Reinterpreting Shannon Entropy under Recursive Temporal Geometry

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1. Shannon Entropy and Its Temporal Premise

Shannon's foundational measure of informational uncertainty,

$$H(X) = -\sum_i p_i \log p_i,$$

rests on the assumption of a *linear, orientable temporal axis* where information is emitted sequentially from a statistically independent source. Each signal is sampled from an ensemble whose probabilistic structure is static in time. The flow of information—measured as bits per second—is meaningful only under the existence of a continuous and unidirectional temporal substrate.

This premise, though extraordinarily powerful, presumes that time itself is a neutral parameter. In Shannon's model, time carries no internal structure—it is simply the coordinate along which entropy is measured.

2. Entropy as Recursive Curvature in UNNS

Within the **Unbounded Nested Number Sequences (UNNS)** framework, time is replaced by recursion depth $n \in \mathbb{N}$. Each state evolves not through transmission, but through recursive generation:

$$a_{n+1} = F(a_n, a_{n-1}, n).$$

Here, F is a non-linear operator defining how information folds through itself over discrete layers of recursion. Entropy is no longer a scalar uncertainty, but a measure of local curvature in this recursion manifold.

We may define a *recursive entropy functional*:

$$H_{\text{UNNS}}(n) = -\int \rho(x) \log |\det J_F(x)|\, dx,$$

where J_F is the Jacobian of F. Entropy now quantifies how information expands or contracts through recursive depth—a measure of geometric distortion rather than stochastic ignorance.

3. Non-Orientability and Reversibility

When the recursion substrate resides on a Klein surface, the global orientation of time becomes topologically undefined. Local patches of recursion preserve directionality, but the entire manifold is *non-orientable*. Thus, the global entropy flux satisfies:

$$\oint_{\partial\Sigma} \nabla H \cdot dl = 0,$$

implying that while local entropy can increase or decrease, the net informational flux is self-canceling. Shannon's unidirectional arrow of information is revealed as a projection of a deeper, non-orientable geometry of recursion.

4. Implications: Folding Rather Than Loss

In the UNNS model, information is not lost; it is *folded*. Noise becomes recursive interference, not randomness. Compression is not reduction in code length, but resonance—the synchronization of nested states.

Entropy coding, in this light, is a search for topological equivalence: different recursive trajectories that yield invariant informational structures. Where Shannon's entropy measures *ignorance about state*, UNNS measures *curvature of causality*. The two coincide only when recursion depth is shallow and the system is locally linear.

5. A Recursive Reformulation of the Shannon Functional

The recursive analog of Shannon's entropy may be expressed as:

$$H[F] = -\sum_{n} P(F_n) \log \left| \frac{\partial F_{n+1}}{\partial F_n} \right|.$$

This expression bridges statistical and geometric interpretations. Where Shannon's functional measures statistical uncertainty, this formulation measures the sensitivity of recursive transformation—a kind of *Jacobian entropy* encoding how much information curvature each recursion layer introduces.

6. Philosophical Reflection

In this framework, entropy ceases to be a measure of disorder. It becomes a quantification of how meaning folds into itself through recursive depth. The arrow of time, once a postulate, now emerges as a local consequence of recursion symmetry breaking.

If Shannon's entropy measured the distance between ignorance and message, UNNS entropy measures the curvature between recursion and coherence. One describes communication between separate entities; the other, communication of a system with its own unfolding.

Shannon gave us the map of signals. UNNS reveals the topology of meaning.

References

1. C.E. Shannon, "A Mathematical Theory of Communication," *Bell System Technical Journal*, 1948.
2. UNNS Research Collective, "On the Possibility of Temporal Recursion in the UNNS Substrate," 2025.
3. UNNS Research Collective, "The Klein Surface and Non-Orientable Temporal Dynamics," 2025.