Assignment#6 – CECS 228

1. In how many different ways can five elements be selected in order from a set with three elements when repetition is allowed?  
  
2. How many positive integers less than 1,000,000 have the sum of their digits equal to 19?  
  
3. How many ways are there to deal hands of seven cards to each of five players from a standard deck of 52 cards?

4. How many ways are there to distribute five balls into seven boxes if each box must have at most one ball in it if

a) both the balls and boxes are labeled?

b) the balls are labeled, but the boxes are unlabeled?

c) the balls are unlabeled, but the boxes are labeled?

d) both the balls and boxes are unlabeled?  
  
5. How many ways are there to travel in xyz space from the origin (0, 0, 0) to the point (4, 3, 5) by taking steps one unit in the positive x direction, one unit in the positive y direction, or one unit in the positive z direction? (Moving in the negative x, y, or z direction is prohibited, so that no backtracking is allowed.)  
  
6. Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.  
  
7. A company stores products in a warehouse. Storage bins in this warehouse are specified by their aisle, location in the aisle, and shelf. There are 50 aisles, 85 horizontal locations in each aisle, and 5 shelves throughout the warehouse. What is the least number of products the company can have so that at least two products must be stored in the same bin?  
  
8. There are 38 different time periods during which classes at a university can be scheduled. If there are 677 different classes, how many different rooms will be needed?  
  
9. How many strings of eight English letters are there

a) that contain no vowels, if letters cannot be repeated?

b) that start with a vowel, if letters cannot be repeated?

c ) that contain exactly one vowel, if letters can be repeated?

d) that start and end with X and contain at least one vowel, if letters can be repeated?  
  
10. One hundred tickets, numbered 1, 2, 3, . . . , 100, are sold to 100 different people for a drawing. Four different prizes are awarded, including a grand prize (a trip to Tahiti).How many ways are there to award the prizes if

a) the person holding ticket 47 wins the grand prize?

b) the person holding ticket 47 does not win a prize?

c) the people holding tickets 19, 47, and 73 all win prizes?

d) none of the people holding tickets 19, 47, 73, and 97 wins a prize?

e) the people holding tickets 19 and 47 win prizes, but the people holding tickets 73 and 97 do not win prizes?

11. Thirteen people on a softball team show up for a game.

a) How many ways are there to choose 10 players to take the field?

b) How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?

c) Of the 13 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?

12. How many different strings can be made from the letters in MISSISSIPPI, using all the letters?

13. Suppose that k and n are integers with 1 ≤ k < n. Prove the **hexagon identity**

which relates terms in Pascal’s triangle that form a hexagon.

14. Let n be a positive integer. Show that