CECS 228 Name:  
LAB #1.3 ID: Date:

Objectives:

* Be able to negate predicate
* Be able to translate from English into logical expressions
* Be able to understand and apply nested quantifiers

Exercise 1: Let P(x) be the statement “x spends more than five hours every weekday in class,” where the domain for x consists of all students. Express each of these quantifications in English.  
a.   
There exists a student who spends more than five hours every weekday in class.  
  
b.  
There exists a student who does not spend more than five hours every weekday in class.

c.   
Every student spends more than five hours every weekday in class.  
  
d.   
Every student does not spend more than five hours every weekday in class.  
No student spend more than five hours every weekday in class.  
  
Exercise 2: Translate in each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class.

a. Everyone in your class has a cellular phone.

, where C(x): x has a cellular phone.

b. Somebody in your class has seen a foreign movie.

, where F(x): x has seen a foreign movie.

c. There is a person in your class who cannot swim.

d. All students in your class can solve quadratic equations.  
, where Q(x): x can solve quadratic equations.  
  
Exercise 3: Negate each statement in Exercise 6 and write complete sentence in the space below  
a.  
: Not everyone in your class has a cell phone.  
 : Someone in your class does not have a cell phone.  
b.  
: There is not a student (No one) in your class who has seen a foreign movie.  
 : Every student in your class has not seen a foreign movie.  
c.  
 There is no one in your class who cannot swim.  
 : Everyone in your class can swim.  
d.  
 Not everyone in your class can solve quadratic equations.

: Someone in your class cannot solve quadratic equations.

Exercise 4: Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase “It is not the case that.”). Assume that the domain is all animals.  
**a)** All dogs have fleas.

-We write this statement as ∀ x (D(x) → F(x)) or ∀ x (¬ D(x) ∧ F(x)).   
-Its negation is ∃ x (D(x) ∨ ¬ F(x))  
-In English it translates into “There is a dog that does not have fleas”.

**b)** There is a horse that can add.

-We write this statement as ∃ x (H(x) ∧ A(x)).   
-Its negation is ∀ x (¬H(x) ∨ ¬A(x)) or, equivalently, ∀ x (H(x) → ¬A(x)).   
-In English: “no horse can add”.

**c)** Every koala can climb.

-We write this statement as ∀ x (K(x) → C(x)).

- Similar to a), its negation is ∃ x (K(x) ∨ ¬ C(x)).

-In English: “there is a koala that cannot climb”.

**d)** No monkey can speak French.

-We write this statement as ∀ x (M(x) → ¬F(x)) or ∀ x (¬M(x)∨ ¬F(x)).

-Its negation is ∃ x (M(x) ∧ F(x)).

-In English: There is a monkey who can speak French.

**e)** There exists a pig that can swim and catch fish

-We write this statement as ∃ x (P(x)∧S(x)∧F(x))).   
-Its negation is ∀x (¬P(x)∨¬S(x)∨¬F(x)) or ∀x (P(x) → (¬ S(x)∨¬ F(x)).  
- In English: “Every pig either can’t swim or it can’t catch fish”.

Exercise 5: Translate these system specifications into English where the predicate *S(x, y)* is “*x* is in state *y*” and where the domain for *x* and *y* consists of all systems and all possible states, respectively.

**a)** ∃*xS(x,* open)

Some system is open.

**b)** ∀*x(S(x,* malfunctioning) ∨ *S(x,* diagnostic))

Every system is either malfunctioning or in diagnostic mode.

**c)** ∃*xS(x,* open) ∨ ∃*xS(x,* diagnostic)

Either some system is open, or some system is in diagnostic mode.

**d)** ∀*x*￢*S(x,* working)

Every system is not working

Exercise 6: Let *C(x, y)* mean that student *x* is enrolled in class *y*, where the domain for *x* consists of all students in your school and the domain for *y* consists of all classes being given at your school. Express each of these statements by a simple English sentence.

**a)** *C*(Randy Goldberg, CS 252)

Randy Goldberg is enrolled in CS252

**b)** ∃*xC*(*x*, Math 695)

There is at least one student in our school who is enrolled in Math695.

**c)** ∃*x*(*C*(*x*, Math 222) ∧*C*(*x*, CS 252))

There is a student in your school who is enrolled in Math222 and CS252

**d)** ∃*x*∃*y*∀*z((x* ≠ *y)* ∧ *(C(x, z)* → *C(y, z)))*

There are at least two students x, y in your school such that y is enrolled in the all the classes that x is in.

Exercise 7: Let *F(x, y)* be the statement “*x* can fool *y*,” where the domain consists of all people in the world.   
i. Use quantifiers to express each of these statements.  
ii. Negate these statements using quantifiers and write them in simple English

**a)** Everybody can fool Fred.

-Statement: ∀x F(x, Fred)

-Negation: ¬∀x F(x, Fred) or ∃x ¬F(x, Fred)

-In English: Not everybody can fool Fred / Someone cannot fool Fred.

**b)** Evelyn can fool everybody.

-Statement: ∀y F(Evelyn, y)

-Negation: ¬∀y F(Evelyn, y) or ∃y ¬F(Evelyn, y)

-In English: Not everyone can be fooled by Evelyn / There is a person whom Evelyn cannot fool.

**c)** Everybody can fool somebody.

-Statement: ∀x∃y F(x, y)

-Negation: ∃x∀y ¬F(x, y)  
-In English: Someone cannot fool everyone.

**d)** There is no one who can fool everybody.

-Statement: ¬∃x∀y F(x, y)

-Negation: ∃x∀y F(x, y)  
-In English: Someone can fool everyone.

**e)** Everyone can be fooled by somebody.

-Statement: ∀y∃x F(x, y)

-Negation: ∃y∀x ¬F(x, y)  
-In English: Someone cannot be fooled by everyone.

**f )** No one can fool both Fred and Jerry.

-Statement: ¬∃x (F(x, Fred) ∧ F(x, Jerry))

-Negation: ∃x (F(x, Fred) ∧ F(x, Jerry))  
-In English: Someone can fool both Fred and Jerry.

**g)** There is exactly one person whom everybody can fool.

-Statement: ∃y [∀x( F(x, y) ∧ ( ∀z ( F(x, z) → y = z)) ]

-Negation: ∀y[∃x(¬F(x, y) ∨ ∃ z (F(x, z) ∧ (y ≠z)))]  
-In English: Everyone cannot be fooled by someone or someone can fool at least two people.

**h)** No one can fool himself or herself.

-Statement: ¬∃x F(x, x)

-Negation: ∃x F(x, x)

-In English: Someone can fool himself or herself.