On the Nature of Space in the UNNS Substrate

Abstract

This paper develops a formal notion of *space* within the UNNS (Unbounded Nested Number Sequences) framework. Unlike classical physics, where space is treated as a continuous container, UNNS views space as an emergent lattice generated by recursive embeddings of number systems. We define UNNS space, analyze its metric and adjacency structure, and show how its recursive nature connects number theory, geometry, and topology.

1 Introduction

Conventional physics and mathematics model space as a smooth continuum or as a manifold. The UNNS perspective is fundamentally different: space is not given a priori but emerges from recursive number structures. Each recurrence relation generates a *lattice layer*, and the aggregation of such layers constitutes the UNNS notion of space.

2 Definition of UNNS Space

Definition 1 (UNNS Space)**.** *The* UNNS space *is defined as*

$$\mathcal{S} = \bigcup_{k=0}^{\infty} \Lambda_k,$$

where Λ_k is the k-th lattice layer generated by an order-k recurrence relation.

Remark 1. *Examples of Λ_k include:*

- $\Lambda_0 = \mathbb{Z}$ *(the integers: a 1D line),*
- $\Lambda_1 = \mathbb{Z}[i]$ *(Gaussian integers: a square lattice),*
- $\Lambda_2 = \mathbb{Z}[\omega]$ *with* $\omega = e^{2\pi i/3}$ *(Eisenstein integers: a hexagonal lattice).*

Thus, space grows richer as recursions inlay new algebraic fields.

3 Adjacency and Distance

Definition 2 (Recursion-based distance)**.** *Let $x, y \in \mathcal{S}$. Their UNNS distance is*

$$d(x, y) = \min\{k \mid x, y \in \Lambda_k \textit{ and align under } k\textit{-step recursion}\}.$$

This metric captures the idea that two points are "close" if they emerge from similar recursive histories, even if they are far apart in Euclidean embedding.

4 Containment Structure

Lemma 1 (Nested Containment)**.** *UNNS space is naturally stratified by embeddings*

$$\mathbb{Z} \subset \mathbb{Z}[i] \subset \mathbb{Z}[\omega] \subset \cdots ,$$

where each inclusion corresponds to an inlaying operator.

Sketch. Each recurrence relation defines an algebraic integer ring, which contains the previous. The inclusions are strict and generate strictly richer lattice geometries. □

5 Visualization

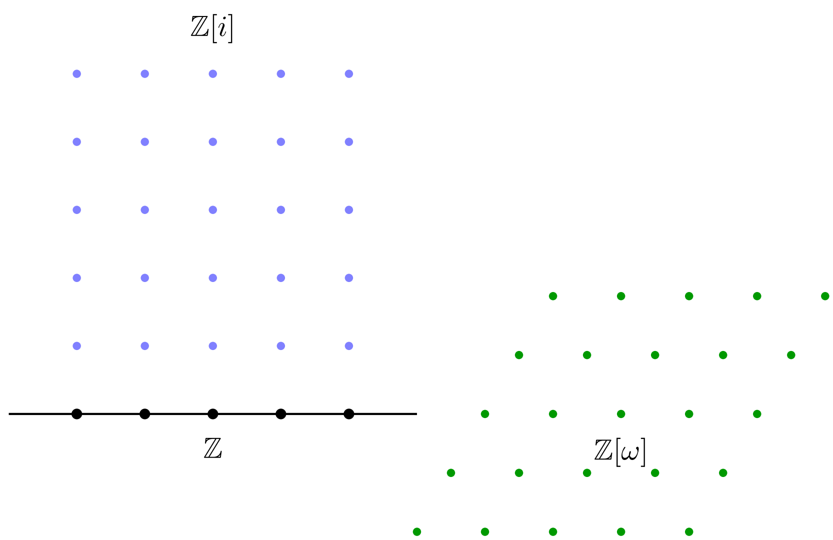

Figure 1: Recursive growth of space: integers (line), Gaussian integers (square lattice), Eisenstein integers (hexagonal lattice).

6 Properties of UNNS Space

- **Discrete but expandable:** Unlike $\mathbb{R}^n$, UNNS space expands with each recursion.
- **Operator-dependent:** Inletting expands the lattice, Inlaying embeds it, Repair contracts it, Collapse removes redundant echoes.
- **Topology from algebra:** Adjacency is determined by algebraic relations, not geometric proximity.

7 Philosophical Implications

UNNS space suggests that geometry is secondary to recursion. "Here" and "there" are not primitive; they emerge from recursive embeddings. Thus space is not a container but a living structure generated by number-theoretic processes.

8 Applications

- **Mathematics:** Bridges algebraic number theory and geometry by interpreting rings of integers as lattice-spatial layers.
- **Physics:** Provides a discrete substrate where spatial expansion and contraction map to operator actions, offering new perspectives on cosmology.
- **Philosophy:** Challenges the notion of absolute space, emphasizing recursion as the generator of "where."

9 Conclusion

Space in UNNS is recursive, lattice-based, and operator-driven. This perspective reframes space not as a continuum but as a construct emerging from recursive grammar. Its implications reach across mathematics, physics, and philosophy.