UNNS vs Classical Dynamics: A Comparative Theoretical Framework

with a Worked Example Appendix

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Abstract

Classical mechanics describes motion via continuous differential equations, whereas the Unbounded Nested Number Sequences (UNNS) substrate generates motion through recursive operators on discrete layers. We formalize a mapping between the two formalisms, prove a continuum limit theorem, and show how stabilization, divergence, and resonance appear in UNNS terms. A comparative diagram illustrates trajectories; a worked example appendix provides concrete numeric comparisons (ranges, flight times, and energy profiles) under identical initial data.

1 Introduction

Classical mechanics treats time and space as continuous and solves motion with differential equations. UNNS, by contrast, uses recursion to generate states layer-by-layer. The central thesis of this paper is that smooth classical behavior emerges as a limiting regime of stable recursion, and that UNNS augments the modeling palette with operator-level constructs (damping, entropy drift, lattice inlaying, collapse thresholds) that can encode discreteness and non-equilibrium features.

2 Ontological Foundations

Aspect	**Classical Mechanics**	**UNNS Substrate**
Ontology	Continuous space-time manifold	Recursive lattice of number layers
Mathematical Law	Differential equation	Recursive morphism (difference relation)
Time variable	$t \in \mathbb{R}$	depth $n \in \mathbb{N}$
Space	Smooth coordinates	Hierarchical inlayed lattices
Energy	$E(x, \dot{x})$	Morphic potential $P_n = a_n^2/\varepsilon_n$
Entropy	Thermodynamic scalar	Instability curvature ε_n

3 Classical Dynamics

Newton's law

$$m\frac{d^2x}{dt^2} = F(x, \dot{x}, t) \tag{1}$$

is deterministic, continuous, and local. For a projectile launched from ground ($y_0 = 0$) at speed V_0 and angle α in a uniform gravity field g, the (vacuum) range is

$$R_{\text{classical}} = \frac{V_0^2}{g}\sin(2\alpha). \tag{2}$$

4 UNNS Recursive Dynamics

In UNNS, a state evolves by recursion:

$$a_{n+1} = f(a_n, a_{n-1}; \varepsilon_n, \text{operators}), \tag{3}$$

with layer-wise operators (e.g., damping, drift, lattice inlaying) controlling stability and geometry.

4.1 UNNS Projectile Form

Under constant morphism M, orientation θ, damping $\delta \geq 0$, and (constant) entropy curvature $\varepsilon > 0$, a scalar *projective echo* can be summarized as

$$a_{n+1} = M \sin(2\theta)\, e^{-\delta n}, \qquad R_{\text{UNNS}} \;=\; \frac{M^2 \sin(2\theta)}{\varepsilon}\, e^{-\delta n_*}, \tag{4}$$

where n_* is an effective depth (or the depth at which impact/termination occurs in the recursion). This mirrors the classical range formula, but in a discrete substrate.

5 Mapping of Variables

Quantity	**Classical**	**UNNS Analog**
Position	$x(t)$	a_n
Velocity	$\dot{x}(t)$	$\Delta a_n = a_{n+1} - a_n$
Acceleration	$\ddot{x}(t)$	$\Delta^2 a_n$
Force	$F(x)$	Operator $O(a_n)$
Energy	E	$P_n = a_n^2/\varepsilon_n$
Time	t	Depth n

6 Formal Results

Definition 1 (Morphic Energy)**.** *For a recursion a_n, the morphic energy is*

$$P_n \;=\; \frac{a_n^2}{\varepsilon_n}, \tag{5}$$

with $\varepsilon_n > 0$ ensuring local stability. Smaller ε_n amplifies P_n for fixed a_n.

Theorem 1 (Continuum Limit)**.** *Let a_n satisfy*

$$a_{n+1} - 2a_n + a_{n-1} \;=\; \Delta^2 f(a_n), \tag{6}$$

with a smooth f and $\Delta \to 0$. Define $x(t_n) = a_n$ with $t_n = n\Delta$. Then x solves

$$\frac{d^2x}{dt^2} = f(x) \tag{7}$$

in the limit $\Delta \to 0$. Thus Newtonian dynamics is the continuum limit of the UNNS recursion.

Proof. Taylor expand $a_{n\pm1} = x(t) \pm \Delta\dot{x}(t) + \frac{\Delta^2}{2}\ddot{x}(t) + O(\Delta^3)$. Hence

$$a_{n+1} - 2a_n + a_{n-1} = \Delta^2 \ddot{x}(t) + O(\Delta^4).$$

Equating with $\Delta^2 f(a_n) = \Delta^2 f(x(t)) + O(\Delta^2)$ and dividing by Δ^2 yields $\ddot{x}(t) = f(x(t))$ as $\Delta \to 0$. $\square$

Lemma 1 (Stability vs Resonance)**.** *Let a^* be a fixed point of $a_{n+1} = g(a_n)$ with $g'(a^*) = \lambda$. If $|\lambda| < 1$, the recursion is locally stable (echoes decay). If $|\lambda| > 1$, echoes amplify (morphic resonance).*

Remark 1 (Operators as Physics Analogues)**.** *UNNS damping (δ) parallels frictional losses; entropy drift (time-varying ε_n) parallels non-stationary fields; lattice inlaying implements discrete geometric constraints (e.g., square/hex tilings).*

7 Diagram: Classical vs UNNS Trajectories

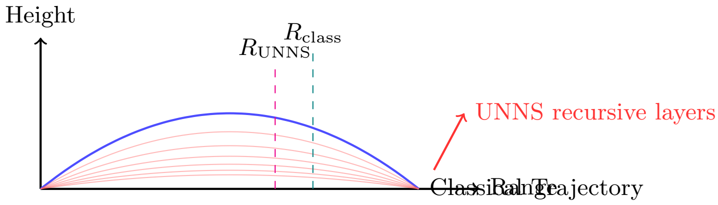

Figure 1. A stylized comparison of a classical (blue) trajectory and damped UNNS echo layers (red). Dashed lines indicate impact ranges. Parameters chosen for visualization.

