# From Bit to $\tau$ on: Toward a Field Theory of Recursive Information

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#### Abstract

Building upon Shannon's probabilistic foundation and the UNNS reinterpretation of information as recursive curvature, this paper develops a complete field-theoretic description of  $\tau$ ons—the elementary quanta of recursive transformation. The  $\tau$ on Field Equations are extended to a Lagrangian formulation, introducing the  $\tau$ -Field Tensor and its associated energy—momentum tensor. This unifies recursion, geometry, and informational flow under a variational principle of recursive symmetry.

#### 1. From Discrete Information to Recursive Fields

The bit, in Shannon's theory, quantifies uncertainty reduction:

$$H = -\sum_{i} p_i \log_2 p_i.$$

In contrast, the UNNS substrate treats information as a continuous curvature process. Local curvature  $\kappa(n)$  evolves with recursion depth n:

$$\tau = \frac{\Delta \kappa}{\Delta n}.$$

A field of such  $\tau$  ons constitutes the dynamic structure of recursive time.

#### 2. The $\tau$ on Field Tensor

Define two primary vector fields:

$$\vec{\kappa}$$
 (curvature field),  $\vec{\tau}$  (torsion field).

These correspond to "potential" and "kinetic" information states within recursion depth. We define the antisymmetric  $\tau$ -field tensor:

$$\mathcal{T}_{\mu\nu} = \partial_{\mu}\Psi_{\nu} - \partial_{\nu}\Psi_{\mu},$$

where  $\Psi_{\mu} = (\Phi, \vec{\Psi})$  is the recursive information 4-potential and indices  $\mu, \nu$  run over spatial and depth coordinates (x, y, z, n).

In differential form notation:

$$\mathcal{T} = d\Psi$$
.

so  $\mathcal{T}$  encodes local curvature of recursion-space.

### 3. Lagrangian Density

We introduce the  $\tau$ on field Lagrangian:

$$\mathcal{L}_{\tau} = -\frac{1}{4} \mathcal{T}_{\mu\nu} \mathcal{T}^{\mu\nu} + \frac{1}{2} \rho_{\tau} \Phi - \vec{J}_{\tau} \cdot \vec{\Psi}.$$

Here:

- $\mathcal{T}_{\mu\nu}$   $\tau$  on field tensor (recursive curvature flux);
- $\rho_{\tau}$  recursive charge density;
- $\vec{J_{\tau}}$  recursive current vector;
- $\Phi$  and  $\vec{\Psi}$  scalar and vector potentials of recursion.

The first term represents intrinsic curvature energy, while the second and third terms couple the field to recursive sources.

## 4. Variational Principle

Stationarity of the recursive action

$$S = \int \mathcal{L}_{\tau} \, d^3x \, dn$$

under variation of  $\Psi_{\mu}$  yields:

$$\partial_{\nu} \mathcal{T}^{\mu\nu} = J^{\mu}_{\tau}.$$

This reproduces the previously proposed  $\tau$  on field equations in covariant form:

$$\nabla \cdot \vec{\kappa} = \rho_{\tau}, \qquad \nabla \times \vec{\tau} - \frac{\partial \vec{\kappa}}{\partial n} = \vec{J}_{\tau},$$

and their dual conditions from the Bianchi identity:

$$\nabla \cdot \vec{\tau} = 0, \qquad \nabla \times \vec{\kappa} + \frac{\partial \vec{\tau}}{\partial n} = 0.$$

## 5. Energy-Momentum Tensor

The canonical energy–momentum tensor of the  $\tau$ on field is:

$$T^{\mu\nu} = \mathcal{T}^{\mu\alpha}\mathcal{T}^{\nu}{}_{\alpha} - \frac{1}{4}g^{\mu\nu}\mathcal{T}_{\alpha\beta}\mathcal{T}^{\alpha\beta}.$$

This tensor describes recursive energy density, depth flux, and curvature stress.

Its 00-component defines the recursive energy density:

$$\mathcal{E}_{\tau} = \frac{1}{2} \left( |\vec{\kappa}|^2 + |\vec{\tau}|^2 \right),\,$$

and its mixed components represent the recursive Poynting vector:

$$\vec{S}_{\tau} = \vec{\kappa} \times \vec{\tau}.$$

Thus, information propagation manifests as curvature—torsion energy flow in recursion-space.

### 6. Gauge Symmetry and Conservation

Under the recursive gauge transformation:

$$\Psi_{\mu} \to \Psi_{\mu} + \partial_{\mu} \Lambda(n),$$

the tensor  $\mathcal{T}_{\mu\nu}$  and hence  $\mathcal{L}_{\tau}$  remain invariant, establishing the conservation of recursive charge:

$$\partial_{\mu}J_{\tau}^{\mu}=0.$$

This gauge invariance implies the universality of recursion symmetry — the geometric analog of information invariance under representation change.

#### 7. Geometric Unification

The  $\tau$ on field tensor links curvature (geometry) and recursion (process):

$$\mathcal{T}_{\mu\nu} \sim \text{Recursion curvature} \leftrightarrow \text{Information flux.}$$

Local distortions correspond to information flow; global topological invariants correspond to persistent structures—recursive "memory attractors."

#### 8. Recursive Action Integral

The total action of the UNNS recursive field becomes:

$$S_{\text{UNNS}} = \int \left( -\frac{1}{4} \mathcal{T}_{\mu\nu} \mathcal{T}^{\mu\nu} + \mathcal{L}_{\text{interaction}} \right) d^3x \, dn.$$

Variations of this action generate  $\tau$  on field equations, while global minimization corresponds to the self-organizing principle of recursive stability—interpretable as the emergence of meaning through topological persistence.

## 9. Philosophical Synthesis

Where Shannon's bit quantified epistemic reduction, the  $\tau$ on Lagrangian describes ontological recursion. Information ceases to be a static quantity and becomes a dynamic field of self-reflective curvature, obeying conservation, coupling, and symmetry principles akin to those of physics—but unfolding in the manifold of time itself.

Information is not transmitted—it evolves. The  $\tau$  on field is the recursive geometry of meaning.

## 10. Conclusion

The  $\tau$ on Field Tensor Lagrangian completes the mathematical transition from probabilistic to recursive information theory. It introduces a variational foundation for curvature-based information dynamics, extending the idea of entropy to a non-orientable manifold of recursion.

In this formalism:

Bit  $\rightarrow$  Resolution of uncertainty,

 $\tau$ on  $\rightarrow$  Quantum of recursive curvature,

 $\mathcal{T}_{\mu\nu} \to \text{Field tensor of recursive information.}$ 

Shannon's equation measured ignorance; the  $\tau$  on Lagrangian measures becoming.