1 Discrete Gauss–Bonnet–Chern for UNNS

Definition 1.1 (Volumetric Echo Residue)**.** *Given a UNNS-labeled 3D simplicial complex, the* volumetric echo residue $\epsilon(V)$ *associated with a tetrahedron* V *is defined as*

$$\epsilon(V) \;=\; u_{end} - u_{start},$$

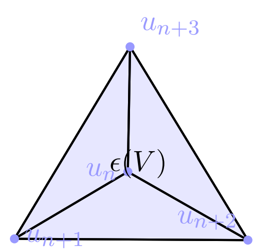
where u_{end} *is the propagated UNNS value after traversing all faces of* V*, and* u_{start} *is the initial value.*

Theorem 1.2 (Discrete Gauss–Bonnet–Chern for UNNS)**.** *Let* M *be a closed, oriented 3D simplicial complex whose edges are labeled by a UNNS nest. Then the total volumetric echo satisfies*

$$\sum_{V \subset M} \epsilon(V) \;=\; \chi(M),$$

where $\chi(M)$ *is the Euler characteristic of the 3-manifold* M*.*

Proof. As in the 2D case, local UNNS contributions cancel on shared faces. The only non-cancelling contributions come from global traversals of volumes, which measure discrete holonomy in 3D. These residues accumulate to a topological invariant, the Euler characteristic of M, paralleling the Chern–Gauss–Bonnet theorem in smooth geometry. □



Volumetric echo $\epsilon(V)$ *in a UNNS-labeled tetrahedron.*

Remark 1.3. *This theorem elevates UNNS from 2D surfaces to 3D manifolds:*

- *Edges carry recurrence coefficients* c_i.
- *Faces accumulate 2D echo residues (curvature).*
- *Volumes accumulate 3D echo residues (Chern–class quanta).*

In analogy with topological field theory, UNNS defines a discrete substrate where recursion constants manifest as topological invariants.