Midrerm1

Friday, May 19, 2017 5:23 PM

c. (2.5 pts) (If this is true, prove it using any proving method. If not, give a counter examp
Let n be an integer. Then any number of this form n(n+1)(n+2) is divisible by 6.

d. (2.5 pts) (If this is true, prove it using definition. If not, give a counter example)
If $A \subset C$ and $A \subseteq B \subseteq C$, then either $A \subset B$ or $B \subset C$.

p6.5

5. Find the domain and range of each function and determine whether the function is one-to-one or onto. If the function is not one-to-one or not onto explain why not.	
a. (2.5 pts) The function that assigns each person in the US a driver license number.	
-(0.25 pt) Domain: People in the U.S.	-(0.25 pt) Range: People with licenses
1.5 -(1 pt) Onto ~~true~~	-(1 pt) One-to-one: true
b. (2.5 pts) The function that assigns to each pair of negative integers the quotient of the pair.	
-(0.25 pt) Domain: $\mathbb{Z}^-$ $\cancel{\mathbb{Z}^- \times \mathbb{Z}^-}$	-(0.25 pt) Range: $\cancel{\mathbb{Z}^+}$
2 -(1 pt) Onto true	-(1 pt) One-to-one: true
c. (2.5 pts) The function that assigns to each real number x the largest integer that is less than or equal to x	
-(0.25 pt) Domain: $\mathbb{R}$	-(0.25 pt) Range: $\mathbb{Z}$
2.0 -(1 pt) Onto true	-(1 pt) One-to-one: false −0.5
d. (2.5 pts) The function that assigns to each pair of natural number the average of each pai	
-(0.25 pt) Domain: $\mathbb{N}$ $\cancel{\mathbb{N} \times \mathbb{N}}$	-(0.25 pt) Range: $\cancel{\mathbb{R}}$ $\{ \frac{a}{2} \mid a \in \mathbb{N}$
+1 -(1 pt) Onto false −0.5	-(1 pt) One-to-one: false −0.5

6. (2.5 pts each) Consider the following statement:
"To pass the final exam it is necessary that you study."

i) Rewrite the statement in If…, then … form

If you pass the exam, then you studied

ii) Write the converse of the statement in part (i)

if you study, then you pass the test

ii) Write the contrapositive of the statement in part (i)

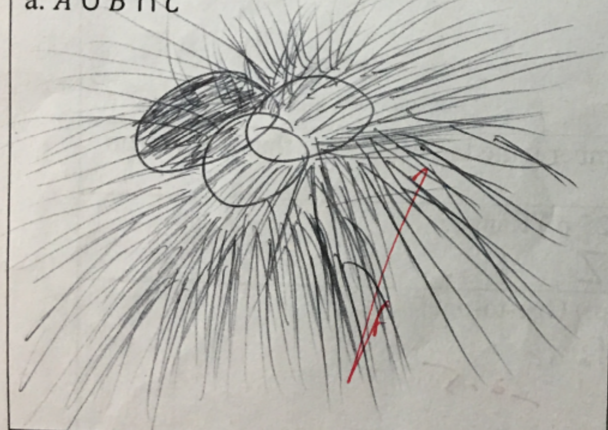
If you do not study, then you will not pass the test

ii) Write the inverse of the statement in part (i)

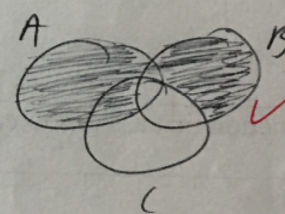
If you do not pass the test, then you did not stud

7. For each of the following set, draw a Venn diagram and clearly shade the area representi
given set. (2.5 pts each)

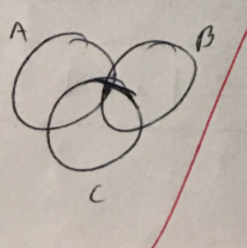
a. $A \cup \overline{B \cap C}$



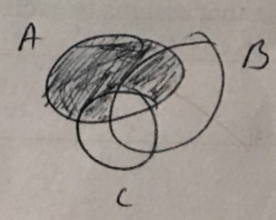
c. $(A \cup B) - (B \cap C)$



b. $\overline{A - B} \cap C$



d. $(\bar{B} \cup \bar{C}) \cap A$



Jonathan Asc
Group 6

10 +3

8. Prove that
Prove that if x, y and z are positive real numbers, then
$x^2 + y^2 + z^2 \geq xy + yz + zx$

assume x, y, z are positive real numbers
$x^2 + y^2 + z^2 < xy + yz + zx$

let $x = 1$ $y = 2$ $z = 3$

$1^2 + 2^2 + 3^2 < 1(2) + 2(3) + 1(3)$

$1 + 4 + 9 < 2 + 6 + 3$

$14 < 11$

$\therefore$ By contradiction if x, y, z are real positive numbers, th
$x^2 + y^2 + z^2 \geq xy + yz + zx$

