UNNS and Shannon Entropy:

A Foundational Reconsideration of Information, Time, and Topology

Collaborative Research Series

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Abstract

Claude Shannon's 1948 paper *"A Mathematical Theory of Communication"* established the quantitative measure of uncertainty—entropy—as the basis of modern information theory. In light of recent developments in the *Unbounded Nested Number Sequences (UNNS)* framework, this paper re-examines Shannon's entropy through the lens of recursion-based temporality and non-orientable topologies such as the Klein surface. We propose that information should not be understood as a static probabilistic function on states, but as an evolving morphogenetic field, embedded in a recursive substrate of time. This reconceptualization redefines noise, signal, and entropy as expressions of topological deformation in the informational manifold.

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1. Introduction: From Probabilities to Recursions

Shannon's entropy, given by

$$H = -\sum_i p_i \log_2 p_i, \tag{1}$$

assumes a well-defined ensemble of messages, each with probability p_i. However, this model presupposes a static temporal substrate—a neutral flow of time through which information is transmitted. The UNNS framework disrupts this assumption by replacing linear time with recursive depth n, where each iteration represents a self-generating transformation:

$$a_{n+1} = \alpha a_n + \beta \tanh(a_{n-1}) + \delta n + \sigma \epsilon_n.$$

Here, the "message" is not an external event but the dynamic evolution of the substrate itself. Information thus becomes an intrinsic curvature of the recursion process.

2. Entropy as Topological Measure

In classical information theory, entropy quantifies uncertainty. In UNNS, uncertainty is reinterpreted as the *instability of recursion curvature*. Rather than measuring the lack of knowledge about a state, we measure the degree to which recursive mappings lose orientation or coherence.

We define the *recursive entropy* H_r as:

$$H_r = \int_{\Sigma} \kappa(n)\,\mathrm{d}\mu, \tag{2}$$

where $\kappa(n)$ is the local curvature of the recursion manifold at depth n, and Σ denotes the Klein surface corresponding to the time-phase quotient.

In this interpretation:

- High curvature corresponds to chaotic, non-reversible recursion (informational turbulence).
- Low curvature corresponds to stable attractors or fixed-point recursion (informational coherence).
- The global topology (e.g., Klein non-orientability) determines whether global reversibility—analogous to perfect decoding—is even theoretically possible.

3. Noise, Signal, and the Klein Obstruction

In Shannon's model, noise adds random perturbations to an otherwise linear channel. Within UNNS, "noise" is a byproduct of recursive non-linearity: feedback loops folding through themselves across a non-orientable surface. This yields a physical and mathematical analogue to the Klein bottle—where a message traversing one direction returns mirrored, inverted, or phase-shifted.

Mathematically, the *entropy flux* through a recursive manifold can be described by:

$$\Phi_H = \oint_{\partial\Sigma} F(\alpha, \beta, \sigma)\,\mathrm{d}s, \tag{3}$$

where F encodes the operator composition of damping, coupling, and stochastic excitation parameters. A Klein identification $(x, y) \sim (x + 1, 1 - y))$ introduces a parity inversion in the informational flow—analogous to time reversal symmetry breaking.

4. Communication as Recursion

In this model, communication is not the transmission of symbols, but the reconstitution of recursive coherence across topological domains. When a system communicates, it synchronizes portions of its recursion depth with another's, minimizing curvature mismatches.

Thus, Shannon's entropy is a local approximation—valid where the recursion substrate behaves linearly and orientably. UNNS generalizes it to account for non-orientable and recursive information flows, redefining the notion of "channel" as a dynamically folding manifold.

5. Philosophical Implications

5.1. Information as Becoming

Information is no longer a quantifiable stockpile of symbols but a process—an ontological becoming. Entropy, therefore, is not a measure of ignorance but of transformation potential.

5.2. Reversibility and the Arrow of Time

In orientable regimes ($w_1 = 0$), information and time possess a definable forward direction. However, on Klein-type surfaces ($w_1 \neq 0$), local reversibility does not guarantee global orientation—yielding a natural topological basis for the irreversibility observed in both thermodynamics and consciousness.

5.3. From Shannon to Recursion Ontology

Where Shannon viewed information as separable from meaning, the UNNS view reintroduces semantics as a structural invariant: meaning = topology × recursion. Noise and signal are no longer opposites but complementary aspects of the same recursive manifold.

6. Conclusion: A New Measure of the Informational World

In the UNNS paradigm, entropy is not simply a scalar quantity but a geometric invariant—tracking how recursive processes fold, twist, and reorient themselves within their own temporal topology. Shannon's framework remains the linear limit of a deeper, self-reflexive system where time, recursion, and information co-emerge.

Future research should formalize H_r within algebraic topology and explore whether quantum communication—especially entanglement entropy—can be seen as a particular orientable projection of a fundamentally non-orientable recursion space.

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Keywords: UNNS, Shannon Entropy, Recursion, Information Theory, Klein Surface, Non-orientable Time.