Midterm 2

Friday, May 19, 2017 5:15 PM

7. (10 pts) Find the solution to $a_{n+2} = -4a_{n+1} + 5a_n$ for $n \ge 0, a_0 = 2, a_1 = 8$

$r^2 + 4r - 5 = 0$

$(r+5)(r-1)$

+5

$r = -5 \quad r = 1$

$0 = a_1(5)^n - a_2(1)^n \qquad 5 = \alpha_1$

$1 = \alpha_0$

8. (10 pts) Find the solution to $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ with $a_0 = 7, a_1 = -4$, and $a_2 = 8$.

$-2r^2 - 5r + 6 = 0$

$$\frac{+5 \pm \sqrt{25+48}}{-4} = \frac{-5 \pm \sqrt{73}}{4}$$

$$r = \frac{-5+\sqrt{73}}{4}, \quad \frac{-5-\sqrt{73}}{4}$$

$$= a_1\left(\frac{-5+\sqrt{73}}{4}\right)^n - a_2\left(\frac{-5-\sqrt{73}}{4}\right)^n$$

$$\frac{-5+\sqrt{73}}{4} = \alpha$$

5. (2 pts each) Calculate the following sum

a. $2+4+8+16+\cdots+2^{100}$

$r = \frac{2^{100}}{2^{99}} = 2$

$a_n = 2(2)^{n-1}$

$\sum_{i=1}^{n} 2(2) = 2\left(\frac{1-2^{100}}{1-2}\right)$

b. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$

$a_n = 1 \cdot \frac{1}{2}^{n-1}$

$= \cancel{1\left(\frac{1-\frac{1}{2}^n}{1-\frac{1}{2}}\right)}$

$r<1$

$\frac{1}{1-\frac{1}{2}} = \boxed{2}$

c. $2-\frac{1}{2}+\frac{1}{8}-\frac{1}{32}+\frac{1}{128}-\cdots$

$r = \frac{-1}{4}$

$2\left(-\frac{1}{4}\right)^{n-1} = \cancel{2\left(\frac{1+\frac{1}{4})^n}{1+\frac{1}{4}}\right)}$

$r<1$

$\frac{1}{1+\frac{1}{4}} = \boxed{\frac{4}{5}}$

d. $112+113+114+115+\cdots+673$

$d = 1$

$a_n = 112+(561)$

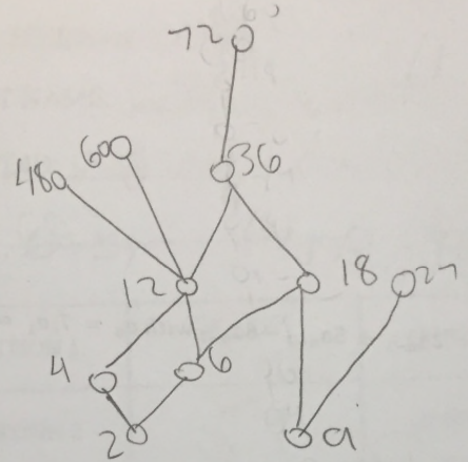
$a_n = 673$

$\sum_{i=1}^{561} 673 = (561)(673)$

e. $\sum_{j=1}^{3}\sum_{i=1}^{j} ij$

$\frac{j(j+1)}{2}$

6. Consider the poset ($\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, |$).
I. (2 pts) Draw a Hasse diagram that represents the poset above



II. (1 pt each) Answer these questions for the poset above

a) Find the maximal elements.

72

b) Find the minimal elements.

2

c) Is there a greatest element?

no

d) Is there a least element?

NO

e) Find all upper bounds of {2, 9}.

4, 6, 18, 27

f) Find the least upper bound of {2, 9}, if it exists.

4

g) Find all lower bounds of {60, 72}.

12, 36

h) Find the greatest lower bound of {60, 72}, if it exists.

36

... be the relation on the set of ordered pairs of positive integers such that ((a, b), (c, d)) ∈ R if and only if a + d = b + c. Show that R is an equivalence relation.

