

Simulation of Crowd Egress with Environmental Stressors

Peng Wang Xiaoda Wang Peter Luh

Christian Wilkie Timo Korhonen Neal Olderman

This article introduces a modeling framework to characterize evacuee response to environmental stimuli during emergency egress. The model is developed in consistency with stress theory, which explains how an organism reacts to environmental stressors. We integrate the theory into the well-known social-force model, and develop a framework to simulate crowd evacuation behavior in multi-compartment buildings. Our method serves as a theoretical basis to study crowd movement at bottlenecks, and further simulate their herding and way-finding behavior in normal and hazardous conditions. The pre-movement behavior is also briefly investigated by using opinion dynamics. The algorithms have been partly tested in FDS+EVAC as well as our simulation platform crowdEgress.

Glossary—

Crowd Egress: A multitude of people with conscious mind, and they move and interact collectively towards a place of safety. In this article the term mainly refers to an evacuation process in a multi-compartment layout of buildings.

Stress: The word stress relates to a mental strain or a harmful environmental stimuli that cause response of organism. The current usage of this term was mainly referred to the interaction between the environment and the living bodies, emphasizing the role of the individual's appraisal of situations in shaping their responses.

Social Force Model: Force-based model of social interaction of individuals, and it is based on Newtonian equations, and it enables agent-based simulation of emergent behavior of a large number of people in normal and emergency situation.

1. Social Force Model and Stress Theory

Historically, stress was initially a physical quantity, which is a measure of the internal forces in a body between its particles. In the 1920s and 1930s, biological and psychological circles occasionally used the term to refer to a mental strain or a harmful environmental stimuli that cause response of organism. The current usage of the word stress was mainly referred to the interaction between environmental stimuli and the living bodies, emphasizing the role of the individual's appraisal of situations in shaping their responses (Selye, 1975).

For human beings stress is normally perceived when we think the demand being placed on us exceed our ability to cope with, and it can be external and related to the environment, and it becomes effective by internal perception. This paper will integrate the stress theory into the well-known social force model (Helbing et. al., 2002 and 2005), which has been widely used in simulation of crowd evacuation in the past two decades. The feasibility of our method relies on certain psychological factors abstracted in the model, such as “desired velocity” and “social force.” For instance, desired velocity is not the commonly-known velocity in physics, but refers to the moving speed and direction that an individual expects to realize, and it actually exists in one's mind, and it could be sometimes equal to physical velocity, but conceptually they are different. This new concept enables us to model how the motivation level in the psychological sense (i.e., desired velocity) leads to behavioral change in the physics world (i.e., actually velocity), and

such motivation level are result of human perception, and are adapted to the environmental stressors. Thus, it is feasible to extend social-force model to characterize individuals' response to their surroundings based on stress theory. As below we present a diagram to describe the interplay between individuals and their surroundings based on the extended social-force model.

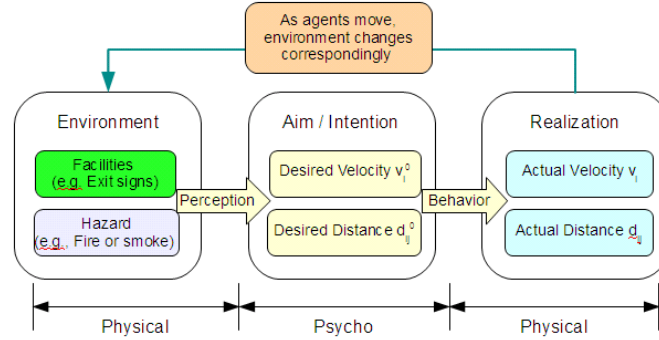


Figure 1. Perception and Behavior in a Feedback Loop: The motivation level v_i^0 and d_{ij}^0 in social force model are the result of human perception, and are adapted to the environmental stressors such as fire and smoke, and v_i^0 and d_{ij}^0 could vary both temporally and spatially, and they lead to behavior change in v_i and d_{ij} . The social-force model is extended to characterize the interplay between individuals and their surroundings.

In the above diagram the stressors include egress facilities (e.g., alarm, guidance) and hazard (e.g., fire and smoke), and people respond to such environmental stressors, moving to the safety. In a psychological sense such stress is caused by mismatch between psychological demand and realistic situation (Staal, 2004), and we summarize the mismatch in terms of velocity and interpersonal distance as listed in Table 1: the psychological demands are abstracted as desired velocity v_i^0 and desired interpersonal distance a_{ij}^0 while the physical reality is described by the physical velocity v_i and distance a_{ij} . The difference of two variables measures how much people feel stressed, by which people are motivated into certain behavior. The motivation is abstracted as the driving force and social force as shown in Table 1. The entire process formalizes the stimuli-reaction model (S-R model) in psychological view of behaviorism.

There are two types of stress listed in Table 1. The first type refers to time-related stress which is commonly known as time-pressure, and it is measured by the difference of desired velocity and actual velocity, i.e., $v_i^0 - v_i$. The second type indicates space-related stress, which refers to proxemics and social norms, and is represented by the gap of desired interpersonal distance and actual interpersonal distance, i.e., $a_{ij}^0 - a_{ij}$.

