1 Interpolation of Tensor Calculus with UNNS

Definition 1.1 (UNNS Nest)**.** *A* UNNS nest of order r *is a sequence* (u_n) *satisfying*

$$u_{n+r} = c_1 u_{n+r-1} + \cdots + c_r u_n,$$

for fixed coefficients $c_i \in \mathbb{Z}$ *(or in an algebraic integer ring). The tuple* $(c_1, \ldots, c_r)$ *is called the* coefficient vector.

Lemma 1.2 (Differentiation as Recurrence Expansion)**.** *Let* $f : \mathbb{Z} \to \mathbb{R}$ *be a discrete field on a lattice with mesh size* h*. The forward difference*

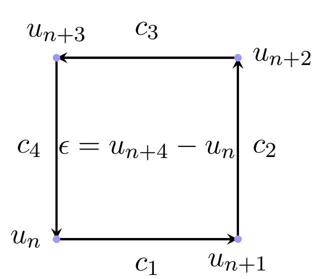
$$\frac{f(x+h) - f(x)}{h}$$

is equivalent to a 2-term UNNS recurrence. Higher derivatives correspond to higher-order recurrences.

Lemma 1.3 (Contraction as Echo Collapse)**.** *Given a tensor* T^{ij} *on a UNNS-labeled lattice, contraction* T^i_i *corresponds to summation of UNNS-propagated components. The resulting sum is accompanied by an* echo residue *measured by the UNNS Paradox Index (UPI), which quantifies the stability of the contraction.*

Theorem 1.4 (Tensor Product as Nest Coupling)**.** *Let* (u_n) *and* (v_n) *be two UNNS nests with coefficient vectors* $(c_1, \ldots, c_r)$ *and* $(d_1, \ldots, d_s)$*. Then their tensor product corresponds to a coupled recurrence of order* $r+s$ *whose coefficients are obtained by Kronecker product of the companion matrices.*

Theorem 1.5 (Curvature as Echo Amplification)**.** *Let* u_n *be a UNNS nest defined on a closed loop in a simplicial complex. Transporting* u_n *around the loop produces an echo residue* ϵ*, given by deviation from initial value. This echo amplification is the discrete analog of curvature* R^i_{jkl} *in tensor calculus.*



Echo amplification around a closed UNNS loop, representing curvature.

Remark 1.6. *In this interpretation:*

- *The metric tensor* g_{ij} *corresponds to the coefficient matrix of a UNNS nest (propagation law).*
- *Christoffel symbols* Γ^k_{ij} *correspond to second-order recurrence couplings across lattice directions.*
- *Curvature* R^i_{jkl} *measures accumulated echo residue along a closed recurrence loop.*