Bayesian trap

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You are not feeling well one day. Not super sick but just not 100%. You don't feel like yourself. So you go to the doc and the doc doesn't know what's going on either, so they order a bunch of tests for you.

A few days later you get the results, it turns out that you tested positive for a rare disease:

- Affects about 0.1% of the population.
- Bad prognosis
- Test correctly identify 99% of people that have this disease.
- Test will incorrectly identify only 1% of people who don't have the disease.

What are the chances that you actually have the disease?

NOT 99% as the test's accuracy suggests.

Let's use baye's Rule:

$$\boxed{P(A_i \mid B) = \frac{P(A_i) \cdot P(B \mid A_i)}{P(B)}}$$

Baye's Rule.

A ← Event that you have the disease. This is what we are hypothesising.

B ← You tested positive for the disease

$P(A \mid B)$ ← Probability that you have the disease given that you tested +tive.

$P(B \mid A)$ ← Probability that the test is positive if someone has the disease.

Do we have the above probabilities?

$P(A)$ = The prior probability ← This is a guess most of the time. In this case a good guess is the occurrance of this disease, which in the problem statement is said to be 0.1% or 0.001

$P(B)$ ← The probability you test positive.
= Probability of having disease ∩ testing positive
+ Probability of not having disease ∩ testing +tive

$$P(B) = (0.1\%)(99\%) + (99.9\%)(1\%)$$

$$P(B) = 0.01098 = 1.098\%$$

$P(A \mid B)$ = This is what we are looking for.

$P(B \mid A)$ = This is the probability that someone who actually has the disease is correctly identified by the test. This is given in the problem statement as 99%.

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(B)} = \frac{(0.001)(0.99)}{0.01098}$$

$$= 0.09016 = 9\%$$

So we really only have 9% chance of actually having the disease.

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