Table 1 On Conception of Stress in Extended Social-Force Model

	<i>Opinion (Psychological Characteristics)</i>	<i>Behavior (Physic-Based Characteristics)</i>	<i>Difference between subjective opinion and objective reality</i>	<i>Forced-Based Term for Newton Second Law</i>
Time-Related Stress: Velocity	desired velocities v_i^0	actual velocities v_i	Time-Related Stress: Velocity $v_i^0 - v_i$ or $ v_i / v_i^0 $	Driving Force $f_i^{drv} = m_i(v_i^0 - v_i)/\tau$
Space-related Stress: Distance	desired distance a_{ij}^0	actual distance a_{ij}	Space-related Stress: Distance $a_{ij}^0 - a_{ij}$ or a_{ij}/a_{ij}^0	Social Force $f_{ij}^{soc} = A_i \exp((a_{ij}^0 - a_{ij})/B_i) n_{ij}$

Usually emergency egress is a stressful situation. When people hear the fire alarm and confirm any kind of threat in their surroundings, they normally begin to escape, and this process is called movement phase in crowd egress. Each individual evacuee's motion is governed by physics laws, and the motion equation of individual i is specified as below.

$$m_i \frac{d\mathbf{v}_i(t)}{dt} = \mathbf{f}_i^{drv} + \sum_{j(\neq i)} \mathbf{f}_{ij} + \sum_w \mathbf{f}_{iw} + \sum_h \mathbf{f}_{ih} \quad , \quad (1)$$

The mass of an individual is denoted by m_i . The change of the instantaneous velocity $\mathbf{v}_i(t)$ of individual i is determined by the total force. The forces motivates people to adapt to surrounding stimuli, and they consist of driving force \mathbf{f}_i^{drv} , interaction force \mathbf{f}_{ij} as well as boundary wall force \mathbf{f}_{iw} . The hazard impact is also abstracted as a kind of force \mathbf{f}_{ih} .

The driving force functions as an energy source that drives people to move to a destination exit, and it is commonly specified by $\mathbf{f}_i^{drv} = \mathbf{F}^{drv}(\mathbf{v}_i^0 - \mathbf{v}_i)$, where $\mathbf{F}^{drv}(\cdot)$ is generally a monotonically increasing function. The interaction force \mathbf{f}_{ij}

describes the social-psychological tendency of two individuals to keep proper distance (as called the social-force), and if people have physical contact with each other, physical forces are also taken into account, i.e., $f_{ij} = f_{ij}^{soc} + f_{ij}^{phy}$. In this article we mainly focus on the social force term. In Helbing et. al., 2002 and 2005 this term is given in an exponential form, and we will integrate a new concept of desired distance d_{ij}^0 into this force term. In a general sense we specify this force term by $f_{ij}^{soc} = F^{soc}(d_{ij}^0 - d_{ij})$. The interaction of an individual with obstacles like walls is treated analogously, and denoted by $f_{iw} = f_{iw}^{soc} + f_{iw}^{phy}$, and this force specifies boundary of people's motion. The hazard force is a special term we take into account for emergency egress and we will briefly discuss it in detail soon later.

Next we will emphasize that f_i^{drv} , f_{ij}^{soc} , f_{iw}^{soc} and f_{ih} are all subjective entities coming from people's opinions, and they are generated intentionally by people through foot-ground friction on physics basis. These forces essentially describe how an individual perceive and react to the outside environmental stimuli. Because f_{iw}^{soc} can be formulated in analog of f_{ij}^{soc} , we will not further discuss f_{iw}^{soc} in this article. In sum, v_i^0 and d_{ij}^0 are non-physics entities for they exist in people's opinion, not in the physical world, and we have well explained the psychological background of the model in Wang, 2016, and critically modified the key concept of social force in consistency of both physical laws and psychological principles.

In the following section we will mainly introduce f_i^{drv} , f_{ij}^{soc} and f_{ih} in the background of evacuation study. Our focus is applying the above model in simulation of crowd egress, especially in modeling how perception of hazard (fire and smoke) and guidance (e.g., exit signs) influence evacuees' escape behavior. The method has been partly tested in FDS+Evac, a well-known open-source evacuation simulator (Korhonen, 2017; Forney 2017; McGrattan, 2018; Hostikka et. al., 2007; Korhonen et. al., 2008) as well as our simulation platform CrowdEgress (Wang et. al., 2023).

Last but not least, an evacuation process is commonly divided into two phases: pre-movement phase and movement phase. In the pre-movement phase evacuees are inclined to collect information rather than actually move to any exit. The pre-movement behavior will be also discussed in this article, where existing model in opinion dynamics are applied to calculate pre-movement time of many individuals. We will especially introduce this model in Section 2 and 3.

(a) Adapting Desired Velocity To Environmental Stressors: Fight-or-Flight Response and Hazard Effect

When the fire/smoke spread towards people, people normally desire moving faster to escape from danger (Proulx , 1993; Ozel, 2001; Kuligowski, 2009). Thus, we suggest that the desired velocity v_i^0 increase when people perceive such danger, and this means increase of arousal, resulting in fight-or-flight response in psychological theory (Cannon, 1932). As a result the driving force is increased, and people are motivated to respond and move to the safety. This process suggests that people transfer their internal energy into kinetic form in order to realize desired motion. Such driving force depends on how much an individual feel stressed and thus increase with $v_i^0 - v_i$.

A widely-used formula of the driving force is given in linear form $f_i^{drv} = m_i(v_i^0 - v_i)/\tau$, and it describes an individual tries to move with a desired velocity v_i^0 and expects to adapt the actual velocity v_i to the desired velocity v_i^0 within a certain time interval. In specific v_i^0 is the target velocity existing in one's opinion while v_i is the physical velocity being achieved in the reality. Thus, $v_i^0 - v_i$ implies the difference between the human subjective wish and realistic situation, and it is scaled by a time parameter τ , to formulate an acceleration term, making the physical velocity v_i approaching towards desired velocity v_i^0 . The convergence rate is determined by m_i/τ . If v_i^0 changes with time, v_i will approach to v_i^0 with a time delay. This mathematical description of driving force was initially used for vehicle traffic flow problem (Payne, 1971; Whitham, 1974), and it was also applied to pedestrian traffic problem in Helbing et. al., 2002 and 2005.

Interestingly, velocity is a time-related concept in physics and thus $v_i^0 - v_i$ reflects a kind of time-related stress. Such stress usually relates to a time-critical situation, or so-called "emergency." This may explains why the model is suitable for simulation of emergency evacuation in the past twenty years. Consequently the driving force is a function of $v_i^0 - v_i$, which motivate people to move to a destination exit. In this modeling method the desired velocity v_i^0 indicates one's arousal level, which relates to one's awareness of the situation, and v_i^0 increases when an individual directly detects the harmful gas or smoke, or indirectly informed by the fire alarms.

