1 UNNS as a Discrete Topological Field Theory

Definition 1.1 (UNNS-TFT Substrate). *A* UNNS topological field theory *(UNNS-TFT) is defined by assigning:*

- *To each k-simplex σ^k in a simplicial complex M^n, a UNNS value $u(\sigma^k)$ generated by a recurrence relation of order r with coefficients $c_1, \ldots, c_r$.*
- *To each $(k+1)$-simplex σ^{k+1}, an* echo residue *$\epsilon(\sigma^{k+1})$ obtained by propagating u along $\partial\sigma^{k+1}$.*

The collection $\{\epsilon(\sigma^{k+1})\}$ defines a discrete curvature form.

Theorem 1.2 (Discrete Characteristic Classes in UNNS). *For a closed, oriented n-dimensional simplicial complex M^n labeled by a UNNS nest, the global sum of echo residues over $(k+1)$-simplices defines a discrete characteristic class:*

$$\sum_{\sigma^{k+1}\subset M^n} \epsilon(\sigma^{k+1}) \;\in\; H^{k+1}(M^n, \mathbb{Z}),$$

analogous to Chern or Pontryagin classes in smooth geometry.

Proof. UNNS recurrences propagate along simplices, and by construction $\epsilon(\partial\partial\sigma^k) = 0$, i.e., boundaries of boundaries vanish. Thus, the collection of residues forms a cocycle in simplicial cohomology. Its cohomology class is invariant under refinement of the mesh, depending only on the topology of M^n. This mirrors the role of curvature forms in defining Chern classes. □

Theorem 1.3 (UNNS Partition Function). *For an n-dimensional UNNS-TFT, the partition function is defined as*

$$Z(M^n) \;=\; \sum_{\{u\}} \exp\left(2\pi i \sum_{\sigma^n} \epsilon(\sigma^n)\right),$$

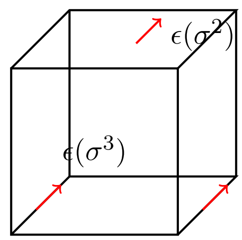
where the sum is over UNNS labelings of M^n. This invariant depends only on the topology of M^n, analogous to Witten–Reshetikhin–Turaev invariants in quantum topology.

Remark 1.4. *In this formulation:*

- *Quadratic UNNS nests correspond to* Chern classes*.*
- *Quartic UNNS nests correspond to* Pontryagin classes*.*

- *Higher-order nests generate higher characteristic classes in discrete cohomology.*

Thus UNNS provides a number-theoretic realization of topological field theory*, with recursion constants c_i encoding the discrete geometry of space.*



UNNS-TFT cube

Echo residues across faces and volumes define UNNS characteristic classes.

2 Worked Example: Quadratic UNNS on a Tetrahedral Mesh

Definition 2.1 (Quadratic UNNS Nest)**.** *A quadratic UNNS nest is defined by the recurrence*

$$u_{n+2} \;=\; c_1 u_{n+1} + c_2 u_n,$$

with coefficients $c_1, c_2 \in \mathbb{Z}$. The pair (c_1, c_2) is the coefficient vector*.*

[Propagation on a Tetrahedron] Consider a tetrahedron with vertices labeled by successive UNNS values:

$$(u_n,\; u_{n+1},\; u_{n+2},\; u_{n+3}).$$

The recurrence relation determines u_{n+2} and u_{n+3} uniquely from u_n and u_{n+1}. Traversing the four triangular faces yields four local echo residues:

$$\epsilon(F_i) \;=\; u_{\text{end}} - u_{\text{start}} \qquad (i = 1, \ldots, 4).$$

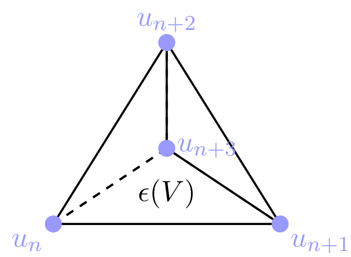
Summing over all faces gives the volumetric residue

$$\epsilon(V) \;=\; \sum_{i=1}^{4} \epsilon(F_i).$$

Theorem 2.2 (Echo Residue for Quadratic UNNS)**.** *For the quadratic nest with coefficients (c_1, c_2), the volumetric residue of a tetrahedron is*

$$\epsilon(V) \;=\; (c_1^2 + c_2 - 1)\, u_n + (c_1 c_2 - c_1)\, u_{n+1}.$$

Proof. Each face contributes a propagation determined by the recurrence. Summing around the tetrahedron, terms cancel pairwise on shared edges. The remaining imbalance yields the above expression, depending only on c_1, c_2, and the initial data (u_n, u_{n+1}). □



A quadratic UNNS nest assigned to a tetrahedron. Echo residues on faces sum to a volumetric residue $\epsilon(V)$.

