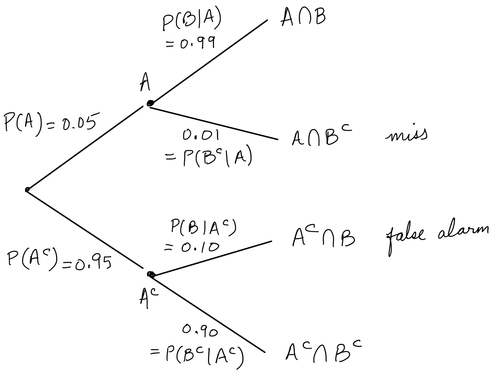
Conditional probabilities: Radar example

Tuesday, April 14, 2020 7:46 PM

Event A : An airplane is flying above.

Event B : Something is detected by the radar.



Can we calculate $P(A \cap B)$?

We know $P(B|A)$ from the tree visual above, and we have the definition of conditional probability, from which we can get :

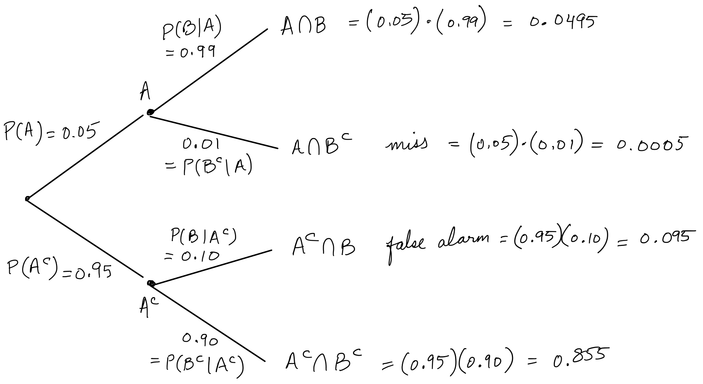
$$P(B|A) = \frac{P(A \cap B)}{P(A)} \qquad P(A|B) = \frac{P(B \cap A)}{P(B)}$$

The A & B can be interchanged so that

We can use the version on the left:

$$P(A \cap B) = P(B|A) \cdot P(A) = (0.99) \cdot (0.05) = 0.0495$$

The above is the same thing as following each tree branch in the above diagram and multiplying it to the end. By doing this we can find all the leaves in the tree. This gets us the following diagram:



The above probabilities add up to 1.

But what is $P(B)$? The probability that something is detected by the radar ?

In the above tree diagram, this comprises $A \cap B$ and $(A^c \cap B)$. We need to add these probabilities:

$$P(B) = P(A \cap B) + P(A^c \cap B) = 0.0495 + 0.095$$
$$= 0.1445$$

But what is the most important question in this example? It is, what is the probability that an airplane is in the sky, given that the radar detected something?

This is given by:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.0495}{0.1445} = 0.34$$

So there is a 34% probability that an airplane is in the sky, given that our radar detects something.