However, increasing v_i^0 does not mean that individuals are able to move faster, especially in the movement towards an exit. There is possibility that people may slow down when passing through a bottleneck or a hazard-filled area (e.g., smoke-filled corridors). As mathematically described by Equation (1), the impact of hazard is indicated by a resistance force which increases with hazard intensity (e.g., smoke density). This hazard force describes how hazardous condition impedes people's motion in egress, and it is denoted by f_{ih} . In the following discussion we usually take smoke for example and f_{ih} is supposed to be a function of smoke density. Other hazard characteristics can also be considered such as gas temperature. In a sense Equation (1) indicates that the hazard condition (e.g., smoke) is a kind of "spreading walls" that impede people's motion. The force from solid walls are expressed by f_{iw} while force from hazardous

condition are denoted by f_{ih} . An individual is able to go through such “spreading walls” if the smoke is not thick. As smoke density increase, people are impeded and cannot get into that area (See Figure 2).

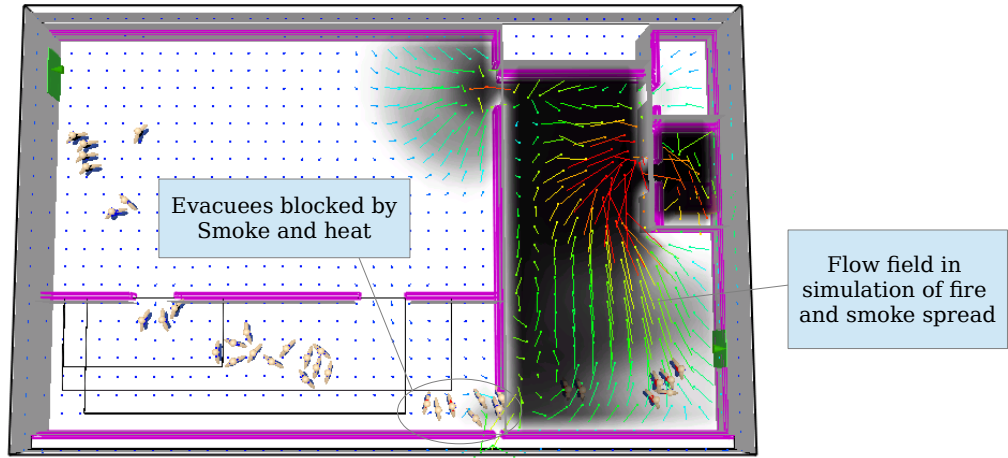


Figure 2. Simulation of Crowd Evacuation with Smoke: Smoke spreads and it is like “moving walls” which block evacuees' movement, and evacuees are not able to pass through such “moving walls” if the smoke is thick.

How to select the direction of f_{ih} is an interesting topic, and it mainly depends on characteristics of hazard. The physics law of smoke transport is not the same as that of heat transfer. A common method is assuming f_{ih} always impedes an evacuee's movement in any direction, and thus f_{ih} is always opposite to the direction of moving velocity v_i . In FDS+Evac we use (-HR%U, -HR%V) to specify the direction of f_{ih} , especially for resistance of smoke, and HR%U and HR%V compose 2D vector for human velocity. Another option is using gradient of hazard intensity. This gradient is useful to represent the direction of heat flux. The gradient points in the direction of the greatest increasing rate of hazard intensity. For example the hazard intensity is described by gas temperature $TMP_G(x, y)$ in a 2-dimensional plane at people's average height, and thus the direction of f_{ih} is opposite to the gradient of $TMP_G(x, y)$, which points in the direction of the greatest decreasing rate of hazard intensity.

$$\text{direction}(f_{ih}) = -\text{direction}(v_i) \quad \text{direction}(f_{ih}) = -\nabla TEM_G(x, y) \quad (2)$$

In sum the driving force and hazard force are conflicting factors, and they function together to give a whole picture of the model. The driving force motivates people to escape, describing how people are motivated into escape motion. In contrast the hazard force is used to describe if the outside condition permits such change or not. The following plot exemplifies the increasing curve of the driving force and smoke resistance when the smoke density increases. When the smoke density increases initially, people are able to speed up in smokiness. As the smoke density keeps increasing, the resistance from smoke is predominant and people have to slow down. As real-world fire drills or experiments have suggested, such motion is mainly due to poor visibility on the path (Jin and Yamada, 1989; Fridolf et. al., 2013; Was, 2018) or reduced percentage of oxygen.

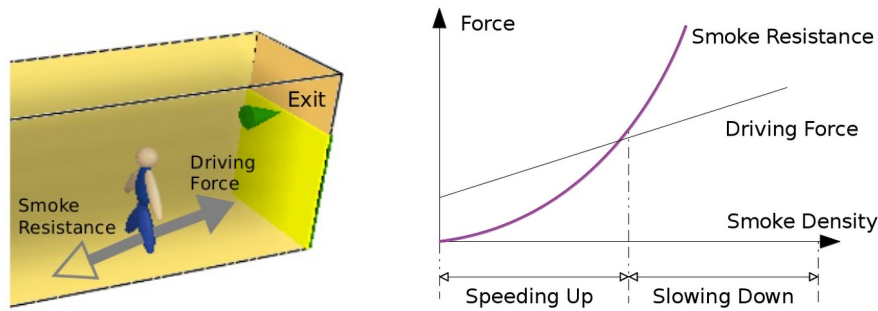


Figure 3. Walking Behavior in Smoke Condition: When the smoke density increases initially, the smoke is not thick so that people are able to speed up. As the smoke density keeps increasing, the resistance from smoke is predominant and people have to slow down even if they desire moving fast in escape.