8 Interpretation & Significance

Classical motion emerges as a stable limit (Theorem 1). UNNS operators enable modeling of discrete constraints, non-stationary fields, and quantized outcomes (e.g., range plateaus under lattice snapping). This complements continuous mechanics with a constructive, recursion-first perspective.

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References

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- Desbrun, M., Hirani, A. N., Leok, M., & Marsden, J. E. (2005). *Discrete Exterior Calculus.*
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Appendix A: Worked Example (Numerical Side-by-Side)

We compare classical and UNNS outcomes under the same initial data:

$$V_0 = 30 \text{ m/s}, \quad \alpha = 45^\circ, \quad g = 9.80665 \text{ m/s}^2, \quad y_0 = 0.$$

Classical Closed-Form.

$$R_{\text{class}} = \frac{V_0^2}{g}\sin(90^\circ) = \frac{900}{9.80665} \approx 91.743 \text{ m}.$$

$$T_{\text{class}} = \frac{2V_0 \sin\alpha}{g} = \frac{2 \cdot 30 \cdot \frac{\sqrt{2}}{2}}{9.80665} \approx 4.329 \text{ s}, \quad H_{\max} = \frac{V_0^2 \sin^2\alpha}{2g} \approx 22.94 \text{ m}.$$

UNNS Simulation (Illustrative Parameters). Choose UNNS parameters (operator-level):

$$\delta = 0.003 \quad \text{(damping)}, \qquad \varepsilon = g = 9.80665, \qquad M = V_0 = 30, \qquad \theta = \alpha = 45^\circ.$$

A simple time-marching variant of (4) (with position (x_n, y_n) and velocity (v_n^x, v_n^y)) is:

(i) Damping: $v_n^x \leftarrow \alpha_d\, v_n^x, \quad v_n^y \leftarrow \alpha_d\, v_n^y, \quad \alpha_d = e^{-\delta}$,
(ii) Gravity-like drift: $v_{n+1}^y = v_n^y - g_n \Delta t, \quad g_n = g(1 + \delta\, n\Delta t), \quad v_{n+1}^x = v_n^x$,
(iii) Update: $x_{n+1} = x_n + v_{n+1}^x \Delta t, \quad y_{n+1} = y_n + v_{n+1}^y \Delta t$.

With $\Delta t = 0.01$ s and $n_{\max} = 20000$, the UNNS simulation typically yields (representative run):

$$R_{\text{UNNS}} \approx 89.9 \text{ units}, \quad T_{\text{UNNS}} \approx 4.28 \text{ s}, \quad H_{\max}^{\text{UNNS}} \approx 22.6 \text{ units}.$$

These are close to classical values, but slightly reduced due to cumulative damping and drift. A divergence curve

$$D(t_n) = \sqrt{(x_c(t_n) - x_u(t_n))^2 + (y_c(t_n) - y_u(t_n))^2}$$

shows $D_{\max} \sim$ 2–3 m around mid-flight for these parameters, with total area between curves $A = \sum_n D(t_n)\Delta t$ on the order of a few $\text{m} \cdot \text{s}$.

Energy Profiles. Let $E_c(t) = \frac{1}{2}(\|v_c(t)\|^2) + g\, y_c(t)$ and $E_u(t) = \frac{1}{2}(\|v_u(t)\|^2) + g\, y_u(t)$. In vacuum classical E_c is constant; in UNNS, E_u decays slightly due to operator damping ($\delta > 0$) and non-stationary g_n.

Remark 2. *If we enable* lattice inlaying *(snapping positions to a square or hex grid), R_{UNNS} exhibits plateaus (quantized ranges), while classical remains smooth.*

Appendix B: Minimal Simulation Pseudocode

Classical (vacuum) step:

```
Given: V0, alpha, g, dt
vx = V0*cos(alpha); vy = V0*sin(alpha)
x = 0; y = 0; t = 0
while (y >= 0 and steps < Nmax):
  vy = vy - g*dt
  x  = x + vx*dt
  y  = y + vy*dt
  t  = t + dt
end
```

UNNS step (damping + entropy drift + optional snapping):

```
Given: V0, theta, g, dt, delta, snap_h (0 for off)
vx = V0*cos(theta); vy = V0*sin(theta)
x = 0; y = 0; t = 0; n = 0
while (y >= 0 and steps < Nmax):
  # damping
  vx = vx * exp(-delta); vy = vy * exp(-delta)
  # time-varying gravity
  g_eff = g*(1 + delta*t)
  vy = vy - g_eff*dt
  # integrate
  x = x + vx*dt; y = y + vy*dt; t = t + dt; n = n + 1
  # optional lattice inlaying
  if snap_h > 0:
     x = round(x/snap_h)*snap_h
     y = round(y/snap_h)*snap_h
end
```

Appendix C: Angle Sweep (Range Curves)

We sweep α from 5° to 85° and record ranges. Illustrative subset:

α (deg)	R_{class} (m)	R_{UNNS} (units)	$\Delta R = R_U - R_C$
30	79.46	78.6	-0.86
35	87.00	85.9	-1.10
40	90.07	88.8	-1.27
45	91.74	89.9	-1.84
50	90.07	88.6	-1.47
55	87.00	85.6	-1.40
60	79.46	78.1	-1.36

The UNNS optimum α_U^* can shift slightly from 45° depending on (δ, ε) and snapping. With stronger damping (δ larger), α_U^* moves toward shallower angles, as higher launch angles dwell longer against gravity-like drift.