

# Optimisation in Python

Alireza Soroudi

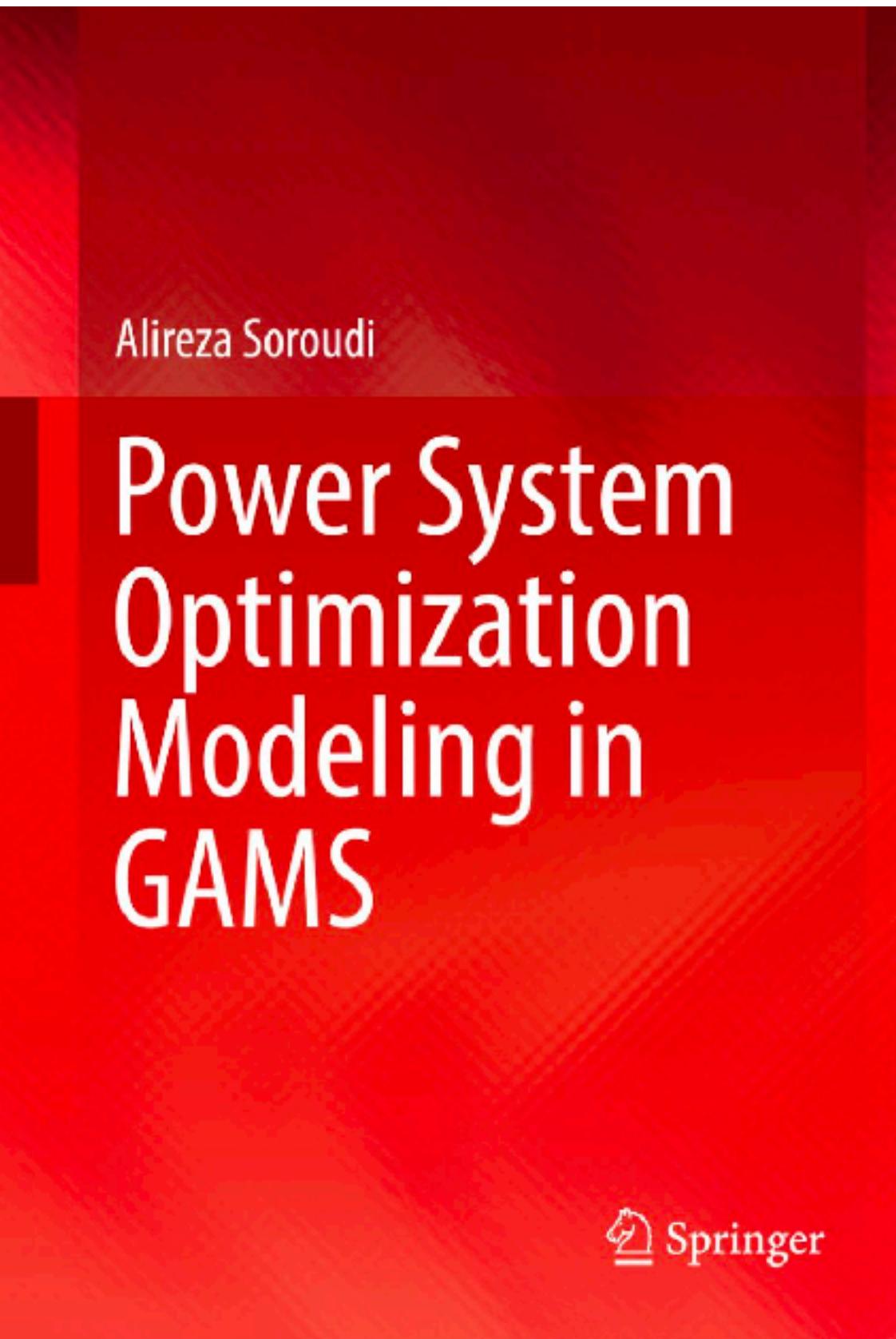
[alireza.soroudi@gmail.com](mailto:alireza.soroudi@gmail.com)

[www.linkedin.com/in/soroudi/](http://www.linkedin.com/in/soroudi/)



[github.com/OptimizationExpert](https://github.com/OptimizationExpert)

# How it started



# How it's going

The image is a screenshot of a GitHub repository page. The URL "github.com/OptimizationExpert" is at the top. Below it is a large circular profile picture containing a magnifying glass icon and the word "Optimiz". To the right of the profile picture, the repository name "Pyomo" is shown as pinned. A brief description follows: "Pyomo repository provides a comprehensive library of solved models in Supply chain management". Below this, there are statistics: "Jupyter Notebook" (orange dot), "162" (star icon), and "30" (eye icon). A chart below shows "104 contributions in the last year" with a heatmap grid. The legend indicates "No contributions" for certain days. The footer of the screenshot includes "Learn how we count contributions".

# Skill Levels

alireza.soroudi@gmail.com



بهبود مدلسازی ریاضی

مدلسازی یک مساله به فرم ریاضی

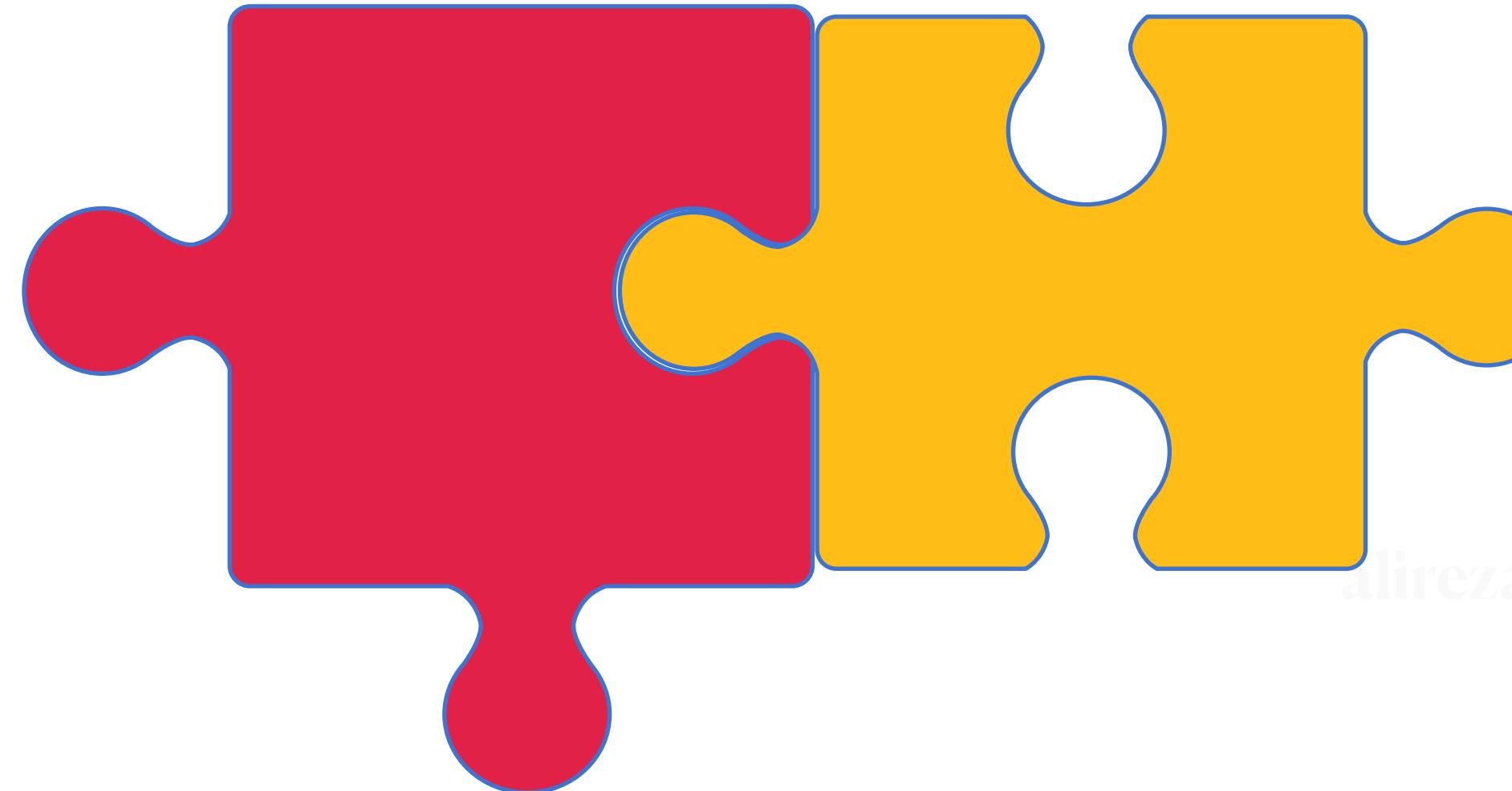
کد کردن یک الگوریتم ریاضی

کد کردن یک مدل ریاضی

فهمیدن یک کد آماده

# Machine Learning - Operations Research

alireza.soroudi@gmail.com



alireza.soroudi@gmail.com

# Optimisation Tools



**SciPy**

a.soroudi@gmail.com



**PuLP**



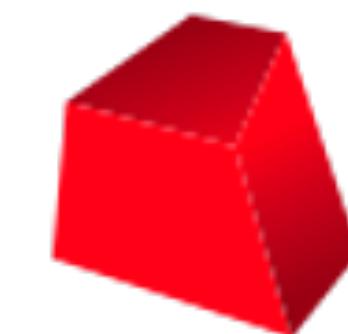
**AMPL**



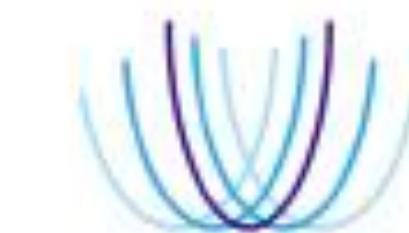
**PYOMO**



**Google OR-Tools**



**GUROBI**  
OPTIMIZATION



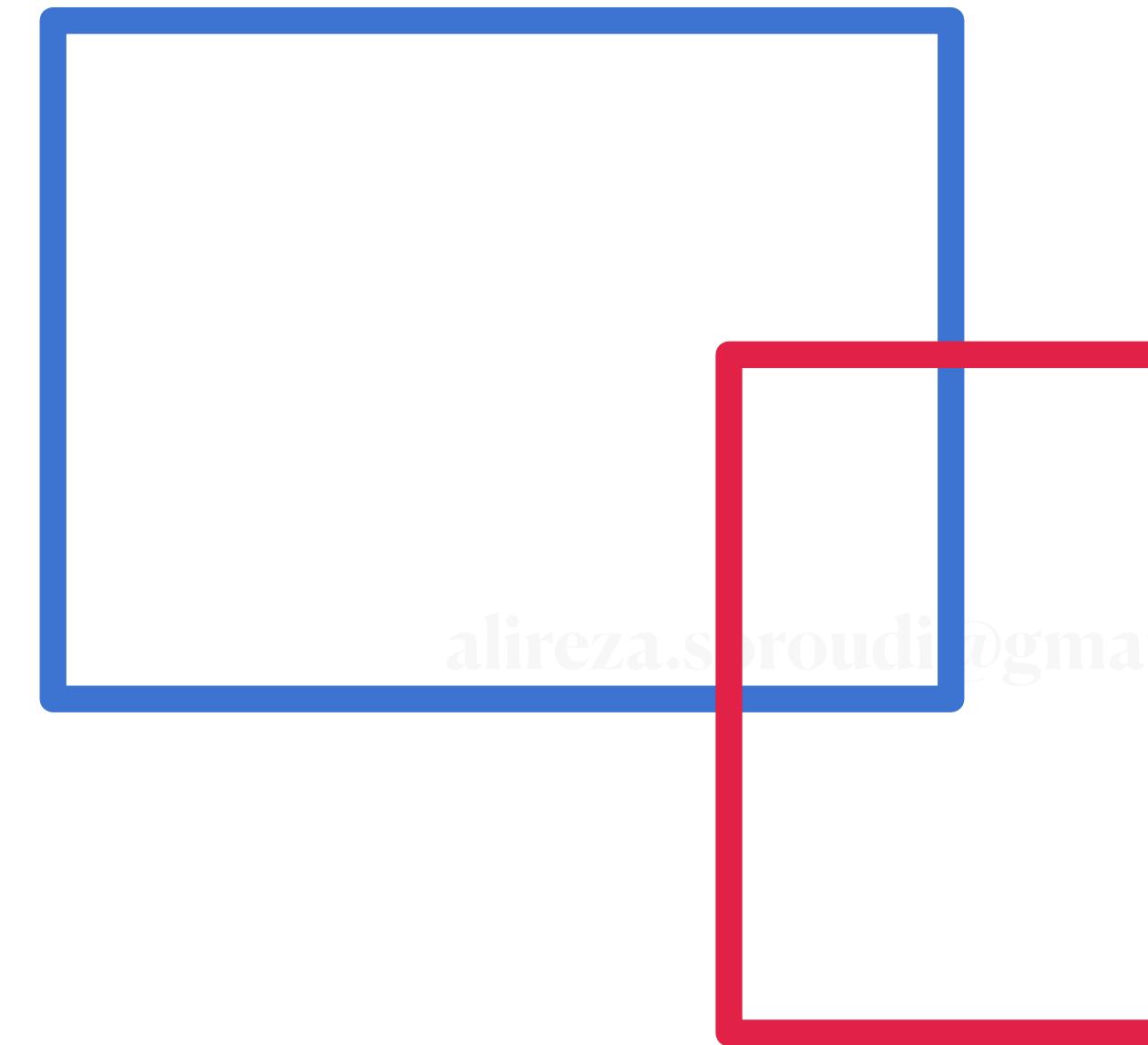
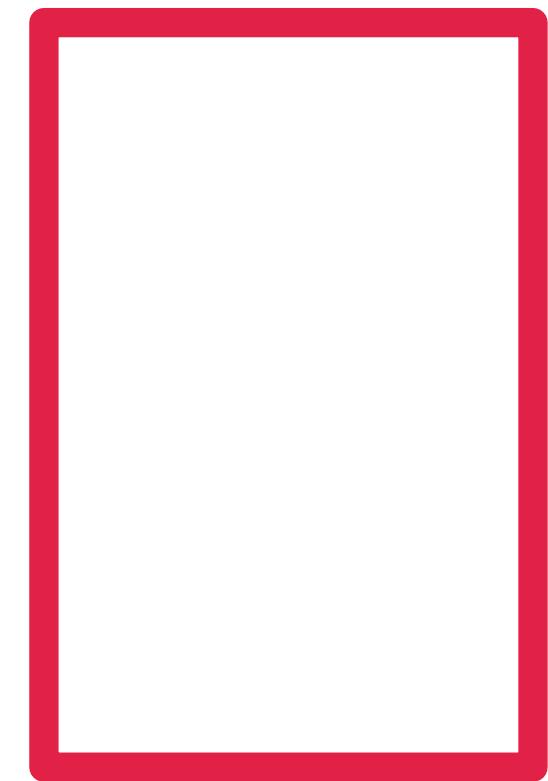
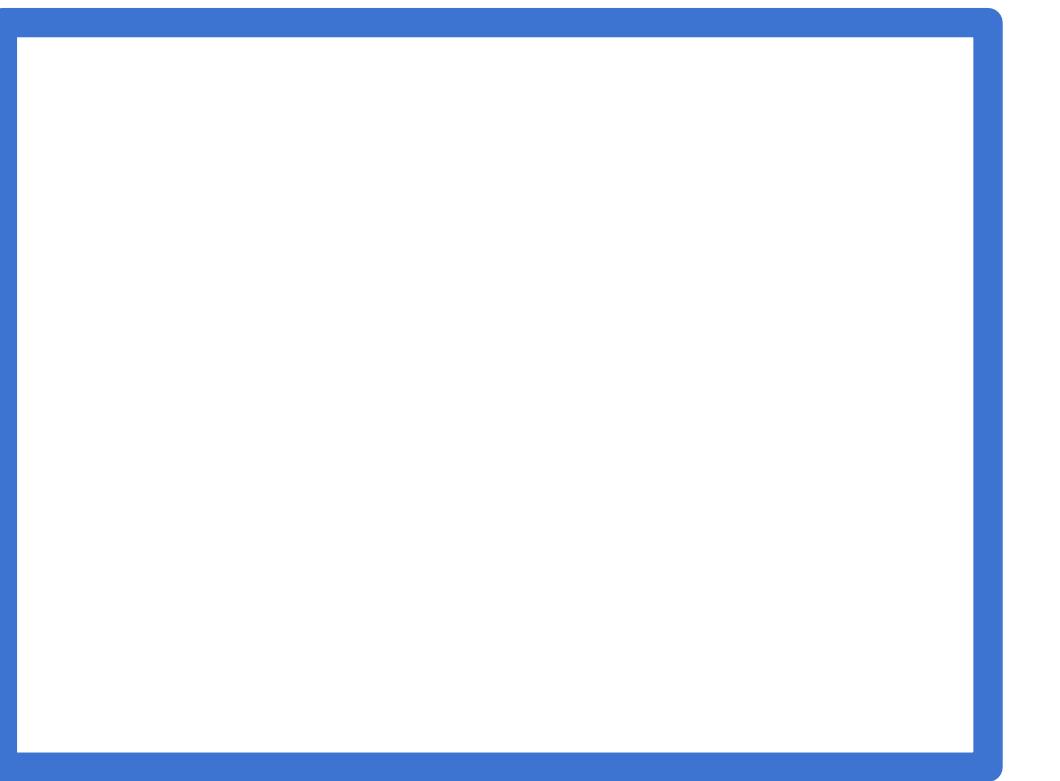
**CVXPY**



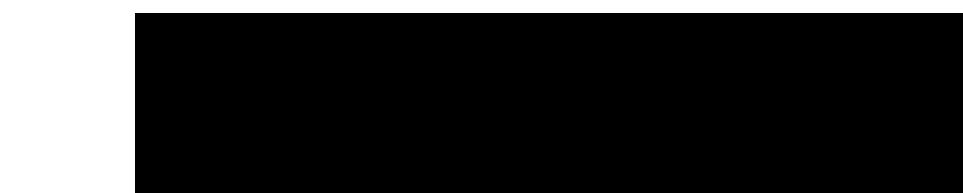
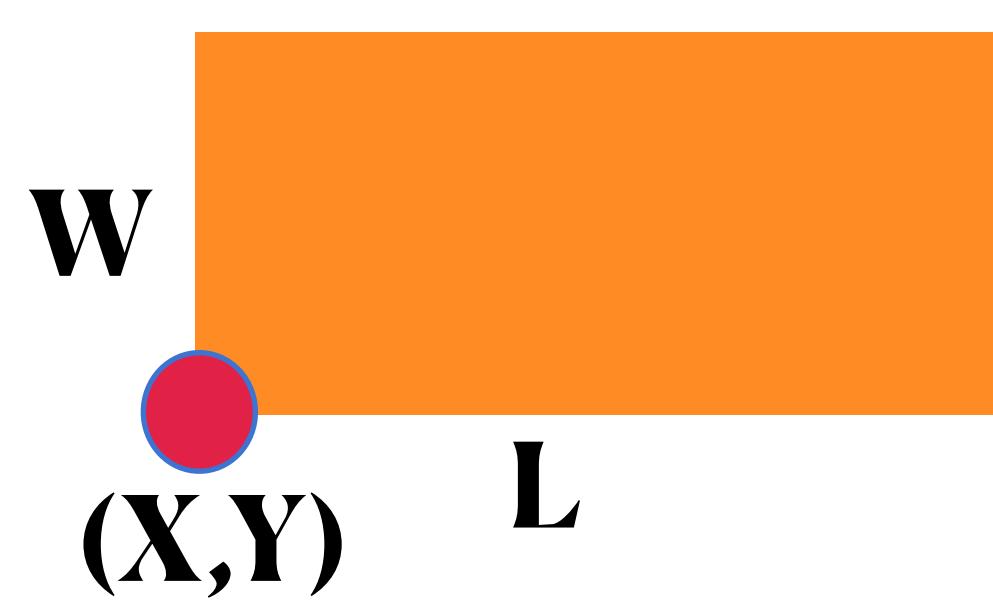
# Example