The difficulty of the above method exists in quantitative analysis. It is not easy to quantify how fast evacuees desire moving or how they perceive threat from smoke or heat. Especially, the hazard force is not commonly a physics-based force, but a subjective entity which is closely related to human perception and cognition. In other words this force does not only depends on the hazard intensity itself, but also how people perceive it. For example, some people are sensitive to smoke inhaled while others may tolerate much. This effect may be inferred from certain clinical database, and theory in psychophysics may contribute valuable viewpoint (Stevens, 1971) because it discusses the relationship of the physics-based things and human perception of things. In brief, hazard force describes human perception of hazardous environmental stimuli. Although we call it “force” in the above model, it is actually different from the common “force” concept in physics, but related more about study subjects in psychology and psychophysics.

However, simulation is still a useful tool for risk analysis of building egress (Gwynne et. al., 2001). It is relatively easy to adjust parameters and observe different scenarios. For instance, from the above simulation we learn how smoke affects people's escape behavior. There are standard examples of FDS+Evac to test walking speed of evacuees in smoke condition. Please refer to the section of supplementary data for details.

(b) Adapting Desired Distance To Environmental Stressors: Proximics and Social Norms

In social psychology social norms are "representations of appropriate behavior" in a certain situation or environment, and it partly refers to a theory of how people use their personal space to interact with surrounding people. In Hall, 1963 the theory was named by proxemics, and it suggests that we surround ourselves with a "bubble" of personal space, which protects us from too much arousal and helps us keep comfortable when we interact with others. People normally feel stressed if such space is compressed. This theory justifies the repulsive social force in Helbing et. al., 2002 and 2005, where the force describes a potential field, and it reflects social repulsion of interacting individuals.

From the perspective of crowd modeling, the proxemics is represented by desired interpersonal distance a_{ij}^0 in our model, and $a_{ij}^0 - a_{ij}$ indicates a kind of space-related stress as shown in Table 1. Usually a_{ij}^0 changes with locations. For example, in elevators, entrance or narrow corridors, people usually accept smaller proximal distance, implying that desired interpersonal distance a_{ij}^0 is accepted to be small in these places. In sum adaption of a_{ij}^0 to different locations is a representation of social norms which are common rules in people's social life.

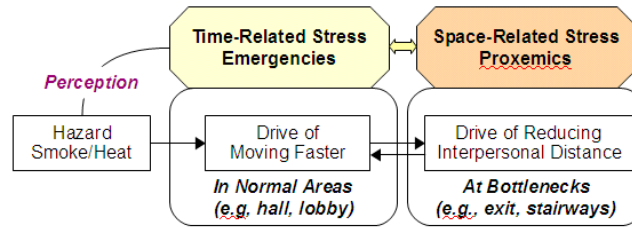


Figure 4. Two Types of Stress Transforming Mutually: The sense of emergencies creates a kind of time-related stress which drives people to move fast in escape. At bottlenecks (e.g., narrow doorways) people cannot speed up as desired, and thus time-related stress is transformed into space-related interpersonal stress in order to pass through the bottleneck quickly.

In emergencies the social norm is commonly adjusted and competitive behavior may emerge, and the model is thus applied to simulation of jamming and stampede at narrow passageways. Such narrowings are commonly identified as bottlenecks for crowd egress. Especially, the sense of emergency initially produces time-related stress that drive people to move fast. At bottlenecks such motion cannot be realized as desired, and the time-related stress is transferred to the space-related stress (See Figure 4). The desired interpersonal distance at bottlenecks is thus reduced, and people would like to “compress” their personal space in order to pass through the bottleneck quickly. This effect actually exhibits a kind of collective intelligence to increase transport efficiency at the bottleneck. The social norm is thus modified such that a_{ij}^0 is scaled down at bottlenecks. The parameter of A_i may also be scaled down so that the social force as a whole is reduced in such an occasion (Korhonen, 2017).

To testify the above effect at bottlenecks, we slightly modify the source program of FDS+Evac to implement the desired interpersonal distance. The social force is modified as below (Wang, 2016).

$$f_{ij}^{soc} = A_i \exp\left[\frac{(d_{ij}^0 - d_{ij})}{B_i}\right] n_{ij} \quad \text{or} \quad f_{ij}^{soc} = \left(\lambda_i + (1 - \lambda_i) \frac{1 + \cos \varphi_{ij}}{2}\right) A_i \exp\left[\frac{(d_{ij}^0 - d_{ij})}{B_i}\right] n_{ij} \quad (3)$$

Here A_i and B_i are positive constants, which affect the strength and effective range about how two individuals are repulsive to each other. The distance of individual i and j is denoted by d_{ij} . n_{ij} is the normalized vector which points from individual j to i . The gap of d_{ij}^0 and d_{ij} implies the difference between the subjective wish in one's mind and objective feature in the reality, and it is an indication of interpersonal stress related to the social space, and it fits well in the social force $f_{ij}^{soc} = F^{soc}(d_{ij}^0 - d_{ij})$ in a general sense.

The geometric features of two individuals are illustrated in Figure 5. Moreover, an anisotropic formula is widely used where Equation (3) is scaled by a function of λ_i . The angle φ_{ij} is the angle between v_i and n_{ij} . If $\lambda_i = 1$, the social force is isotropic and $0 < \lambda_i < 1$ implies that the force is larger in front of an individual than behind.

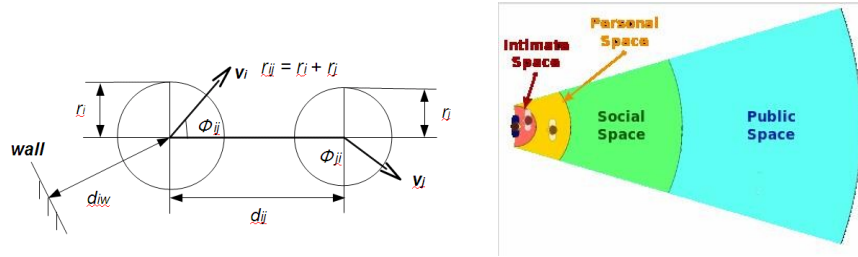


Figure 5. A Schematic View of Two Individual Evacuees (See Equation 3).

