CECS 228 Name:

Lab 3.2 ID: Date:  
Objective:

* Be able to prove set identities
* Be able to understand Cartesian products
* Be able to recognize partition of a set

Exercise 1: Let A, B, and C be sets. Show that = ( ∪ ) ∩ .  
  
Solution: We have   
= ∩ by the first De Morgan law  
 = ∩ ( ∪ ) by the second De Morgan law  
 = ( ∪ ) ∩ by the commutative law for intersections  
 = ( ∪ ) ∩ by the commutative law for unions.

Exercise 2: Let A, B, and C be sets. Show that ∪ (A ∩ B) = ∪ B.  
Solution: We have  
 ∪ (A ∩ B) = ( ∪ A) ∩ ( ∪ B) Distributive law  
 = U ∩ ( ∪ B) Complement law  
 = ∪ B Identity law  
  
Exercise 3: Use set builder notation and logical equivalences to establish the first De Morgan law = ∪ ∪ .  
= {x | x /∈ A ∩ B ∩ C } by definition of complement  
 = {x | ¬(x ∈ (A ∩ B∩ C))} by definition of does not belong symbol  
 = {x | ¬(x ∈ A ∧ x ∈ B)   
 ∧ (x ∈ C) } by definition of intersection   
 = {x | ¬(x ∈ A)∨¬(x ∈ B)  
 ∨¬(x ∈ C)} by the first De Morgan law for logical equivalences  
 = {x | x A ∨ x B  
 ∨ x C } by definition of does not belong symbol  
 = {x | x ∈ ∨ x ∈  
 ∨ x ∈} by definition of complement  
 = {x | x ∈ ∪ ∪ } by definition of union  
 = ∪ ∪ by meaning of set builder notation

Exercise 4:  
Let A = {a, b, c}, B = {x, y}, and C = {0, 1}. Find  
a) A × B.   
A × B = {(a, x), (a, y), (b, x), (b, y), (c, x), (c, y)}  
  
b) C × B × A.  
C × B × A = {(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b), (0, y, c),  
 (1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)}

Exercise 5:  
Which of these collections of subsets are partitions of the set of integers? If not, explain why.  
a) the set of even integers and the set of odd integers  
Yes, this is a partition.

b) the set of positive integers and the set of negative integers  
This is not a partition, since 0 is in neither set.

c) the set of integers divisible by 3, the set of integers leaving a remainder of 1 when divided by 3, and the set of integers leaving a remainder of 2 when divided by 3  
Yes, this is a partition

d) the set of integers less than −100, the set of integers with absolute value not exceeding 100, and the set of integers greater than 100  
This is a partition, since the second set mentioned is the set of all number between −100 and 100, inclusive.

e) the set of integers not divisible by 3, the set of even integers, and the set of integers that leave a remainder of 3 when divided by 6  
The first two sets are not disjoint (4 is in both), so this is not a partition.