UNNS Repair Rules and Normalization:
DNA-Inspired Stability Protocols for Recursive Substrates

UNNS Research Notes

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Abstract

Unbounded Nested Number Sequences (UNNS) provide a recursive substrate for modeling algebraic, combinatorial, and topological structures. Like DNA, which maintains fidelity through sophisticated repair mechanisms, UNNS requires stability protocols to preserve coherence under perturbations, noise, and paradox-inducing growth. This paper formalizes a system of *repair rules and normalization* for UNNS, explicitly paralleling DNA proofreading, excision repair, mismatch correction, homologous recombination, and global normalization. Mathematical operators, stability lemmas, and algorithmic recipes are given, together with biological analogies that motivate their design.

Contents

1 Introduction

DNA encodes life through recursive structures (nucleotide sequences), yet its survival depends critically on repair mechanisms. Without proofreading, excision, mismatch repair, and normalization, errors would accumulate and collapse genomic stability. Similarly, the UNNS substrate—a recursive medium of number sequences, nested lattices, and echo residues—requires mechanisms to detect and correct instability.

The purpose of this paper is to present a detailed, formal system of *UNNS repair rules and normalization* inspired by DNA repair. This provides both mathematical stability and a conceptual bridge between computation, biology, and topology.

2 Preliminaries

Definition 2.1 (UNNS substrate). *A* UNNS substrate $\mathcal{U}$ *consists of sequences or mesh-labeled structures generated by finite-order recurrences of the form*

$$u_{n+r} = \sum_{j=1}^{r} c_j u_{n+r-j},$$

together with derived diagnostics such as local residues, growth factors, and the UNNS Paradox Index (UPI).

Definition 2.2 (Residue). *Given predicted value $\widetilde{u}_i$ from recurrence coefficients and actual stored value u_i, the* residue *is $r_i = \widetilde{u}_i - u_i$. Large residues indicate instability or corruption.*

3 DNA-Inspired Repair Rules

3.1 Proofreading

Definition 3.1 (Proofreading operator). *For index i and parameter $\eta \in (0, 1]$, define*

$$\mathcal{R}_{pf}(i, \eta) : \quad u_i \leftarrow (1 - \eta)u_i + \eta\widetilde{u}_i.$$

Proposition 3.2 (Residue contraction). *After $\mathcal{R}_{pf}(i, \eta)$, the residue becomes $r_i^{new} = (1 - \eta)r_i$. Thus $|r_i|$ decreases strictly unless $\eta = 0$.*

Remark 3.3. *Biological analogy: DNA polymerase proofreading replaces a mismatched base with the correct prediction.*

3.2 Excision and Refit

Definition 3.4 (Excision operator). *Let $I = [a, b]$ be a contiguous index set. The operator $\mathcal{R}_{ex}(I)$ removes values $u_j, j \in I$ and reconstructs them by fitting a local recurrence to data outside I (regularized least squares).*

Remark 3.5. *Biological analogy: Nucleotide excision repair cuts out a damaged DNA fragment and resynthesizes it.*

3.3 Mismatch Projection

Definition 3.6 (Projection operator). *Given ring R (e.g. Gaussian integers $\mathbb{Z}[i]$), define*

$$\mathcal{R}_{pr}(c) = \operatorname*{argmin}_{r \in R} |c - r|.$$

Remark 3.7. *Biological analogy: Mismatch repair aligns mispaired bases to the canonical alphabet $\{A, T, C, G\}$. In UNNS, coefficients are projected to admissible algebraic integers.*

3.4 Homologous Replacement

Definition 3.8 (Homologous replacement)**.** *Given stable motif P and unstable region L, the operator $\mathcal{R}_{hr}(P \to L)$ replaces L with an aligned copy of P (possibly scaled/rotated).*

Remark 3.9. *Biological analogy: Homologous recombination uses an intact sister chromatid to repair a damaged region.*

3.5 Global Renormalization

Definition 3.10 (Global normalization)**.** *Let $\{u_1, \dots, u_N\}$ be a finite block. Define scaling*

$$u_j \leftarrow \lambda u_j, \qquad \lambda = \frac{1}{\sqrt{\frac{1}{N}\sum_{j=1}^{N} |u_j|^2}}.$$

Remark 3.11. *Biological analogy: Chromatin remodeling normalizes packing density, preventing runaway growth of transcriptional noise.*

4 Mathematical Properties

Lemma 4.1 (Monotone residue reduction)**.** *If $\mathcal{R}_{pf}(i, \eta)$ is applied repeatedly, residue r_i converges geometrically to zero.*

Proposition 4.2 (Stability of global normalization)**.** *If $\{u_j\}$ has bounded variance, then after $\mathcal{R}_{gn}$ the block has unit variance and average energy is normalized.*

Remark 4.3. *Excision and homologous replacement can be shown to decrease global energy norms under mild assumptions, though their exact effect depends on chosen fitting models.*

5 Algorithmic Repair Policy

1. Monitor residues and UPI.
2. If local residue $|r_i| > \tau_1$, apply proofreading $\mathcal{R}_{pf}$.
3. If block I exceeds threshold τ_2, apply excision $\mathcal{R}_{ex}$ or homologous replacement $\mathcal{R}_{hr}$.
4. If coefficients drift outside ring R, apply projection $\mathcal{R}_{pr}$.
5. Periodically apply global normalization $\mathcal{R}_{gn}$ to prevent divergence.

6 Unified Diagram

7 Conclusion

UNNS repair rules, inspired by DNA stability protocols, provide a robust framework for recursive number substrates. Proofreading, excision, projection, homologous replacement, and normalization collectively prevent instability, just as biological repair preserves genomic fidelity. This analogy is not merely metaphorical: both DNA and UNNS are recursive information media, and their survival depends on active correction.

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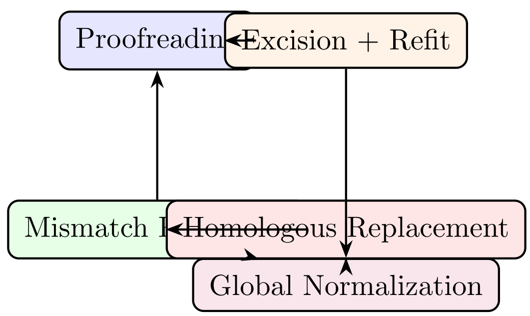

Figure 1: DNA-inspired UNNS repair and normalization rules in cyclic interplay.