Below is the simulation result by using FDS+Evac, and the example is based on IMO door flow test (IMO, 2007), where the door width is $1m$, and it is also the door width used in Helbing et. al., 2002 and 2005. Let r_{ij} denote the sum of the radii of individual i and j , namely, $r_{ij} = r_i + r_j$. The left diagram corresponds to large d_{ij}^0 , where we specify $d_{ij}^0 = 3 \cdot r_{ij}$, while the middle diagram corresponds to relatively small d_{ij}^0 , where $d_{ij}^0 = 2 \cdot r_{ij}$ is used. The comparative results suggest that decreasing desired distance d_{ij}^0 moderately will increase the egress flow rate at the bottleneck. This result explains why people tend to reduce their interpersonal distance at the entrance or exit because such behavior increases the egress flow rate and thus reduce egress time.

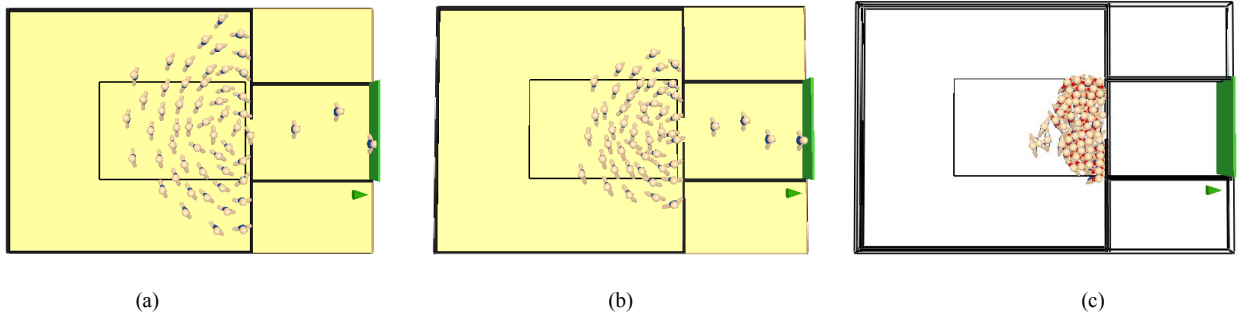


Figure 6. Social Force and Blocking Effect: (a) Use large d_{ij}^0 in normal situation such that people obey social norm of large interpersonal distance. The result is decrease of flow rate and less chance of physical interaction. (b) Use small d_{ij}^0 in emergency egress such that people follow the social norm of small interpersonal distance. Flow rate thus increases and physical interaction increase in a stochastic sense. (c) As d_{ij}^0 continues to decrease, the physical interaction causes someone to fall down, and the doorway is thus blocked by those falling-down people.

By introducing desired distance d_{ij}^0 we suggest that two types of stressors could transform as shown in Figure 4. The emergencies creates a kind of time-pressure which motivates one to move fast. Such time-related stress is transformed to space-related stress at bottlenecks. In current version of FDS+Evac such transformation happens to be modeled by tuning parameter A_i , where $A_i = 2000 \max(0.5, |v_i|/|v_i^0|)$ is a function of $|v_i|/|v_i^0|$. This method reduces the strength of social force if $|v_i| < |v_i^0|$. As a result, people will get closer to each other if $|v_i| < |v_i^0|$, indicating that the time-related stress is transformed into a space-related issue. Here the ratio of actual speed and desired speed partly indicates time-related stress, and it is analogous to the difference $v_i^0 - v_i$ in term of only speed, and the direction of velocity is ignored. In a similar manner we bring another formula by tuning d_{ij}^0 as below $d_{ij}^0 = d_{ij}^{0-\text{preset}} \max(0.2, |v_i|/|v_i^0|)$, where $d_{ij}^{0-\text{preset}}$ is the desired interpersonal distance in normal situation and 0.2 is the lower bound factor. As a result d_{ij}^0 is a function of $|v_i|/|v_i^0|$.

v_i^0 , and it decreases if $|v_i| < |v_i^0|$, implying that the time-related stress also impacts the desired distance d_{ij}^0 and the social force. As a result, the social force decreases at bottlenecks. However, the trade-off of reducing a_{ij}^0 at bottlenecks is increase of possibility of physical interaction among people, which is the major cause of crowd disasters such as stampede.

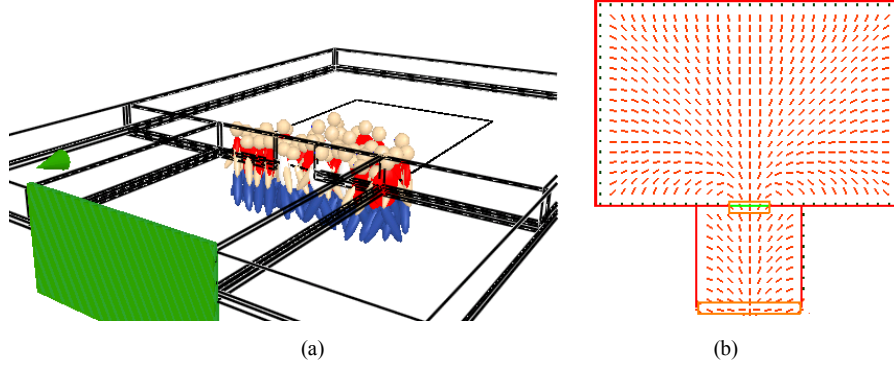


Figure 7. Crowd Escape at Bottleneck with Falling-Down Model: The white agents are falling down people who cannot move and are considered as obstacle to other agents. They fall down because the physical force exceeds a given threshold. The red agents are moving toward exit, and they have to get over the white ones to reach the door and the egress flow rate is thus significantly decreased.