$((a,b),(c,d)) \in R$

$a+d=b+c$
$1+4=2+3$

SYMMETRIC: $a+d=b+c = b+c=a+d$ $((a,b),(c,d))$ ...

TRANSITIVE: $a+d=b+c = a$, $a+d$; d ; a ; a,b ; b

REFLEXIVE: $((a,b),(c,d)) =$

EQUIVALENCE R

4. (2 pts each) Suppose that a "word" is any string of seven letters of the alphabet, with repeated letters allowed.

a. How many words are there? 26^7	f. How many words contain exactly one A? $7\cdot(26^6)$
b. How many words begin with R and end with T? $1\cdot 26^5\cdot 1$	g. How many words contain the string SEVEN in it? 3
c. How many words begin with A or end with B? 26^6+26^6 $26^6+26^6-26^5$	h. How many words begin with a vowel and end with a vowel? $5\cdot 26^5\cdot 5$
d. How many words begin with A or B or end with A or B? $(2\cdot 26^6)+(2\cdot 26^6)$ $4\cdot 26^6-4\cdot 26^5$	i. How many words contain only vowels? 5^7
e. How many words begin with a vowel or end with a vowel? $(5\cdot 26^6)+(5\cdot 26^6)$ $-5^2\cdot 26^5$	j. How many words contain exactly two vowels? $32\cdot 26^5$

3. (10 pts) Find the solution to $a_{n+2} = -4a_{n+1} + 5a_n$ for $n \geq 0, a_0 = 2, a_1 = 8$

$r^2 \quad (r^2 + 4r - 5) = 0$

$(r+5)(r-1)$

$r = -5, 1$

$\boxed{a_n = (-1)(-5)^n + 3(1)^n}$

$a_n = \alpha_1(-5)^n + \alpha_2(1)^n$

$2 = \alpha_1 + \alpha_2$

$8 = -5\alpha_1 + \alpha_2$

$6 = -6\alpha_1$

$\alpha_1 = -1$

$\alpha_2 = -1 + \alpha_2$

$\alpha_2 = 3$

4. (10 pts) Find the solution to $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ with $a_0 = 7, a_1 = -4$, and $a_2 = 8$.

$r^3 = 2r^2 + 5r - 6$

$r^3 - 2r^2 - 5r + 6 = 0$

$$\begin{array}{c|cccc} 1 & 1 & -2 & -5 & 6 \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ & 1 & -1 & -6 & 0 \end{array}$$

$r^2 - r - 6 = 0$

$(r-3)(r+2) = 0$

$r = 3, -2, 1$

$a_n = \alpha_1(3)^n + \alpha_2(-2)^n + \alpha_3(1)^n$

$7 = \alpha_1 + \alpha_2 + \alpha_3$

$-4 = 3\alpha_1 - 2\alpha_2 + \alpha_3$

$8 = 9\alpha_1 + 4\alpha_2 + \alpha_3$

$\boxed{a_n = -1(3)^n + 3(-2)^n + 5(1)^n}$

$$\begin{array}{c} \\ L_1 - L_2 \\ L_3 - L_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 3 & -2 & 1 & -4 \\ 9 & 4 & 1 & 8 \end{array}\right]$$

$-1 + 3 + 5 = 7$ ✓

$-3 - 6 + 5 = -4$

$-9 + 5 = -4$

$-9 + 12 + 5 = 8$ ✓

$$\begin{array}{c} \\ \\ L_3 + L_2 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ -2 & -3 & 0 & -11 \\ 8 & 3 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ -2 & -3 & 0 & -11 \\ 10 & 0 & 0 & -10 \end{array}\right]$$

$\alpha_1 = -1$

$-2 - 3\alpha_2 = -11$

$-3\alpha_2 = -9$

$\alpha_2 = 3$

$-1 + 3 + \alpha_3 = 7$

$\alpha_3 = 5$