Countable additivity axiom

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Discreet but infinite sample space :

$P(n) = \frac{1}{2^n}$ $n = 1, 2, \ldots, \infty$ Sample Space $= \{1, 2, 3, \ldots\}$

$$\sum_{i=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^\infty}$$

$r = \frac{1}{2}$, $a = \frac{1}{2}$

$$\sum_{i=1}^{\infty} a_i = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

$P(\text{outcome is even}) = P(\{2, 4, 6, \ldots\})$

$= P(\{2\} \cup \{4\} \cup \{6\} \cup \ldots) = P(2) + P(4) + P(6) + \ldots$

$= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \ldots = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \ldots$

$r = \frac{1}{4}$ $a = \frac{1}{4}$

$$\sum_{i=1}^{\infty} \frac{1}{4^n} = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{4}{12} = \frac{1}{3}$$

$P(\text{outcome is even}) = \frac{1}{3}$

But is this correct? We