In fact scaling down a_{ij}^0 at bottleneck implies a sort of competitive behavior among people. In other words reducing a_{ij}^0 motivates people to reduce their physical distance a_{ij} , and this trend leads to high-density crowd, where the physical force might become effective and people may have more physical interaction. If physical force is intensified, someone may fall down. The falling-down people further impedes surrounding ones and slow down the egress flow significantly, and they may cause others to fall down again and stampede becomes a possible result. In sum the social force model with a_{ij}^0 is useful to study crowd behavior when jointly used with a falling-down model. As below FDS+Evac is used to simulate a falling-down event where evacuees fall down when the physical force exceeds a threshold (See Figure 6(c) and Figure 7).

In the above simulation a_{ij}^0 is manually changed to compare different scenarios. In fact a_{ij}^0 can be better adapted to a flow field as shown in Figure 7(b), where a_{ij}^0 is in inverse ratio of flow density. This setup may require a compressible fluid model where flow density varies with locations (Wang, 2016). In a psychological sense such a flow field can be better understood as a kind of social field (Helbing et. al., 2005; Lewin, 1951), which is used to adapt interpersonal distance in emergency egress.

2. Herding Effect and Pre-Movement Behavior

The phenomenon of herding widely exists in nature, not only for human crowd, but also among many socialized animals (e.g., a herd of sheep, a flock of birds). Herd members benefits from joining large groups, many of which refers to keep safety from outside environment such as predators. In this section we will mainly discuss herding effect in emergency evacuation for human crowd, and an opinion dynamics is formulated as a linear model, and the model is applied in simulation of pre-movement behavior in crowd egress.

(a) Herding Effect

Herding is especially evident when people are responding to an emergency (Low, 2000). Emergency implies time-pressure as mentioned before, and excessive time-pressure weakens the ability of logical thinking and reasoning, and independent decision making is more difficult in stressful conditions. Thus, people are more inclined to follow others (e.g., surrounding others' decision) rather than make decisions by themselves, and such herding behavior help people to gain a sense of safety when facing uncertainty. This is evident in social psychological study. Such herding effect is a rooted nature in many species of social animal such as in sheep herd, bird flock or fish school, and it is thus a base instinct for many individuals to keep safety against the environment with uncertainty. Based on opinion dynamics and social-force model, the computational model of herding behavior is generally given as below (Deffuant et. al., 2000).

$$opinion_i(t+1) = (1-p_i)opinion_i(t) + p_i[others(t)]_{\rightarrow i} \quad [others(t)]_{\rightarrow i} = \sum_j c_{j \rightarrow i} opinion_j(t) \quad \sum_j c_{j \rightarrow i} = 1 \quad (4)$$

where $opinion_i(t)$ represents opinion of individual agent i at time point t , and it is a continuous real number or vector. Here $c_{j \rightarrow i}$ is non-zero if individual i is able to perceive or acquire opinion of individual j such that individual i is thus influenced by individual j . A simple example is assuming that $c_{j \rightarrow i}$ becomes non-zero if the physical distance is less than R_i , namely, $d_{ij} < R_i$. In other words individual i is able to know surrounding people's opinions within the range of R_i by talking or observing their behavior. A more useful method is using the value of $c_{j \rightarrow i}$ to represent the social relationship of individuals. Usually one's opinion is more affected by those who have close social relationship, and the value of $c_{j \rightarrow i} > 0$ is larger for such individuals, such as family members or friends. By normalizing parameter of $c_{j \rightarrow i}$ with $\sum_j c_{j \rightarrow i} = 1$, it is reasonable to define a matrix $C = [c_{ij}]_{n \times n}$ for quantitative measurement of such social relationship. Such a matrix C could be represented by a directed graph (DAG). Take three individuals for example (See Figure 8), where the directed arc from individual 1 to 2 means that individual 2's opinion is impacted by individual 1. As individuals move and interact in the agent-based simulation, the graph is dynamically updated and the social topology is time-varying.

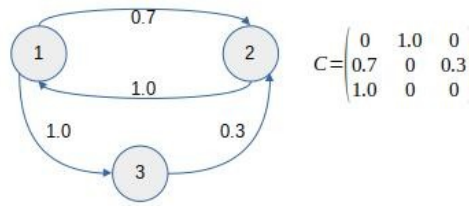


Figure 8. Social Topology of Individuals

An issue to be emphasized is on the diagonal element of $C = [c_{ij}]_{n \times n}$. We define $c_{i \rightarrow i} = 0$ if there exists any input arc to individual i in the social topology. If not, we will let $c_{i \rightarrow i} = 1$ to meet the requirement of $\sum_j c_{j \rightarrow i} = 1$. Thus, for practical computing, Equation (4) is directly applicable to an individual who is socially connected to others. As for any isolated individual who has no input connection from other individuals, it implies that $c_{j \rightarrow i} = 0$ for all possible j ($j \neq i$), and we will define $c_{i \rightarrow i} = 1$ on the self-loop instead such that $opinion_i(t)$ is not changing with time. This definition looks a little intricate, but it is useful and consistent when we discuss time-variant topology.

Now it is feasible to integrate parameter p_i in the matrix, describing how much the opinion of individual i is socially influenced by individual j , and such influence is determined by p_{ij} in matrix $P = [p_{ij}]_{n \times n}$. Based on Equation (4) it gives $p_{ij} = p_i \cdot c_{j \rightarrow i}$ if $i \neq j$. If $i = j$, we have $p_{ii} = (1-p_i) + p_i \cdot c_{i \rightarrow i}$, where $p_{ii} = (1-p_i)$ for any individual with input social connection from others, namely $c_{i \rightarrow i} = 0$ if there exists any input arc to individual i in the topology. As for an isolated individual, it implies $p_{ii} = (1-p_i) + p_i \cdot c_{i \rightarrow i} = 1$ because we define $c_{i \rightarrow i} = 1$ for any isolated individual as mentioned before.

