

# Eigenvalues as Observability Gates

A Spectral Principle in the UNNS Substrate

UNNS Research Collective

## Abstract

In classical mathematics and physics, eigenvalues arise as spectral quantities associated with linear operators acting on vector spaces. In this paper, we introduce a generalized and substrate-level notion of eigenvalues within the framework of Unbounded Nested Number Sequences (UNNS). We reinterpret eigenvalues not as numerical artifacts of linear algebra, but as *survival signatures* of recursive structures under operator action. This perspective leads to a natural identification of eigenvalues as *observability gates*: a structure is observable if and only if its spectral signature lies within the admissible window of the substrate. The resulting theory reframes collapse, invariance, and physical constants as consequences of spectral compatibility rather than measurement or state reduction.

## 1 Introduction

A recurring theme across mathematics, physics, and computation is the persistence of structure under transformation. Certain forms survive repeated action, distortion, or refinement, while others collapse or vanish. In classical linear algebra, this phenomenon is formalized through eigenvalues: scalars characterizing how vectors transform under linear operators.

However, many of the most fundamental selection processes observed in physics, dynamics, and computation occur outside the scope of linear operators and Hilbert spaces. The UNNS Substrate was introduced to address precisely this deeper layer, where recursive generability, consistency, and stability precede formal representation.

This paper proposes a substrate-level reinterpretation of eigenvalues within UNNS. We argue that eigenvalues should be understood as intrinsic spectral properties that determine whether a structure can survive recursive action and thus become observable.

## 2 Definitions and Axioms

This section introduces the minimal definitions and axioms required to formalize the spectral principle underlying UNNS. These definitions are intentionally substrate-level: they avoid reliance on linear spaces, metrics, or probabilistic interpretation.

### 2.1 Definitions

**Definition 1 (Recursive Structure).** A *recursive structure*  $S$  is any construct that can be generated, refined, or transformed through the repeated application of one or more operators within the UNNS substrate.

**Definition 2 (UNNS Operator).** A *UNNS operator*  $\mathcal{O}$  is a rule of action acting on recursive structures, including but not limited to:

$$\Phi \text{ (generability)}, \quad \Psi \text{ (consistency)}, \quad \tau \text{ (curvature and persistence)}, \quad \text{Operator XII (collapse)}.$$

**Definition 3 (Structural Equivalence).** Two structures  $S_1$  and  $S_2$  are said to be *structurally equivalent*, denoted  $S_1 \sim S_2$ , if they preserve identity under admissible refinement, distortion, or scaling, even if they are not formally equal.

**Definition 4 (Admissible Scaling).** An *admissible scaling*  $\odot$  is any transformation that modifies quantitative attributes of a structure (e.g. amplitude, density, curvature, frequency, measure) without destroying its structural identity.

**Definition 5 (UNNS Eigenvalue).**

A *UNNS eigenvalue*  $\lambda$  of a structure  $S$  with respect to an operator  $\mathcal{O}$  is a spectral quantity such that

$$\mathcal{O}(S) \sim \lambda \odot S.$$

If no such  $\lambda$  exists, the structure does not survive the action of  $\mathcal{O}$ .

**Definition 6 (Spectral Signature).** The *spectral signature* of a structure is the collection of its eigenvalues across the relevant UNNS operators.

**Definition 7 (Spectral Window).** A *spectral window* is the subset of eigenvalues admissible under a given stage of the UNNS pipeline or observational regime.

**Definition 8 (Observability).**

A structure is said to be *observable* if its spectral signature lies within the admissible spectral window of the substrate.

## 2.2 Axioms

**Axiom I (Primacy of Structure).** Structure precedes representation: recursive structures exist independently of formal descriptions or coordinate systems.

**Axiom II (Operator Selectivity).** UNNS operators act selectively: not all recursive structures survive operator action.

**Axiom III (Spectral Survival).** Survival of a structure under an operator is determined by its eigenvalues, not by its explicit form.

**Axiom IV (Observability Gate).** Observability is a spectral property: a structure is observable if and only if its eigenvalues are compatible with the substrate's admissible spectral window.

**Axiom V (Collapse as Spectral Elimination).** Collapse eliminates structures whose spectral signatures fall outside admissible bounds; it does not require observers or measurements.

