CECS 228 Name:  
LAB #1.6 ID: Date:

Objectives:

* Be able to apply different methods of proof and strategy

Exercise 1:

a. Show that the product of two of the numbers 651000 −82001 + 3177,

791212 − 92399 + 22001, and 244493 −58192 + 71777 is nonnegative. Is your proof constructive or nonconstructive? [Hint: Do not try to evaluate these numbers!]

Of these three numbers, at least two must have the same sign (both positive or both negative), since there are only two signs. (It is conceivable that some of them are zero, but we view zero as positive for the purposes of this problem.) The product of two with the same sign is nonnegative. This was a nonconstructive proof, since we have not identified which product is nonnegative. (In fact, a computer algebra system will tell us that all three are positive, so all three products are positive.)

b. Prove that there exists a pair of consecutive integers such that one of these integers is a perfect square and the other is a perfect cube. Is your proof constructive or nonconstructive?

8 and 9 where 8 = 23 and 9 = 32

Exercise 2: Prove that *n*2 + 1 ≥ 2nwhen *n* is a positive integer with 1 ≤ *n* ≤ 4.

(Proof by cases)

Case 1: n = 1. Then 12 ­+ 1 ≥ 21 🡪 2 ≥ 2

Case 2: n = 2. Then 22 ­+ 1 ≥ 22 🡪 5 ≥ 4

Case 3: n = 3. Then 32 ­+ 1 ≥ 23 🡪 10 ≥ 8

Case 4: n = 4. Then 42 ­+ 1 ≥ 24 🡪 17 ≥ 16

Exercise 3: Prove or disprove that if a and b are rational numbers, then ab is also rational.

Let a = 2 and b = 1/2  
The statement is incorrect since 2 and 1/2 are rational numbers but 21/2 is not.

Exercise 4: Prove that there are no solutions in integers x and y to the equations 2x2 + 5y2 = 14

If |y| ≥ 2, then 2x2 + 5y2 ≥ 2x2 + 20 ≥ 20, so the only possible values of y to try are 0 and ±1. In the former case we would be looking for solutions to 2x2 = 14 and in the latter case to 2x2 = 9. Clearly there are no integer solutions to these equations, so there are no solutions to the original equation.

Exercise 5:  
The **quadratic mean** of two real numbers *x* and *y* equals . By computing the arithmetic and quadratic means of different pairs of positive real numbers, formulate a conjecture about their relative sizes and prove your conjecture.  
Let x = 1 and y = 10. Then their arithmetic is 5.5 and their quadratic mean is 7.11. Similarly, if x = 5 and y = 8, then the arithmetic mean is (5+8)/2 = 6.5 and the quadratic mean is

So we conjecture that the quadratic mean is always greater than or equal to the arithmetic mean. Thus we want to prove that

for all positive real numbers x and y .Doing some algebra, we find that this inequality is equivalent to the true statement

In fact, our argument also shows that equality holds if and only if x = y .