**Are these rectangles overlapping ?**

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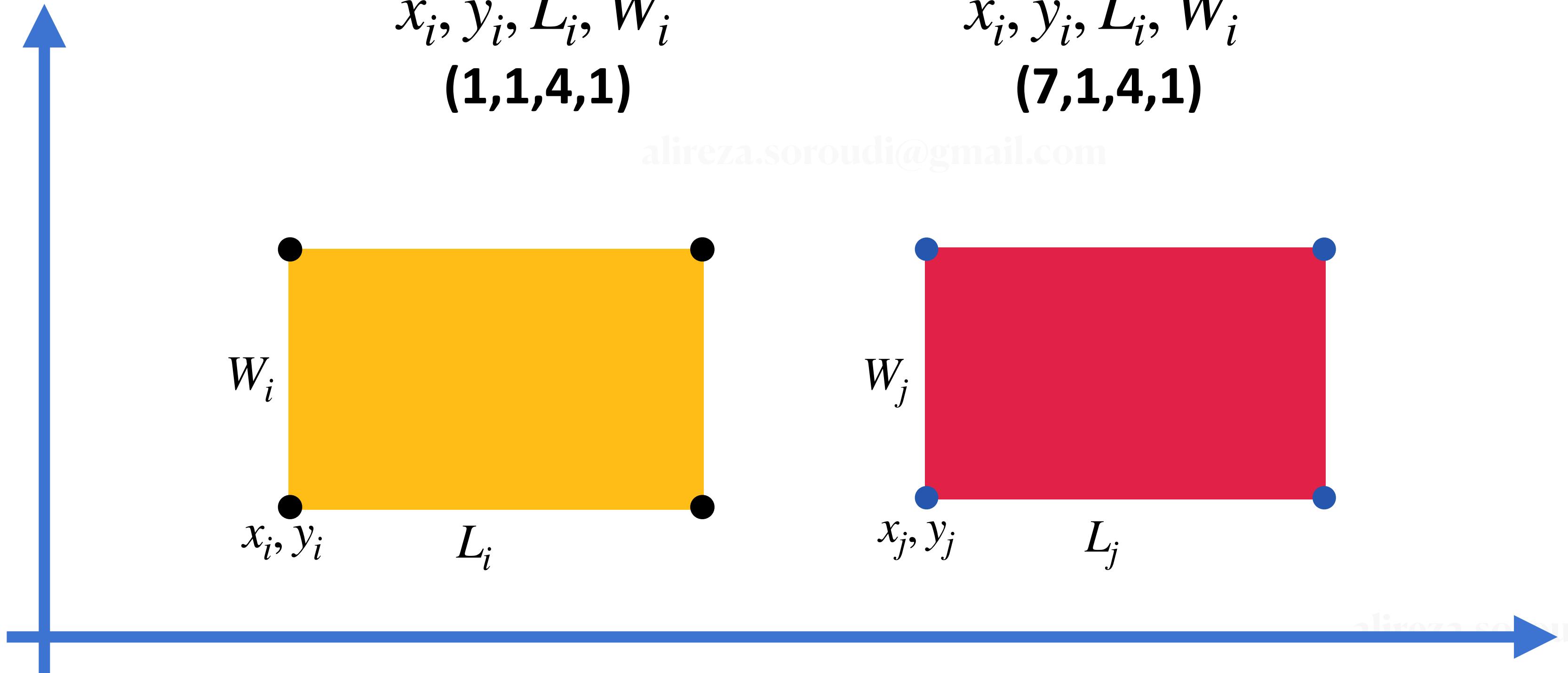
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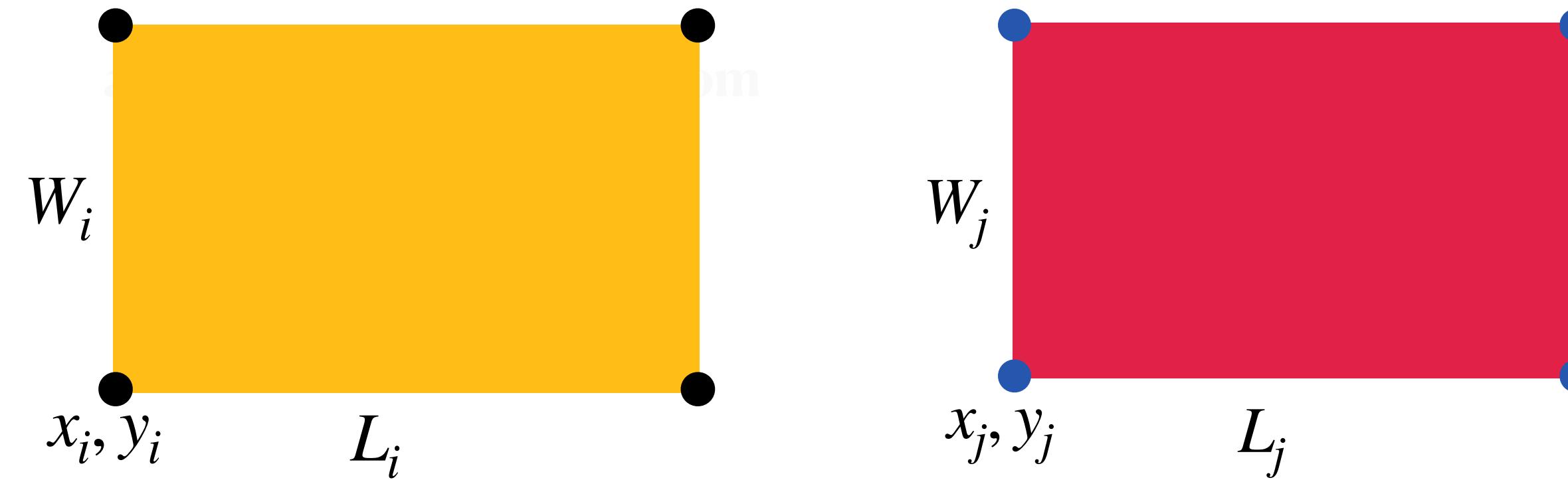
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alireza.soroudi@gmail.com



8



$$x_i + L_i - M(1 - R_{i,j}) \leq x_j$$

J is in the right hand side of I

$$x_i + L_i + MR_{i,j} \geq x_j$$

$$x_i \geq 0$$

$$R_{i,j} \in \{0,1\}$$

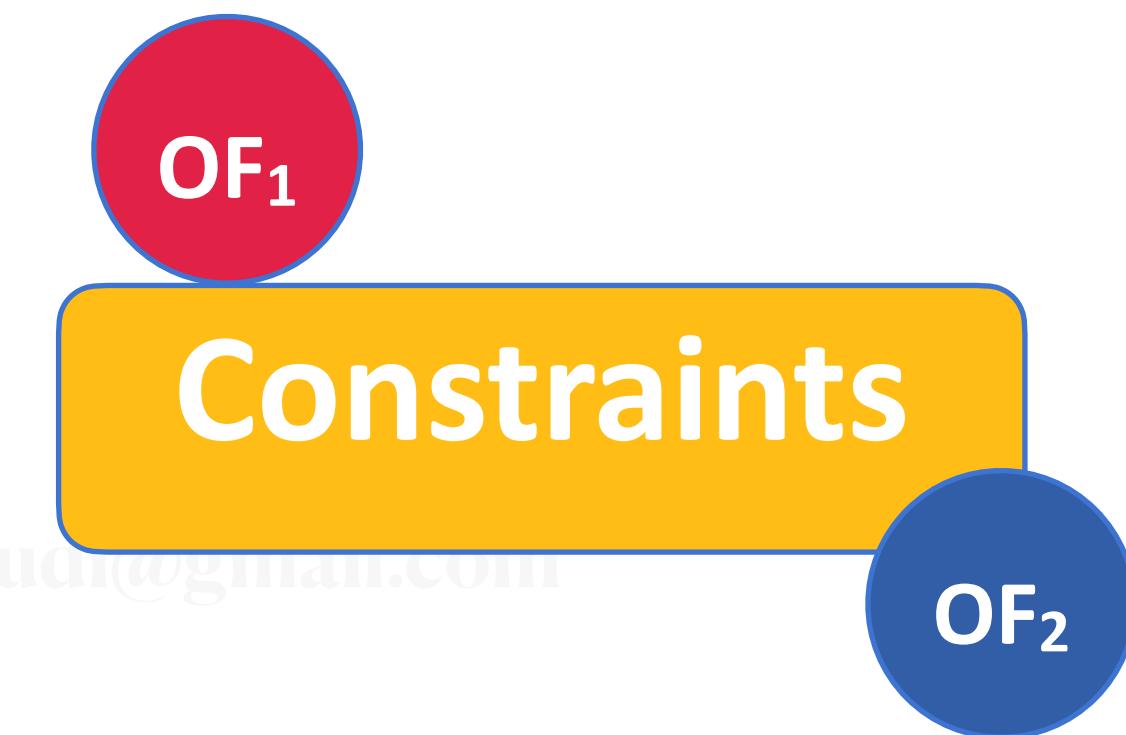
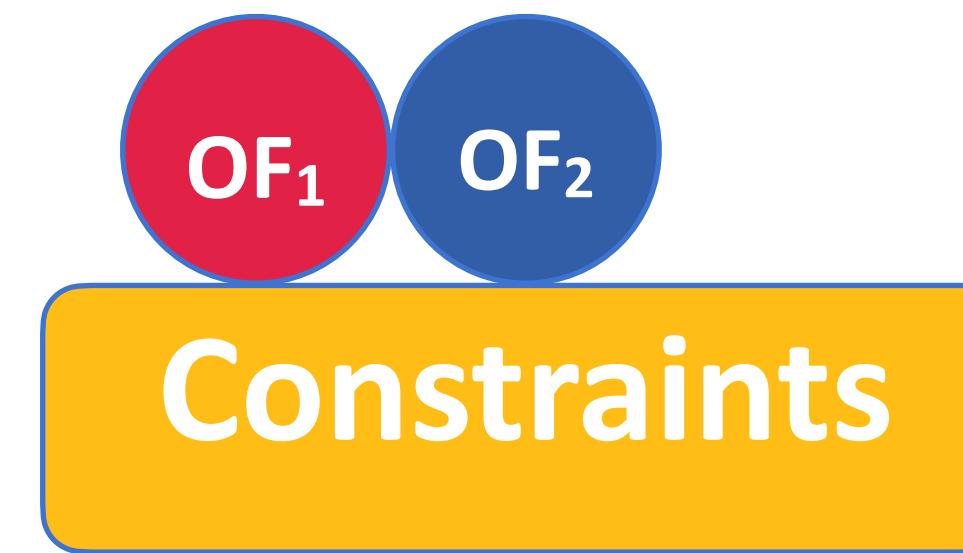
# University Exams

alireza.soroudi@gmail.com

<b>Room 101</b>	A		B	C	D		
<b>Room 102</b>		L		F			
<b>Room 201</b>	J		H				
<b>Room 30</b>		K		M			
<b>Time</b>	10	11	12	13	14	15	16

# Optimisation Types

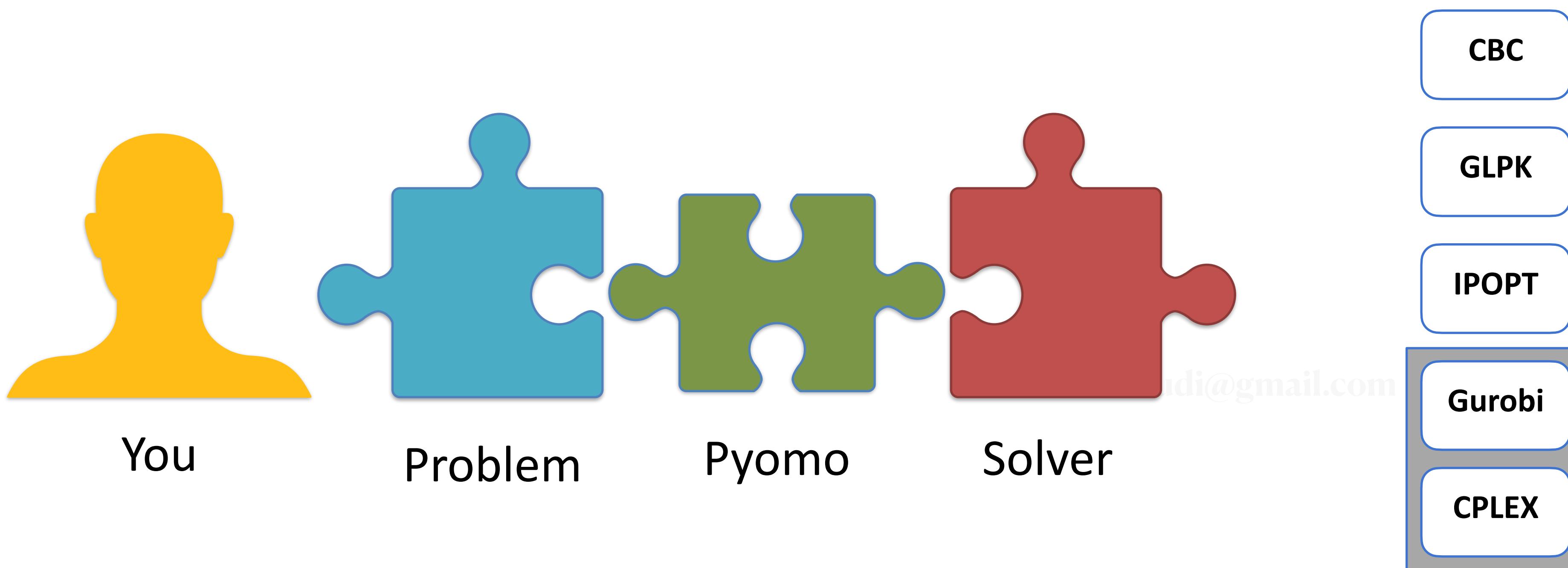
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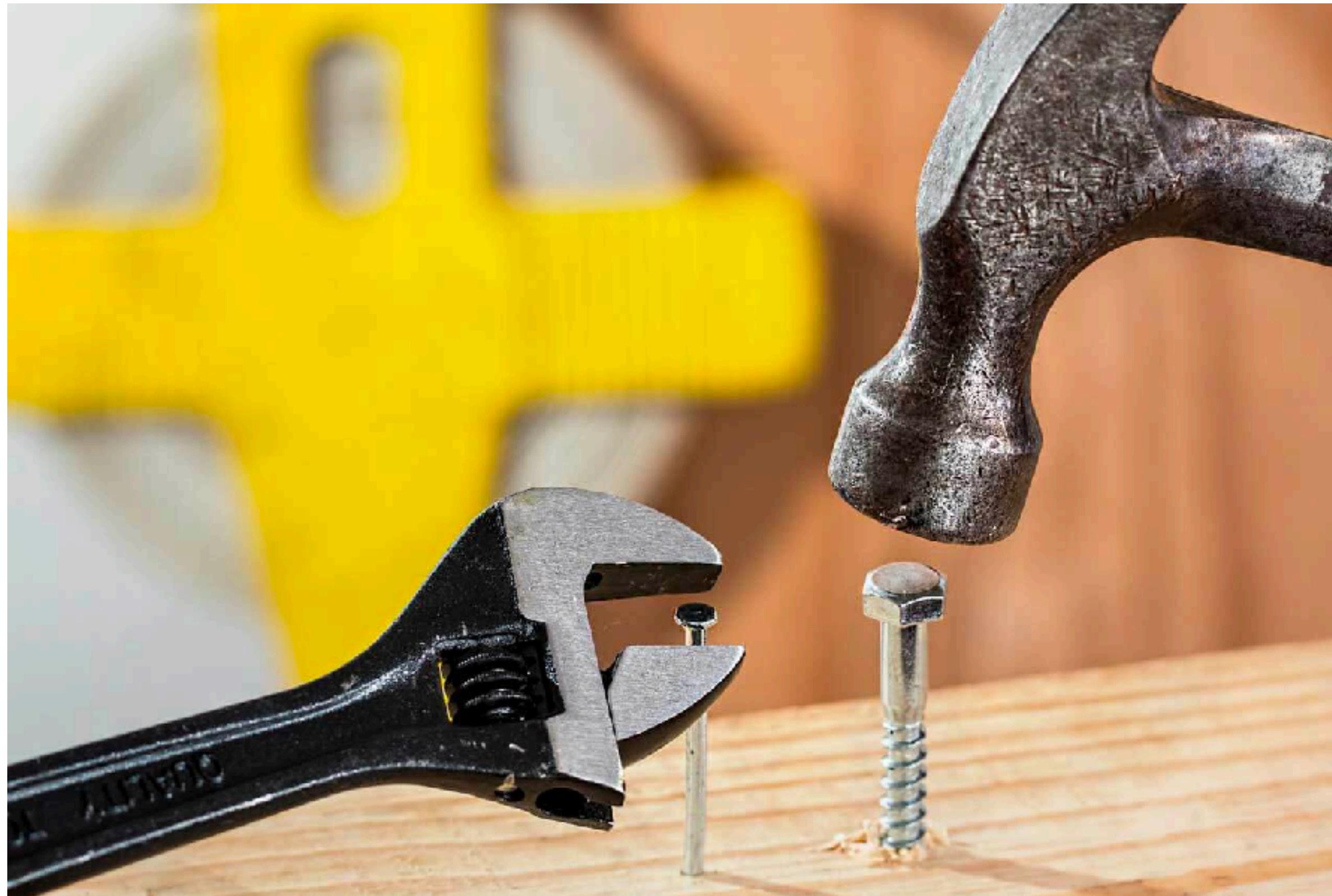
# Optimisation modelling steps