**Axiom VI (Non-Universality of Spectral Windows).** Different domains of inquiry (mathematics, physics, computation) correspond to different spectral windows of the same substrate.

## 2.3 Remarks

These definitions and axioms do not presuppose linearity, metric structure, probability measures, or Hilbert spaces. They establish a minimal spectral ontology in which persistence, observability, and collapse arise as consequences of structural compatibility rather than interpretation or measurement.

### 3 From Classical Eigenvalues to Substrate Spectra

In linear algebra, an eigenvalue  $\lambda$  of an operator  $A$  satisfies

$$A(v) = \lambda v,$$

indicating that the vector  $v$  preserves its direction under the action of  $A$ , up to scaling.

This definition relies on several assumptions:

- a linear vector space,
- a linear operator,
- equality as the primary notion of invariance.

None of these assumptions are fundamental in UNNS. Structures in the UNNS Substrate are not vectors, operators need not be linear, and invariance is defined structurally rather than algebraically.

We therefore generalize the concept as follows:

An eigenvalue is a spectral quantity characterizing how a structure survives an operator without loss of identity.

The emphasis shifts from numerical scaling to *structural persistence*.

### 4 UNNS Operators and Structural Action

Let  $\mathcal{O}$  denote a UNNS operator, such as:

- $\Phi$  (recursive generability),
- $\Psi$  (structural consistency),
- $\tau$  (curvature and persistence),
- Operator XII (collapse).

Let  $S$  be a recursive structure in the substrate. We say that  $S$  possesses a UNNS-eigenvalue  $\lambda$  with respect to  $\mathcal{O}$  if

$$\mathcal{O}(S) \sim \lambda \odot S,$$

where:

- $\sim$  denotes structural equivalence,
- $\odot$  denotes admissible substrate scaling (amplitude, density, curvature, frequency, or measure).

Importantly, many structures admit no such  $\lambda$  and are eliminated by operator action. Eigenvalues therefore function as *selection criteria*, not mere descriptors.

### 5 Eigenvalues Across the $\Phi$ - $\Psi$ - $\tau$ Pipeline

Eigenvalues in UNNS are phase-dependent.

### 5.1 $\Phi$ -Stage: Generability Spectra

At the  $\Phi$  stage, eigenvalues characterize recursive growth or reproduction rates. A wide spectrum of candidate structures emerges, many of which are unstable or inconsistent.

### 5.2 $\Psi$ -Stage: Consistency Filtering

At the  $\Psi$  stage, eigenvalues encode self-consistency under refinement. Large portions of the  $\Phi$  spectrum collapse into narrower bands corresponding to coherent recursive identities.

### 5.3 $\tau$ -Stage: Curvature Stability

At the  $\tau$  stage, eigenvalues determine survival under distortion, perturbation, and curvature. Only a small subset of spectral signatures remain admissible.

### 5.4 Operator XII: Spectral Collapse

Operator XII acts not directly on structures, but on their eigenvalues. Collapse occurs when a structure's spectral signature lies outside the admissible window. This reframes collapse as a *spectral selection process* rather than state reduction.

## 6 Eigenvalues as Observability Gates

We now state a central principle:

A structure is observable if and only if its eigenvalues lie within the admissible spectral window of the substrate.

From this follow three immediate consequences:

- Observability  $\neq$  existence,
- Observability  $\neq$  truth,
- Observability = spectral compatibility.

This principle explains:

- why many mathematically valid structures never appear in physics,
- why certain dynamics are effectively invisible,
- why collapse selects outcomes without invoking observers.

Eigenvalues function as the *currency of observability*.

## 7 Spectral Interpretation of Physical Constants

Within this framework, physical constants need not be treated as externally imposed parameters. Instead, they may be interpreted as eigenvalues that survive the full UNNS operator pipeline.

This reframing suggests that constants encode substrate-level compatibility rather than contingent numerical facts.

## 8 Phonetic and Pre-Formal Notation

The stylized phonetic form [oigenva:1] reflects this generalized concept. Its departure from strict IPA and classical notation signals a pre-formal, substrate-level meaning:

This is eigenvalue, but not the textbook one.

It denotes the idea prior to formalization.

