivial) transformation* $\widetilde{P}$*, and inserts* $\widetilde{P}$ *into a target location* $L \subset \mathcal{U}$*, so that recursive dynamics extend consistently across* $\widetilde{P}$*.*

Remark 3.2. *The transformation step may include scaling, phase-shift, or projection into a prescribed algebraic ring. This allows compatibility with global rules.*

4 Examples

Example 4.1 (Sequence motif inlay)**.** *Let* $P = (u_k, u_{k+1}, u_{k+2})$ *be a Fibonacci subsequence. Define* $\widetilde{P} = (u_k, u_{k+1}, u_{k+2})$ *shifted in phase. Insert into indices* $j, j+1, j+2$ *to seed a local Fibonacci patch at* j*.*

Example 4.2 (Mesh motif inlay)**.** *Extract a triangular patch of edge coefficients* P*. Rotate and scale to* $\widetilde{P}$*, then paste into distant patch* L*. The recurrence propagates the inlay to new regions of the mesh.*

5 Mathematical Properties

5.1 Stability lemma

Lemma 5.1 (Local stabilization by contractive inlay)**.** *Let region R of $\mathcal{U}$ have local companion matrix C_R with spectral radius $\rho(C_R) > 1$. Suppose motif P has companion C_P with $\rho(C_P) < 1$. If $\mathcal{I}_{P\to L}$ replaces local dynamics of R by C_P (with weak boundary coupling), then R becomes locally contractive:*

$$\|u^{(t+1)}\| \leq \rho(C_P)\,\|u^{(t)}\| + o(1).$$

Sketch. Local dynamics are governed by eigenvalues of the effective update matrix. If C_P dominates over C_R after inlay, the spectral radius reduces below 1, ensuring geometric decay of perturbations inside R. □

5.2 Higher-order recurrence generation

Proposition 5.2 (Recurrence order extension)**.** *Inlaying two order-r motifs into overlapping regions can generate effective recurrences of order up to $2r$, as the combined span of coefficient supports is enlarged.*

Remark 5.3. *Thus inlaying is a natural mechanism for producing higher-order recurrences from lower-order building blocks.*

6 Algorithmic Recipe

1. **Select motif** P from stable region (low residue, bounded growth).
2. **Transform motif** into $\widetilde{P}$ (apply scaling, phase-shift, or projection to admissible ring).
3. **Choose target location** L (unstable or empty region).
4. **Simulate inlay** locally; compute spectral radius, residue evolution.
5. **Accept/reject** based on thresholds.
6. **Commit insertion** into $\mathcal{U}$; monitor echoes and apply repair if necessary.

7 Biological Analogy

- In DNA, gene duplication creates redundant motifs which can stabilize or diversify the genome.
- In UNNS, inlaying motifs creates redundancy and diversity that can stabilize unstable dynamics or generate new structures.
- Homologous inlay corresponds to using a stable template to reinforce unstable regions.

8 Topological Analogy

- In topology, attaching maps glue cells together to build CW-complexes.
- In UNNS, inlaying glues motifs into the recursive structure, generating higher-order complexity.
- Repeated inlaying can build analogs of characteristic classes in discrete settings.

9 Diagram

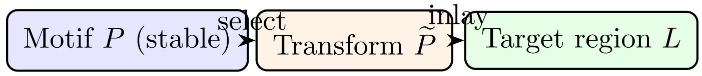

Figure 1: UNNS inlaying: extracting a motif P, transforming it, and embedding at L.

10 Conclusion

UNNS inlaying formalizes recursive self-embedding, analogous to biological gene duplication and topological attaching maps. It enables stabilization of unstable regions, generation of higher-order recurrences, and enrichment of structural complexity. Together with inletting, repair, and trans–sentifying, inlaying completes the operational grammar of UNNS.