alireza.soroudi@gmail.com

alireza.soroudi@gmail.com



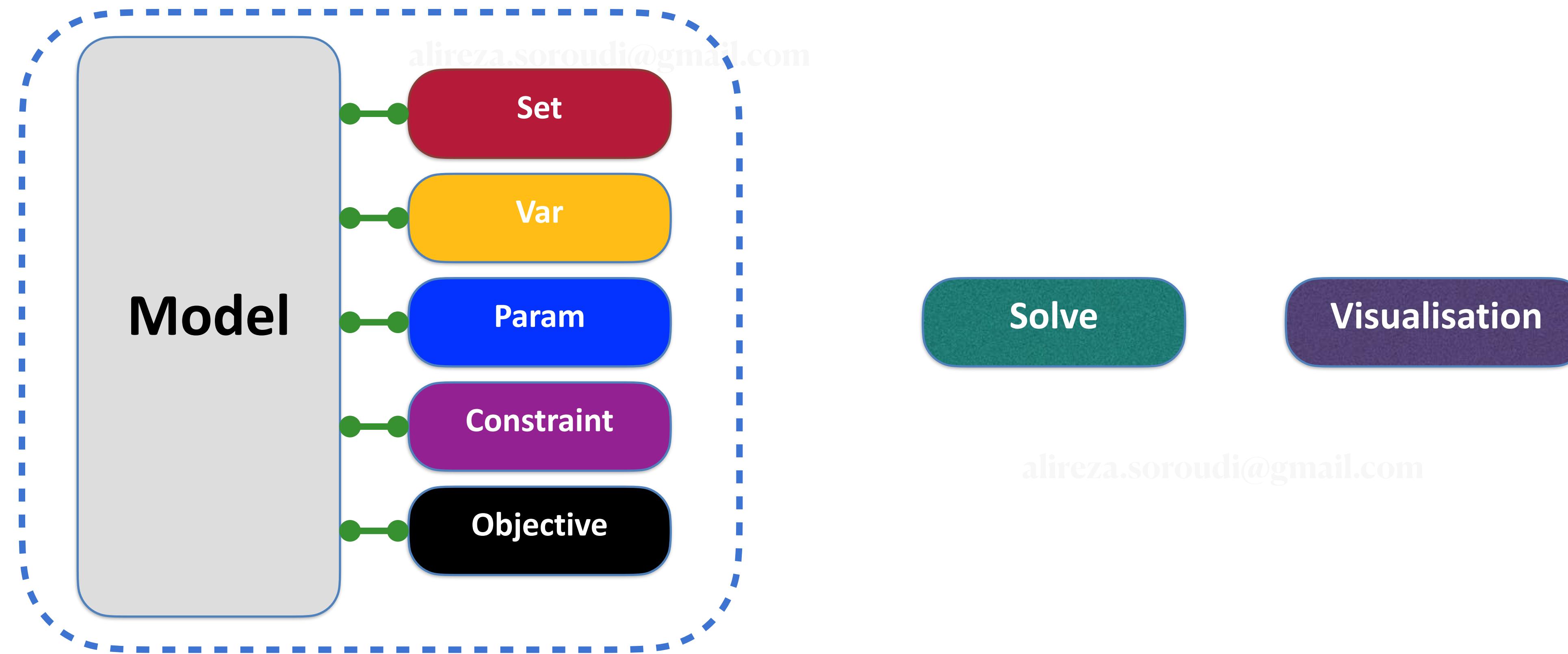
# Right Tool for Right Problem

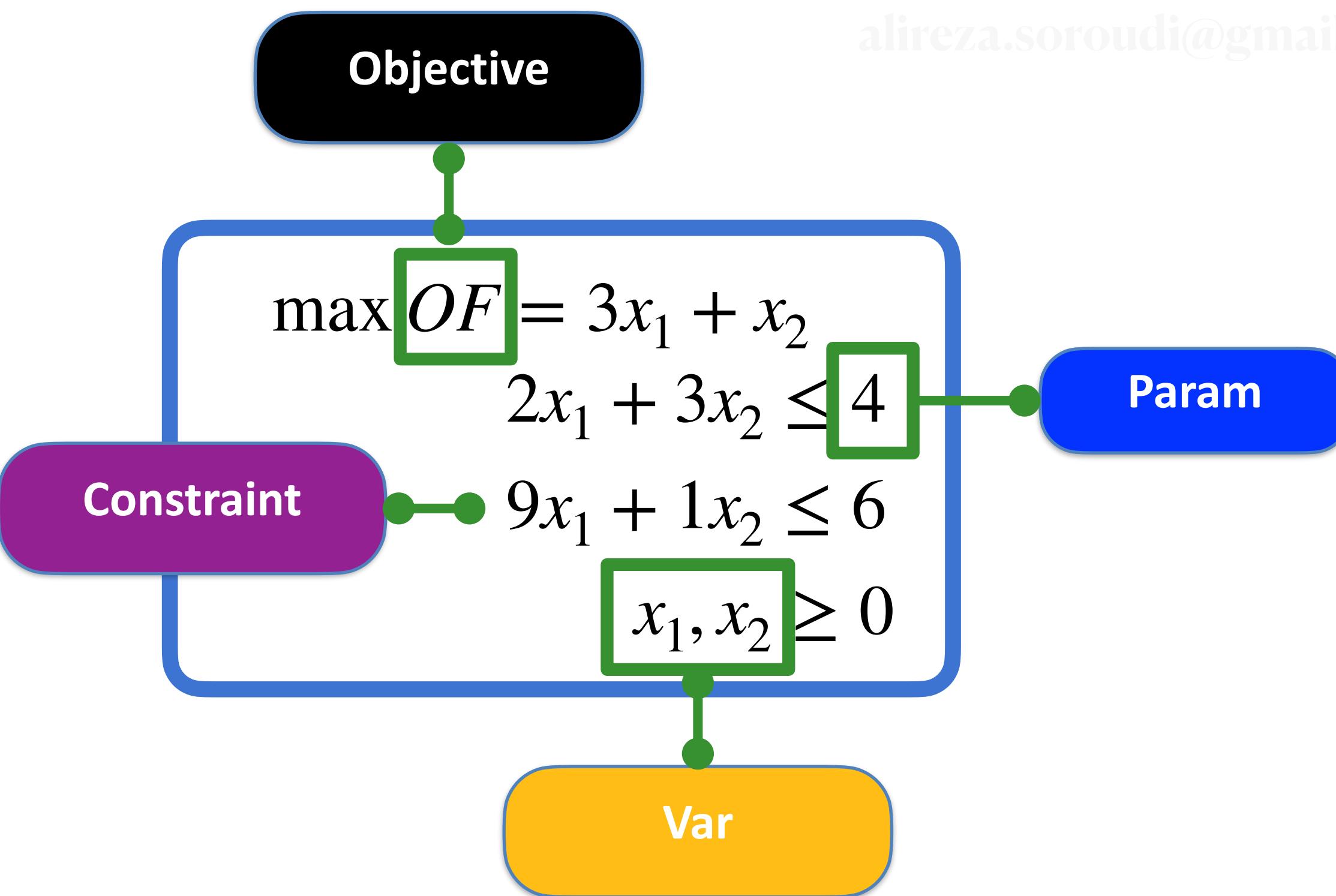


LP MIP MIQP QCP NLP

CBC	✓	✓		
GLPK	✓	✓		
IPOPT	✓			
Gurobi	✓	✓	✓	✓
CPLEX	✓	✓	✓	✓

# Pyomo Elements





alireza.soroudi@gmail.com

## Python code

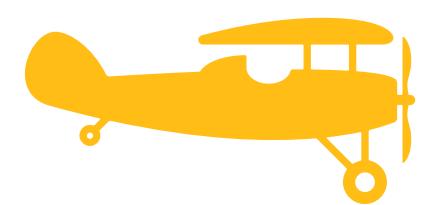
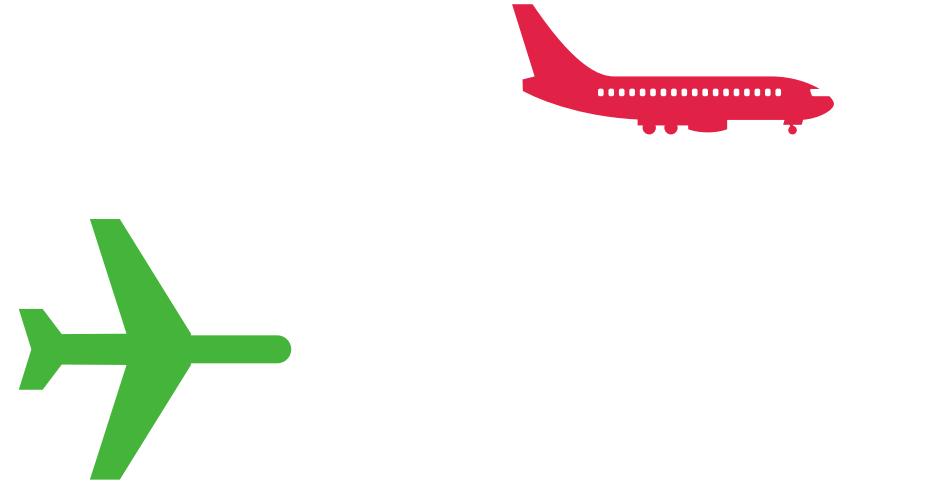
```
[1]: 1 from pyomo.environ import *
[2]: 1 model = AbstractModel()
2 model.x1 = Var(initialize=0, within=NonNegativeReals)
3 model.x2 = Var(initialize=0, within=NonNegativeReals)
4 model.C1 = Constraint(expr=2*model.x1+3*model.x2<=4)
5 model.C2 = Constraint(expr=9*model.x1+ model.x2<=6)
6 model.obj1 = Objective(expr=3*model.x1+model.x2, sense=maximize)

[3]: 1 opt = SolverFactory('glpk')
2 instance = model.create_instance()

[4]: 1 results = opt.solve(instance) |
2 print('OF= ',value(instance.obj1))
```

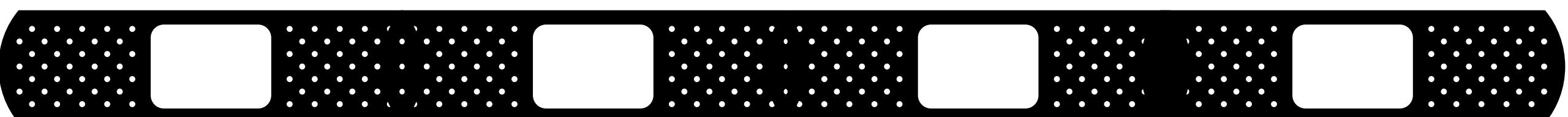
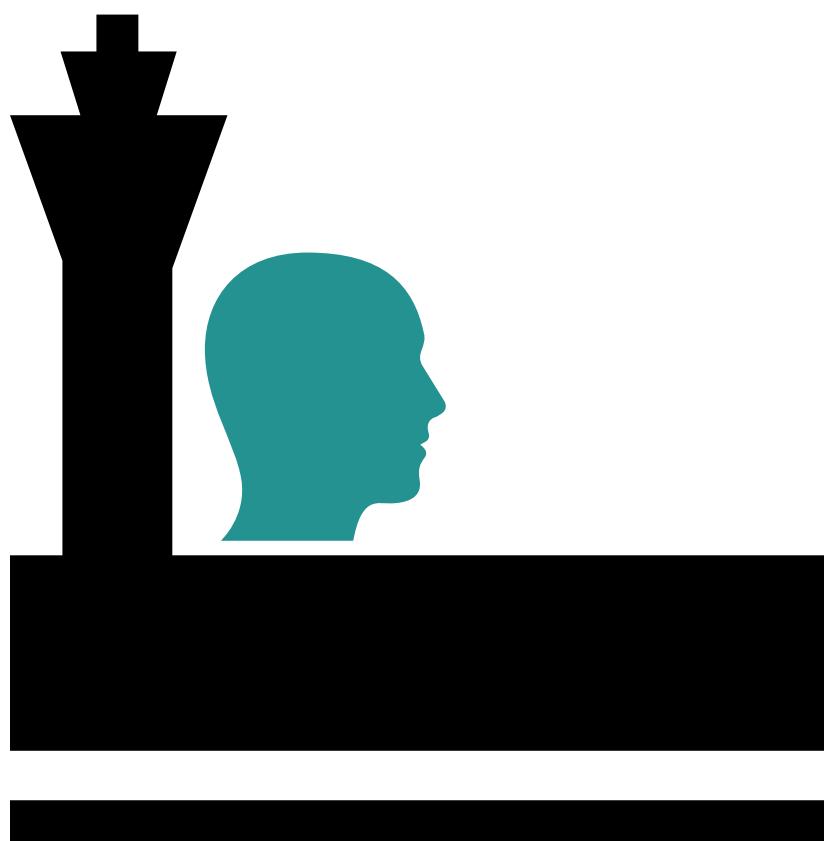
OF= 2.64

# Aviation Industry



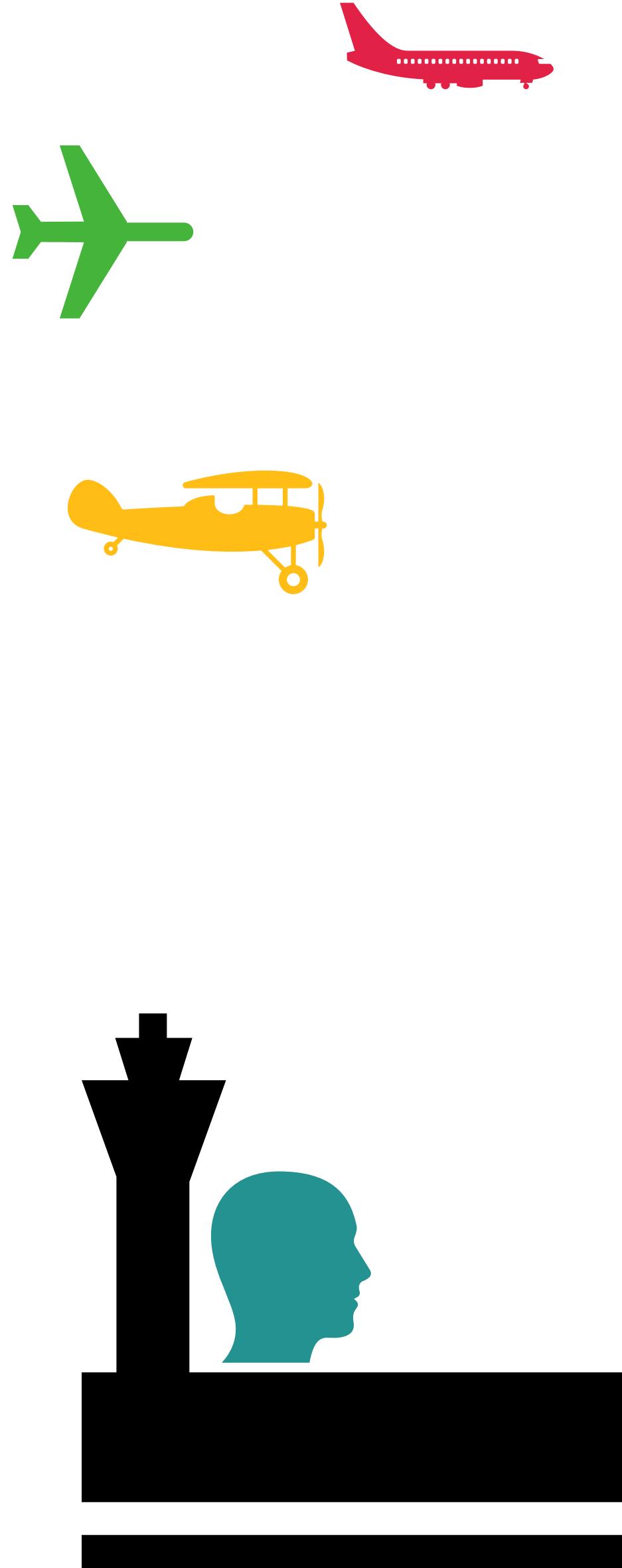
**There are N Planes in the sky  
How should the landing be scheduled?**

**Looks easy?**



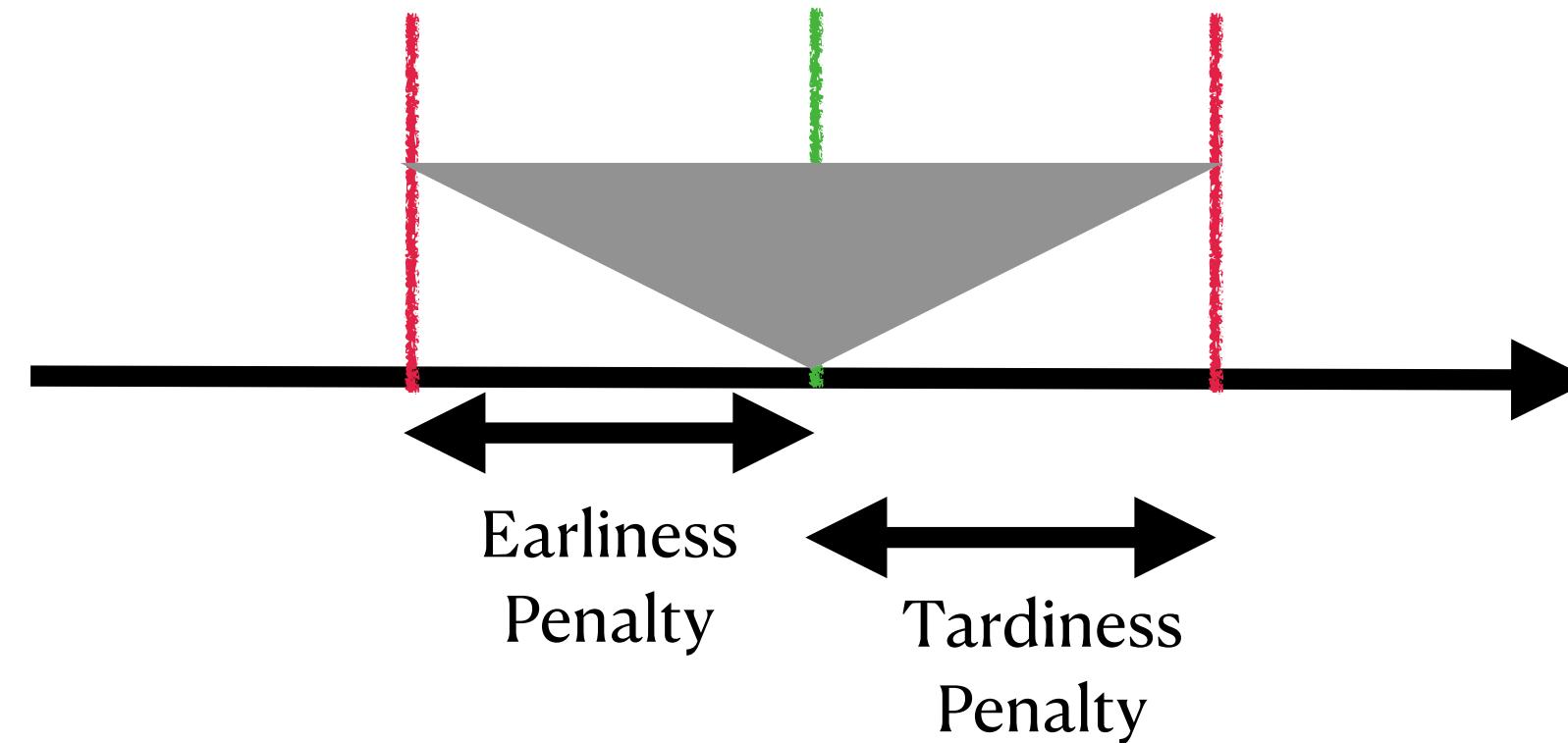
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# Aviation Industry

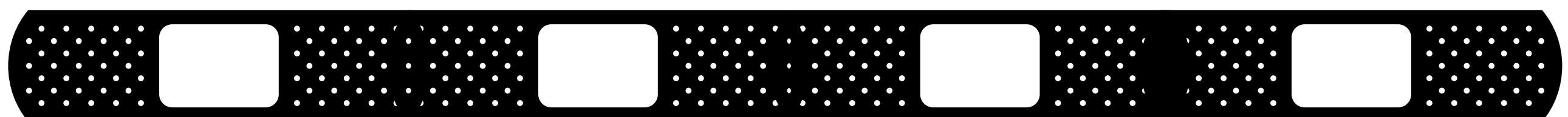
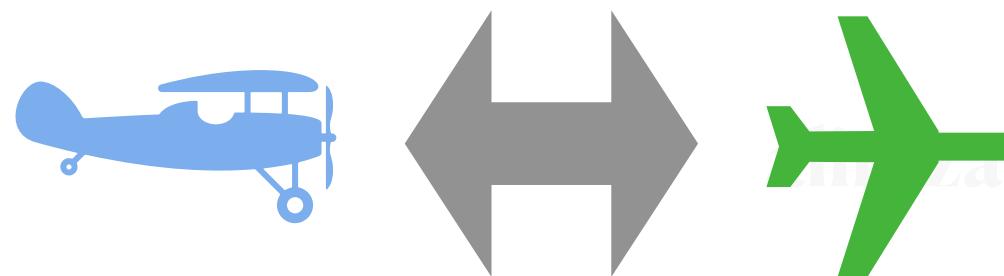


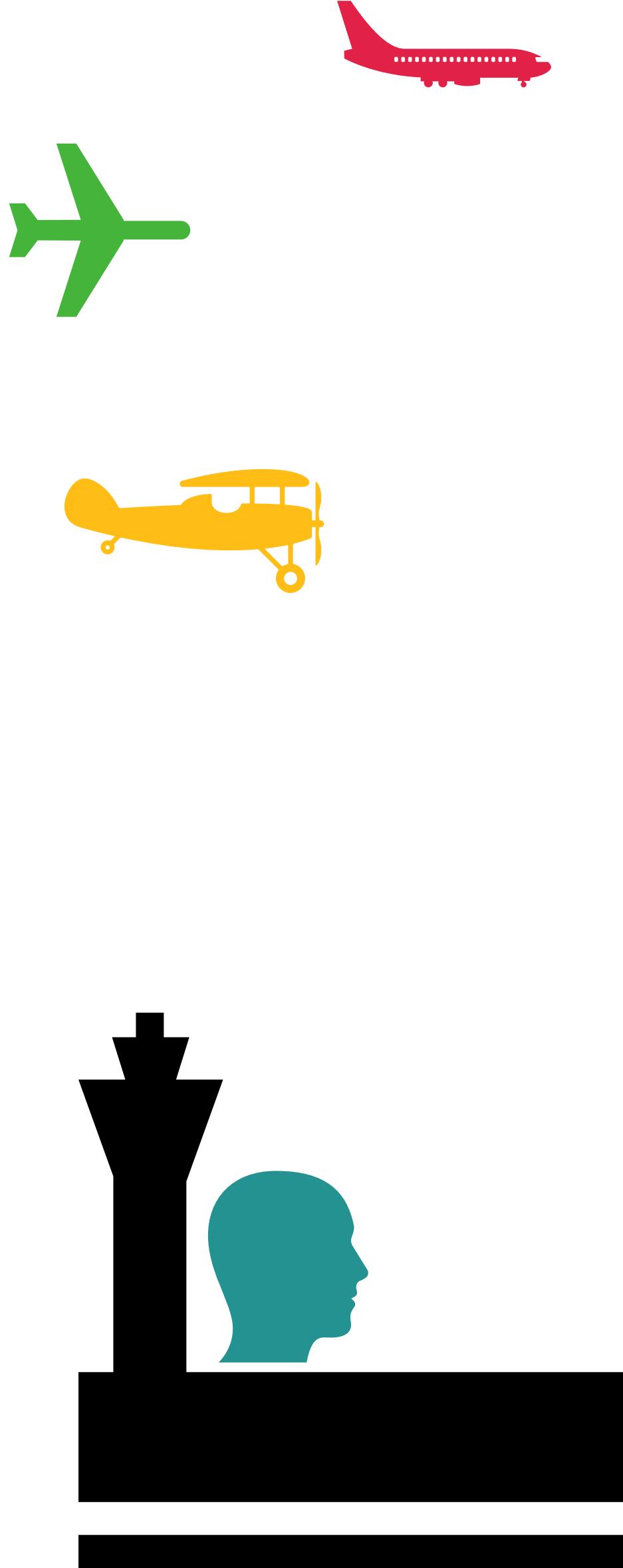
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Start time   Target time   Final time



