Homework 5

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1. $1.1! + 2.2! + \dots + n.n! = (n+1)! - 1$

Basis
$1 \cdot 1! + (1+1)! - 1$

Inductive
$1.1! + 2.2! + \dots + k \cdot k! = (k+1)! - 1$
add $(k+1)(k+1)!$

$1.1! + 2.2! + \dots + (k+1)(k+1)! = (k+1)! - 1 + (k+1)(k+1)!$

2. $\bigcap_{K=1}^{\bar{K}} K_n = \bigcup_{K=1}^{K} \bar{A}K$

Def

$= \left(\bigcap_{K=1}^{K} 1^K\right) \cup A\bar{K} \neq 1$

3. $2n+3 \leq 2^n$

Basis $n = 4$
$2(4)+3 \leq 2^4$
$11 \leq 16$

Inductive $n = k$
$2k+3 \leq 2^k$
$n = k+1$
$2(k+1)+3 = 2k+5 \leq 2^k + 2$

$2^k + 2 \leq 2^k + 2^k$
$= 2^k \cdot 2$
$= 2^{k+1} \rightarrow 2(k+1)+3 \leq 2^{k+1}$

4. $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$A^n = \begin{bmatrix} a^n & 0 \\ 0 & b^n \end{bmatrix}$

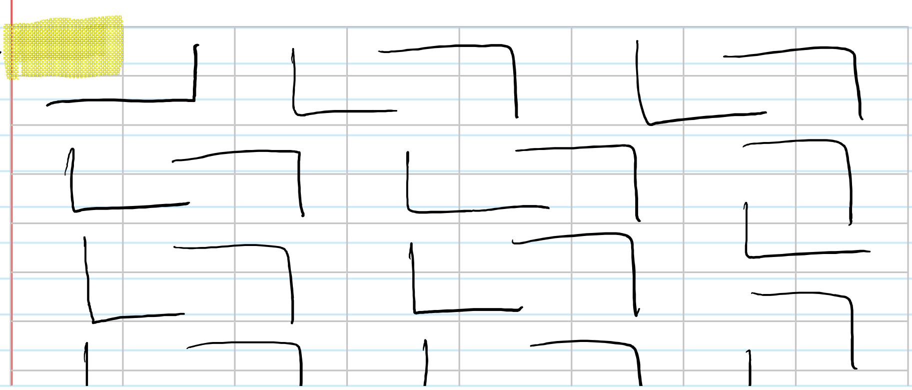
$n = k$ $A^k = \begin{bmatrix} a^k & 0 \\ 0 & b^n \end{bmatrix}$

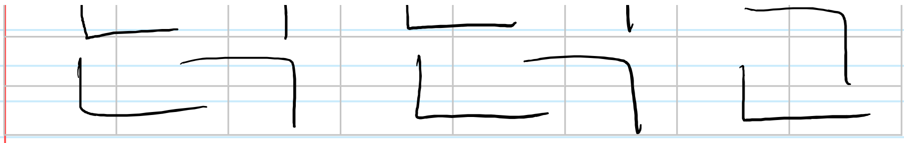
$A^{k+1} = \begin{bmatrix} a^{k+1} & 0 \\ 0 & b^{k+1} \end{bmatrix}$ TRUE

$A^k = \begin{bmatrix} a^k & 0 \\ 0 & b^k \end{bmatrix}$ TRUE for A^n

5. Even $n = 2p$ Alway +
$n^2 + n = (2p)^2 + 2n = 4p^2 + 2p = 2(p^2+p)$ True

6.





7. $a_n = 7a_{n-2} + 6a_{n-3}$

$(-1)^3 - 7(-1) - 6 = 0$

$r = 1, 3, -2$

$a_0 = 9$

$9 = \alpha_1 + \alpha_2 + \alpha_3$

$a_1 = 10$

$10 = -\alpha_1 + 3\alpha_2 + (-2\alpha_3)$

$a_2 = 30$ $\quad 5 - 4 = 1$

$32 = \alpha_1 + 9\alpha_2 + 4\alpha_3$

$\boxed{8(-1)^n + 4(3)^n + (-3)(-2)^n}$

8. $a_n = 5a_{n-1} - 6a_{n-2}$

$5^2 - 4(1)(6) = 2$

$r^2 - 5r + 6 = 0$

$(r-2)(r-3) = 0$

$r = 2, 3$

$1 = \alpha_1 + \alpha_2$

$0 = 2\alpha_1 + 3\alpha_2$

$a_n = 3(2^n) - 2(3^n)$