CECS 228 Name:  
LAB #1.4 ID: Date:

Objectives:

* Be able to apply rules of inference
* Be able to translate more complex English

sentences into logical expression

Exercise 1: For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

**a)** “If I take the day off, it either rains or snows.” “I took Tuesday off or I took Thursday off.” “It was sunny on Tuesday.” “It did not snow on Thursday.”

D(x) : I take x day off. R(x): It rains on x day D: all days of a week

S(x) : It snows on x day  
1. ∀x(D(x) →(R(x) S(x)) (hypothesis)

2. D(Tue) D(Thu) (hypothesis)

3. R(Tue) ∧ S(Tue) (R(Tue) S(Tue)) (hypothesis)  
4. S(Thu) (hypothesis)  
5. D(Tue) →(R(Tue) S(Tue)) D(Tue) (R(Tue) S(Tue)) (Universal Instantiation on 1)

6. D(Thu) →(R(Thu) S(Thu)) D(Thu) (R(Thu) S(Thu)) (Universal Instantiation on 1)

7. D(Thu) R(Thu) (Disjunctive Syllogism on 4 and 6)

8. D(Tue) R(Thu) (Resolution on 2 and 7)

9. **D(Tue)** (Modus Tollens on 3 and 5)

10. R(Thu) (Disjunctive Syllogism on 8 and 9)

**b)** “Every computer science major has a personal computer.” “Ralph does not have a personal computer.” “Ann has a personal computer.”

P(x) : x has a personal computer D: all students

C(x) : x is a CS major  
1. ∀x(C(x) → P(x)) (hypothesis)

2. P(Ralph) (hypothesis)

3. P(Ann) (hypothesis)  
4. C(Ann) → P(Ann) (Universal Instantiation on 1)  
5. C(Ralph) → P(Ralph) (Universal Instantiation on 1)

6. **(Ralph)** (Modus Tollens on 2 and 5)

**c)** “All foods that are healthy to eat do not taste good.” “Tofu is healthy to eat.” “You only eat what tastes good.” “You do not eat tofu.” “Cheeseburgers are not healthy to eat.”

H(x) : x is health to eat Y(x): You eat x D: all food

T(x) : x tastes good   
1. ∀x(H(x) → T(x)) (hypothesis)

2. H(tofu) (hypothesis)

3. ∀x(T(x) → (x)) (hypothesis)

4. (tofu) (hypothesis)

5. (cheeseburger) (hypothesis)  
6. H(tofu) → T(tofu) (Universal Instantiation on 1)

7.T(tofu) (Modus Ponens on 2 and 6)

8. T(tofu) → Y(tofu) (Universal Instantiation on 3)

9**.T(tofu)** (Modus Tollens on 4 and 8)

Exercise 2: Suppose the variable x represents people, and

F(x): x is friendly T(x): x is tall A(x): x is angry.

(i) Write the statement using these predicates and any needed quantifiers.

(ii) Write the negation of these statements using these predicates and any needed quantifiers.

(iii) Translate the negation into English.

1. Some people are not angry.

Original:

∃x A(x)

Negation:

∀x A(x)

Translation of negation:

All people are angry.

1. All tall people are friendly but not angry.

Original:

∀x [T(x) → (F(x) ∧ A(x)]

Negation:

∃x [T(x) ∧ (A(x) F(x))]

Translation of negation:

Some tall people are not-friendly or angry.

1. No friendly people are angry.

Original:

∃x [F(x) ∧ A(x) ]

Negation:

∃x [F(x) ∧ A(x)]

Translation of negation:

Some people are both friendly and angry.

1. All friendly people are tall and angry.

Original:

∀x [F(x) → (T(x) ∧ A(x))].

Negation:

∃x [F(x) ∧ (A(x) T(x))]

Translation of negation:

Some friendly person is not-tall or not-angry.

Exercise 3: Suppose the variable x represents students and y represents courses,

z represents faculty and:

U(y): y is an upper-level course M(y): y is a math course   
F(x): x is a freshman A(x): x is a part-time student

T(x, y): student x is taking course y.

R(x,z): x respects z

(A) Write the statement using these predicates and any needed quantifiers.

a. Every student is taking at least one course.

∀x∃y T(x, y).

b. There is a part-time student who is not taking any math course.

∃x [A(x) ∧ ∀y (M(y) → T(x, y))].

c. Every part-time freshman is taking some upper-level course.

∀x∃y [F(x) → (U(y) ∧ T(x, y))].

(B) Write the statement in good English without using variables in your answers.

a. ∀x∃y T(x, y).

Every student is taking a course

b. ∃x∀y T(x, y).

A student is taking all courses

c. ∀x∃y [F(x) → (U(y) ∧ M(y) ∧ T(x, y))].

Every freshman is taking an upper-level math course

d. ∀x [(A(x) ∧ F(x)) →∃z R(x,z))].

All part-time freshman respect a faculty member.

Exercise 4: Identify the error or errors in this argument that supposedly shows that if  
 ∀*x(P(x)* ∨ *Q(x))* is true then ∀*xP(x)* ∨ ∀*xQ(x)* is true. Explain your reasoning

1. ∀*x(P(x)* ∨ *Q(x))* Premise

2. *P(c)* ∨ *Q(c)* Universal instantiation from (1)

3. *P(c)* Simplification from (2)

4. ∀*xP(x)* Universal generalization from (3)

5. *Q(c)* Simplification from (2)

6. ∀*xQ(x)* Universal generalization from (5)

7. ∀*x(P(x)* ∨ ∀*xQ(x))* Conjunction from (4) and (6)

Error at step 3- cannot use simplification. The rule should be applied to P(c) ∧ Q(c)  
Error at step 5- cannot use simplification. The rule should be applied to P(c) ∧ Q(c)  
Error at step 7- cannot use conjunction. The rule should result ∀*x(P(x)* ∧ ∀*xQ(x))*