Assignment#4 –CECS228

1. Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, asymmetric, and/or transitive, where (x, y) ∈ R if and only if (Please explain your reasoning)

a) x + y = 0.  
b) x = ±y.  
c) xy ≥ 0.  
d) x = 1.

e) x = 1 or y = 1.  
  
2. How many nonzero entries does the matrix representing the relation R on A = {1, 2, 3, . . ., 100} consisting of the first 100 positive integers have if R is

a) {(a, b) | a > b}?   
b) {(a, b) | a = b + 1}?   
c) {(a, b) | ab = 1}?  
  
3. Represent the following relation by using directed graph and matrix (with the elements of this set listed in increasing order.)

{(a, a), (a, b), (b, c), (c, b), (c, d), (d, a), (d, b)}.

4.

R1 = {(a, b) ∈ R2 | a > b}, the “greater than” relation,

R2 = {(a, b) ∈ R2 | a ≥ b}, the “greater than or equal to” relation,

R3 = {(a, b) ∈ R2 | a < b}, the “less than” relation,

R4 = {(a, b) ∈ R2 | a ≤ b}, the “less than or equal to” relation,

R5 = {(a, b) ∈ R2 | a = b}, the “equal to” relation,

R6 = {(a, b) ∈ R2 | a ≠ b}, the “unequal to” relation.

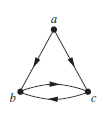
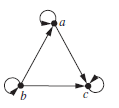
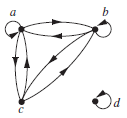
Find  
a) R2 ◦ R1.   
b) R2 ◦ R2.

c) R3 ◦ R5.   
d) R4 ◦ R1.

e) R5 ◦ R3.   
f ) R3 ◦ R6.

g) R4 ◦ R6.   
h) R6 ◦ R6.

5. Show that the relation R consisting of all pairs (x, y) such that x and y are bit strings of length three or more that agree in their first three bits is an equivalence relation on the set of all bit strings of length three or more.

6. i) List the ordered pairs in the relations represented by the directed graphs.  
 ii) Determine whether the relations represented by the directed graphs are reflexive, symmetric, antisymmetric, and/or transitive  
a) b) c)

7. Which of these relations on the set of all functions from Z to Z are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a) {(f, g) | f (1) = g(1)}

b) {(f, g) | f (0) = g(0) or f (1) = g(1)}

c) {(f, g) | f (x) − g(x) = 1 for all x ∈ Z}

d) {(f, g) | for some C ∈ Z, for all x ∈ Z, f (x) − g(x) = C}

e) {(f, g) | f (0) = g(1) and f (1) = g(0)}  
  
8. Which of these relations on {0, 1, 2, 3} are partial orderings?

Determine the properties of a partial ordering that the others lack.

a) {(0, 0), (1, 1), (2, 2), (3, 3)}

b) {(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)}

c) {(0, 0), (1, 1), (1, 2), (2, 2), (3, 3)}

d) {(0, 0), (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)}

e) {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)}  
  
9. Draw the Hasse diagram for the “less than or equal to” relation on {0, 2, 5, 10, 11, 15}.  
  
10. Answer these questions for the poset ({3, 5, 9, 15, 24, 45}, |).

a) Find the maximal elements.

b) Find the minimal elements.

c) Is there a greatest element?

d) Is there a least element?

e) Find all upper bounds of {3, 5}.

f ) Find the least upper bound of {3, 5}, if it exists.

g) Find all lower bounds of {15, 45}.

h) Find the greatest lower bound of {15, 45}, if it exists.