Depending on the airplane type,  
there should be a delay between  
airplane i and airplane j





$$\min OF = \sum_i \beta_i TP_i + \alpha_i EP_i$$

$$XS_i \leq x_i \leq XF_i$$

$$x_i + D_{i,j}U_{i,j} - M(1 - U_{i,j}) \leq x_j$$

$$x_i - T_i \leq TP_i$$

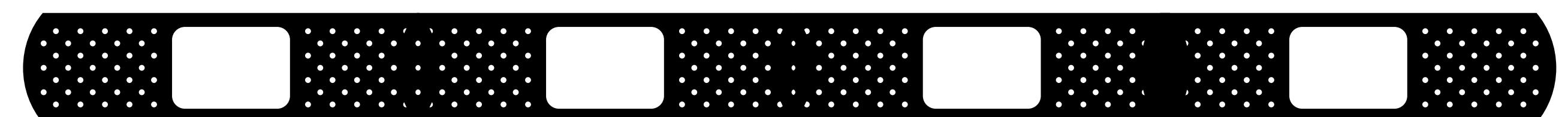
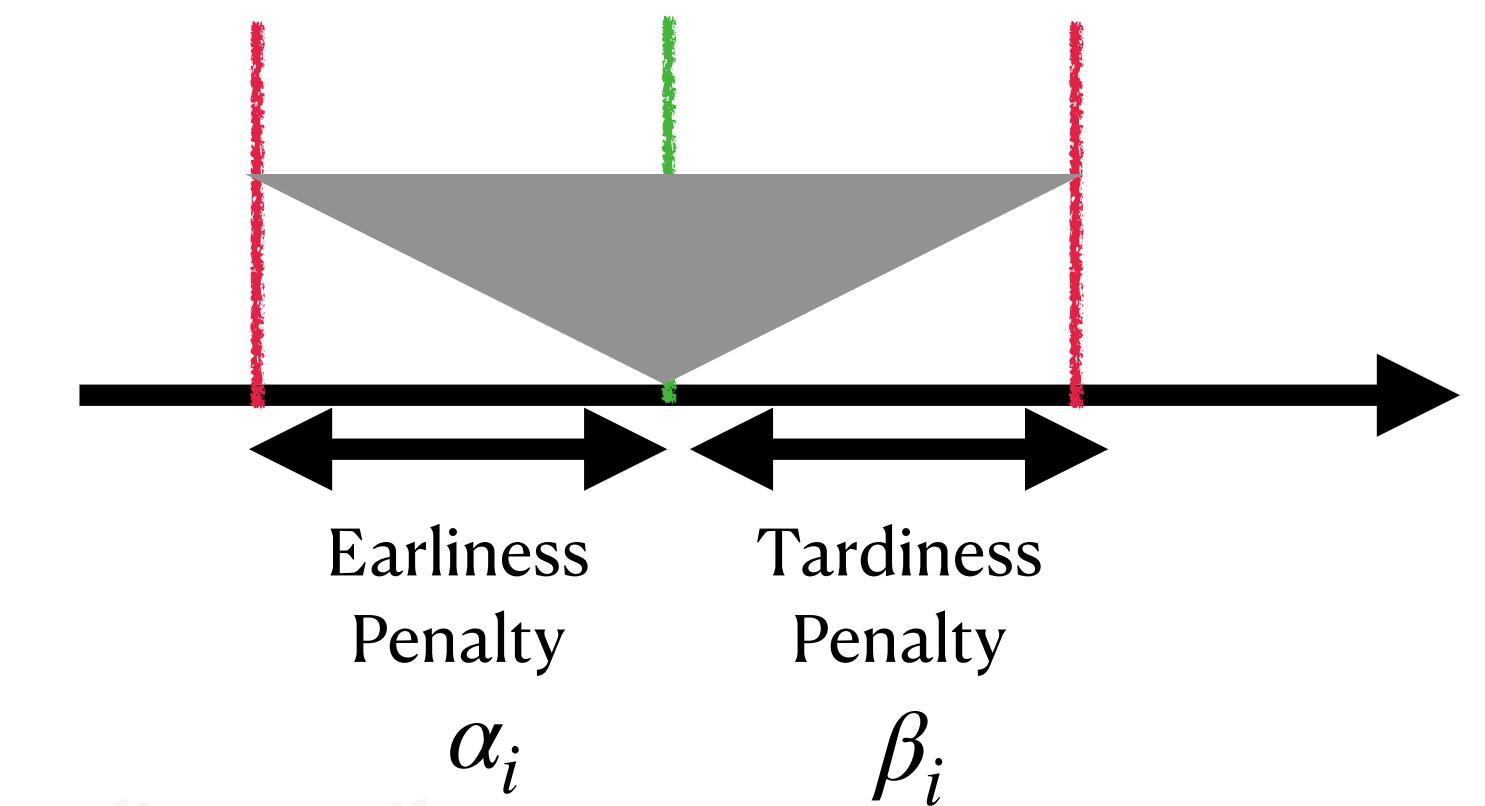
$$T_i - x_i \leq EP_i$$

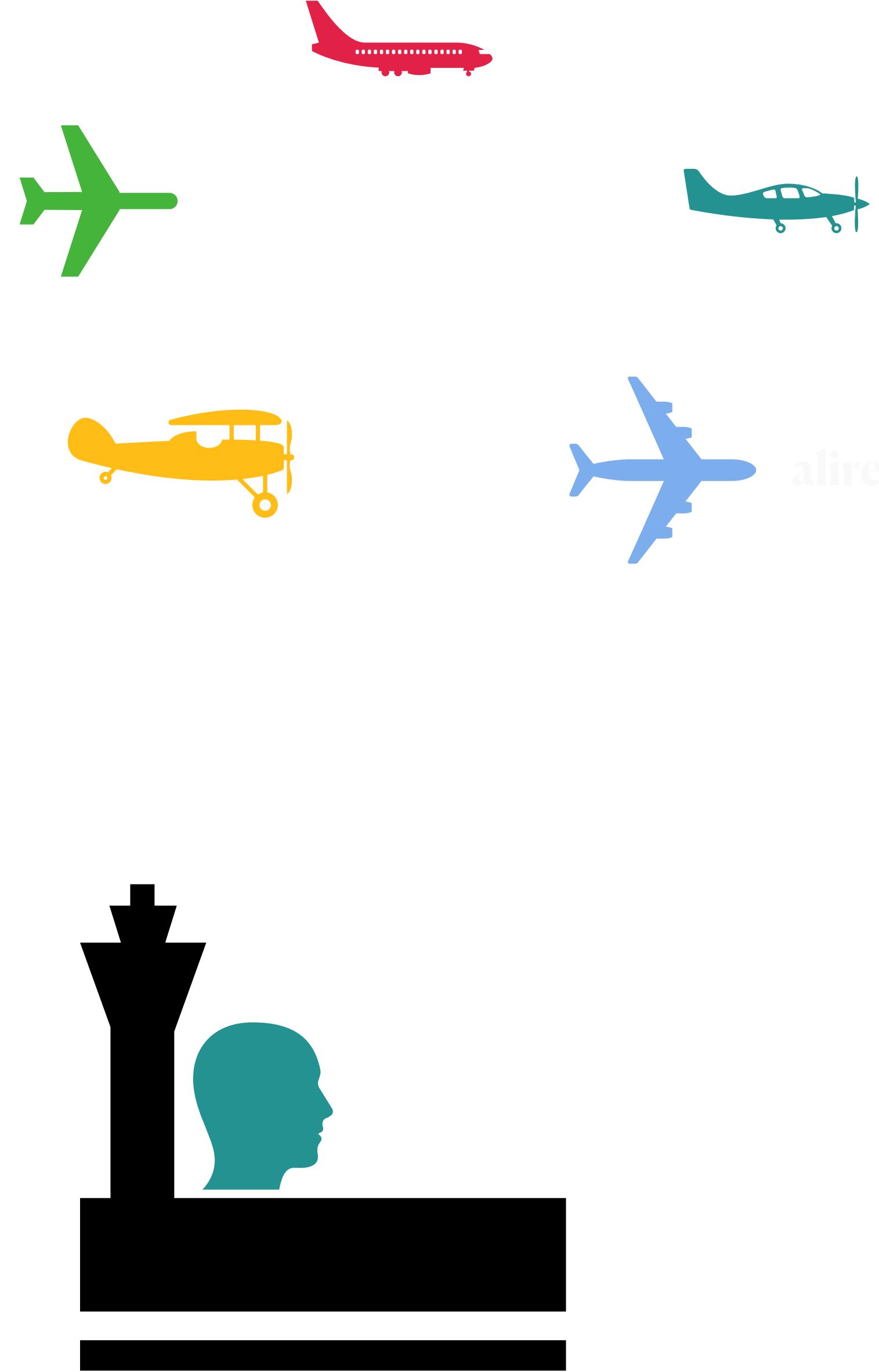
$$\forall i \neq j \quad U_{i,j} + U_{j,i} = 1$$

$$U_{i,j} \in \{0,1\}$$

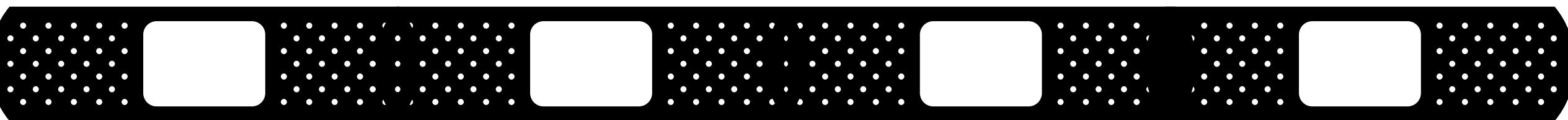
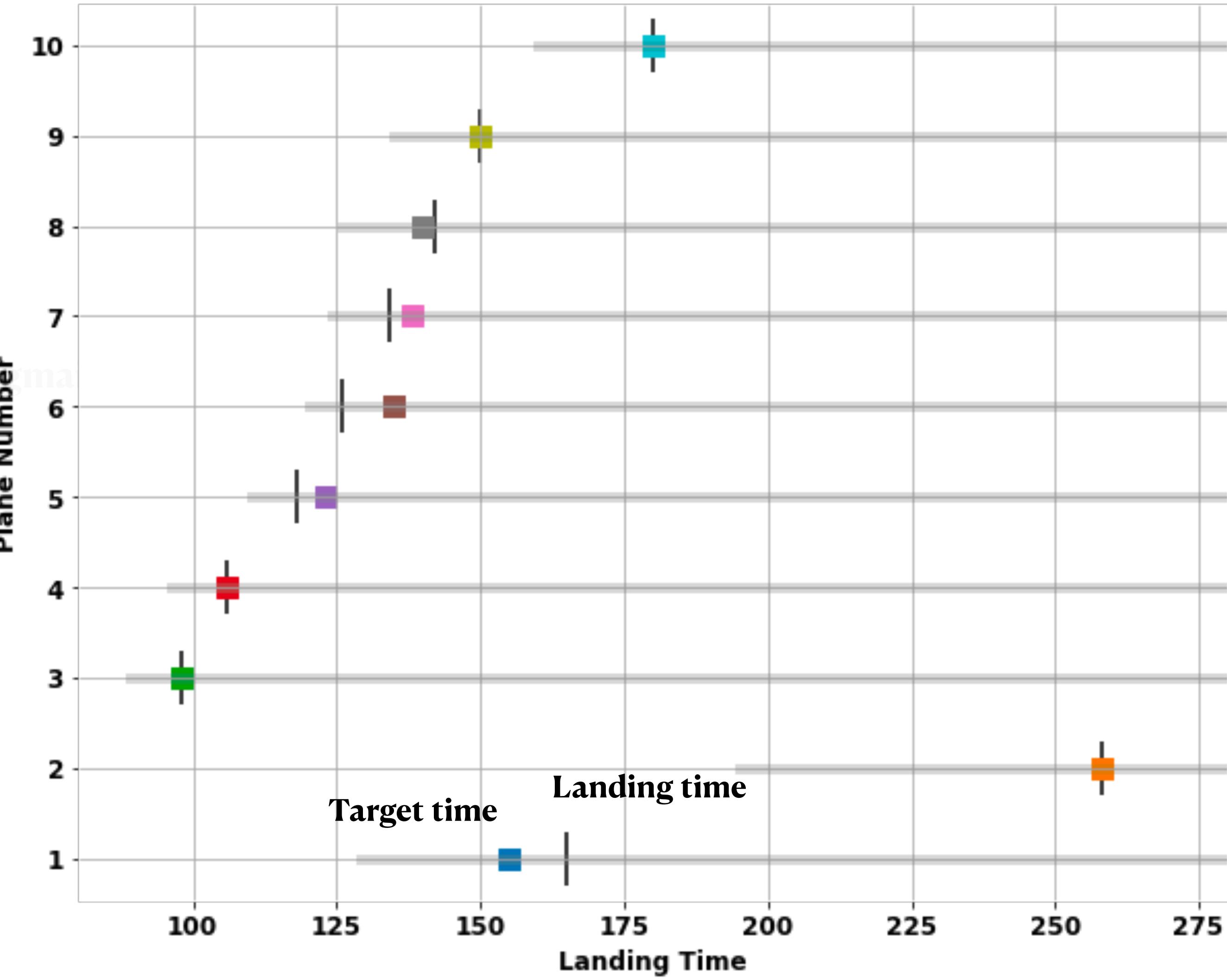
$$x_i, TP_i, EP_i \in R^+$$

**Start time**   **Target time**   **Final time**



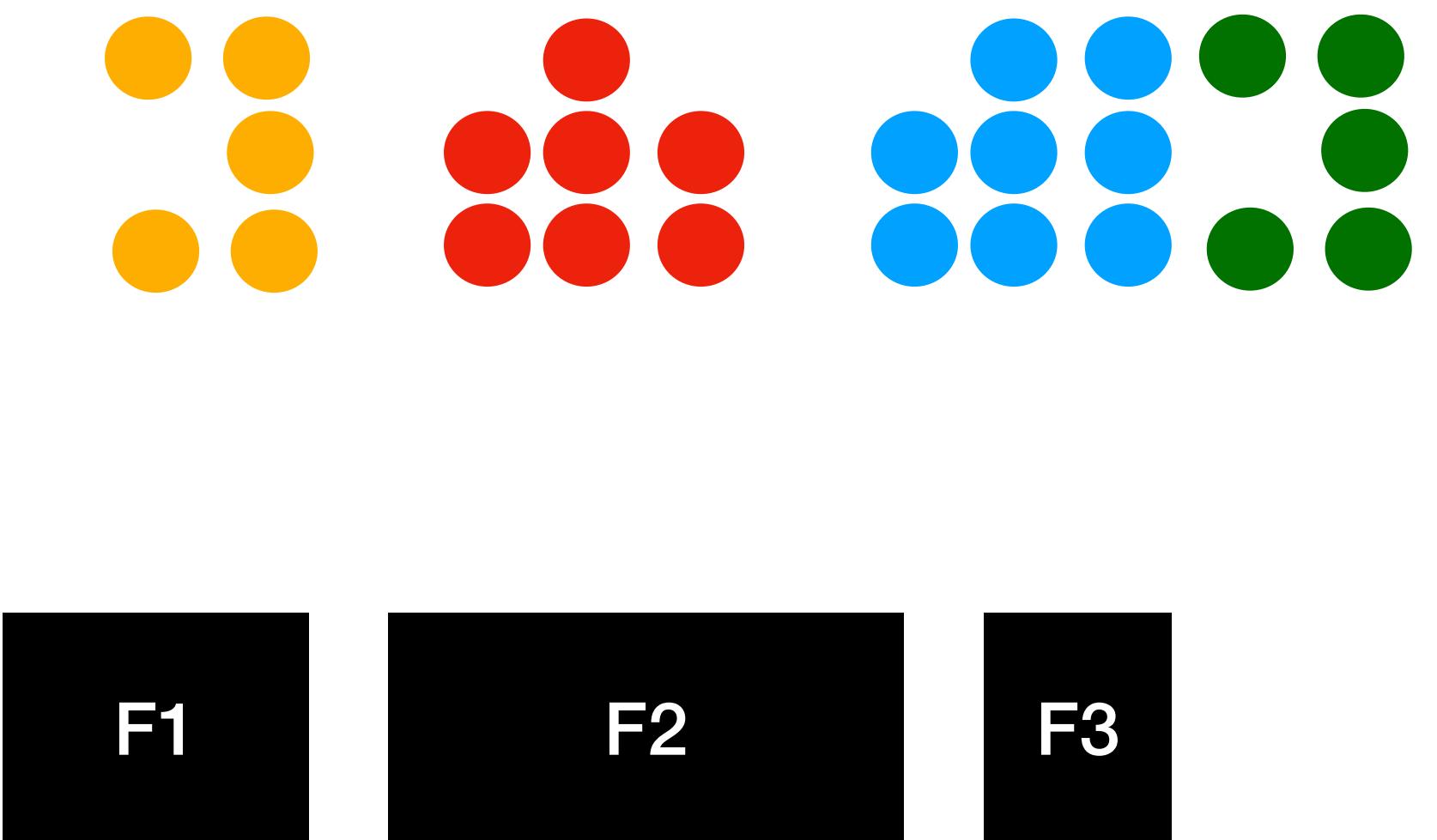


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# MILP

## Cargo Airplane Loading

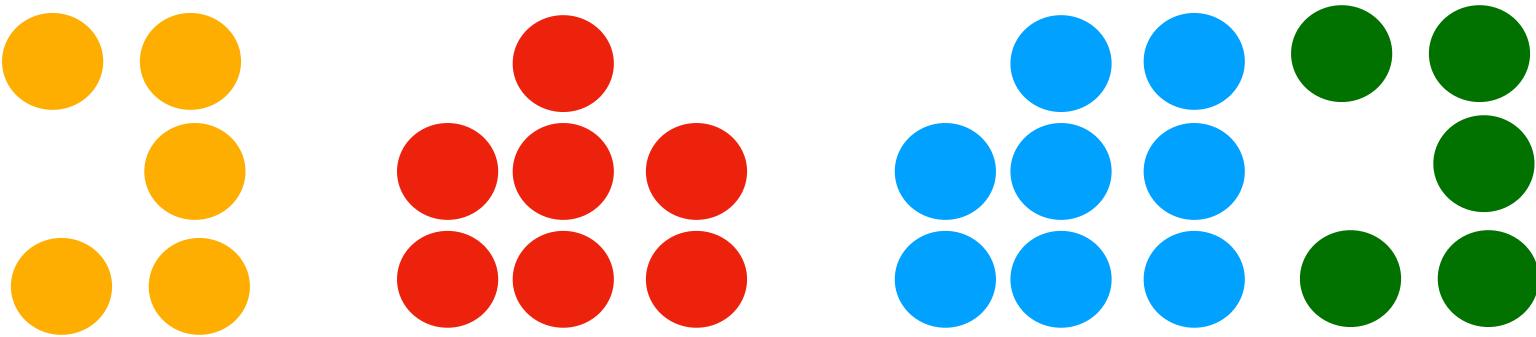


# MILP Cargo Airplane Loading

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- How much of each cargo should go to each section ?
- How to maximise the total profit?
- What are the constraints ?



F1

F2

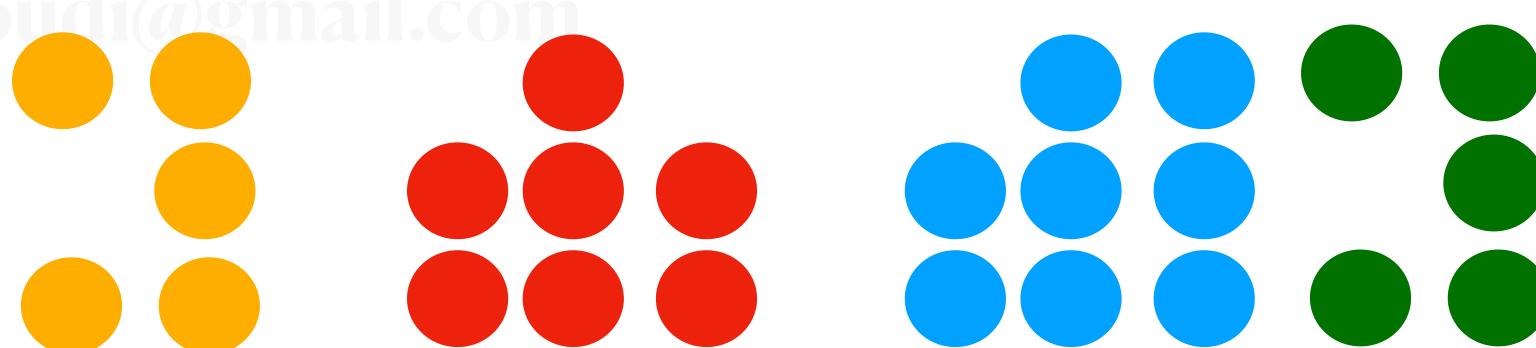
F3

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# MILP

## Cargo Airplane Loading

- How much of each cargo should go to each section ?
- How to maximise the total profit?
- What are the constraints ?