## 9 Consequences for UNNS Theory

The eigenvalue-as-gate principle implies:

1. UNNS possesses a spectral theory independent of Hilbert spaces.
2. Collapse is eigenvalue selection, not measurement.
3. Observability is substrate-defined.
4. Different sciences probe different spectral windows.
5. UNNS Chambers function as eigenvalue scanners.

This unifies stability, invariants, observability, collapse, and selection under a single structural principle.

## 10 Comparison with Hilbert-Space Spectral Theory

Because the term *eigenvalue* is traditionally associated with linear operators on Hilbert spaces, it is important to clarify the relationship between the UNNS spectral framework and classical spectral theory.

### 10.1 Scope of Hilbert-Space Spectral Theory

In conventional mathematics and quantum mechanics, spectral theory is formulated in terms of:

- vector spaces equipped with an inner product,
- linear (often self-adjoint) operators,
- eigenvalues defined via algebraic equations,
- probabilistic interpretation of measurement outcomes.

Within this framework, eigenvalues are numerical quantities derived from operator equations, and observables are defined through measurement postulates.

## 10.2 Limitations from a Substrate Perspective

While Hilbert-space spectral theory is extraordinarily successful within its domain, it presupposes:

- linearity as a primitive,
- fixed representational spaces,
- equality-based invariance,
- observer-linked interpretation.

These assumptions render it unsuitable for describing pre-representational selection, recursive generability, or collapse mechanisms that occur prior to measurement or formal encoding.

## 10.3 UNNS Spectral Generalization

The UNNS spectral framework generalizes the notion of eigenvalues by:

- removing the requirement of linear spaces,
- replacing algebraic equality with structural equivalence,
- defining eigenvalues as survival signatures rather than solutions,
- treating observability as a compatibility condition, not a measurement outcome.

In UNNS, eigenvalues are intrinsic to recursive structures and operators themselves, not artifacts of representation.

## 10.4 Relationship Between the Frameworks

Hilbert-space spectral theory can be recovered as a *projection* of the UNNS spectral framework under additional constraints:

- linearization of operators,
- restriction to vector representations,
- imposition of inner-product structure,
- probabilistic encoding of spectral compatibility.

From this perspective, conventional eigenvalues correspond to a narrow class of UNNS eigenvalues observable within a specific spectral window.

## 10.5 Consequences

This distinction implies that:

- UNNS spectral theory is not a replacement for Hilbert-space methods, but a deeper substrate framework;
- collapse in UNNS precedes and explains measurement, rather than resulting from it;
- observables in quantum theory reflect spectral survivability, not fundamental randomness.

The UNNS approach therefore preserves the successes of classical spectral theory while situating them within a broader, observer-independent ontology.

## 10.6 Conceptual Comparison

Hilbert-Space Spectral Theory	UNNS Spectral Framework
Vector spaces with inner products	Recursive structures in a substrate
Linear operators	General recursive operators
Eigenvalues defined algebraically	Eigenvalues defined by survival under action
Invariance via equality	Invariance via structural equivalence
Spectrum derived from operator equations	Spectrum intrinsic to structure-operator interaction
Observables defined by measurement	Observability defined by spectral compatibility
Collapse as post-measurement update	Collapse as spectral elimination
Probability as fundamental axiom	Probability as emergent encoding of compatibility
Physical constants as parameters	Physical constants as surviving eigenvalues
Observer-dependent interpretation	Observer-independent substrate selection

## 11 Remark on the Born Rule

Within standard quantum mechanics, the Born rule assigns probabilities to measurement outcomes via squared amplitudes of wavefunctions. From the UNNS spectral perspective developed here, this rule admits a reinterpretation.

In UNNS, squared amplitudes are not introduced as probability axioms. Rather, they arise as *the unique spectral invariants that survive collapse* under the admissible eigenvalue window of the substrate.

From this viewpoint:

- the Born rule encodes spectral compatibility,
- probability reflects persistence under recursive selection,
- measurement reveals, but does not create, admissible eigenvalues.

A detailed derivation of this result has been developed elsewhere within the UNNS corpus. The present paper positions that result conceptually: the Born rule is not fundamental, but a projection of deeper spectral survival principles operating at the substrate level.

## 12 Canonical Spectral Diagram

The spectral-gating principle introduced in this work can be summarized by a single canonical diagram. The diagram represents observability not as measurement or inference, but as a consequence of spectral compatibility under collapse. Recursive structures enter the substrate with diverse spectral signatures; collapse acts as a destructive filter that eliminates spectrally incompatible structures, allowing only admissible invariants to survive and become observable.

