Assignment #1 CECS 218

1 a) $r \wedge \sim p$

b) $\sim p \wedge q \wedge r$

c) $r \rightarrow (q \leftrightarrow \sim p)$

d) $\sim q \wedge (\sim p \wedge r)$

e) $(q \rightarrow (\sim p \wedge \sim r)) \wedge ((\sim p \wedge \sim r) \rightarrow q)$

f) $(p \wedge r) \rightarrow \sim q$

2 a) If the wind blows from the northeast, then it snows.

b) If it stays warm for a week, then the apple trees will bloom

c) If the pistons win the championship, then they beat the lakers

d) If you get to the top of Long's peak, then you walked 8 miles

e) If you are world famous, then you are tenure as professor.

f) If you drive more than 400 miles, then you will need to buy gasoline.

g) If you bought your CD player less than 90 days ago, then your gudrantee is good.

h) If the water is not too cold, Jan will go swimming

a) Truth Table for $((p \to q) \to r) \to s$

p	q	r	s	$(p \to q)$	$(p \to q) \to r$	$((p \to q) \to r) \to s$
T	T	T	T	T	T	T
T	T	T	F	T	T	F
T	T	F	T	T	F	T
T	T	F	F	T	F	T
T	F	T	T	F	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	T	T
F	F	T	F	T	T	T
F	F	F	T	T	F	T
F	F	F	F	T	F	T

b) Truth table $(p \oplus q) \to (p \oplus \neg q)$

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \to (p \oplus \neg q)$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

Truth table for $(p \to q) \wedge (\sim p \to r)$

c)

p	q	r	$p \to q$	$\sim p$	$\sim p \to r$	$(p \to q) \wedge (\sim p \wedge r)$
T	T	T	T	F	T	F
T	T	F	T	F	T	F
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

4 a) $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ are logically equivalent

Truth table for $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth table for $(p \to q) \wedge (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

b) $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent

$(\sim p \vee q) \wedge (\sim p \vee r)$

$\sim p \vee (q \wedge r)$ — Implication law

$\equiv \sim p \vee (q \wedge r)$ — Distributive law

c) $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are not logically equivalent

$(\sim p \vee q) \to (\sim r \vee s) \not\equiv (\sim p \vee r) \to (\sim q \vee s)$ — implication law

$\sim(\sim p \vee q) \vee (\sim r \vee s) \not\equiv \sim(\sim p \vee r) \vee (\sim q \vee s)$ — Implication law

$(p \wedge \sim q) \vee (\sim r \vee s) \not\equiv (p \wedge \sim r) \vee (\sim q \vee s)$ — Simplified, not equal

5a) $\forall n.(n+1 > n)$

TRUE, b/c $n = 3$

$3+1 > 3$

$4 > 3$ ✓

b) $\exists n\,(2n = 3n)$

TRUE, b/c $n = 0$

$2(0) = 3(0)$

$0 = 0$ ✓

c) $\exists n\,(n = -n)$

TRUE, b/c $n = 0$

$0 = -(0)$

$0 = 0$ ✓

d) $\forall n\,(3n \leq 4n)$

FALSE, b/c $n = -1$

$3(-1) \nleq 4(-1)$

$-3 \nleq -4$

6. $P(x)$: x is the correct place

$Q(x)$: x is in excellent condition Dom_x = all tools

$R(x)$: x is a tool

a) $\exists x \sim P(x)$

b) $\forall x\,(R(x) \rightarrow (P(x) \wedge Q(x)))$

c) $\forall x\,(P(x) \wedge Q(x))$

d) $\forall x \sim (P(x) \wedge Q(x))$

e) $\exists x\,(R(x) \rightarrow (\sim P(x) \wedge Q(x)))$

7. a) Sarah Smith has visited the website www.att.com

b) there exists a student who has visited the website www.imdb.org. $\equiv$ Atleast one student has visited the website www.imdb.org.

c) There exists a website that has been visited by Jose Orez ≡ Jose Orez has visited at least one website

d) There exists a website that has been visited by Ashok Puri and Cindy Yoon ≡ There is a website that has been visited by Ashok Puri and Cindy Yoon.

e) There exists a person that is not David Belcher and has visited all the websites, if David Belcher has visited those websites ≡ There is a person other than David Belcher who has visited same websites as David Belcher.

f) There exists an x and y, where x and y are not the same and has visited the z website at the same time ≡ There are two different people who visited the same website at same time

8a) $S(x)$: x is a student
$F(x)$: x is a faculty member
$A(x,y)$: x has asked y a question
$Dom_{x,y}$: all people with school

a) $A(\text{Lois}, \text{Professor Michaels})$
b) $\forall x \; A(x, \text{Professor Gross})$
c) $\forall x \; (F(x) \rightarrow A(x, \text{Professor Miller}) \leftrightarrow A(\text{Professor Miller}, x))$
d) $\exists x \forall y \; (S(x) \wedge (F(y) \rightarrow \sim A(x,y)))$
e) $\exists x \forall y \; (F(x) \wedge (S(y) \rightarrow \sim A(y,x))$
f) $\exists x \forall y \; (S(x) \wedge F(y) \rightarrow A(x,y))$
g) $\exists x \forall y \; (F(x) \wedge ((F(y) \wedge x \neq y) \rightarrow A(x,y)))$
h) $\exists x \forall y (S(x) \wedge F(y) \rightarrow \sim A(y,x)))$

9a) $\forall x \exists y (x^2 = y)$

TRUE b/c $x = 2$

$2^2 = 4$

$4 = 4 \rightarrow$ so this is the y that equals that number of x.

b) $\forall x \exists y (x = y^2)$

FALSE, b/c when x is negative, y can't be negative

c) $\exists x \forall y (xy = 0)$

TRUE, b/c $x = 0$ will make any xy value 0

d) $\exists x \exists y (x + y \neq y + x)$

FALSE, b/c it's against commutative property of addition

e) $\forall x (x \neq 0 \rightarrow \exists y (xy = 1))$

TRUE, b/c $x = 1, y = 1$ will make this true.

f) $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$

FALSE, b/c it can't be true for all y

suppose $x = 1, y = 2$, the $xy = 2$

g) $\forall x \exists y (x + y = 1)$

TRUE, b/c y value depends on x so the value of y would be $(1 - x)$

h) $\exists x \exists y (x+2y=2 \wedge 2x+4y=5)$

FALSE, because the systems don't have a solution to it and by multiplying first equation by (-2) gives you 1 not 0 by adding both equations.

i) $\forall x \exists y (x+y=2 \wedge 2x-y=1)$

FALSE, because value of y from 1st equation is different from 2nd equation

1st equation $y = 2-x$

2nd " $y = 2x-1$

j) $\forall x \forall y \exists z (z = (x+y)/2)$

TRUE because any value of x and y will result in a value which will be z