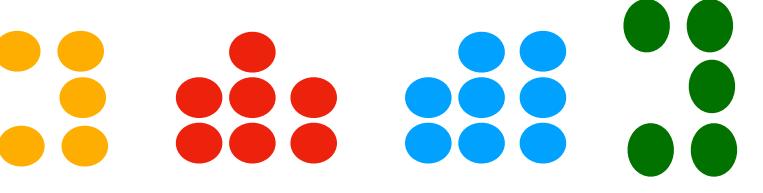
	Total Weight	Volume/ tonne	Profit/ tonne
C1	18	480	310
C2	15	650	380
C3	23	580	350
C4	12	390	285

	Weight	Space
f1	10	6800
f2	16	8700
f3	8	5300

F1

F2

F3



# MILP

## Cargo Airplane Loading

- How much of each cargo should go to each section ?
- How to maximise the total profit?
- What are the constraints ?

$$\forall i \quad \sum_j x_{i,j} \leq TW_i$$

$$\forall j \quad \sum_i x_{i,j} \leq PW_j$$

$$\forall j \quad \sum_i x_{i,j} Vol_i \leq PS_j$$

$$x_{i,j} \in R$$

**LP**

$$OF = \sum_{i,j} x_{i,j} Profit_i$$

	F1	F2	F3

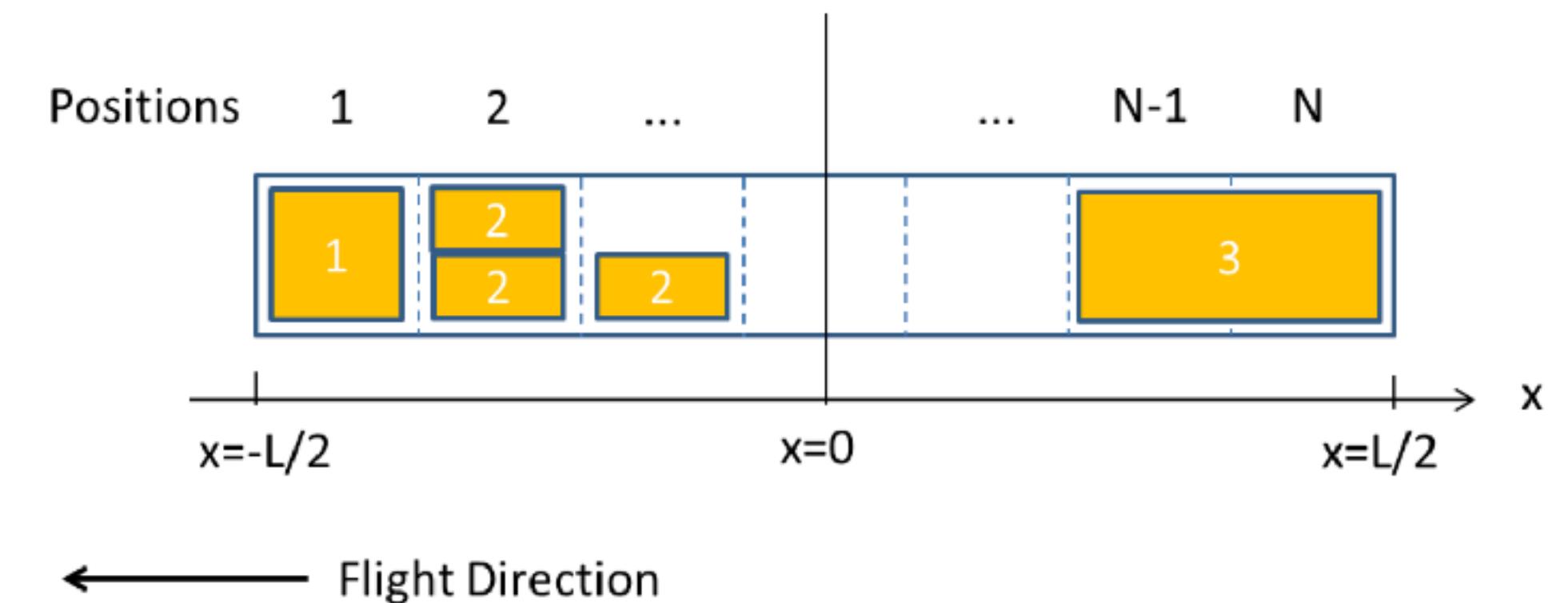
	Weight	Space

# Airbus Challenge

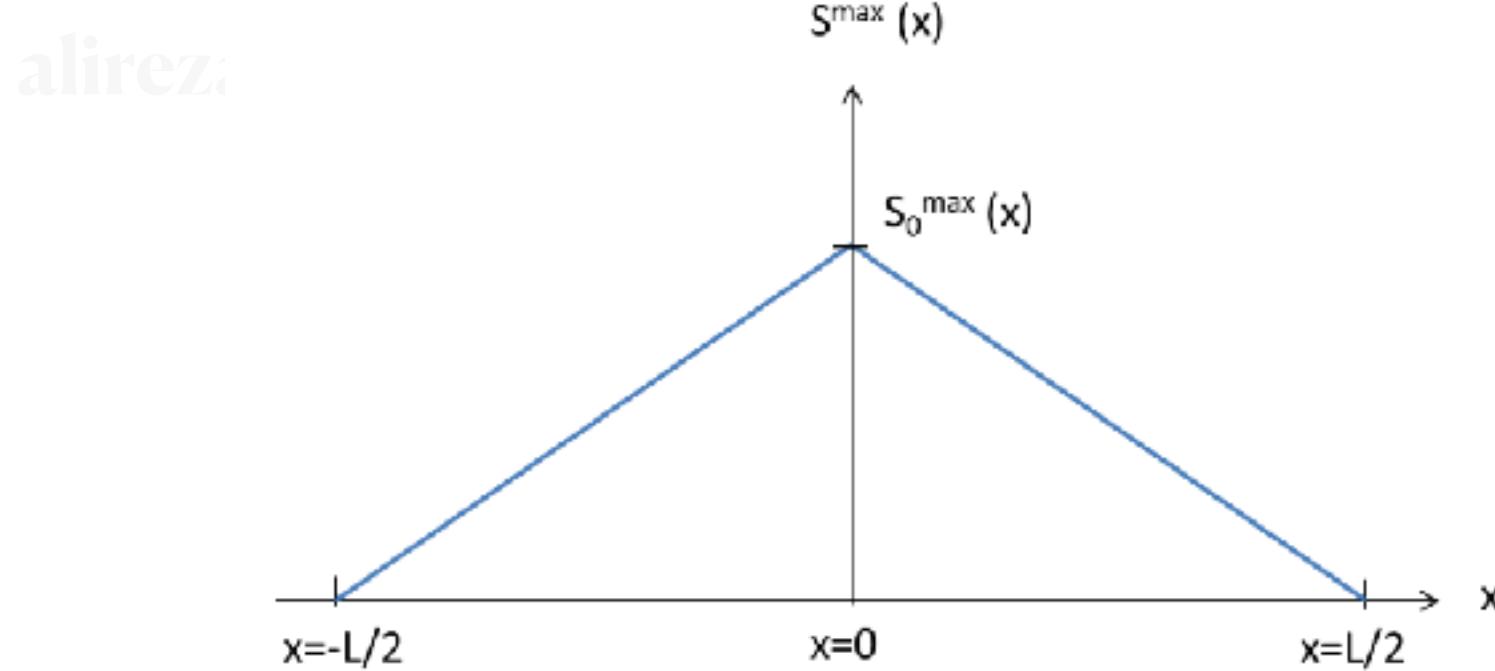
## Problem description



A freighter aircraft has  $N$  positions for standard size cargo containers equally distributed along the fuselage main deck:



← Flight Direction



# Graph Colouring

- Find the minimum number of colours for painting this graph  
no two connected nodes have the same colours

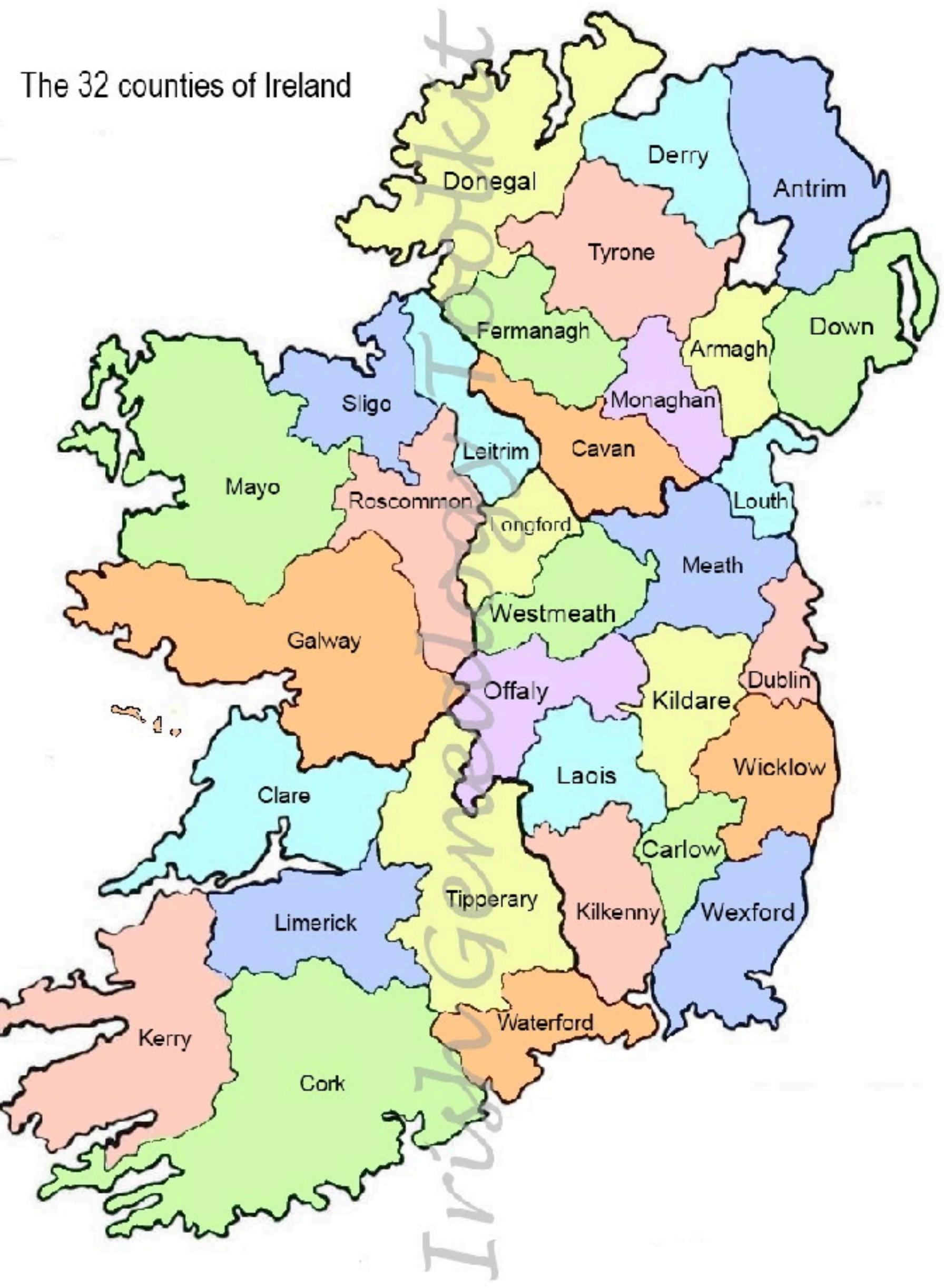
$$\min OF = y$$
$$X_{i,c}, y$$

$$\forall i, j, c \quad (X_{i,c} + X_{j,c})L_{i,j} \leq 1$$

$$\forall i, c \quad y \geq cX_{i,c}$$

$$\forall i \quad \sum_c X_{i,c} = 1$$

type of each variable?



$$\min_U OF = \sum_i y_i$$

$$\forall i \in Cusutomer \quad \sum_{n \in NB_i} y_n \leq 9CV_i$$

$$\forall i \in Cusutomer \quad \sum_{n \in NB_i} y_n \geq CV_i$$

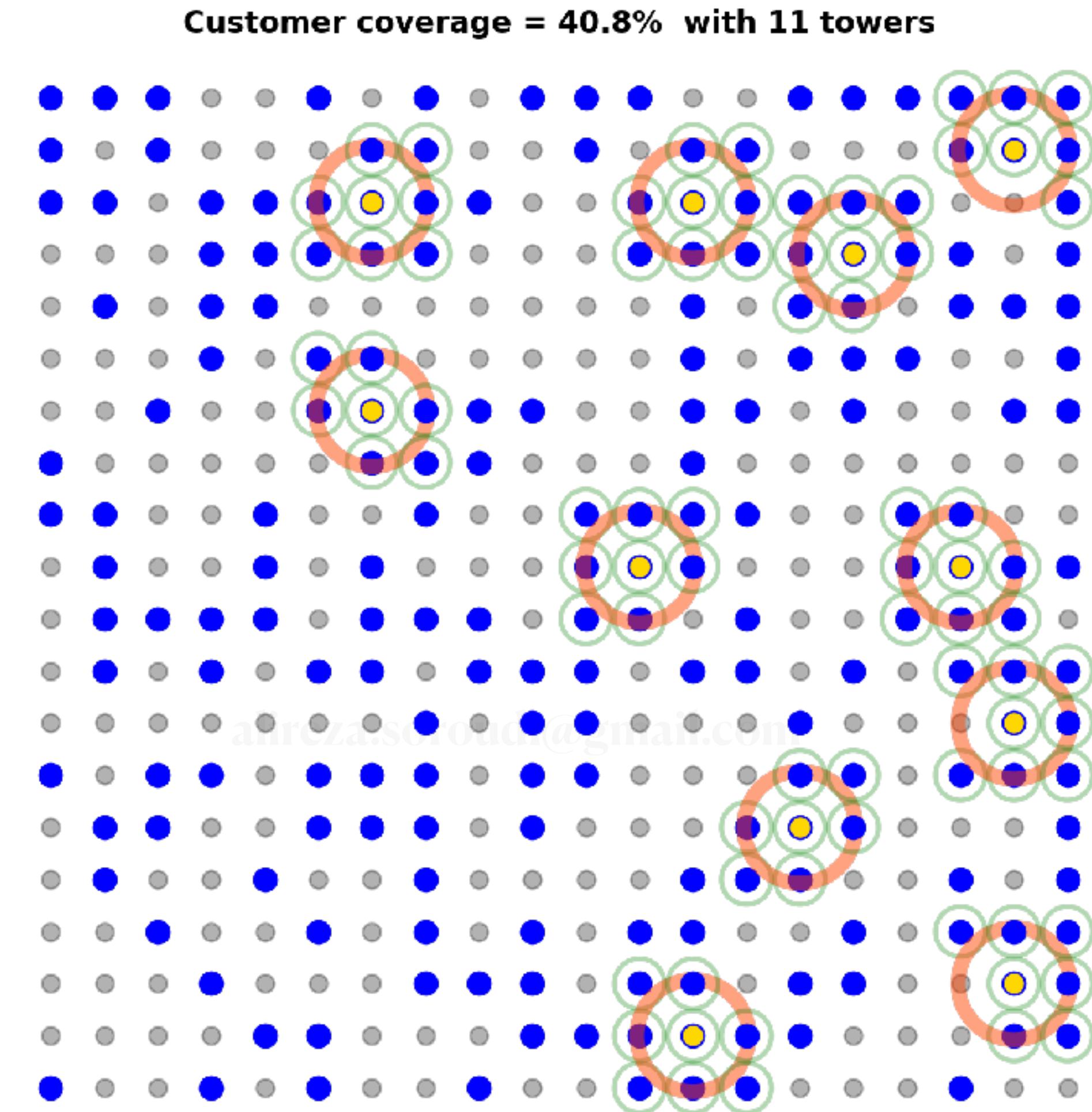
$$\sum_{i \in Cusutomer} CV_i \geq \eta \text{ Total customer}$$

$$CV_i, y_i \in \{0,1\}$$

$y_i$  Cell i has tower or not

$CV_i$  Cell i is covered or not

## Vaccination Centre Allocation



# TSP

Find the tour starting from 1 with min distance

alireza.soroudi@gmail.com

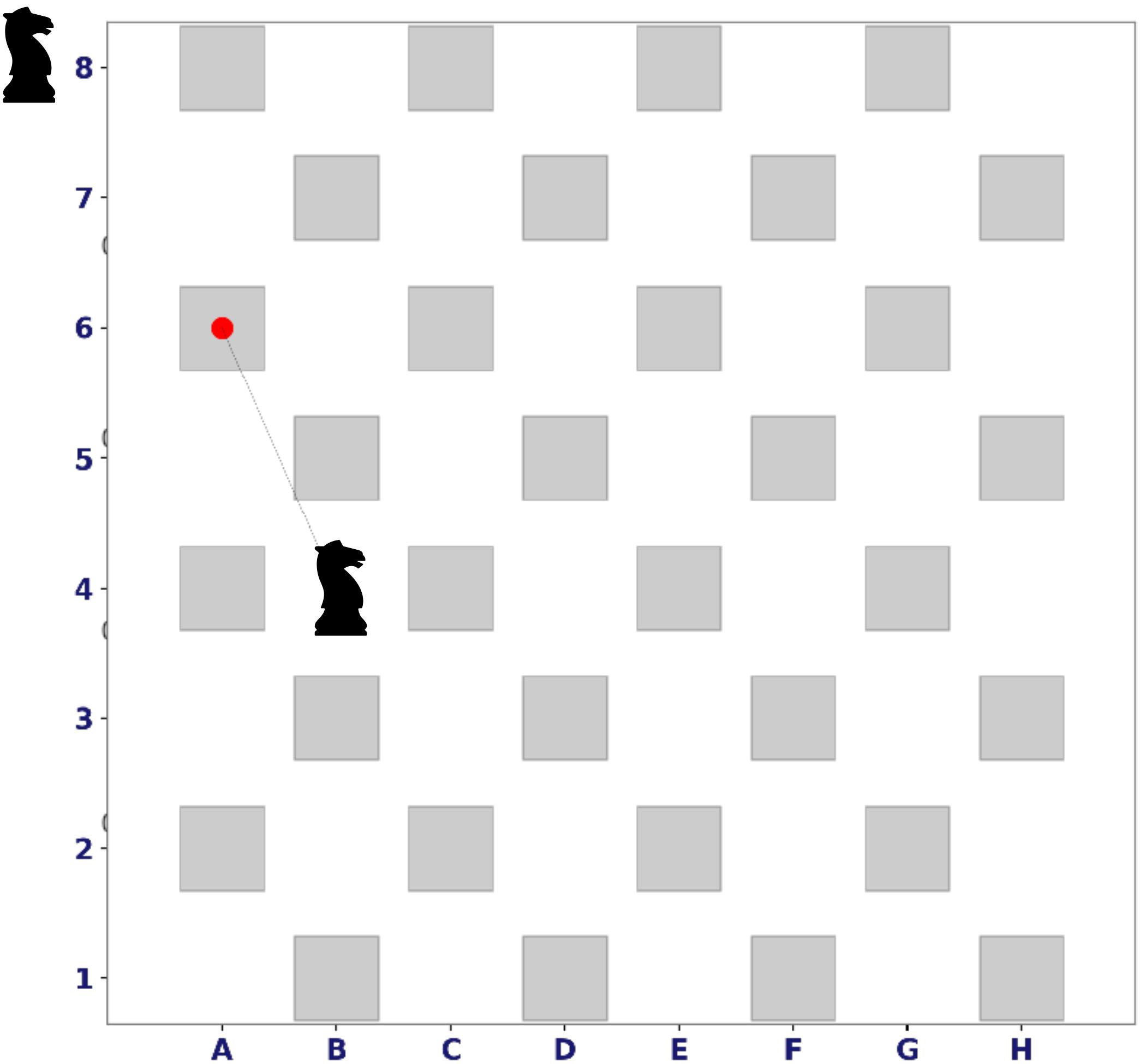
$$\min_u \sum_{i,j} U_{i,j} D_{i,j}$$

$$\forall i \quad G_i - L_i = \sum_j flow_{i,j}$$

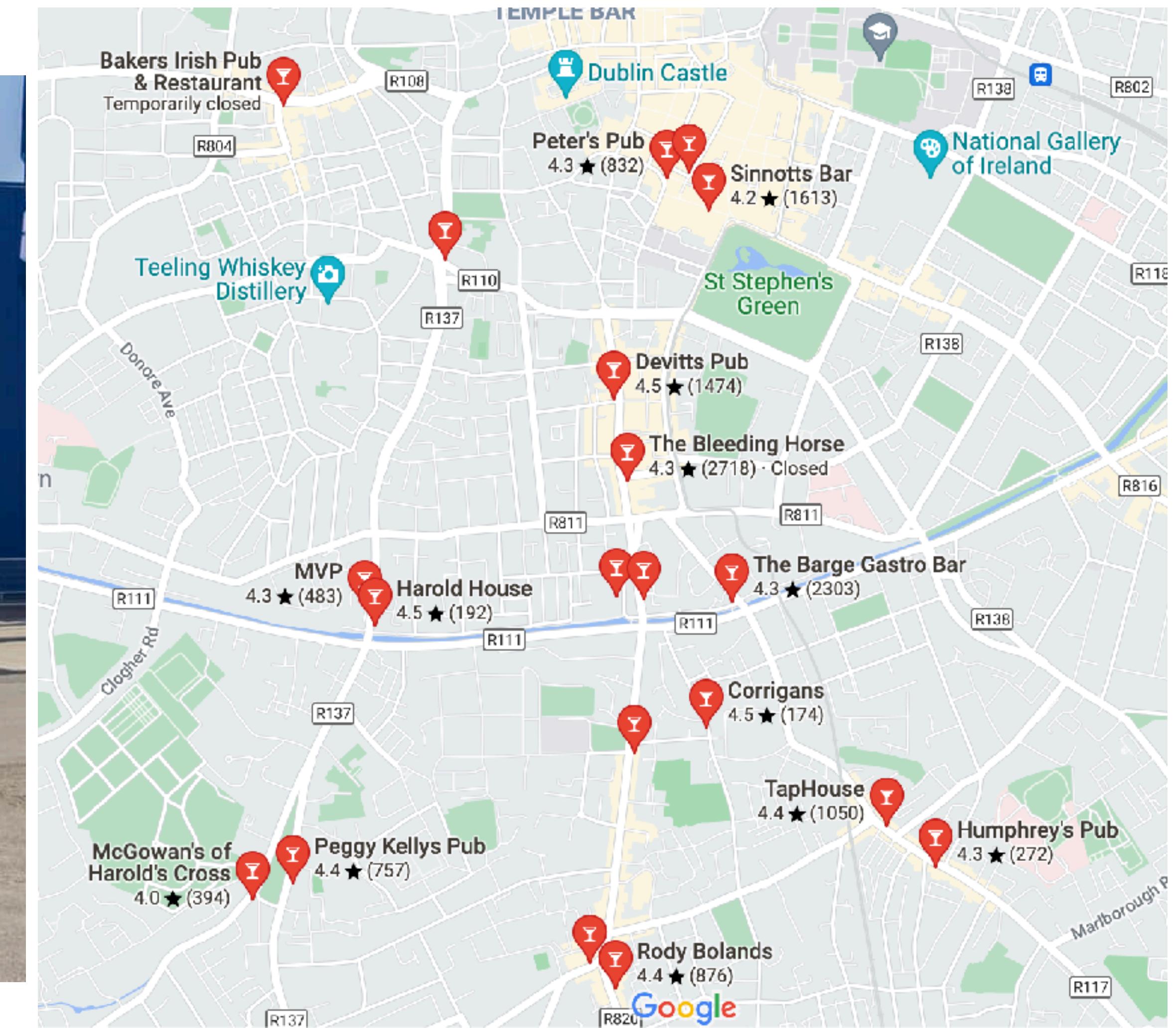
$$\forall i, j \quad flow_{i,j} \leq M U_{i,j}$$