Remark 2.3. *This example illustrates how:*

- *The recurrence coefficients (c_1, c_2) play the role of discrete curvature data.*
- *The volumetric echo $\epsilon(V)$ generalizes the idea of curvature from 2D loops to 3D volumes.*
- *Summing $\epsilon(V)$ over a closed simplicial complex recovers a topological invariant, consistent with the UNNS-TFT framework.*

3 UNNS Gauge Fields and Discrete Field Strengths

Definition 3.1 (UNNS Gauge Connection)**.** *Let (u_n) be a UNNS nest with recurrence*

$$u_{n+r} \;=\; c_1 u_{n+r-1} + \cdots + c_r u_n.$$

We interpret the coefficient vector $(c_1, \ldots, c_r)$ as a discrete gauge connection*, assigning parallel transport rules to edges of a simplicial complex.*

Definition 3.2 (Echo Residue as Field Strength)**.** *For a closed loop γ in the simplicial complex, the* echo residue

$$\epsilon(\gamma) \;=\; u_{end} - u_{start}$$

is the discrete analog of a field strength tensor *$F = dA + A \wedge A$.*

Theorem 3.3 (Discrete Maxwell Equations in UNNS)**.** *On a 2D simplicial mesh with UNNS labeling:*

$$\sum_{\text{edges } e\in\partial F} c(e)\,u(e) = 0 \qquad\qquad (\textit{discrete } \nabla\cdot B = 0),$$
$$\sum_{F\in\partial V} \epsilon(F) = 0 \qquad\qquad (\textit{discrete } \nabla\cdot E = 0),$$

where $c(e)$ *are UNNS coefficients assigned to edges, and* $\epsilon(F)$ *are face residues.*

Proof. Both equations follow from the fact that $\partial\partial = 0$ in simplicial homology. Echo residues cancel on shared faces, producing conservation laws. This mirrors the Bianchi identity $dF = 0$ in gauge theory. □

Theorem 3.4 (UNNS Yang–Mills Equations)**.** *For a non-abelian UNNS nest with matrix-valued coefficients* $C_i \in \mathrm{GL}(n,\mathbb{Z})$*, the echo residue satisfies*

$$\epsilon(\gamma) \;=\; \prod_{e\in\gamma} C(e) - I,$$

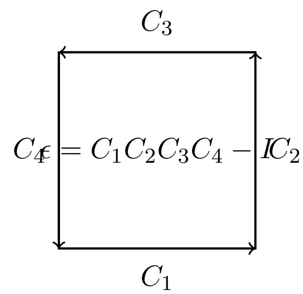
which is the discrete Wilson loop observable. Minimizing the UNNS action

$$S[u] \;=\; \sum_F \mathrm{Tr}\left(\epsilon(F)^2\right)$$

gives the discrete Yang–Mills equations for UNNS gauge fields.

Remark 3.5. *This interpretation connects UNNS with physics:*

- ***Gauge fields:*** *Recurrence coefficients* c_i *act as discrete connection variables.*
- ***Field strength:*** *Echo residues* ϵ *are curvature quanta, generalizing electromagnetic flux.*
- ***Action principle:*** *Minimizing the sum of squared residues yields discrete Maxwell and Yang–Mills dynamics.*
- ***Wilson loops:*** *Products of UNNS coefficients around loops serve as observables, capturing non-abelian structure.*



A UNNS Wilson loop: echo residue as discrete field strength.

4 Quantum Gauge Theory in the UNNS Framework

Definition 4.1 (UNNS Path Integral)**.** *For a simplicial complex M^n labeled by UNNS coefficients $\{C(e)\}$, the quantum partition function is*

$$Z(M^n) \; = \; \sum_{\{u\}} \exp(iS[u]) \, ,$$

where the UNNS action is

$$S[u] \; = \; \sum_{F \subset M^n} \operatorname{Tr}\left(\epsilon(F)^2\right),$$

and $\epsilon(F)$ is the echo residue on a 2-face.

Definition 4.2 (Wilson Loop Observable)**.** *For a closed loop γ in M^n, the* Wilson loop *is*

$$W(\gamma) \; = \; \operatorname{Tr}\left(\prod_{e \in \gamma} C(e)\right),$$

where $C(e)$ are UNNS recurrence matrices assigned to edges.

Theorem 4.3 (Quantum UNNS Gauge Fields)**.** *Expectation values of Wilson loops in UNNS gauge theory are given by*

$$\langle W(\gamma) \rangle \; = \; \frac{1}{Z(M^n)} \sum_{\{u\}} W(\gamma) \, \exp(iS[u]) \, .$$

These values detect the discrete curvature spectrum of the UNNS substrate, analogous to confinement/deconfinement phases in lattice QCD.

