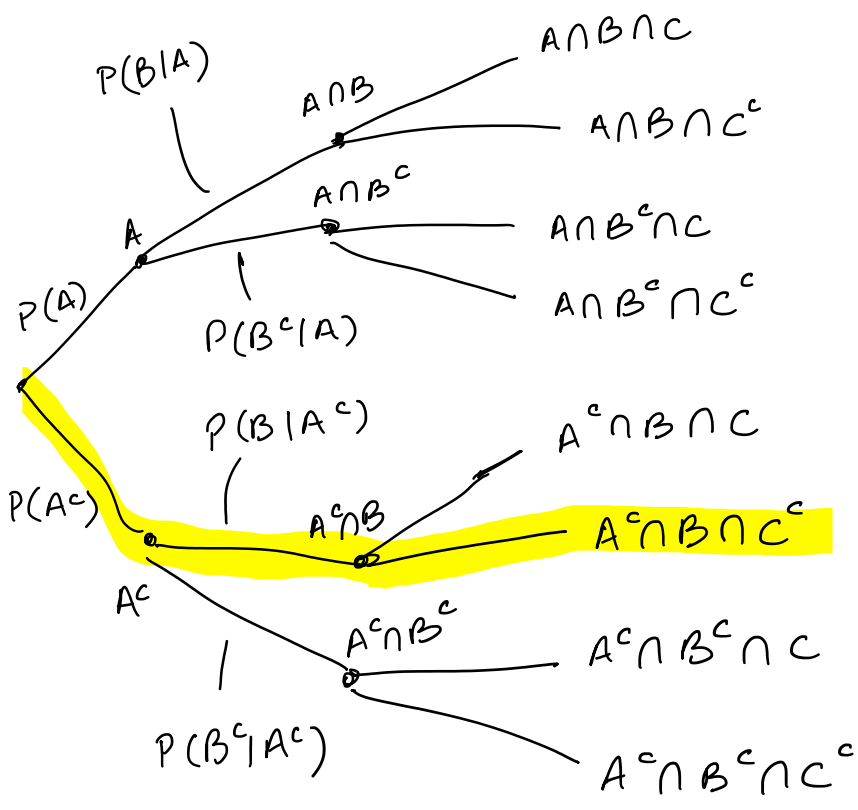
Multiplication rule

Thursday, April 16, 2020 3:36 AM

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(B) \cdot P(A|B)$$
$$= P(A) \cdot P(B|A)$$



Let's try to use multiplication rule to get the highlighted route above:

$$P(A^c \cap B \cap C^c) = P((A^c \cap B) \cap C^c) = P(A^c \cap B) \cdot P(C^c | (A^c \cap B))$$

This can be further simplified

$$P(A^c) \cdot P(B|A^c) \cdot P(C^c | (A^c \cap B))$$

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① $P(A \cap B \cap C^c) = P(A) P(C^c|A) P(B | A \cap C^c)$?

$$P(A \cap B \cap C^c) = P(A \cap C^c \cap B) = P(A \cap C^c) \cdot P(B | (A \cap C^c))$$
$$= P(A) \cdot P(C^c|A) \cdot P(B | (A \cap C^c)) \quad \checkmark$$

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② $P(A \cap B \cap C^c) = P(A) \underline{P(C^c \cap A | A)} P(B | A \cap C^c)$?

$$P(C^c \cap A | A) = \frac{P(C^c \cap A \cap A)}{P(A)} = \frac{P(C^c \cap A)}{P(A)}$$

$$= P(C^c | A)$$

Hence this is the same as ①

$$P(A \cap B \cap C^c) = P(A) P(C^c|A) P(B | A \cap C^c)$$

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③ $P(A \cap B | C) = P(A|C) P(B | A \cap C)$?

$$\frac{P(A \cap B \cap C)}{P(C)} = \frac{P(\cancel{A \cap C})}{P(C)} \cdot \frac{P(A \cap B \cap C)}{P(\cancel{A \cap C})}$$

$$\frac{P(A \cap B \cap C)}{P(C)} = \frac{P(A \cap B \cap C)}{P(C)} \quad \checkmark$$