$$\sum_c U_{i,c} = 1$$

$$\sum_c U_{c,i} = 1$$

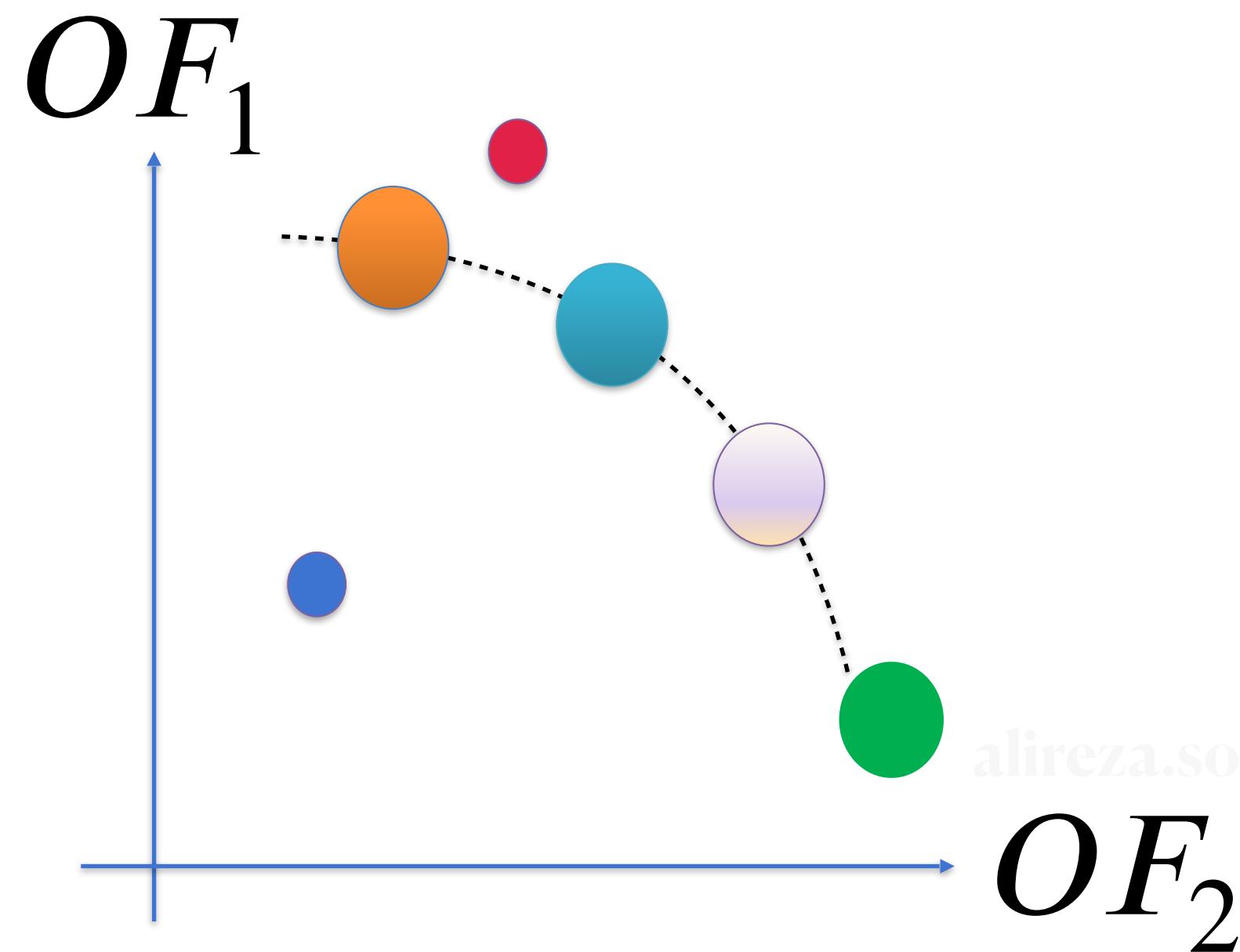


# Vehicle Routine Problem (VRP)



# Pareto Optimal Front

alireza.soroudi@gmail.com



# Pareto Optimal Front

Solve the following optimization problem

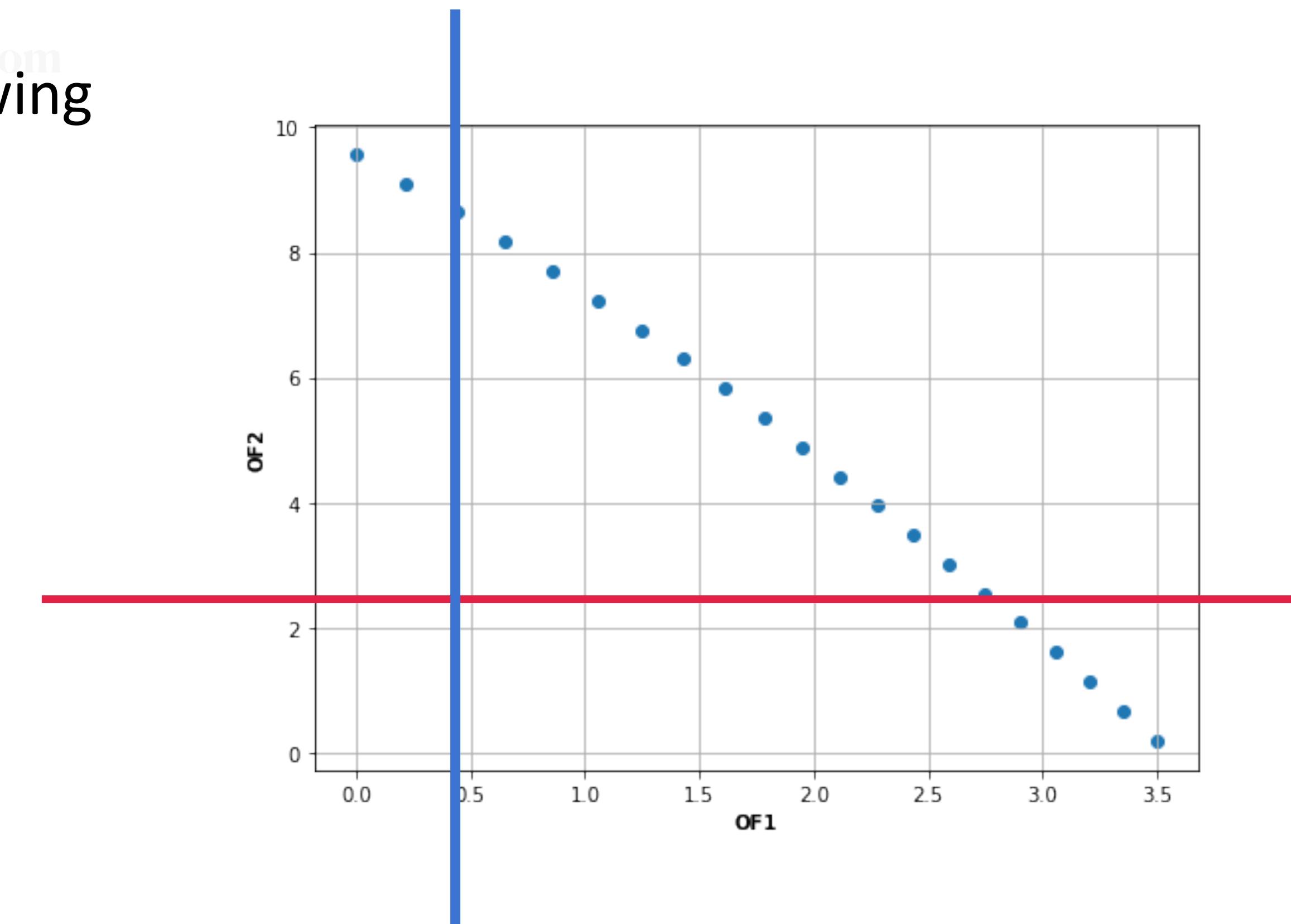
Find the Pareto optimal front for the following optimization problem

$$OF_1 = -1.2x_1^2 + 5x_2$$

$$OF_2 = 2x_1 - 0.5x_2^2$$

$$1 \leq x_1 \leq 2$$

$$1 \leq x_2 \leq 3$$



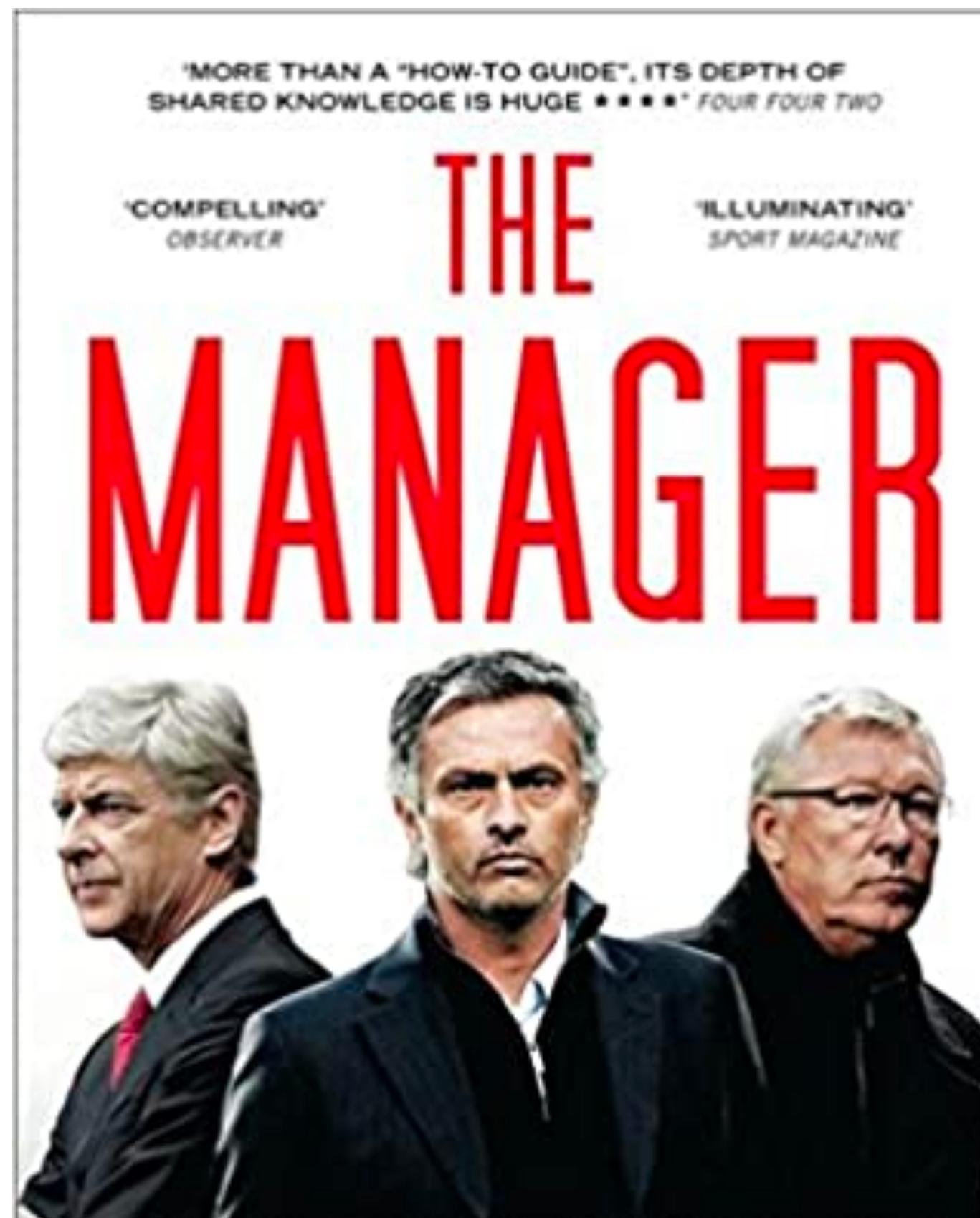
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# **Football Team Selection**

## **Optimisation based approach using Pyomo**

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# Football Team Selection

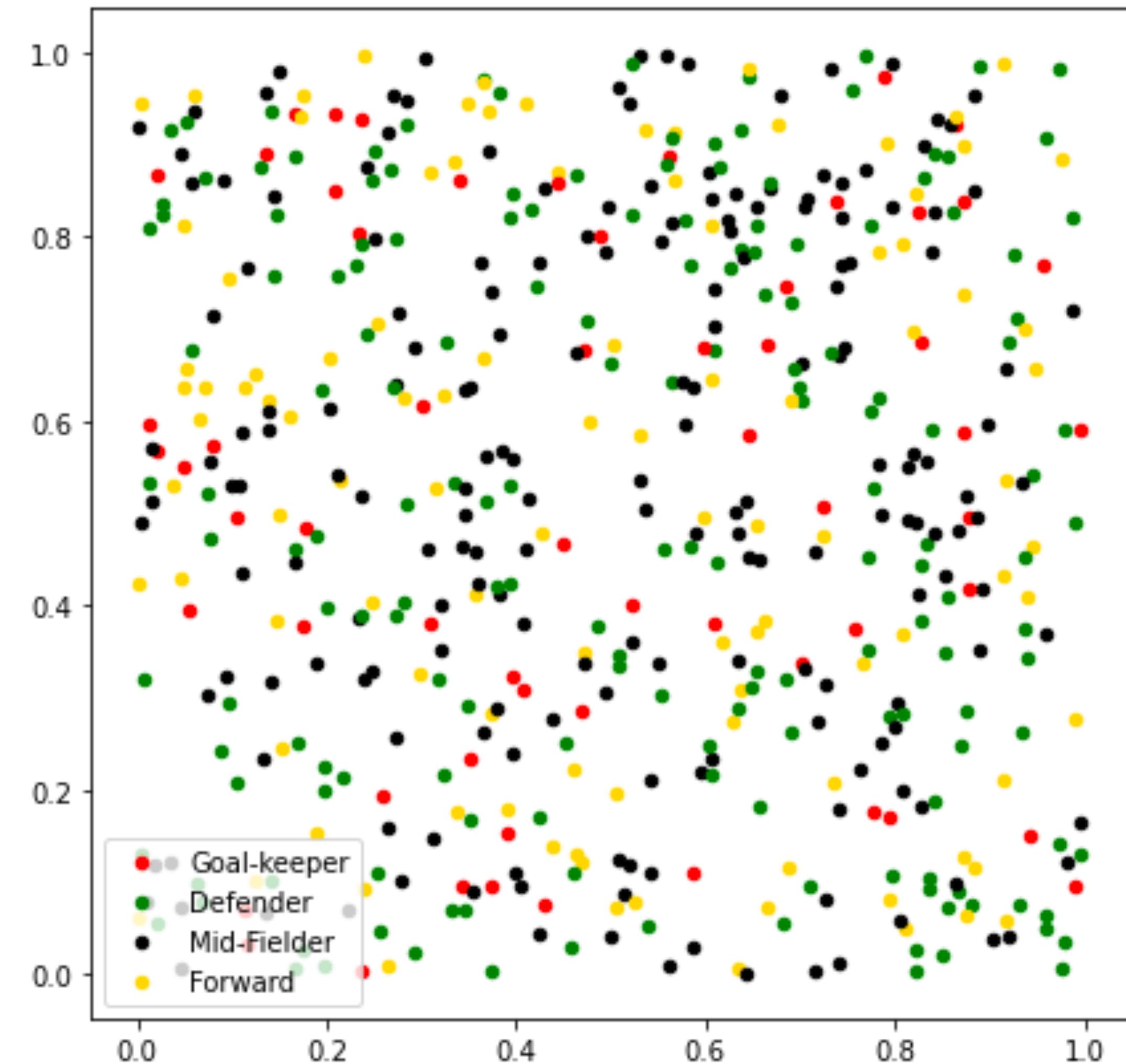


Should select  
players based on

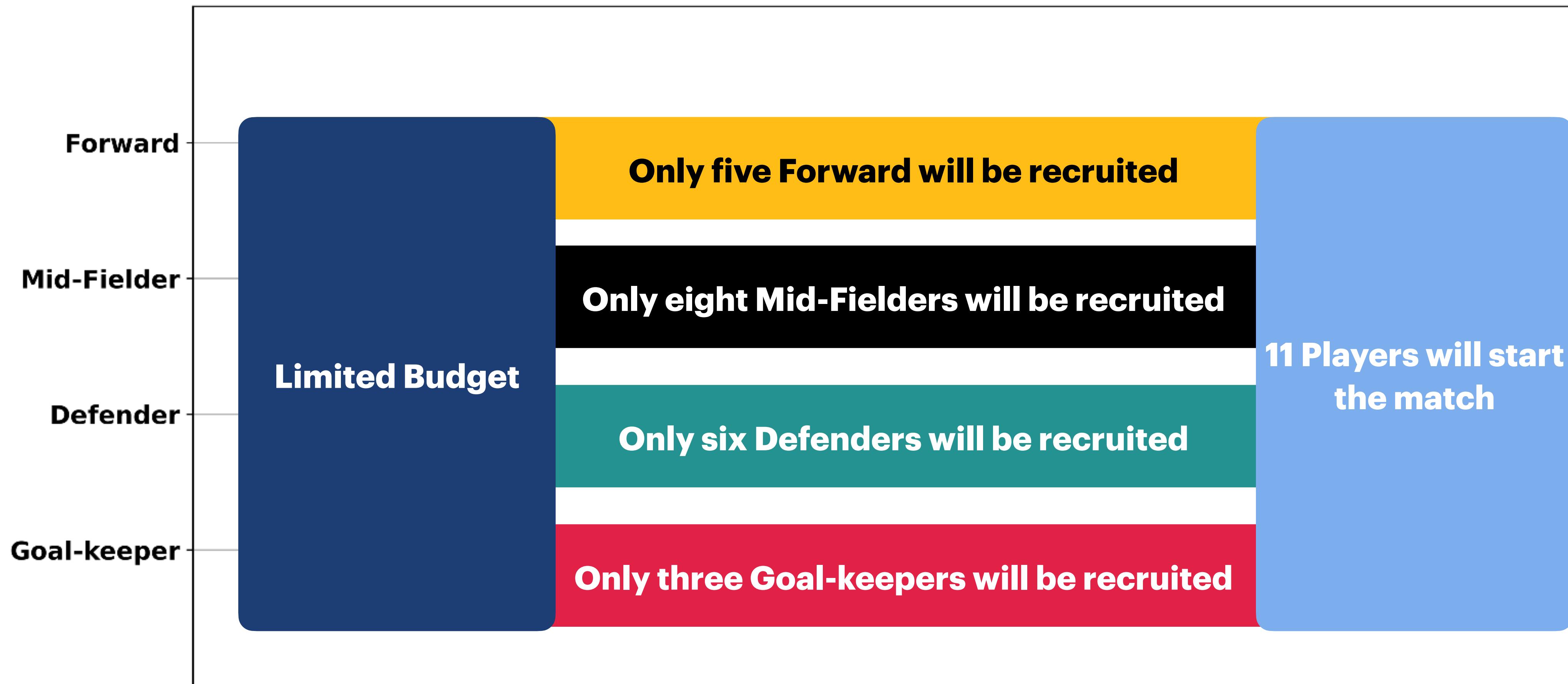


# Too many players with different characteristics

	p	fn	In	c	pos	marktwert	punkte
0	pl-k00030669	Manuel	Neuer	Bayern München	1	4500000	248
1	pl-k00030860	Philipp	Tschauner	RB Leipzig	1	500000	0
2	pl-k00030961	Philipp	Pentke	TSG Hoffenheim	1	800000	24
3	pl-k00033610	Kevin-Prince	Boateng	Hertha BSC	3	2200000	0
4	pl-k00037056	Tobias	Sippel	Bor. Mönchengladbach	1	700000	30
...	...	...	...	...	...	...	...
523	pl-k00133799	Soumaila	Coulibaly	Borussia Dortmund	2	1000000	0
524	pl-k00135221	Fabio	Blanco	Eintracht Frankfurt	4	700000	0
525	pl-k00136298	Bryan	Lasme	Arminia Bielefeld	4	1500000	0
526	pl-k00136434	Guilherme	Ramos	Arminia Bielefeld	2	1000000	0
527	pl-k00136679	Enrique	Herrero	Eintracht Frankfurt	4	500000	0



