CECS 228 Name:

Lab 6.2 ID: Date:  
Objective:

* Be able to identify equivalence relation
* Be able to compute composition of relation
* Be able to identify partial ordering relation

Exercise 1: Which of these relations on the set of all people are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a) {(a, b) | a and b are the same age}

Yes. It is an equivalent relation

b) {(a, b) | a and b speak a common language}

Not an equivalent relation since it is not transitive

c) {(a, b) | a and b have met}

Not an equivalent relation since it is not transitive

Exercise 2:  
Let R be the relation {(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)}, and let S be the relation {(2, 1), (3, 1), (3, 2), (4, 2)}. Find S ◦ R.  
Since (1, 2) R and (2, 1) S , we have (1, 1) S ◦ R. We use similar reasoning to form the rest of the pairs in the composition, giving us the answer {(1, 1), (1, 2), (2, 1), (2, 2)}.

Exercise 3:   
R1 = {(a, b) ∈ | a > b}, the “greater than” relation,

R2 = {(a, b) ∈ | a ≥ b}, the “greater than or equal to” relation,

R3 = {(a, b) ∈ | a < b}, the “less than” relation,

R4 = {(a, b) ∈ | a ≤ b}, the “less than or equal to” relation,

R5 = {(a, b) ∈ | a = b}, the “equal to” relation,

R6 = {(a, b) ∈ | a ≠ b}, the “unequal to” relation.  
a) R1 ◦ R4.

For (a, c) to be in R1 ◦ R4, we must find an element b such that (a, b) R4 and (b, c) R1. This means that a ≤ b and b > c. Clearly this can always be done simply by choosing b to be large enough. Therefore we have R1 ◦ R4 = , the relation that always holds.

b) R1 ◦ R6.

For (a, c) to be in R1 ◦ R6, we must find an element b such that (a, b) R6 and (b, c) R1. This means that a ≠ b and b > c. Clearly this can always be done simply by choosing b to be large enough. Therefore we have R1◦ R6 = , the relation that always holds.

c) R2 ◦ R3.  
For (a, c) to be in R2 ◦ R3, we must find an element b such that (a, b) R3 and (b, c) R2. This means that a < b and b ≥ c. Clearly this can always be done simply by choosing b to be large enough. Therefore we have R2 ◦ R3 = , the relation that always holds.

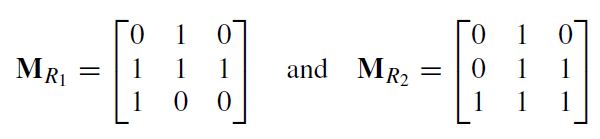
d) R3 ◦ R3.

For (a, c) to be in R3 ◦ R3, we must find an element b such that (a, b) R3 and (b, c) R3. This means that a < b and b < c. Clearly this can be done if and only if a < c to begin with. But that is precisely the statement that (a, c) R3. Therefore we have R3 ◦ R3= R3. We can interpret (part of) this as showing that R3 is transitive.

e) R1 ◦ R5.

For (a, c) to be in R1 ◦ R5, we must find an element b such that (a, b) R5 and (b, c) R1. This means that a = b and b > c. Clearly this can be done if and only if a > c to begin with (choose b = a). But that is precisely the statement that (a, c) R1. Therefore we have R1◦ R5 = R1. One way to look at this is to say that R5, the equality relation, acts as an identity for the composition operation (on the right—although it is also an identity on the left as well).

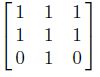
Exercise 4: Let R1 and R2 be relations on a set A represented by the matrices



Find the matrices that represent

a) R1 ∪ R2.   
  
  
b) R1 ∩ R2.   
  
  
c) R2 ◦ R1.

  
  
d) R1 ◦ R1.



Exercise 5: Which of these relations on {0, 1, 2, 3} are partial orderings?

Determine the properties of a partial ordering that the others lack.

a) {(0, 0), (2, 2), (3, 3)}

This relation is not reflexive because 1 is not related to itself. Therefore R is not a partial ordering. The relation is antisymmetric, because the only way for a to be related to b is for a to equal b. Similarly, the relation is transitive, because if a is related to b, and b is related to c, then necessarily a = b = c ≠ 1 so a is related to c.

b) {(0, 0), (1, 1), (1, 2), (2, 2), (3, 1), (3, 3)}

This is not a partial ordering, because it is not transitive: 3R1 and 1R2, but 3 is not related to 2. It is reflexive and the pairs (1, 2) and (3, 1) will not introduce any violations of antisymmetry.

c) {(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (3, 3)}  
The relation is clearly reflexive, but it is not antisymmetric (0R1 and 1R0, but 0 ≠ 1) and not transitive (2R0 and 0R1, but 2 is not related to 1).

Exercise 6: Answer these questions for the poset ({{1}, {2}, {4}, {1, 2}, {1, 4}, {2, 4}, {3, 4}, {1, 3, 4}, {2, 3, 4}}, ⊆).

a) Find the maximal elements.

{1,2} , {1, 3, 4}, {2, 3, 4}

b) Find the minimal elements.

{1}, {2}, {4}

c) Is there a greatest element?

No

d) Is there a least element?

No

e) Find all upper bounds of {{2}, {4}}.

{2, 4}, {2, 3, 4}

f) Find the least upper bound of {{2}, {4}}, if it exists.

{2*,* 4}

g) Find all lower bounds of {{1, 3, 4}, {2, 3, 4}}.

{3, 4}, {4}

h) Find the greatest lower bound of {{1, 3, 4}, {2, 3, 4}}, if it exists.

{3, 4}