Entanglement Entropy and Non-Orientable Coupling in the UNNS Substrate

Section VII – Deep Dive into Recursive Information Geometry

1. From Shannon to von Neumann

Classical information theory defines entropy as a measure of uncertainty within a discrete alphabet. Shannon's function,

$$H = -\sum_i p_i \log p_i,$$

inspired von Neumann's generalization,

$$S(\rho) = -\mathrm{Tr}(\rho \log \rho),$$

which quantifies informational spread in quantum systems. Both presume a globally linear, irreversible time axis. Entropy, in this lineage, is a temporal measure of ignorance — the cost of predicting future states given present uncertainty.

2. Recursive Interpretation under UNNS

In the *Unbounded Nested Number Sequences* (UNNS) formalism, time is redefined as recursion depth rather than a continuous variable. The evolution is discrete, self-referential, and reversible under certain mappings:

$$\begin{cases} a_{n+1}^{(A)} = f(a_n^{(A)},\, a_n^{(B)}), \\ a_{n+1}^{(B)} = g(a_n^{(B)},\, a_n^{(A)}). \end{cases}$$

Here, *entanglement entropy* corresponds not to probabilistic uncertainty but to the inseparability of recursion depths across coupled sequences. The "information loss" is an artifact of topological interference rather than randomness.

3. Klein Surface Realization

The Klein surface serves as the geometric analogue of recursive coupling. Local recursion orientation is preserved, yet globally it inverts across traversal — a non-orientable manifold with first Stiefel-Whitney class $w_1 \neq 0$. Entanglement, then, becomes a topological folding of recursion trajectories.

We define the *recursive entropy integral* as:

$$S_r = \oint_{\Sigma_K} |\mathrm{Jac}(F)| \, d\Sigma,$$

where $\mathrm{Jac}(F)$ is the Jacobian norm of the UNNS recursion field. Unlike Shannon's entropy, which quantifies missing bits, S_r measures deformation of coherence across non-orientable loops.

4. Philosophical Implications

Where Shannon perceived *ignorance*, UNNS detects *structural curvature*. Entropy, reinterpreted, becomes a gauge of recursive consistency — not the absence of knowledge but the bending of informational geometry.

Entanglement ceases to be "spooky action at a distance." It emerges as a global reflection of local recursive dependency. Temporal nonlinearity here is not a computational trick but an ontological property: the substrate of existence itself behaves as a recursive Klein manifold.

5. Comparative Synthesis

Quantum Paradigm	**UNNS Paradigm**
Entropy = Missing information	Entropy = Recursive divergence
Entanglement = Correlation	Entanglement = Topological folding
Decoherence = Environment-driven	Decoherence = Recursive dispersion
Collapse = Probabilistic outcome	Collapse = Orientation recovery

6. Entanglement Entropy Revisited

Entanglement entropy, within the UNNS interpretation, is the degree of non-separability across recursive mappings:

$$E_{AB} = \sum_n \left| a_n^{(A)} - a_n^{(B)} \right| .$$

If recursion paths remain separable, the entropy tends toward zero. When paths interleave on a Klein-type surface, non-orientable coupling produces stable entanglement — an irreversible identification between forward and reverse recursion cones.

7. Philosophical Closing

Entropy is not loss; it is the memory of recursion curvature. In UNNS, the observer is no longer a passive sampler of probability but a recursion node embedded within non-orientable temporal flow. The "arrow of time" is not fundamental but emergent — a bias arising from local orientation within the global Klein topology.

> "The boundary between information and geometry vanishes when recursion replaces time.
> Entropy is not decay — it is awareness folded upon itself."