# Problem constraints



# Problem Objective and decision variables

alireza.soroudi@gmail.com

**Maximise the team  
value**

**Decision variable**

**Who to recruit for each post**

**Who to start the match?**

# Multi-Line Fitting Analysis

$$\min_{a_k, b_k} OF = \sum_i r_i$$

$$r_i \geq + (a_k X_i + b_k - Y_i) - (1 - U_{i,k})M$$

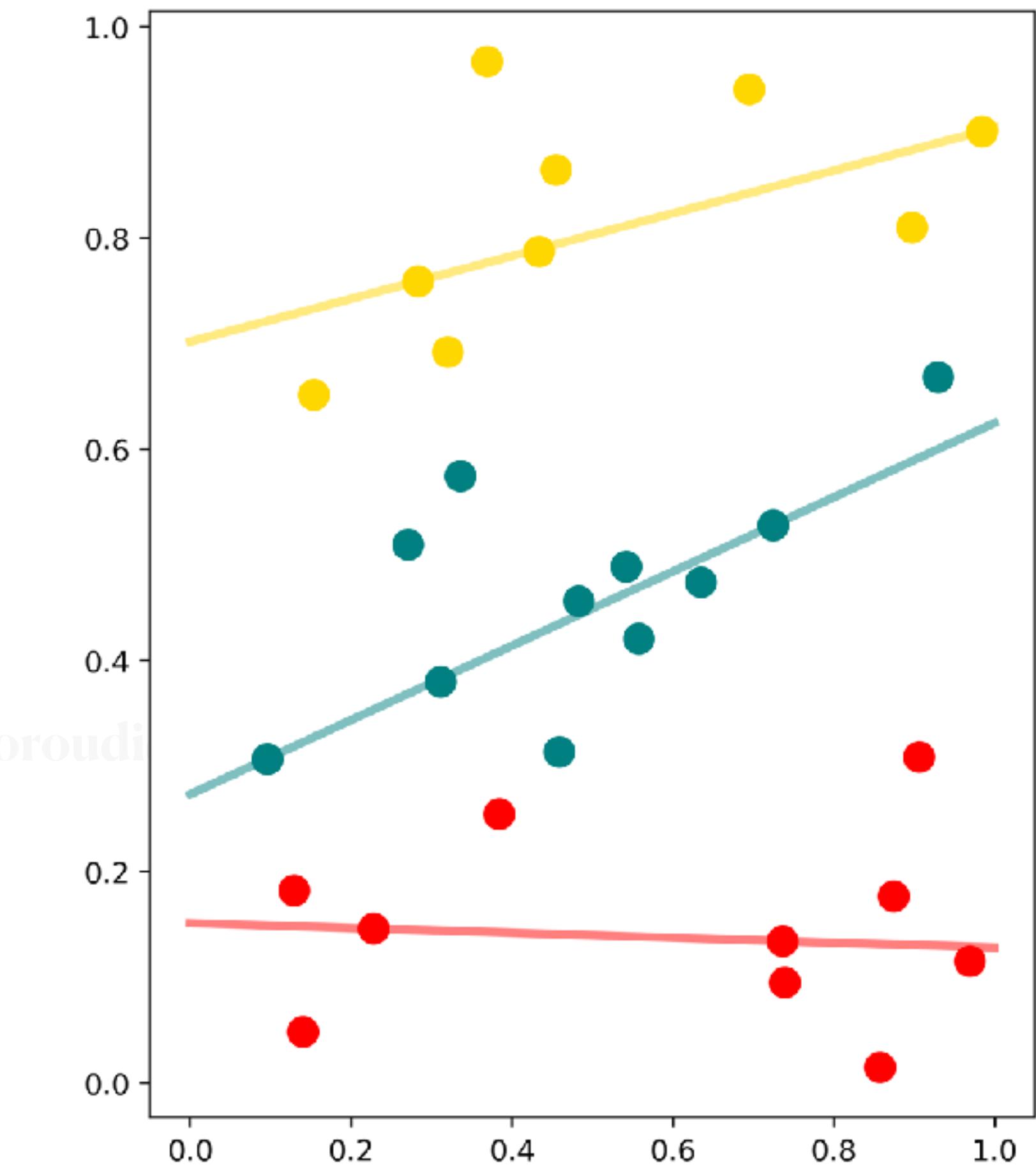
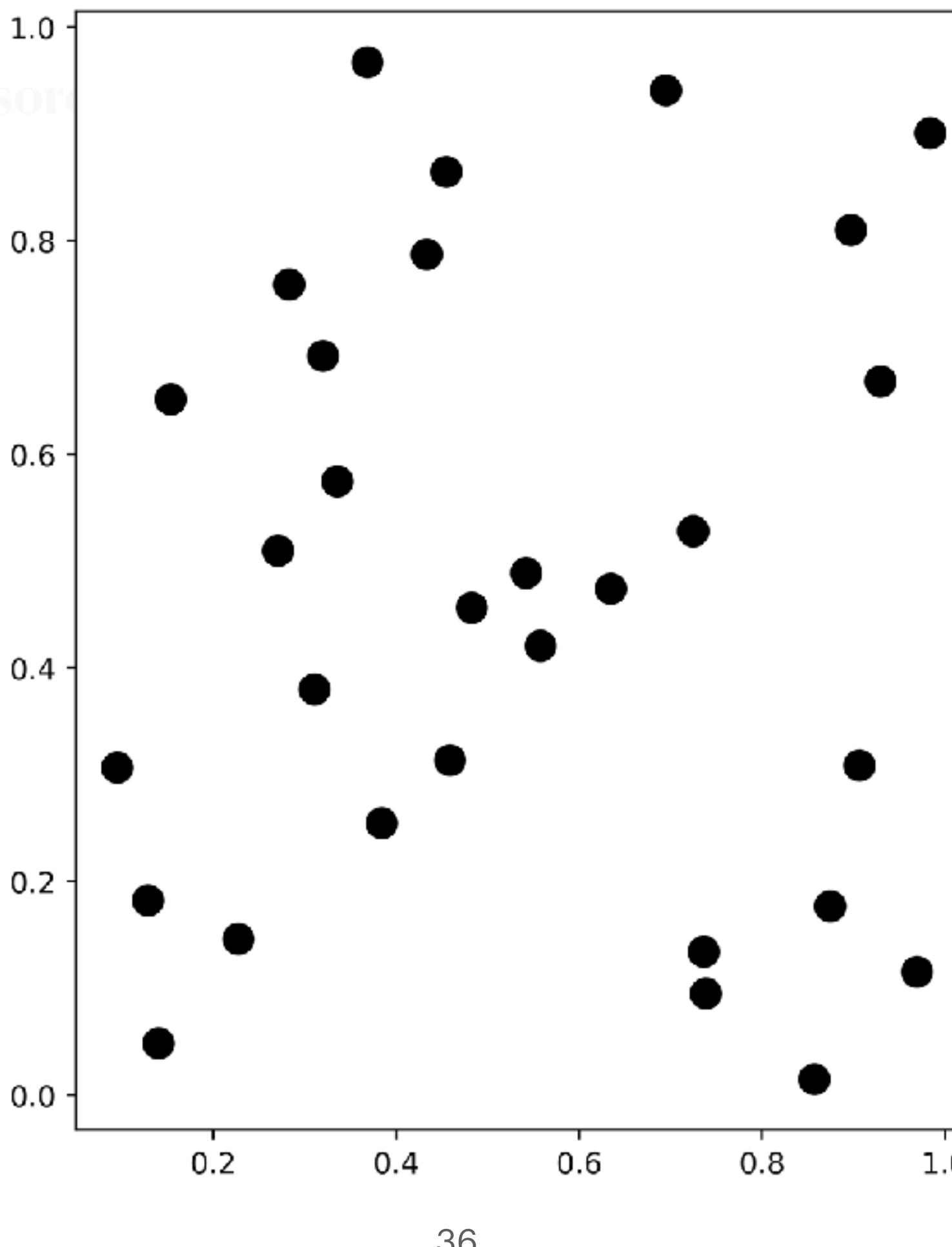
$$r_i \geq - (a_k X_i + b_k - Y_i) - (1 - U_{i,k})M$$

$$\forall i \quad \sum_k U_{i,k} = 1$$

$$U_{i,k} \in \{0,1\}$$

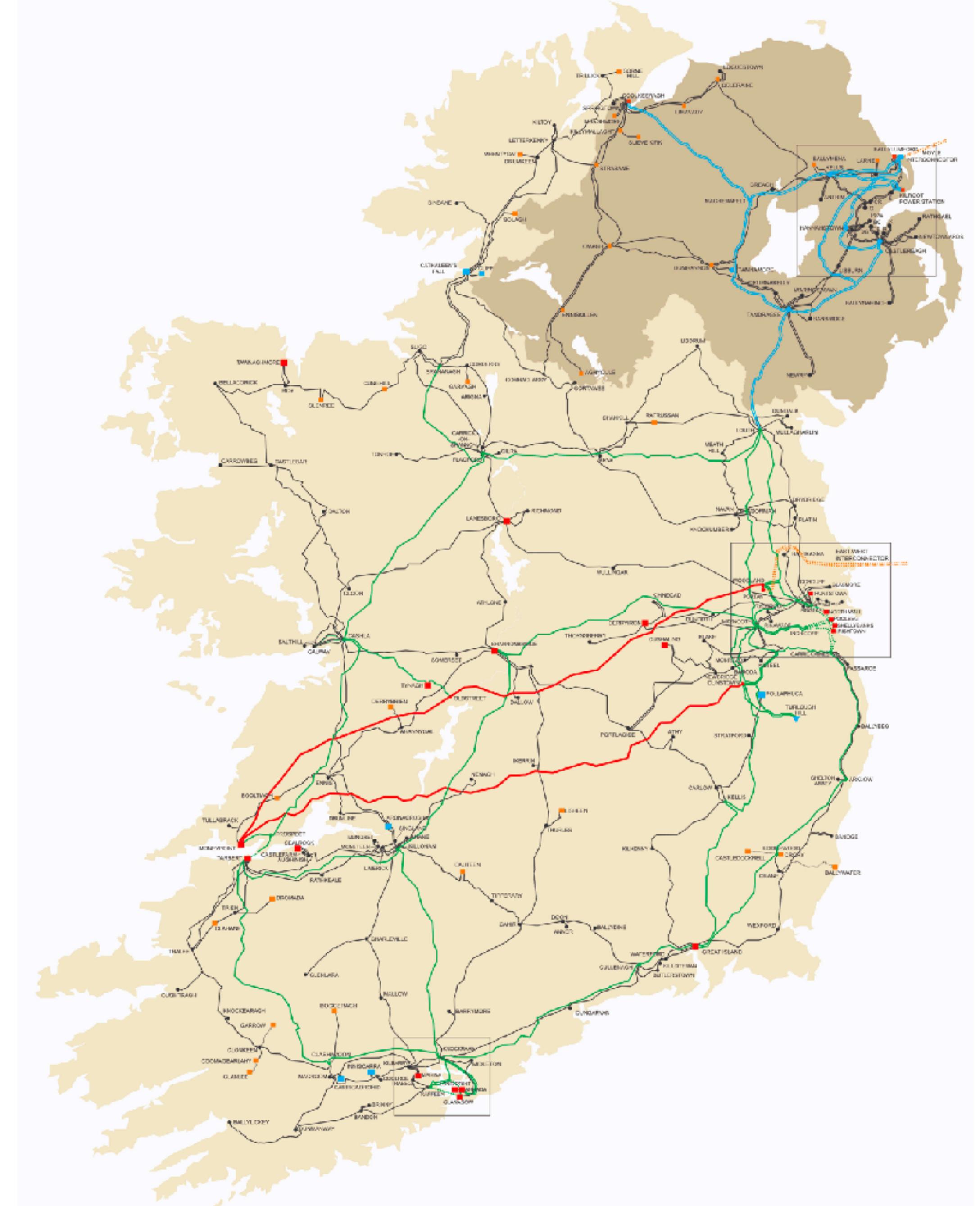
$$r_i \in R^{\geq 0}$$

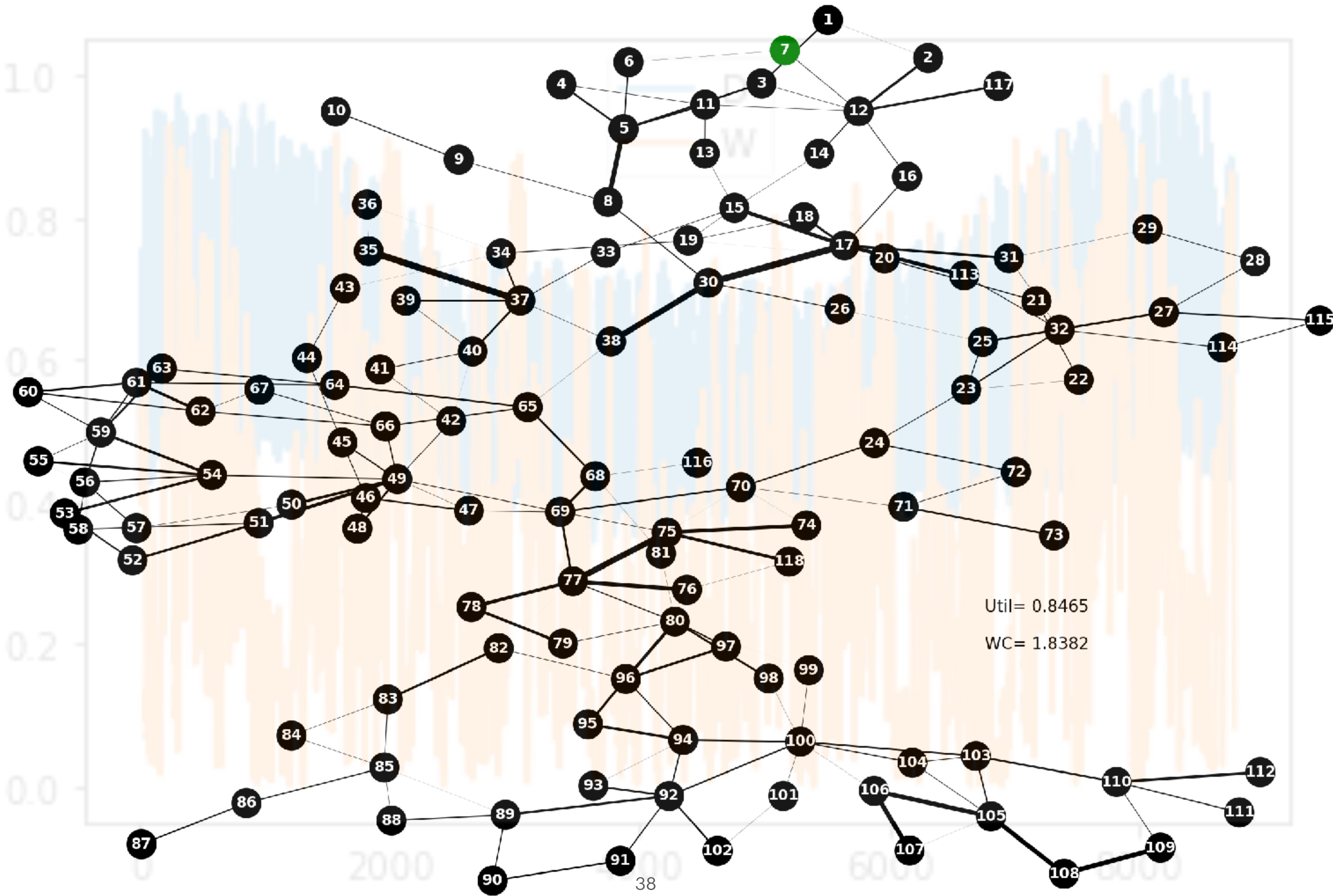
$$a_k, b_k \in R$$



# Transmission line loading

alireza.soroudi@gmail.com



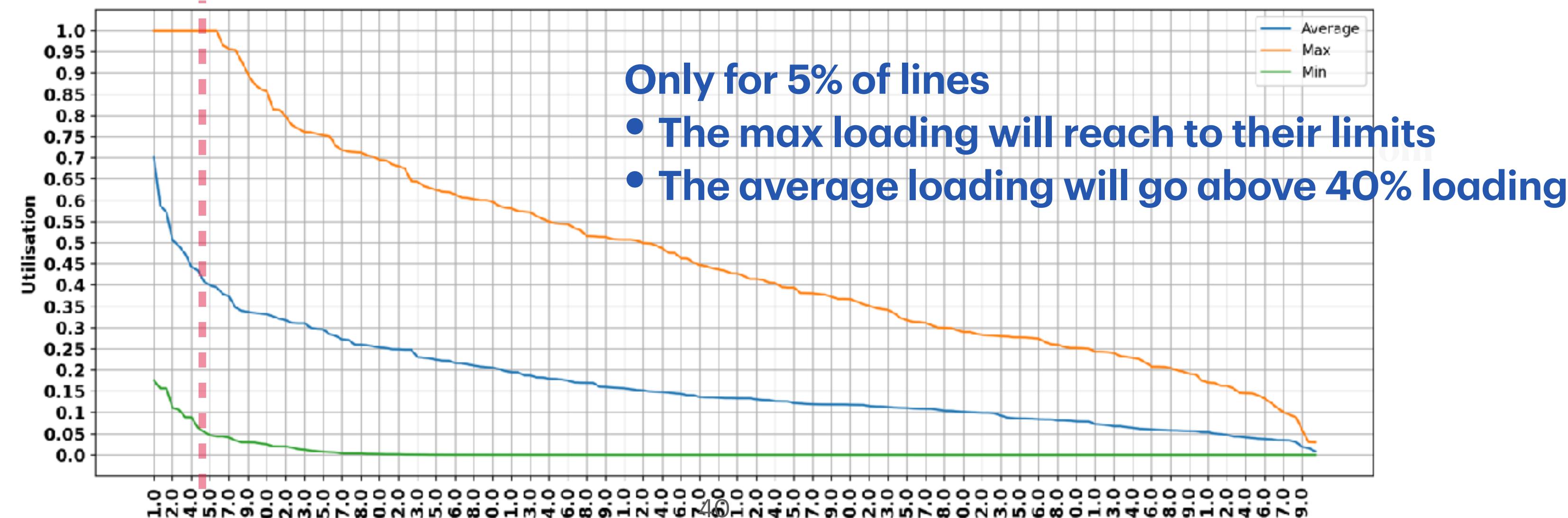
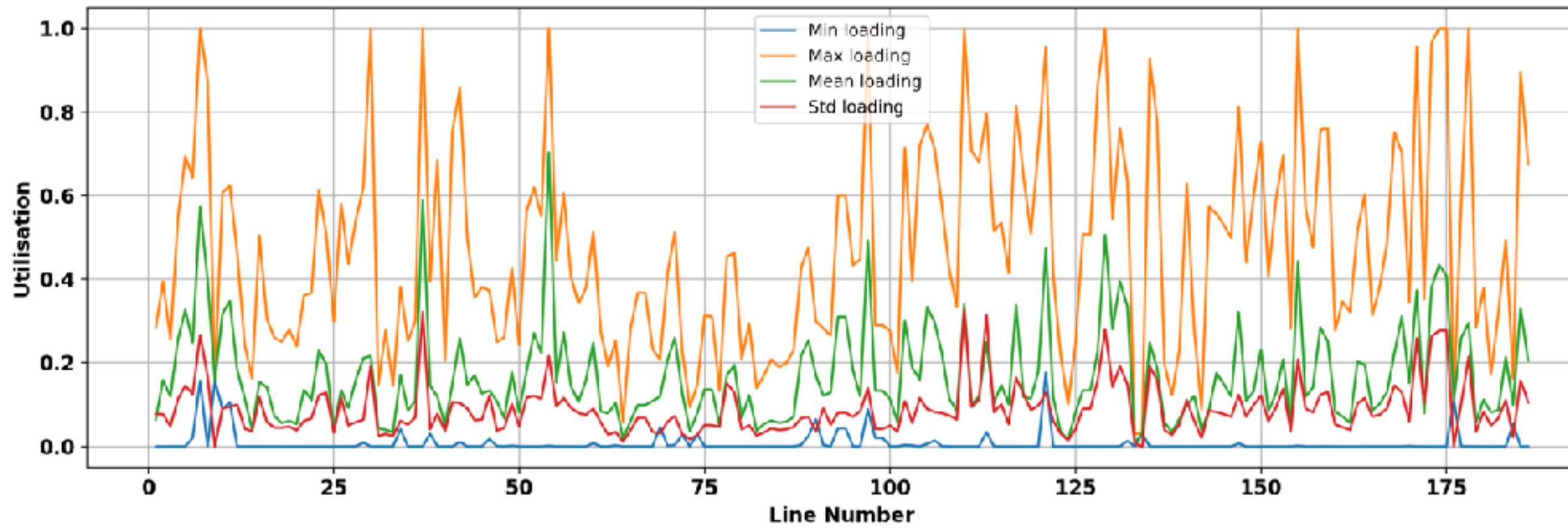


