Divide-and-Conquer

Practical Exercises

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Exercise 1

Given any one-dimensional array `A`[1..*n*] of integers, the **maximum sum subarray problem** tries to find a contiguous subarray of `A`, starting with element *i* and ending with element *j*, with the largest sum: $max \sum_{x=i}^{j} A[x]$ with $1 \leq i \leq j \leq n$. Consider the `maxSubSequence` function below.

```
int maxSubSequence(int A[], unsigned int n , int &i, int &j)
```

The function returns the sum of the maximum subarray, for which *i* and *j* are the indices of the first and last elements of this subsequence (respectively). The function uses an exhaustive search strategy (*i.e.*, Brute-force) so as to find a subarray of `A` with the largest sum, and updates the arguments *i* and *j*, accordingly.

Input example: `A = [−2, 1, −3, 4, −1, 2, 1, −5, 4]`
Expected result: [0, 0, 0, 1, 1, 1, 1, 0, 0], as subsequence `[4, −1, 2, 1] (i = 3, j = 6)` produces the largest sum, 6.

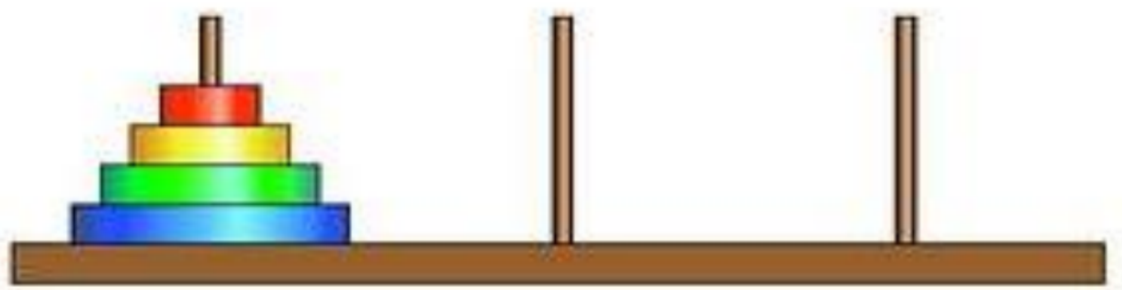
a) Propose in pseudo-code a divide-and-conquer strategy for this problem.
b) Using the master theorem, indicate and justify the time complexity of the proposed algorithm.
c) Implement *maxSubsequenceDC* using the proposed algorithm.

```
int maxSubsequenceDC(int A[], unsigned int n , int &i, int &j)
```

Exercise 2

In the **Hanoi towers problem**, the goal is to move a stack of n disks of decreasing size from one peg to another, with the following constraints:

- Three pegs are available: A, B and C. The stack of n disks begins at one of the pegs.
- Only one disk can be moved at a time.
- At any time, the disk stack of any peg must be ordered in decreasing size order, with the largest disk at the bottom and the smallest one at the top.



a) Propose an algorithm in pseudo-code that solves this problem using a divide-and-conquer strategy. The solution must minimize the number of disk movements.
b) Using induction, prove that for n pegs, at most (2^n - 1) moves are needed. Using this result indicate and justify the algorithm's time complexity, with respect to the number of disks, n.
c) Implement the *hanoiDC*, which implements the proposed algorithm.

```
std::string hanoiDC(unsigned int n, char src, char dest)
```

<u>Input example</u>: **n** = 4, **src** = 'A', **dest** = 'B'
<u>Expected result</u>:
"A→C,A→B,C→B,A→C,B→A,B→C,A→C,A→B,C→B,C→A,B→A,C→B,A→C,A→B,C→B"

Exercise 3

Suppose you are choosing between the following three algorithms:

- Algorithm A solves problems by dividing them into five subproblems of half the size, recursively solving each subproblem, and then combining the solutions in linear time.
- Algorithm B solves problems of size n by recursively solving two subproblems of size n-1 and then combining the solutions in constant time.
- Algorithm C solves problems of size n by dividing them into nine subproblems of size n/3, recursively solving each subproblem, and then combining the solutions in $O(n^2)$ time.

What are the running times of each of these algorithms (in big-O notation), and which would you choose if your problem instances exhibited large values of n?