CECS 228 Name:

Lab 4.1 ID: Date:  
Objective:

* Be able to understand definition of functions
* Be able to determine whether a function is bijective, surjective or injective.
* Be able to understand important functions such as ceiling, floor functions

Exercise 1: Why is f not a function from to if  
a) f (x) = 1/x?  
Not a function because f is not well-defined at x = 0

b) f (x) = ?  
Not a function because f is not defined if x < 0

c) f (x) = ± ?  
Not a function because for each value of x, there are two different values of f(x).

Exercise 2: Determine whether f is a function from **Z** to **R.** If not, explain why.   
a) f (n) = ±n.  
Not a function since for each value of n, there are two values of f(n).

b) f (n) =  
Yes.

c) f (n) = 1/(n2 − 4).  
No since n = 2 or n = -2 makes the function undefined  
  
Exercise 3: Give an example of a function from **N** to **N** that is  
a) one-to-one but not onto.  
f(n) = n + 1  
  
  
  
b) onto but not one-to-one.  
f(n) = ⌈x/2⌉  
  
  
  
  
c) both onto and one-to-one (but different from the identity function).  
We let f(n) = n − 1 for even values of n, and f(n) = n + 1 for odd values of n. Thus we have f(1) = 2, f(2) = 1, f(3) = 4, f(4) = 3, and so on. Note that this is just one function, even though its definition used two formulae, depending on the parity of n.  
  
  
d) neither one-to-one nor onto.  
f(n) = 10

Exercise 4:   
Find these values.  
a. ⌊1/2 + ⌊1/2⌋⌋ = ⌊1/2 + 0⌋=⌊1/2 ⌋=0

b. ⌈⌊1/2 + ⌊1/2⌋⌋⌉ = ⌈⌊1/2 + ⌊1/2⌋⌋⌉ = 0  
  
  
c. ⌊1/2 + ⌈3/2⌉⌋ = ⌊1/2 + 2⌋ = ⌊2.5⌋ = 2

Exercise 5:  
Prove or disprove each of these statements about the floor and ceiling functions  
a. ⌈xy⌉ = ⌈x⌉ ⌈y⌉ for all real numbers x and y.  
Pick x = ½ and y = 2  
⌈(1/2)2⌉ =1   
⌈1/2⌉ ⌈2⌉ = 2

b. ⌈x⌉ + ⌈y⌉ - ⌈x + y⌉ = 0 or 1 whenever x and y are real numbers.  
 True; if x or y is an integer, the difference is 0.   
If neither x nor y is an integer, then x = n + and y = m + δ, where n and m are integers and and δ are positive real numbers less than 1. Then m + n < x + y < m+ n + 2, so ⌈x + y⌉ is either m+n+1 or m+n+2. Therefore, the given expression is either (n + 1) + (m + 1) − (m + n + 1) = 1 or (n + 1) + (m + 1) − (m + n + 2) = 0, as desired.

Exercise 6:  
Sets A and X are defined as:  
A = { a, b, c, d }  
X = { 1, 2, 3, 4 }  
A function f: A → X is defined to be  
f = { (a, 3), (b, 1), (c, 4), (d, 1) }  
a. What is the target (or co-domain) of function f? {1, 2, 3, 4}  
b. What is the range of function f? {1, 3, 4}  
c. What is f(c)? 4  
d. What is the domain of function f? A  
  
Exercise 7: Consider three functions f, g, and h, whose domain and target are **Z**. Let

*f*(*x*)=*x*2           *g*(*x*)=2x           *h*(*x*)=⌈*x5*⌉

a. Evaluate g ο h ο f(4)  
16  
  
b. Give a mathematical expression for f ο g.  
22x

Exercise 8: For each of the following functions, indicate whether the function has a well-defined inverse. If the inverse is well-defined, give the input/output relationship of f-1.

a. f: Z → Z. f(x) = 2x + 3  
The function f is not onto, so f-1 is not well-defined.  
  
b. f: R → R. f(x) = 2x + 3  
f-1(x) = (x - 3)/2