

Usman Khan

251 036 415

CS 2210

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### Concept Assignment 1

#### Question 1) and 2)

1) We need to find constants such that  $c > 0, n_0 \geq 1$   $[f(n) \times h(n) \leq c g(n) \times h(n) \text{ for all } n \geq n_0]$

$\rightarrow \frac{f(n) \times h(n)}{g(n) \times h(n)} \leq c \frac{g(n) \times h(n)}{g(n) \times h(n)} \text{ for all } n \geq n_0$

$\rightarrow \frac{f(n)}{g(n)} \leq c \text{ for all } n \geq n_0$

$\therefore$  After simplifying this inequality, we are left with the definition of "big Oh", leaving us with this definition proves that  $f(n) \times h(n)$  is  $O(g(n) \times h(n))$

2) We will use proof by contradiction to prove that  $n$  is not  $O(\frac{1}{n})$

$\rightarrow$  Assume that the claim is false and get  $n$  is  $O(\frac{1}{n})$

$\rightarrow n \leq c \frac{1}{n} \text{ for all } n \geq n_0$

$\rightarrow n \cdot (n) \leq c \frac{1}{n} \cdot (n) \text{ for all } n \geq n_0$

$\rightarrow n^2 \leq c \text{ for all } n \geq n_0$

$\therefore$  After simplification we are left with  $n^2 \leq c$  for all  $n \geq n_0$ , right away we can conclude that this inequality can't be true, because it says all large values of  $n$  are not bigger than any constant, which is false because there are an infinite number of values that are bigger than any constant.

### **Question 3**

Java File

### **Question 4**

Part 1)

- a. This algorithm can terminate when it either returns false or returns true. This can be understood because while both loops are being executed, the array at index "i" and index "j" are the same, this means there are two values which are similar, which will return false and exit both loops. On the other hand, the algorithm will return true if no values which are similar are found, causing the program to come to termination.
- b. We need to prove that the algorithm has values stored in array "A" which are different, or similar. In this algorithm, we are always comparing two values "i" and "j" and checking the similarity between the two. Now let's say for this instance that all values in array A are different, this means there is no "i" value such that  $A[i] = A[j]$ , meaning that when "i" gets to one less than n, it means we will go through our final iteration of this loop and this means that every value in the array was different and the algorithm will correctly return true. This algorithm will always produce the correct answer because if it ever finds two values that are the same, it will return false and terminate. But in this case we go through all values in the array, which means the values were not the same, therefore returning true and terminating.

Part 2)

The worst case for the algorithm is to iterate through the whole array and not finding two values that match.

Part 3)

4) Algorithm: areDifferent(A, n)

Input: Array A storing n integer values

Output: true if all values in A are different; false if otherwise

for  $i \leftarrow 1$  to  $n-1$  do

for  $j \leftarrow 0$  to  $i-1$  do

$C_1 \rightarrow$  if  $A[j] = A[i]$  then return false

$C_2 \rightarrow$  return true

		# of iterations
$i=1$	$j=0$	1
$i=2$	$j=0, 1$	2
$i=3$	$j=0, 1, 2$	3
$i=4$	$j=0, 1, 2, 3$	4
$\vdots$	$\vdots$	$\vdots$
$i=n-1$	$j=0, 1, 2, \dots, i-1$	$n-1$

$$\begin{aligned}\# \text{ of iterations} &= 1 + 2 + 3 + 4 + \dots + n-1 \\ &= \sum_{i=1}^{n-1} i = \frac{n(n-1)}{2} \\ &= \frac{1}{2}n^2 - \frac{1}{2}n\end{aligned}$$

$$f(n) = C_2 + C_1 \left( \frac{1}{2}n^2 - \frac{1}{2}n \right)$$

$$= (C_2 + n^2 - n)$$

$$= n^2 - n$$

$$= n^2$$

$$\therefore \text{ is } O(n^2)$$