1 UNNS Field Extensions and Classical Sequences

1.1 1. Introduction

The UNNS (Unbounded Nested Number Sequences) framework can be viewed as a universal substrate in which both rational and real fields embed naturally. Each classical recurrence sequence (e.g., Fibonacci, Pell, Tribonacci, Padovan) generates algebraic extensions of the rationals through the characteristic polynomials that define their growth ratios.

1.2 2. Definitions

1.3 Definition 2.1.

The UNNS Kernel is the algebraic generator of nests, defined by recurrence relations with coefficients in Q (rationals).

1.4 Definition 2.2.

A UNNS Field Extension is the minimal field extension of Q obtained by adjoining the dominant root(s) of the recurrence characteristic equation.

1.5 3. Theorem (Many Faces of UNNS)

Every classical linear recurrence sequence defined over Q corresponds to a UNNS nest. The growth constant (dominant root) generates a number field extension $Q(\alpha)$, where α is the algebraic number given by the recurrence's characteristic equation.

1.6 4. Examples

1.7 Example 4.1 (Fibonacci).

The Fibonacci sequence satisfies $F_n = F_{n-1} + F_{n-2}$ with characteristic polynomial $r^2 - r - 1 = 0$. The dominant root $\phi = (1 + \sqrt{5})/2$ generates the quadratic extension $Q(\sqrt{5})$.

1.8 Example 4.2 (Pell)

. Pell numbers satisfy $P_n = 2P_{n-1} + P_{n-2}$ with polynomial $r^2 - 2r - 1 = 0$. The dominant root $\delta = 1 + \sqrt{2}$ generates $Q(\sqrt{2})$.

1.9 Example 4.3 (Tribonacci)

. Tribonacci satisfies $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ with polynomial $r^3 - r^2 - r - 1 = 0$. The dominant root $\psi \approx 1.839$ generates a cubic field extension of Q.

1.10 Example 4.4 (Padovan)

. The Padovan sequence satisfies $P_n = P_{n-2} + P_{n-3}$ with polynomial $r^3 - r - 1 = 0$. The dominant root $\rho \approx 1.3247$ (Plastic Number) generates a cubic field extension of Q.

1.11 5. General Lemma Template for Linear Recurrences

Lemma 5.1 (General UNNS Extension Lemma). Let S_n be a linear recurrence of order k defined by $S_n = a_1 S_{n-1} + a_2 S_{n-2} + ... + a_k S_{n-k}$ with coefficients $a_i \in Q$. The characteristic polynomial is $r^k - a_1 r^{k-1} - ... - a_k = 0$. Let α be the dominant real root of this polynomial. Then the UNNS nest corresponding to S generates the field extension $Q(\alpha)$, with degree $[Q(\alpha) : Q] \leq k$, and α serves as the growth constant in the UNNS framework.

1.12 6. Implications

Thus UNNS nests act as generators of number fields, showing that UNNS embeds both the rational field and its infinite hierarchy of algebraic extensions. This provides a universal algebraic substrate unifying seemingly different sequences under one structural theorem.