The matrix $P = [p_{ij}]_{n \times n}$ summarizes how much individuals will keep his or her own opinion or follow others' opinion, and it enables us to formulate the opinion model as a linear dynamical system. Suppose the opinions of n individuals are vertically stacked into a column vector $OPIN(t) = (opinion_1(t), opinion_2(t), \dots, opinion_n(t))^T$, and Equation (4) could be compactly written by

$$OPIN(t+1) = F \cdot OPIN(t) \quad (5)$$

Here $OPIN(t)$ is the opinion profile of n individuals at time t , and their opinions are updated in an iterative way based on Equation (5). Matrix P is given by $F = (I - \Pi) + \Pi \cdot C$ and $\Pi = diag(p_1, p_2, \dots, p_n)$ is a $n \times n$ square matrix with its diagonal element defined by p_1, p_2, \dots, p_n , and I represents the identity matrix. In specific $\sum_j p_{ij} = 1$ means that $F = [p_{ij}]_{n \times n}$ is a stochastic matrix, i.e., a non-negative matrix with all its rows summing up to 1. If $F = [p_{ij}]_{n \times n}$ is time-invariant, the opinion dynamics of Equation (5) is a homogeneous Markovian process. The existing theory in linear algebra suggests that opinion $OPIN(t)$ converges if matrix F satisfies certain conditions. With the classical case of constant weights a typical phenomenon is consensus (French 1956, DeGroot 1974, Lehrer 1975), and consensus of all n individuals requires that $F = [p_{ij}]_{n \times n}$ is primitive, namely, the power of matrix F is strictly positive (i.e., does not contain any zero). A more complicated case is for time-variant $F(t)$, where $p_{ij}(t) \geq 0$ is dynamically changing as time proceeds. This situation implies that the connectivity of the graph is updated as agents move and interact, and the social topology among agents thus become time-variant.

There is also an interesting variation of this model developed by (Friedkin and Johnsen 1990, 1999). This model assumes that agent i will always keep some prejudice to a certain extent $1 - g_i$ and by a susceptibility of g_i the agent is socially connected with other agents.

$$opinion_i(t+1) = g_i \sum_j p_{ij} \cdot opinion_j(t) + (1-g_i) u_i(t) \text{ with } \sum_j p_{ij} = 1 \quad OPIN(t+1) = G \cdot P \cdot OPIN(t) + (I - G) u(t) \quad (5)$$

Here G is also a diagonal matrix, namely, $G = \text{diag}(g_1 \ g_2 \ \dots \ g_n)$ with $0 \leq g_i \leq 1$, and I is the identity matrix. The prejudice $u_i(t)$ is a continuous real or vector which is similar to $opinion_i(t)$, and $OPIN(t)$ and $u(t)$ are thus vectors in the same dimension. It is often assumed that $u_i(t)$ is given as a constant in the simulation process (Friedkin and Johnsen 1990, 1999). In a more general framework of linear system $u(t)$ is an input signal in a dynamical process, and it could be considered as a noise signal which is randomly generated. The resulting random signal $u(t)$ is could be interpreted as certain amount of irrational opinion in social interaction. As a result, the term of “panic” and irrational behavior are possibly modeled and simulated by integrating this noise signal $u(t)$ into opinion dynamics.

In brief the social relationship of n individuals is described by n -dimensional matrix C and matrix P . As listed in Table 1, we can replace such opinions by desired velocity or desired interpersonal distance, and an example is given as below by replacing opinions by desired velocity.

$$v_i^0(t+1) = (1-p_i)v_i^0(t) + p_i[v_{others}^0(t)]_{\rightarrow i} \quad [v_{others}^0(t)]_{\rightarrow i} = \sum_{R_i} c_{j \rightarrow i} v_j^0(t) \quad (6)$$

Based on Equation (6) the desired velocity v_i^0 is updated by mixing itself with the average velocities of others within radius R_i . Both options are weighted with some parameter $(1-p_i)$ and p_i , and two options follow two-point distribution with probability $(1-p_i)$ and p_i , and v_i^0 is updated by the statistical average. As a consequence, individualistic behavior is dominant if p_i is low whereas herding behavior dominates if p_i is high.

In principle parameter p_i indicates how an individual keeps balance between his or her own opinion and opinions of others, namely, whether an individual is stubborn or open-minded. Matrix C describes whether an individual has access to acquire opinion of others and how much social confidence one will count on them. In the following discussion we will assume that both matrix C and P are time-variant. However, an interesting issue is that matrix C and P cannot be mathematically expressed as an explicit function of time t , but an explicit function of states of n individuals, and in return they also determine their state in the next time step. Consequently, the opinion dynamics of n individuals becomes nonlinear in principle. As below we will briefly introduce some basic models which have been widely studied and used in existing research.

One model refers to how p_i is dynamically adjusted through the simulation process. For example, in Helbing, Farkas and Vicsek, 2000 and Helbing et al., 2002, p_i is considered to indicate one's panic level, which is given by ratio of $(v_i^0 - v_i)/v_i^0$, and is called a “nervousness” parameter. This ratio critically affects several testing results in their work. As mentioned before the gap of $(v_i^0 - v_i)$ is understood as an indicator of one's stress level (See Table 3.1), and it is further normalized by dividing $(v_i^0 - v_i)$ by v_i^0 . As a result, the “nervousness” parameter $(v_i^0 - v_i)/v_i^0$ can be explained as a normalized stress indicator, and it shows that people are more inclined to follow others when they feel more stressed in an emergency situation. However, it not suitable to directly use $(v_i^0 - v_i)/v_i^0$ to calculate desired velocity based on Equation (6) because $(1-p_i)v_i^0$ is simply v_i if we assume $p_i = (v_i^0 - v_i)/v_i^0$. In fact the stress perceived for us is not instantaneous, but accumulating with a period of time, and it is necessary to model such accumulating stress in timeline. Thus, a general formula to update parameter p_i is given by a function such as $p_i(t) = h(t, v_i^0 - v_i, a_{ij}^0 - a_{ij})$. As a result, parameter p_i is an indicator of one's stress level (See Table 1). In sum how to compute parameter p_i in a dynamical process is an interesting study topic and we will further discuss this issue in future.

