# On the Notion of Structure in the Logic of UNNS

## Introduction

In classical mathematical logic, particularly in model theory, a *structure* is a set equipped with functions, relations, and constants that serve as an interpretation of a given formal language.

The Unbounded Nested Number Sequences (UNNS) Substrate introduces a different notion of structure: not static but recursive, not externally imposed but internally emergent. This note formalizes this concept.

## 1 Classical vs. UNNS Structures

### 1.1 Classical Definition

**Definition 1** (Classical Structure). *Given a formal language $\mathcal{L}$, a structure $\mathcal{M}$ for $\mathcal{L}$ consists of a nonempty set $M$ together with:*

1. *interpretations of constant symbols of $\mathcal{L}$ as elements of $M$,*

2. *interpretations of function symbols of $\mathcal{L}$ as functions $M^k \to M$,*

3. *interpretations of relation symbols of $\mathcal{L}$ as subsets of $M^k$.*

*This is a static notion: the set $M$ and its interpretations are fixed.*

### 1.2 UNNS Definition

**Definition 2** (UNNS Structure). *A UNNS-structure is a quadruple*

$$\mathcal{S} = (A, \mathcal{O}, \mathcal{N}, \mathcal{R})$$

*where:*

- *$A$ is a base alphabet of initial values (seeds),*

- *$\mathcal{O}$ is a set of UNNS operators (Collapse, Inlaying, Inletting, Evaluate, Normalize, etc.),*

- *$\mathcal{N}$ is a nesting-depth function that assigns recursion levels,*

- *$\mathcal{R}$ is a resonance map that measures stabilization or divergence of echoes.*

**Lemma 1** (Existence of Structure). *A UNNS-structure exists if $(A, \mathcal{O}, \mathcal{N}, \mathcal{R})$ admits at least one stable attractor orbit (a fixed point or cycle under recursion).*

## 1.3 Comparison

- Classical structures are **interpretations of axioms**.

- UNNS structures are **attractors of recursion grammars**.

# 2 Dynamics of Structure

In UNNS logic, "structure" is inseparable from the dynamics of recursion:

1. **Divergence**: instability reveals fragility of a structure.

2. **Stabilization**: repair and normalization expose hidden attractors.

3. **Resonance**: operator symmetries reveal spectral invariants.

# 3 Diagrammatic View

# Conclusion

In the logic of UNNS, "structure" is no longer a static set-theoretic interpretation but a dynamic recursive entity. It is an attractor of operators, a resonance basin of recursion, and a stability zone for echoes. This perspective extends mathematical logic into a grammar of recursion, where interpretation and evolution merge into one.