# Using countable additivity

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Let the sample space be the set of positive integers and suppose $P(n) = \frac{1}{2^n}$ for $n = 1, 2, \ldots$

Find the probability of the set $\{3, 6, 9, \ldots\}$

↑ +ive integers that are multiples of 3.

The sequence is countably additive.

$$P(n) = \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9} + \ldots$$

$$= \frac{1}{8} + \frac{1}{64} + \frac{1}{512} + \ldots$$

$$= \frac{1}{8^1} + \frac{1}{8^2} + \frac{1}{8^3} + \ldots$$

$$= \sum_{i=1}^{\infty} \frac{1}{8^n} \longleftarrow \sum_{i=1}^{\infty} a_i = \frac{a}{1-r}$$

$a = \frac{1}{8} \quad r = \frac{1}{8}$

$$= \frac{\frac{1}{8}}{1 - \frac{1}{8}} = \frac{\frac{1}{8}}{\frac{7}{8}} = \frac{8}{42} = \frac{1}{7}$$