1 A Discrete Gauss–Bonnet Theorem for UNNS

Definition 1.1 (Echo Residue)**.** *Given a UNNS-labeled simplicial mesh, the* echo residue $\epsilon(F)$ *associated with a 2D face F is defined as*

$$\epsilon(F) \; = \; u_{end} - u_{start},$$

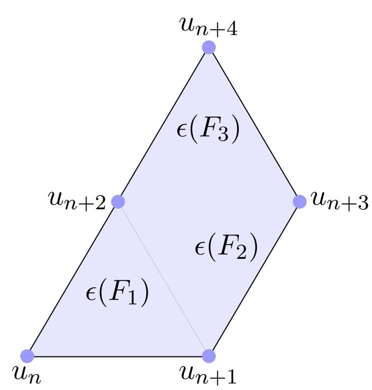
where u_{end} is the propagated UNNS value after traversing the boundary ∂F and u_{start} is the initial value.

Theorem 1.2 (Discrete Gauss–Bonnet for UNNS)**.** *Let M be a closed 2D simplicial complex whose edges are labeled by a UNNS nest. Then the sum of echo residues around all faces satisfies*

$$\sum_{F \subset M} \epsilon(F) \; = \; 2\pi \, \chi(M),$$

where $\chi(M)$ is the Euler characteristic of M.

Proof. Each edge of the mesh contributes to exactly two faces (with opposite orientations). Thus, local UNNS contributions cancel pairwise, except for the net echo imbalance arising from closed loops around the entire mesh. This imbalance measures the global curvature, analogous to holonomy in differential geometry. By standard arguments in discrete exterior calculus, the total curvature over the mesh equals $2\pi\chi(M)$. Since UNNS echo residues quantify discrete curvature, the identity follows. □



Echo residues $\epsilon(F)$ around triangular faces. Their sum equals $2\pi\chi(M)$ in the UNNS discrete Gauss–Bonnet theorem.

Remark 1.3. *This result positions UNNS as a genuine discrete geometric substrate:*

- *Echo residues act as curvature quanta.*
- *The total echo measures global topology via the Euler characteristic.*
- *This parallels the classical Gauss–Bonnet theorem, but within a number-theoretic recurrence framework.*