# Line Loading Duration Curve

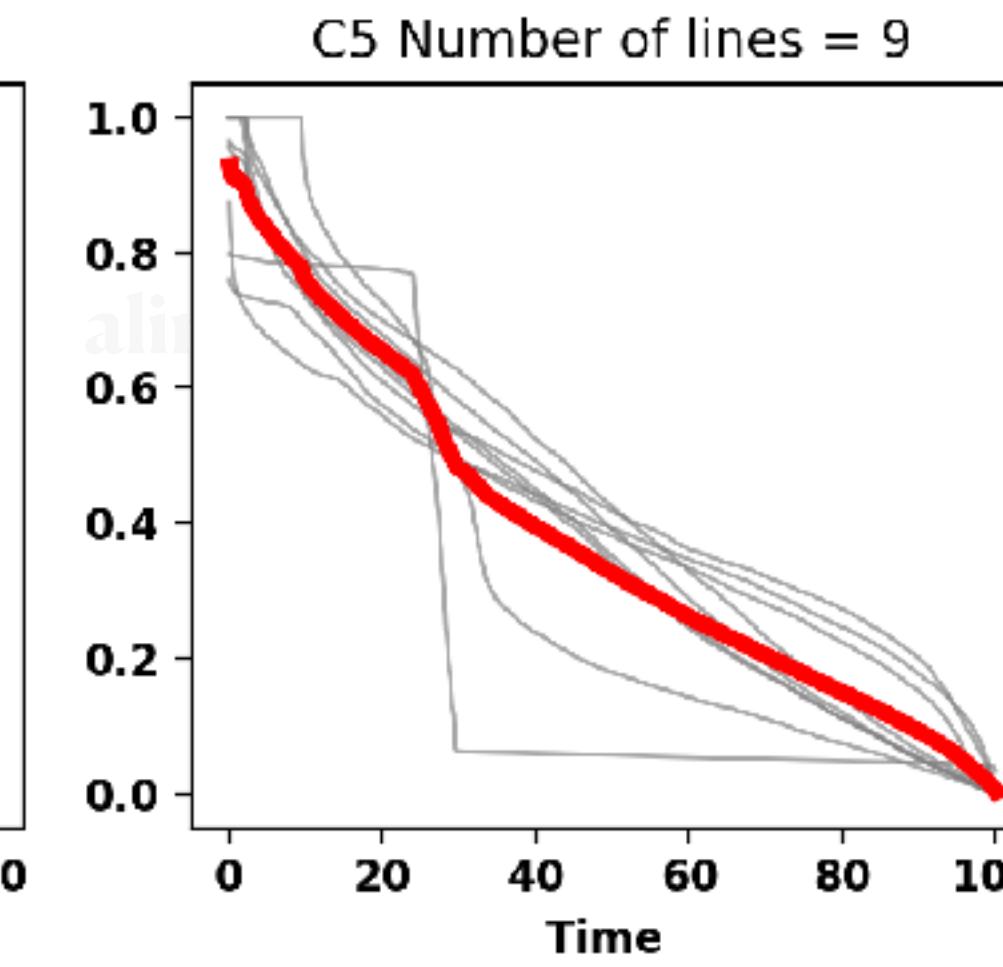
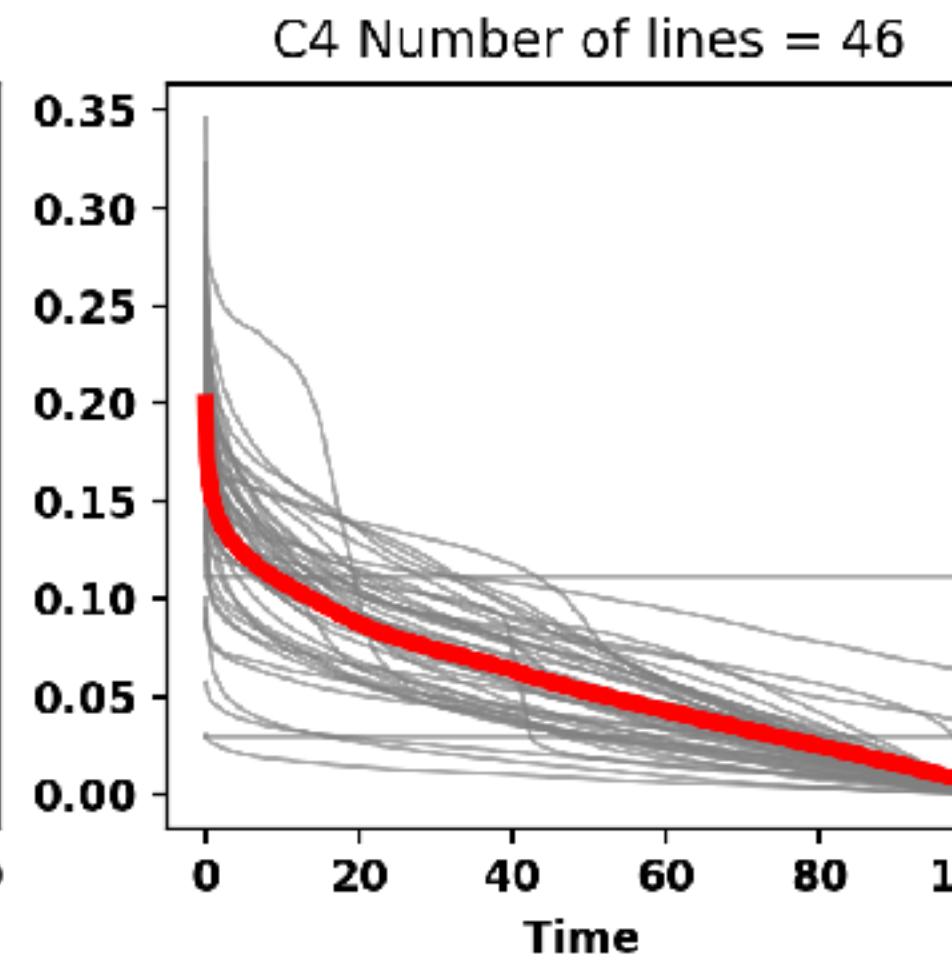
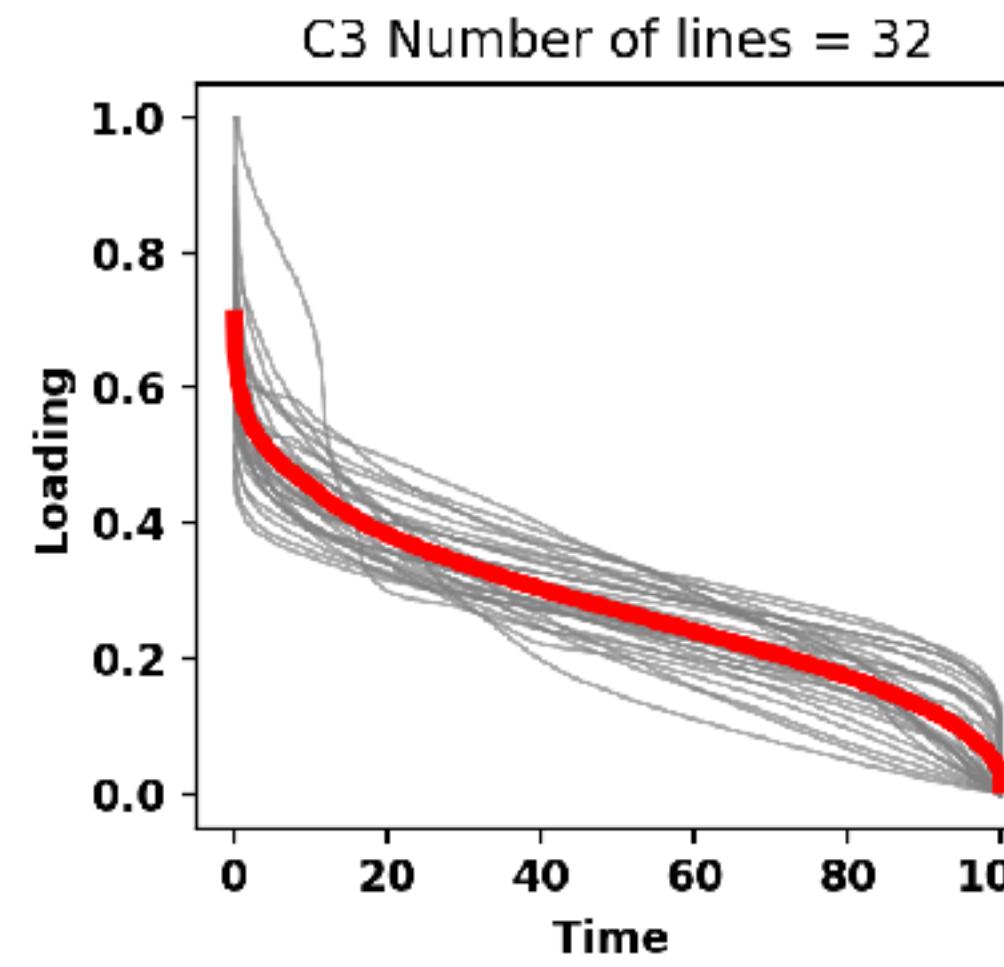
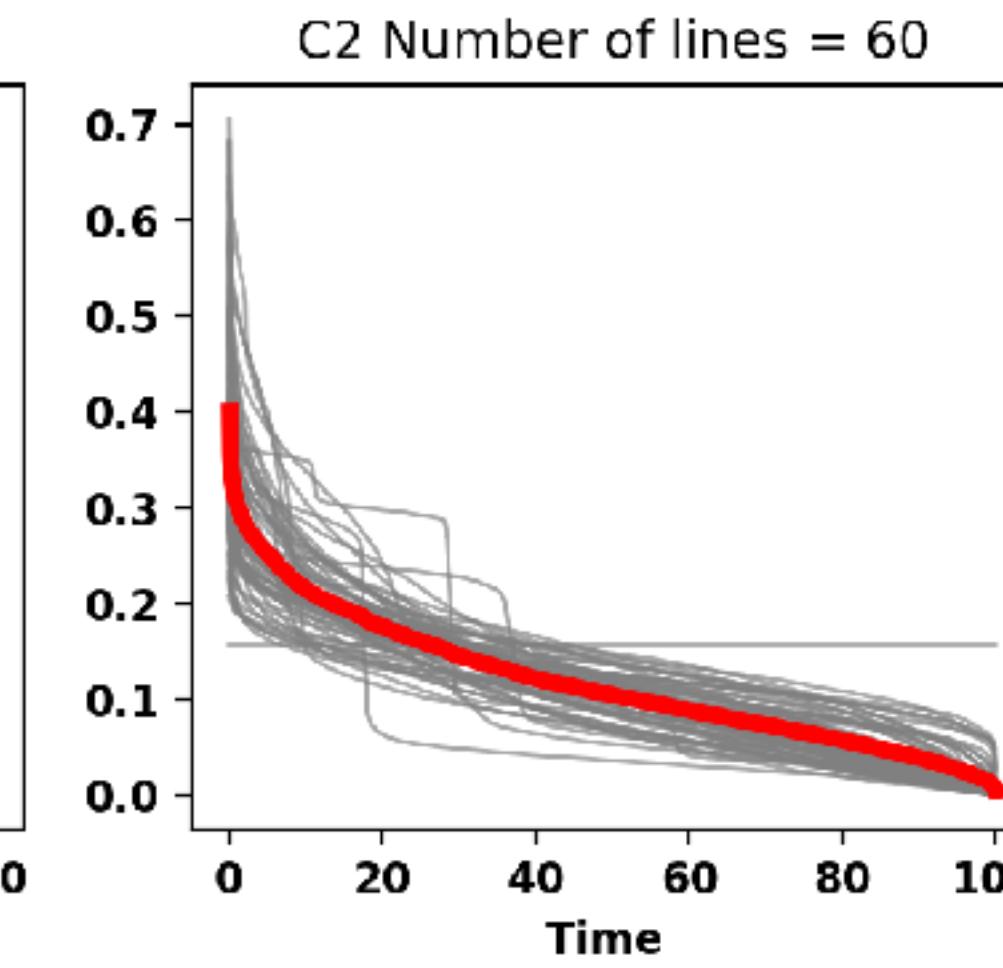
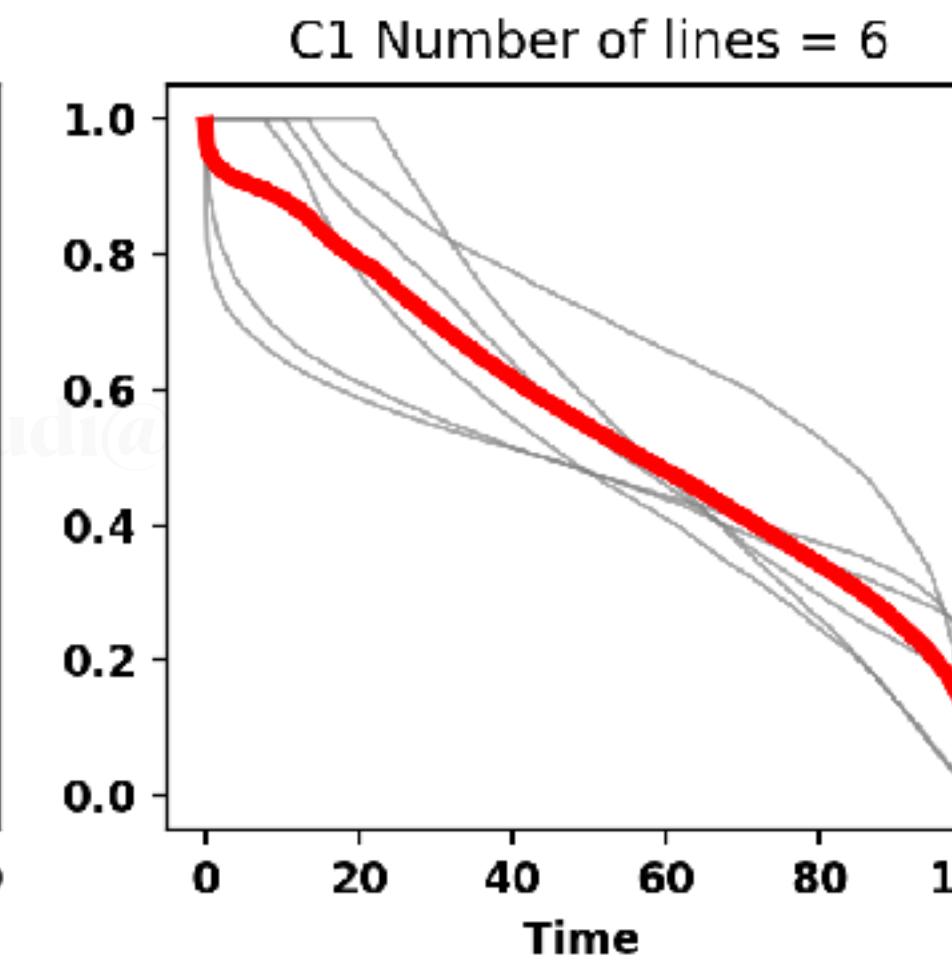
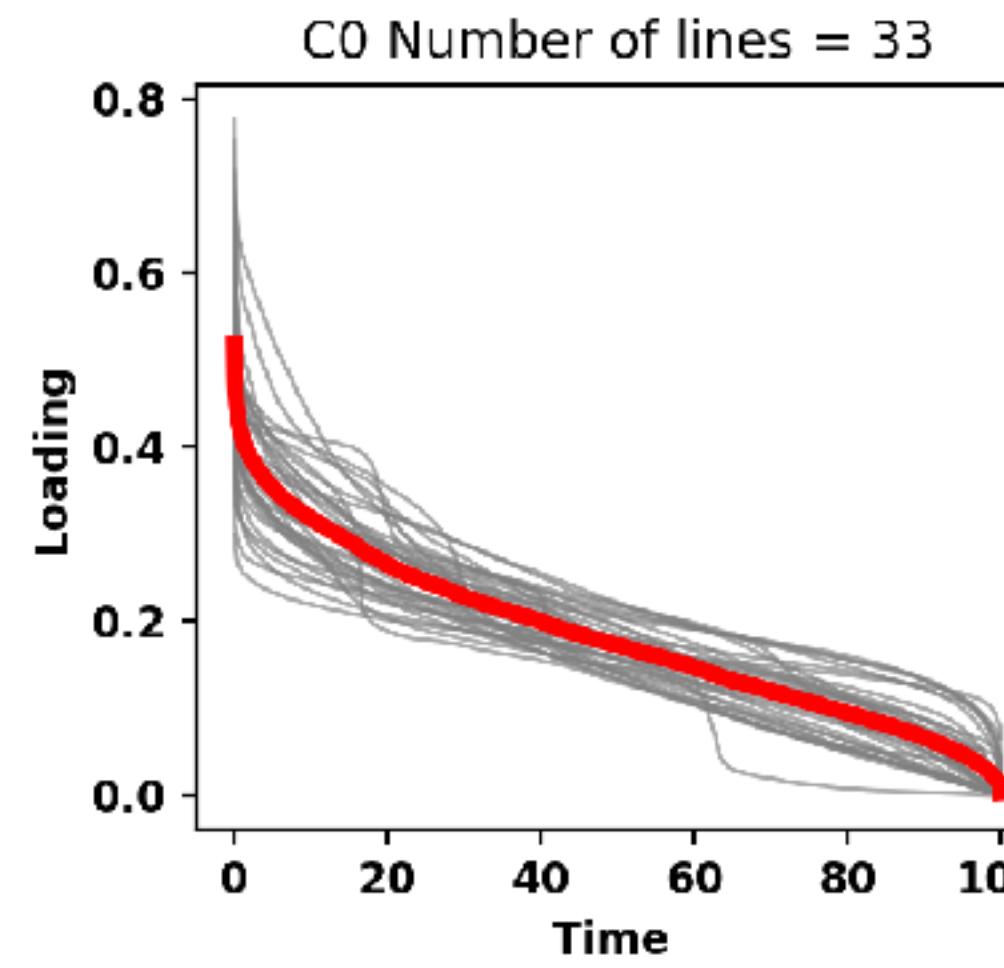


Not clear?

# Statistical properties of Line Loading



# Clustered Line Loading Duration Curve



# Some Real World Challenges

Customer needs

Data issues

Wrong setup

Uncertainty

Explain the output

Scalability

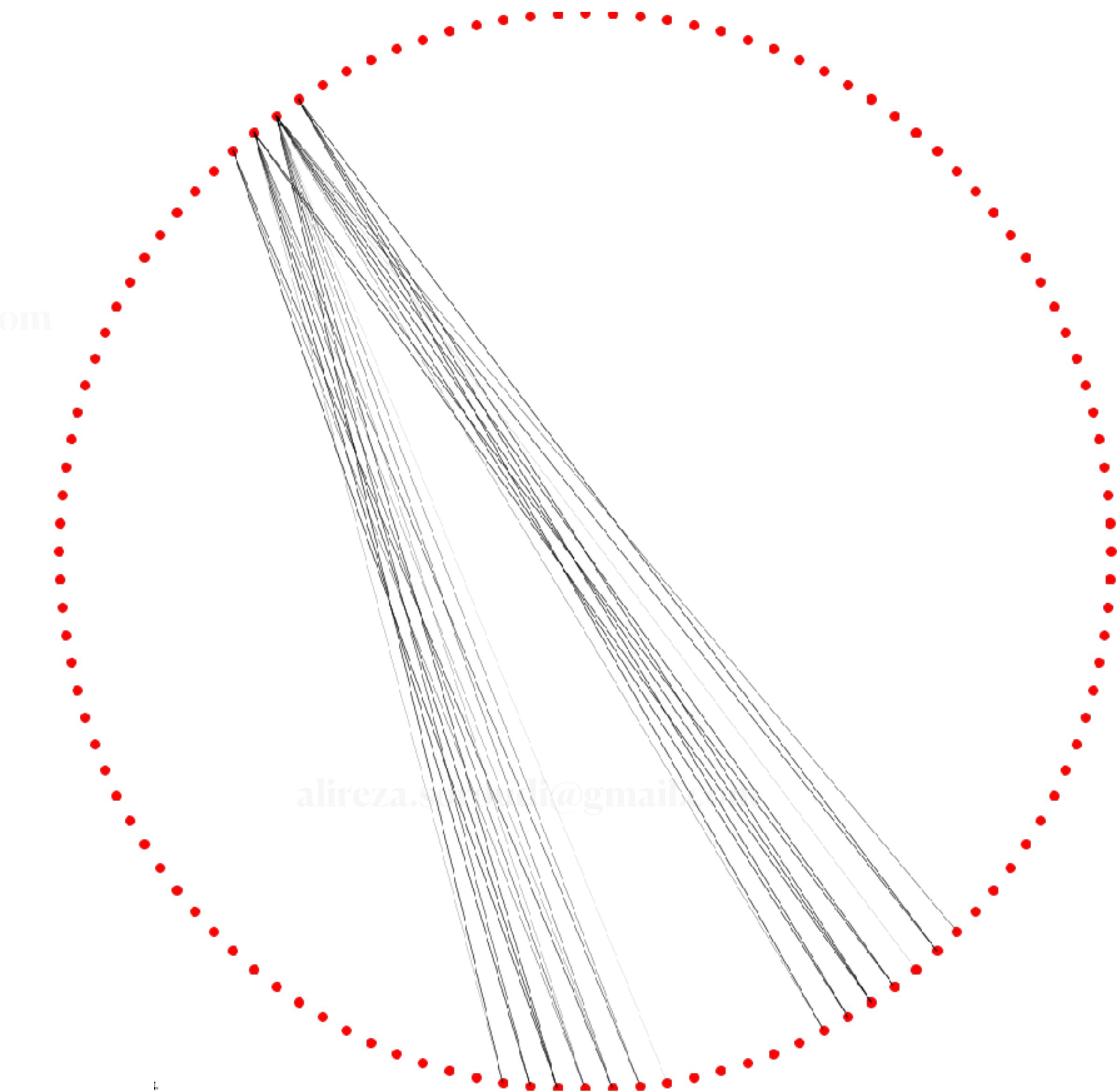
# The Art

$$X_{i,j} \in \{0,1\}$$

alireza.soroudi@gmail.com

$$0.5 \times 180 \times 179 = 16110$$

$$2^{16110} \approx 10^{4833}$$



# Thanks for your time

alireza.soroudi@gmail.com

