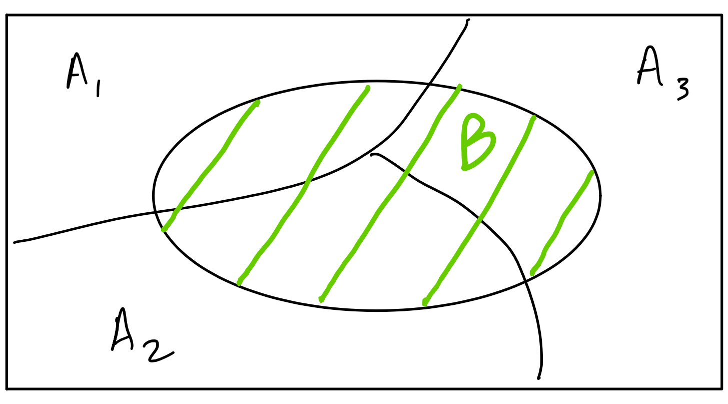
Total probability theorem

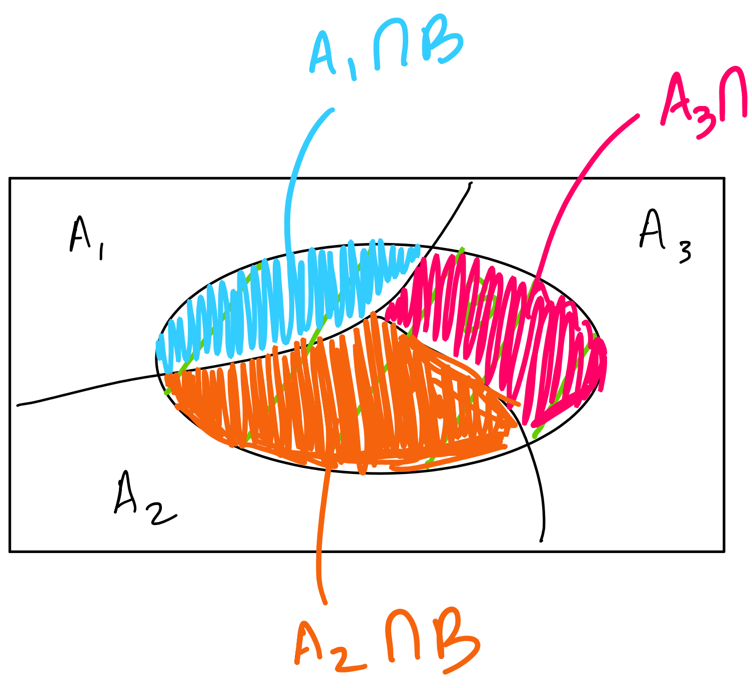
Friday, April 17, 2020 5:20 PM

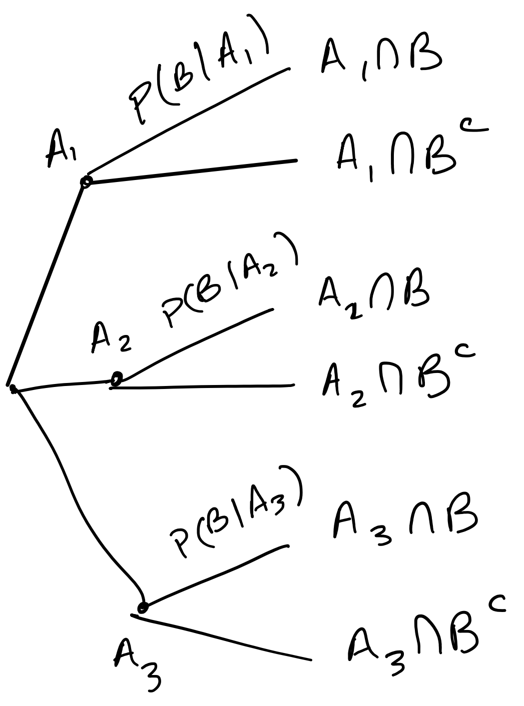


Sample space is A partitioned into A_1, A_2, A_3.

B is a subset of A inside the oval.

Event A can happen in 3 ways, in each of these ways event B may or may not occur.





We have $P(A_i)$ for each i

We have $P(B|A_i)$ for each i

Can we calculate $P(B)$ with this info?

<u>YES:</u>

$$P(B) = P(B \cap A_1) + P(B \cap A_2) + P(B \cap A_3)$$

$$= P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)$$

This is the same as saying:

$$\boxed{P(B) = \sum_{i=1}^{n} P(A_i)P(B|A_i)}$$

This formula is a weighted average. $P(A_i)$ are the weights.

The probability that event B occurs

is the

Weighted avg of the conditional probabilities $P(B|A_i)$ where the probabilities of the individual scenarios $P(A_i)$ are the weights.

<u>Problem</u>:
- We have infinite # of biased coins.
- The coins are indexed by the positive integers so they are countably infinite set.
- Coin i has 2^{-i} chance of being selected.
- A flip of coin i results in heads with probability 3^{-i}

We select a coin and flip it. What is the probability that it is heads?

According to total probability theorem,

Event A = selection of coin i.
- Since there are i coins, event A is partitioned into i parts. $A_1, A_2, A_3, \dots, A_i$.
- $P(A) = \sum_{i=1}^{\infty} P(A_i) = \sum_{i=1}^{\infty} \frac{1}{2^i}$

Event B = getting a heads on the coin flip.
- This event's probability is conditional on A_i occurring.
- $\sum_{i=1}^{\infty} P(B|A_i) = \sum_{i=1}^{n} \frac{1}{3^i}$

$$P(B) = \sum_{i=1}^{\infty} P(A_i) \cdot P(B|A_i) = \sum_{i=1}^{\infty} \frac{1}{2^i} \cdot \frac{1}{3^i}$$

$$= \sum_{i=1}^{\infty} \left(\frac{1}{6}\right)^i = \frac{\frac{1}{6}}{1 - \frac{1}{6}} = \frac{\frac{1}{6}}{\frac{5}{6}} = \frac{6}{30} = \frac{1}{5} = 0.2$$