CECS 228 Name:

Lab 6.1 ID: Date:  
Objective:

* Be able to identify a relation
* Be able to identify properties of a relation
* Be able to represent relation using digraphs and matrices.

Exercise 1: List the ordered pairs in the relation R from A = {0, 1, 2, 3, 4} to B = {0, 1, 2, 3}, where (a, b) ∈ R if and only if

a) a = b.

{(0, 0), (1, 1), (2, 2), (3, 3)}

b) a + b = 4.

{(1, 3), (2, 2), (3, 1), (4, 0)}

c) a | b

{(1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (3, 0), (3, 3), (4, 0)}

d) gcd(a, b) = 1.

{(0, 1), (1, 0), (1, 1), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2), (4, 1), (4, 3)}

Exercise 2: Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, asymmetric, and/or transitive, where (x, y) ∈ R if and only if (explain your reasoning)

a) xy ≥ 0.

This relation is reflexive since squares are always nonnegative. It is clearly symmetric (the roles of x and y in the statement are interchangeable). It is not antisymmetric, since (2, 3) and (3, 2) are both in R. It is not transitive; for example, (1, 0) R and (0,−2) R, but (1,−2) R.

b) x = 1 or y = 1.

This is not reflexive, since (2, 2) R. It is clearly symmetric (the roles of x and y in the statement are interchangeable). It is not antisymmetric, since (2, 1) and (1, 2) are both in R. It is not transitive; for example, (3, 1) R and (1, 7) R, but (3, 7) R.

c) x – y = rational number  
The relation is reflexive, since x − x = 0 is a rational number. The relation is symmetric, because if x − y is rational, then so is −(x − y) = y − x. Since (1,−1) and (−1, 1) are both in R, the relation is not antisymmetric. To see that the relation is transitive, note that if (x, y) R and (y, z) R, then x − y and y − z are rational numbers. Therefore their sum x − z is rational, and that means that (x, z) R.

Exercise 3: For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, whether it is asymmetric, whether it is irreflexive and whether it is transitive.

a) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}  
Transitive

b) {(2, 4), (4, 2)}

Symmetric, Irreflexive

c) {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}

Irreflexive  
  
Exercise 4:  
R1 = {(a, b) ∈ | a > b}, the “greater than” relation,

R2 = {(a, b) ∈ | a ≥ b}, the “greater than or equal to” relation,

R3 = {(a, b) ∈ | a < b}, the “less than” relation,

R4 = {(a, b) ∈ | a ≤ b}, the “less than or equal to” relation,

R5 = {(a, b) ∈ | a = b}, the “equal to” relation,

R6 = {(a, b) ∈ | a ≠ b}, the “unequal to” relation.  
a) R1 ∪ R3.   
The union of two relations is the union of these sets. Thus R1 ∪ R3 holds between two real numbers if R1 holds or R3 holds (or both, it goes without saying). Here this means that the first number is greater than the second or vice versa—in other words, that the two numbers are not equal. This is just relation R6.  
  
b) R2 ∩ R4.   
The intersection of two relations is the intersection of these sets. Thus R2 ∩ R4 holds between two real numbers if R2 holds and R4 holds as well. Thus for (a, b) to be in R2 ∩ R4, we must have a ≥ b and a ≤ b. Since this happens precisely when a = b, we see that the answer is R5.

c) R1 − R2.

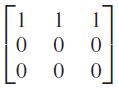
Recall that R1 − R2 =. But = R3, so we are asked for R1 ∩ R3. It is impossible for a > b and a < b to hold at the same time, so the answer is Ø, i.e., the relation that never holds.

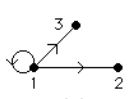
Exercise 5:   
A. Represent each of these relations on {1, 2, 3} with a matrix (with the elements of this set listed in increasing order).  
B. Draw the digraphs representing each of the relations below.  
C. Determine whether the relations represented by the directed graphs in part B are reflexive, symmetric, antisymmetric, asymmetric, irreflexive and/or transitive.

a) {(1, 1), (1, 2), (1, 3)}

A. B C

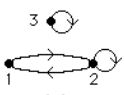
|  |  |
| --- | --- |
| Not reflexive | Not irreflexive |
| Not symmetric | Not asymmetric |
| Antisymmetric | Transitive |



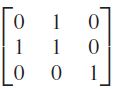


b) {(1, 2), (2, 1), (2, 2), (3, 3)}

A. B C



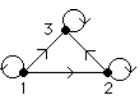
|  |  |
| --- | --- |
| Not reflexive | Not irreflexive |
| Symmetric | Not asymmetric |
| Not antisymmetric | Not Transitive |

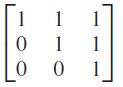


c) {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)}

A. B C

|  |  |
| --- | --- |
| Reflexive | Not irreflexive |
| Not symmetric | Not asymmetric |
| Antisymmetric | Transitive |





d) {(1, 3), (3, 1)}

A. B C

|  |  |
| --- | --- |
| Not reflexive | Irreflexive |
| Symmetric | Not asymmetric |
| Not antisymmetric | Not transitive |