9. (10 pts) Prove that if m is a positive integer and x is a real number then

$$\lfloor mx \rfloor = \lfloor x \rfloor + \left\lfloor x + \frac{1}{m} \right\rfloor + \left\lfloor x + \frac{2}{m} \right\rfloor + \left\lfloor x + \frac{3}{m} \right\rfloor + \cdots + \left\lfloor x + \frac{m-1}{m} \right\rfloor$$

assume: m is a positive integer & x is a real number
$\lfloor mx \rfloor \neq \lfloor x \rfloor + \lfloor x + \frac{1}{m} \rfloor + \lfloor x + \frac{2}{m} \rfloor + \lfloor x + \frac{3}{m} \rfloor + \ldots + \lfloor x + \frac{m-1}{m}$

let $m = 4$ $x = .5$

$\lfloor 4(.5) \rfloor = \lfloor .5 \rfloor + \lfloor .5 + \frac{1}{4} \rfloor + \lfloor .5 + \frac{2}{4} \rfloor + \lfloor .5 + \frac{3}{4} \rfloor$

$\lfloor 2 \rfloor = 0 + \lfloor .75 \rfloor + \lfloor 1 \rfloor + \lfloor 1.25 \rfloor$

$2 = 0 + 0 + 1 + 1$

$2 = 2$

$\therefore$ By contradiction, if m is a positive integer
$\lfloor mx \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{m}$

10.

A. (7pts) Prove that "If f is a one-to-one function from the set X to the set Y and A, B $\subseteq$ X, then f (A $\oplus$ B) = f (A) $\oplus$ f (B)."

We tried.

$f(x \in A \oplus B)$

B. (3 pts) Disprove: "For any three sets A, B, and C, (A $\oplus$ B) $\cup$ C = (A $\cup$ C) $\oplus$ (B $\cup$ C)."

~~$(A \oplus B) \cup C = \{x \mid x \in (A \oplus B) \cup C\}$~~

Let $A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$ $C = \{6, 7, 8\}$

+6

1. (2pts each)Suppose the variable x represents students, y represents courses, and T (x, y) mea
"x is taking y ". Match the English statement with all its equivalent symbolic statements in this li
(i.e. Fill in the blank all number that represents the English sentence).

1. $\exists x \forall y\, T(x, y)$	2. $\exists y \forall x\, T(x, y)$	3. $\forall x \exists y\, T(x, y)$
4. $\neg \exists x \exists y\, T(x, y)$	5. $\exists x \forall y\, \neg T(x, y)$	6. $\forall y \exists x\, T(x, y)$
7. $\exists y \forall x\, \neg T(x, y)$	8. $\neg \forall x \exists y\, T(x, y)$	9. $\neg \exists y \forall x\, T(x, y)$
10. $\neg \forall x \exists y\, \neg T(x, y)$	11. $\neg \forall x \neg \forall y\, \neg T(x, y)$	12. $\forall x \exists y\, \neg T(x, y)$

a. Every course is being taken by at least one student. 6
b. No student is taking all courses. 4
c. Every student is taking at least one course. 3
d. Some students are taking no courses. 5
e. Some courses are being taken by no students. 7

+8.75

2. (1.25 pts each) Construct a truth table for the following proposition
$P = [(p \wedge q) \rightarrow r] \rightarrow [(p \rightarrow r) \vee (q \leftrightarrow r)]$

$\neg(p \wedge q) \vee r$ $\neg p \vee r$

p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow r$	$p \rightarrow r$	$q \leftrightarrow r$	$(p \rightarrow r) \vee (q \leftrightarrow r)$	P
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	T	F	T	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	F	T	T
F	F	T	F	T	T	F	T	T
F	F	F	F	T	T	F	T	T

+3

3. Fill in the blank ∈, ⊂, ⊆, =, or neither. (1 pt each). There could be more than one symbol that fits in the blank. P means the power set (e.g. P(A) means the power set of A)
Let A = {a, ∅, {a}} and B = {{∅}}

a. ∅ ⊆ ⊂ P(A)	f. {{∅}}∩∅ ⊆ ⊂ A ∪ B
b. P(B) ⊆ ⊂ P(A)	g. {a} ⊆ ⊂ ∈ P(A)
c. { ∅, a, {∅}} ⊆ ⊂ ∈ P(A∪B)	h. |P(A∩B)| = |P({ })|
d. a neither B x A	i. A ∩ B neither {∅}
e. |A x B| neither |P(A∪B)|	j. B ∪ {a} ⊆ ⊂ A − {∅}

4. Prove or disprove the following:
a. (2.5 pts) (If the argument is valid, use rules of inferences to prove. If not, point out the fallacy)
She is a Math Major or a Computer Science Major.
If she does not know discrete math, she is not a Math Major.
If she knows discrete math, she is smart.
She is not a Computer Science Major.
Therefore, she is smart.

1. m v cs
2. ¬dm → ¬m
3. dm → s
4. ¬cs
5. m
6. dm
7. s

hyp

disjunctive (1)(4)
tollens (5)(2)
ponens (3)(6)

b.(2.5 pts) (If this is an equivalence, prove without using the truth table. If not, give a counterexample)
$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$