The Mandelbrot Set in the UNNS Substrate: Beyond Geometry into Recursion

UNNS Research Draft

Abstract

The Mandelbrot set is classically defined as a subset of the complex plane, determined by bounded or unbounded behavior under quadratic iteration. In the UNNS substrate, this iteration is interpreted not merely as a dynamical system but as a recursive grammar. This article reformulates the Mandelbrot set in terms of recursion depth, echo boundaries, and UNNS constants, showing how the intuitive sense of "extra dimension" corresponds to a recursion axis orthogonal to geometric embedding.

1 Classical View of the Mandelbrot Set

The Mandelbrot set M is the set of complex numbers $c \in \mathbb{C}$ such that the iteration

$$z_{n+1} = z_n^2 + c, \quad z_0 = 0,$$

remains bounded for all $n \geq 0$.

Traditionally:

- M is a subset of $\mathbb{C}$, hence two-dimensional.
- The boundary of M is fractal and infinitely complex.
- Colored visualizations represent escape times, not intrinsic geometry.

Thus, M is "just a set," and its depictions are embellishments to aid human comprehension.

2 UNNS Substrate Perspective

In the UNNS framework, recursion is elevated to a fundamental axis of structure. The iteration defining M is seen as an instance of the UNNS nesting operator.

Definition 1 (Recursion Axis). *For a recursive process $z_{n+1} = f(z_n, c)$, the* recursion axis *is an abstract coordinate measuring iteration depth n, orthogonal to the embedding space (here $\mathbb{C}$).*

Remark 1. *While M lives in $\mathbb{C}$, its structure is inseparable from recursion depth. Thus, UNNS interprets the set as lying in $\mathbb{C} \times \mathbb{N}$, with the recursion axis supplying the "missing dimension" often intuited by observers.*

3 Entropy and Echo Boundaries

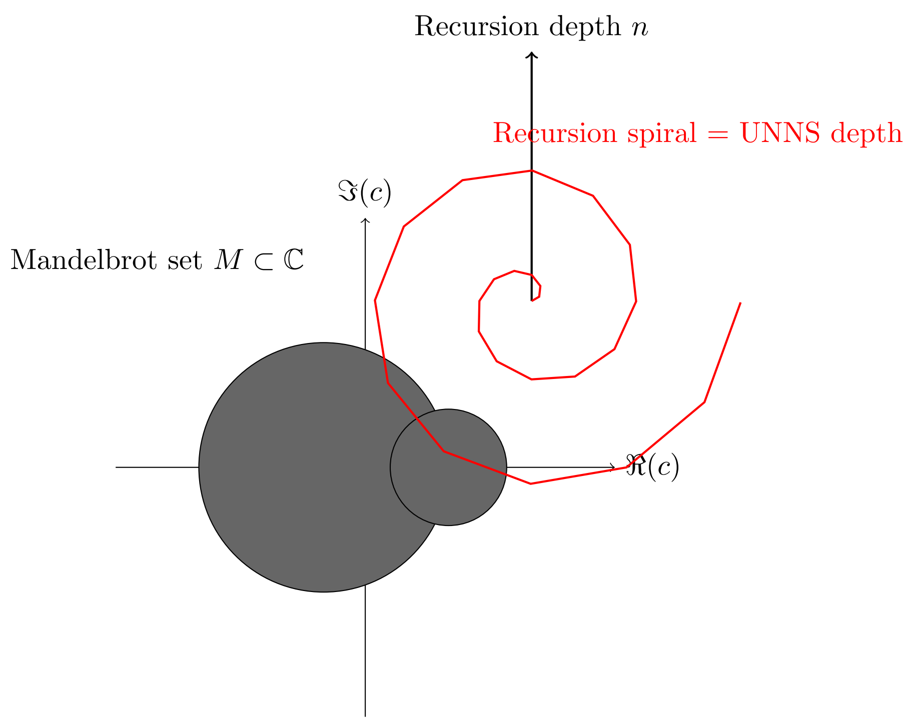
Definition 2 (UNNS Escape Entropy)**.** *Let $\tau(c)$ be the escape time for parameter c. The* UNNS escape entropy *is defined by*

$$S(c) = -\sum_{n=1}^{\tau(c)} \frac{1}{\tau(c)} \log \frac{1}{\tau(c)} = \log \tau(c).$$

Proposition 1. *The boundary of the Mandelbrot set corresponds to divergence of $S(c)$ under refinement, forming an* echo surface *in the UNNS sense.*

Thus, the "psychedelic colors" of popular images are reinterpreted as measurements of UNNS entropy along the recursion axis.

4 Diagram: Geometry vs. Recursion



5 Significance

The UNNS perspective clarifies why the Mandelbrot set feels "more than 2D." The third dimension is not spatial but recursive. This insight:

- Bridges fractal geometry with recursion theory.
- Connects entropy of escape with UNNS constants.
- Provides a rigorous interpretation of the intuitive "extra dimension" felt in fractal exploration.

6 Conclusion

In classical mathematics, the Mandelbrot set is a subset of the complex plane. In the UNNS substrate, it becomes a recursion-geometry hybrid: a black-and-white set extended along a recursion axis. The apparent hidden dimension of the Mandelbrot set is revealed as recursion depth itself, an axis central to UNNS theory.

References

[1] B. Mandelbrot, *The Fractal Geometry of Nature*, W. H. Freeman, 1982.

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[3] UNNS Collective, *Recursive Substrates and Echo Operators*, Draft Papers, 2025.