Propagation of Electric Field from a Suddenly Appearing Charge

Introduction

The electric field due to a point charge is often treated as if it exists instantly everywhere in space. However, according to special relativity and Maxwell's equations, no physical influence—including changes in the electric field—can travel faster than the speed of light, c. This project explores how the electric field of a charge "appears" in space when the charge is suddenly created.

Concept Overview

If a charge q is suddenly introduced at a point in space at time $t = 0$, its electric field does not instantaneously fill space. Instead, the information about the existence of the charge propagates outward at speed c.

Key Ideas

- A newly created charge sends out a disturbance in the electromagnetic field.
- This disturbance travels outward at speed c, forming a spherical front.
- Inside the front, the field becomes the familiar Coulomb field.
- Outside the front, no information about the charge has arrived yet.
- The electromagnetic disturbance is not like a continuous wave (such as light from an antenna), but it is a **field change** governed by Maxwell's equations.
- Once this "message" is delivered to a region of space, it leaves behind a **steady electric field**. The EM wave carries the information about the charge's creation or change.

Mathematical Framework

From Maxwell's equations, the fields from a point charge in arbitrary motion are described by the **Liénard–Wiechert potentials**. For a stationary charge that appears at $t = 0$:

$$\vec{E}(\vec{r},t) = \begin{cases} \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\hat{r}, & \text{if } r < ct \\ 0, & \text{if } r > ct \end{cases} \tag{1}$$

This tells us that the electric field only exists within the sphere of radius $r = ct$ at time t.

Diagram of Field Propagation

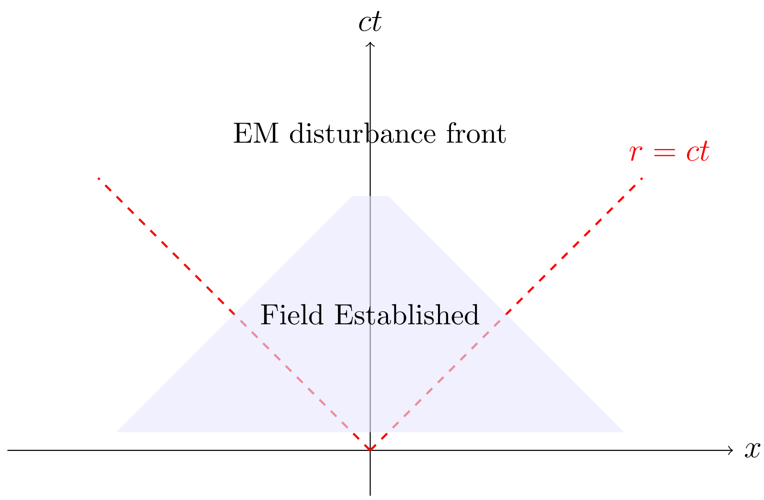

Figure 1: Spacetime diagram showing the light-speed propagation of the electric field after a charge appears at $t = 0$.

Wave Propagation as Information Carrier

When a charge suddenly appears, the information about it—i.e., the change in the electromagnetic field—travels outward like an electromagnetic wave at the speed of light. This information is a **disturbance** in the electromagnetic field, not necessarily a sinusoidal wave like light, but a step-like change. This disturbance is governed by the same set of Maxwell equations and acts as a messenger.

The electromagnetic wave thus carries the information about the creation (or change) of the charge. Once this message is delivered to a point in space, it leaves behind a steady electric field. In this way:

> *The EM wave is the messenger; the static electric field is the message.*

This aligns perfectly with causality: no part of space learns about the new charge until the signal (the EM field change) arrives at light speed.

Comparison with an Accelerating Charge

An accelerating charge behaves differently from a stationary one. In this case, the change is not just in the presence of the charge but in its motion. The acceleration causes a time-varying dipole field that radiates energy outward in the form of electromagnetic waves.

This radiation is described by the far-field (radiative) term of the Liénard–Wiechert potentials, and it produces real electromagnetic waves — sinusoidal or otherwise — that can travel through space and be detected far away.

Key differences:

- A **suddenly appearing stationary charge** creates a step-like change in the field, which propagates outward at speed c, eventually settling to a static Coulomb field.
- An **accelerating charge** continuously sends out changes in its field, producing a real EM wave that carries energy and momentum.
- In both cases, the change in the field propagates at c, but only the accelerating charge produces **continuous radiation**.

Thus, while both situations involve field propagation, only the accelerated charge leads to persistent electromagnetic radiation. The case of a suddenly appearing charge is more like a one-time information signal.

Physical Interpretation

This shows that:

- The electric field does not exist outside the light cone.
- Changes in the electromagnetic field always respect the light-speed limit.
- The "wave" of field propagation is like an EM step-front, not a sinusoidal wave.
- Once the wavefront passes, a static Coulomb field remains.
- In contrast, an accelerating charge produces a continuous electromagnetic wave.

Conclusion

We often imagine static fields as being ever-present, but the physical reality is that they are established over time via light-speed-limited propagation. Once established, they remain static unless the source changes. The initial field disturbance is transient — it brings the message of the charge's presence and leaves behind a static field. In this view, electromagnetic waves act as the carriers of physical information, ensuring causality and consistency with the laws of special relativity.

Furthermore, in the case of an accelerating charge, this field disturbance is not transient — it becomes a continuous radiation field, carrying energy across space and time.