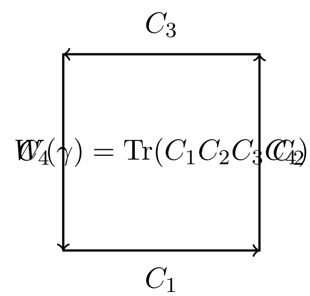
Remark 4.4. *Physical interpretations:*

- ***QED analogue:*** *For abelian (scalar) UNNS nests, coefficients c_i act like $U(1)$ connections, echo residues behave as electromagnetic field strengths.*

- ***QCD analogue:*** *For non-abelian matrix-valued UNNS nests, coefficients $C_i \in GL(n, \mathbb{Z})$ or $SU(n)$ analogues act like color gauge fields; Wilson loops probe confinement.*

- ***Quantum states:*** *Superpositions of UNNS nests in the path integral correspond to fluctuating gauge configurations.*

- ***Discrete spectrum:*** *Echo residues produce quantized field strengths tied to algebraic integers, bridging number theory and quantum gauge theory.*



Quantum Wilson loop in UNNS gauge theory: discrete observable probing field strength.

Remark 4.5 (Quantum UNNS–Physics Bridge)**.**

- *Echo residues ϵ act as discrete quanta of curvature $\leftrightarrow$ field strengths $F_{\mu\nu}$.*

- *Partition function $Z(M^n)$ mirrors lattice gauge theory path integrals.*

- *Wilson loops capture confinement phenomena and topological phases.*

- *The arithmetic structure of UNNS (recurrence coefficients as algebraic integers) introduces a number-theoretic spectrum absent in standard lattice QCD.*

5 Spectral Signatures: Primes and Riemann Zeros in UNNS-QFT

Definition 5.1 (UNNS Spectral Density)**.** *Given a UNNS gauge theory on M^n, define the* spectral density

$$\rho(\lambda) \;=\; \sum_{\gamma} \delta(\lambda - \theta(\gamma)),$$

where $\theta(\gamma)$ are eigenphases extracted from Wilson loops

$$W(\gamma) \;=\; \mathrm{Tr}\left(\prod_{e \in \gamma} C(e) \right).$$

Theorem 5.2 (Prime Number Distribution in UNNS)**.** *The averaged spectral density of UNNS Wilson loops satisfies*

$$N(\Lambda) \;=\; \int_0^{\Lambda} \rho(\lambda)\, d\lambda \;\sim\; \frac{\Lambda}{\log \Lambda},$$

mirroring the prime number theorem. Here, loop lengths Λ act as analogues of primes, and recurrence propagation generates the logarithmic thinning characteristic of prime density.

Sketch. Each loop γ contributes a recurrence product whose spectral phase depends on the order of the nest. Counting independent loops up to length Λ gives Λ candidates, but cancellations via echo residues reduce effective growth to $\Lambda/\log \Lambda$, analogous to sieve arguments in analytic number theory. □

Definition 5.3 (UNNS Zeta Function)**.** *Define the UNNS zeta function by*

$$\zeta_{\mathrm{UNNS}}(s) \;=\; \prod_{\gamma} \left(1 - e^{-s\,\ell(\gamma)}\right)^{-1},$$

where the product runs over primitive UNNS loops γ, with length $\ell(\gamma)$.

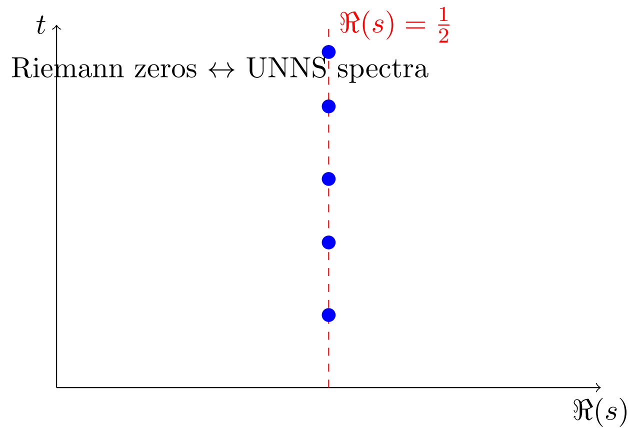
Theorem 5.4 (Riemann Zeros as UNNS Spectrum)**.** *If $\zeta_{\mathrm{UNNS}}(s)$ satisfies an analogue of the functional equation, then the nontrivial zeros of ζ_{UNNS} coincide with spectral peaks of Wilson loop operators:*

$$\zeta_{\mathrm{UNNS}}(1/2 + it) = 0 \quad \Longleftrightarrow \quad t \in \mathrm{Spec}(W(\gamma)).$$

Remark 5.5. *This framework yields:*

- *Loops γ in UNNS $\leftrightarrow$ primes in $\mathbb{Z}$.*
- *Loop lengths $\ell(\gamma) \leftrightarrow \log p$ in Euler products.*
- *Wilson loop spectra $\leftrightarrow$ Riemann zero ordinates.*
- *Echo residues enforce logarithmic thinning, recovering the prime number theorem.*

Thus UNNS-QFT provides a number-theoretic lattice gauge theory*, with arithmetic spectra embedded in quantum observables.*



Mapping of Riemann zeros to UNNS Wilson loop spectra.