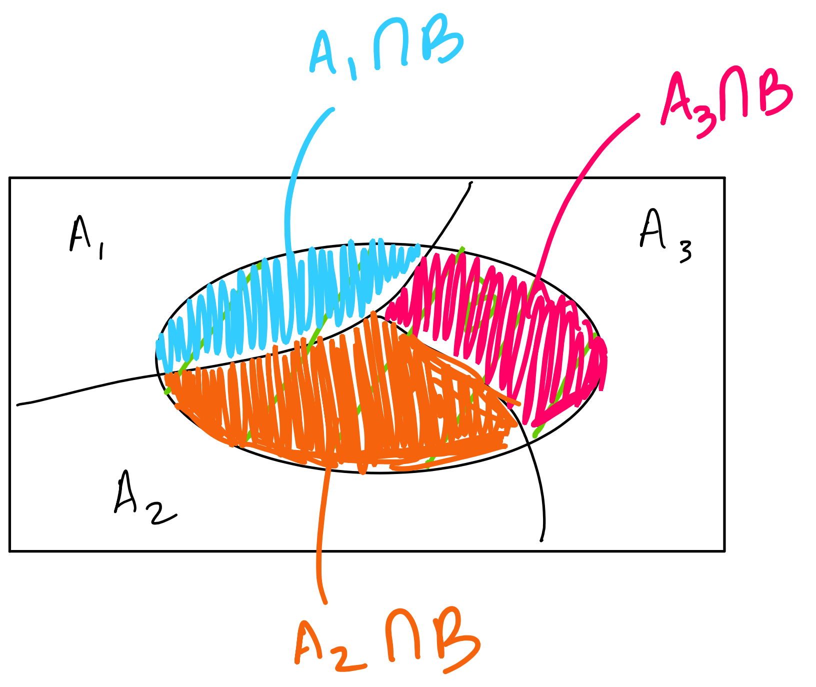
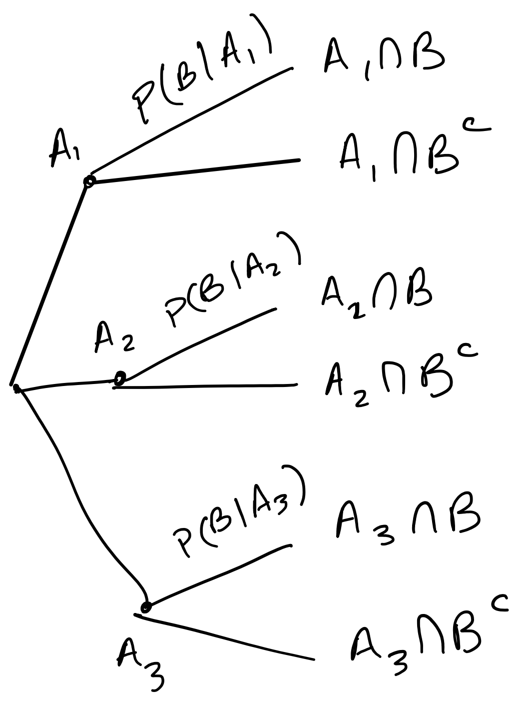
# Bayes Rule

Friday, April 17, 2020 6:10 PM

Following from Total Probability theorem.




The scenario is the same.

- We have the $P(A_i)$ for all our scenarios $P(A_1)$, $P(A_2)$, $P(A_3)$. We think of these as "initial beliefs".
- We also have the probability that an event B of interest occurs.
- If we run the experiment and find out that event B does indeed occur, we can revise our probabilities of Events $A_i$ given the new info. Perhaps certain scenarios are more likely than others.

How do we revise our beliefs? By calculating conditional probabilities:

1) start from the definition of conditional Prob:

$$P(A_i | B) = \frac{P(A_i \cap B)}{P(B)}$$

We can calculate the numerator using the multiplication rule.

We can calculate the denominator using the total probability theorem.

$$\boxed{P(A_i | B) = \frac{P(A_i) \cdot P(B | A_i)}{\sum_j P(A_j) \cdot P(B|A_j)}}$$

Baye's Rule.

## What is Bayesian Inference?

It is a systematic way to incorporte new evidence.

① We start with a probabilistic model
- Model has # of possible scenarios $A_i$
- We have some initial beliefs about the likelihood of each scenario $P(A_i)$
- There is also some particular event B that may occur under each scenario and we know how likely it is under each scenario. $P(B|A_i)$

This is our current model:

$$A_i \xrightarrow[P(B|A_i)]{\text{model}} B$$

② If we observe the B really occurs, then we use this info to draw conclusions about the possible causes of B — we infer how likely the scenarios $A_i$ will be.

$$B \xrightarrow[P(A_i|B)]{\text{inference}} A_i$$

So Baye's Rule allows us to calculate conditional probabilities going in the other direction.. It allows us to revise the probabilities of the different scenarios taking into account the different scenarios.