1 Cantor Sets in the UNNS Framework

Cantor sets, traditionally defined by recursive removal of middle thirds, can be interpreted in the UNNS substrate as recursive nests with digit restrictions. This provides a bridge between classical fractals and recursive constants.

1.1 Set-Theoretic Cantor Set

The standard Cantor set $C \subset [0, 1]$ is defined by:

$$C = \left\{ \sum_{n=1}^{\infty} \frac{a_n}{3^n} \;\middle|\; a_n \in \{0, 2\} \right\}.$$

Equivalently, C is obtained by repeatedly removing the open middle third interval at each recursive step.

1.2 UNNS Interpretation

Definition 1.1 (Cantor UNNS Nest)**.** *Let (u_n) be a UNNS generated by the ternary recurrence*

$$u_{n+1} = \frac{u_n}{3} + \frac{d_n}{3}, \quad d_n \in \{0, 2\}.$$

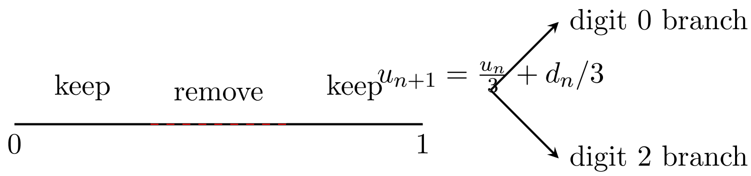
The set of all limit points of such sequences corresponds exactly to the classical Cantor set C.

Remark 1.1. *In this view, Cantor sets arise as* branching nests *within UNNS, where allowed coefficients d_n encode the ternary digit restriction. The "fractal dust" corresponds to the resonant residues of recursive propagation.*

1.3 Constants Involved

- **Nest Depth Constant (D):** depth of removal $\leftrightarrow$ depth of recursion.
- **Coefficient Constants (c_i):** here $c = (1/3)$ with digit restriction $\{0, 2\}$.
- **Paradox Index (UPI):** grows with branching density; ambiguity (e.g. $0.222\ldots = 1.000\ldots$) is quantified.
- **Gödel Constant:** guarantees undecidable membership cases at boundaries.

1.4 Illustration



Conclusion

In the UNNS framework, Cantor sets are not just topological curiosities but constructive nests: recursive restrictions on coefficients yield fractal attractors, where constants such as D, c_i, UPI, and the Gödel constant quantify their structure.