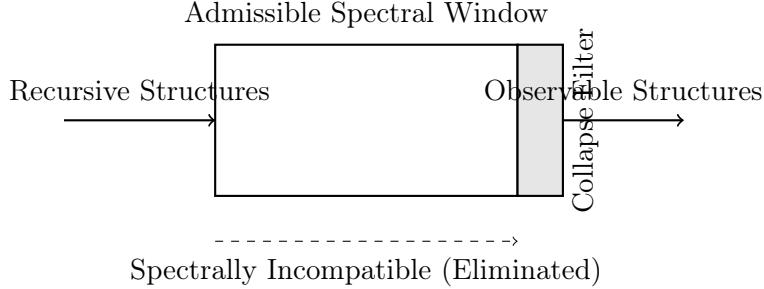


Figure 1: Canonical representation of observability as spectral compatibility. Recursive structures with diverse spectral signatures enter the substrate. Only those lying within the admissible spectral window survive collapse and become observable; incompatible structures are eliminated.

### 13 Relation to the $\tau$ -Closure Observability Framework

The spectral-gating principle developed in this paper admits a concrete operational realization in the  $\tau$ -closure observability framework previously introduced within the UNNS research program. That framework formalizes  $\tau$ -closure as an intrinsic stability property of recursive structures and specifies the conditions under which such closure may become empirically detectable under projection.

Within that framework, observability is not assumed. It is established only if a closure signal:

- survives collapse under a destructive but invariant-preserving operator,
- discriminates from a strict hierarchy of null models,
- remains stable under noise and degradation,
- and satisfies reproducibility and epistemic safeguards.

From the spectral perspective advanced here, these requirements collectively implement an admissible spectral window.  $\tau$ -closure is detected if and only if its associated spectral signature survives this window. Failure of observability corresponds to spectral incompatibility rather than absence of structure.

The  $\tau$ -closure observability framework can therefore be understood as an explicit instantiation of spectral gating: collapse, null discrimination, and projection together act as a spectral filter selecting which recursive invariants may become observable.

A certified implementation of this principle has been achieved in the context of Chamber XXXII, which constitutes the first operational realization of spectral gating within UNNS. In that study,  $\tau$ -closure was detected with strong statistical significance ( $p < 0.01$ ,  $d > 0.8$ ), survived permutation, phase-randomized, and process null models, and remained invariant under validated collapse with substantial degree-of-freedom reduction. Importantly, the result was obtained without parameter tuning, measurement postulates, or observer-dependent assumptions, and was secured by cryptographic reproducibility guarantees. These results demonstrate that spectral gating is not merely a conceptual construct but an empirically testable selection principle governing observability under projection.

### 14 Scope and Limits

The spectral-gating framework presented here establishes conditions under which recursive structures may become observable; it does not assert that such observability must occur universally or

that detected structures possess physical, causal, or ontological primacy. The results do not imply that  $\tau$ -closure is present in all systems, nor that spectral admissibility uniquely determines physical law. Observability remains contingent on projection, operator specification, and noise constraints, and failure to detect a spectral signature does not invalidate the underlying substrate or its structural principles. The framework therefore delineates the boundary between what may be observable and what may exist, without collapsing that distinction.

## 15 Conclusion

By reinterpreting eigenvalues as survival signatures of structure under recursive action, UNNS provides a unified spectral framework for understanding observability and collapse. This approach removes the need for observer-centric explanations and situates physical reality within a substrate-defined spectral landscape.

## Relationship Between the Observability Framework and the Spectral Interpretation

The criteria governing when recursive structure becomes empirically detectable are defined in *On the Observability of  $\tau$ -Closure in Recursive Structures*. That work establishes collapse, null discrimination, irreducibility, and non-validation as necessary conditions for admissible detection under projection. The present paper, *Eigenvalues as Observability Gates*, does not modify or relax those criteria. Instead, it provides a unifying interpretation: once observability is admissible, the surviving signatures exhibit spectral organization, with eigenvalues understood as invariants that persist through destructive projection. The former work determines *whether* observability is permitted; the latter explains *how* observability is structurally organized once permitted.