Another model discusses in what conditions one's opinion will be influenced by another. In others words, Equation (4) only describes a class of opinion model, but it does not explicitly tell when it will be effective as individuals move in two-dimensional space. Here we may assume that this effect takes place only if the physical distance is less than R_i , namely, $a_{ij} < R_i$. This is a reasonable and relatively straightforward assumption, and it is mainly obtained by observing a group of social animals (e.g., pigeons) in collective motion (Vicsek et. al., 1995), and such group members are relatively close to each other. Based on existing model in opinion dynamics, there are some models that inspire us to improve this assumption for human crowd. For example, an existing theory suggests that interactions bring opinion closer to each other if they are already close sufficiently, and thus one's opinion is inclined to selectively follow similar opinions of others, and this algorithm will generate several small groups rather than one large group of consensus. Moreover, the social relationship of individuals is also of significant importance, and the interaction range is not only determined by physical distance a_{ij} , but also refer to social relationship of individuals, which is partly measured by the desired distance a_{ij}^0 . As a result, one tends to follow those in close social relationship, but not only those in close positions. In existing literature various criteria are discussed to extend the basic rule of $a_{ij} < R_i$, and the resulting system can be much more complex than one would intuitively expect. We will further discuss this important issue in the following section of crowd group dynamics.

(b) Pre-Movement Behavior

In brief the evacuation behavior in building egress is commonly divided into two phase: the pre-movement phase and movement phase. In the pre-movement phase evacuees try to interpret the alarm signal received. They normally collect and exchange information, searching for new cues to confirm what really happens instead of directly moving to an exit or safe place. Existing egress research suggests that pre-movement behavior critically affects the total egress time in building evacuation, and it further consists of recognition and response to environmental stressors as shown in Figure 9. The recognition time refers to the time interval from the first cues detected by an evacuee (i.e. fire alarm, warnings, etc.) to the time of awareness that one need to respond to the new situation. During this time interval, evacuees normally seek further information to interpret the perceived cues and try to find out what exactly happens. The response time is the time interval from awareness of potential threats to the time to start leaving for a safety place. Different actions can be performed during this interval such as calling for firefighters, collecting personal belongings or socially connecting with others. According to fight-and-flight response (Cannon, 1932; Adam and Gaudou, 2017), leaving is one option, but seldom the first option.

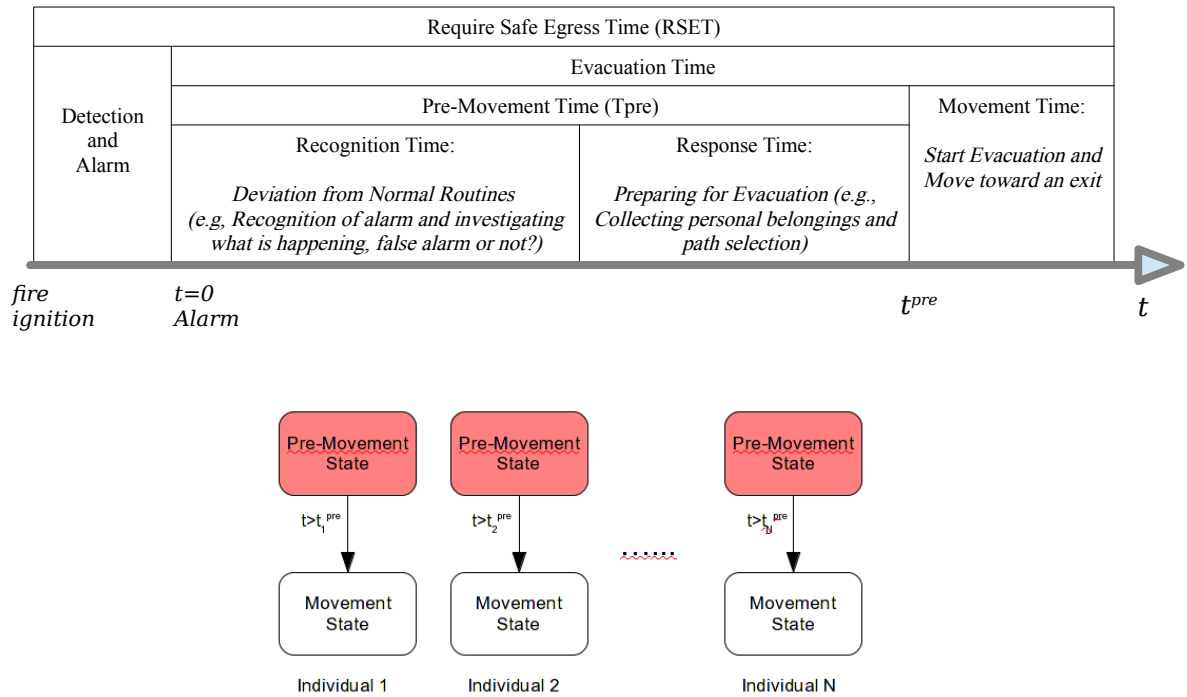


Figure 9. State-Transition Model from Pre-movement Phase to Movement Phase: Such state transition takes place if simulation time t exceeds the an individual's pre-movement time t^{pre}_i . Each individual's t^{pre}_i is different, and social interaction could bring t^{pre}_i to converge in certain conditions.

Very importantly, if many individuals are involved in evacuation, leaving for safety is rarely an individualistic action, but a group decision in a collective sense. In other words, people are inclined to be socially tied with others to respond to the situation as a group, and such social attachment behavior helps to relieve stress, providing group members with a sense of safety. However, the trade-off is that it often takes certain time for people to form a social group, and opinions of the group members may vary to some extent such that it also takes time for them to reach an agreement. Here the group size and social relationship of the group members are among the key factors to determine the recognition and response time (Sorensen, 1991), and things often become better if there is an effective leader in the group, for example in occasions of office buildings or schools. The reason is that there is commonly no responsibility diffused or inhibited in these place, and it becomes easier to form the leader-and-follower pattern in collective behavior.

In the past 30 years various behavioral models were proposed in the circles of psychological and behavior science (Sime, 2001; Kuligowski, 2009). These models are usually simplified in the engineering field for practical use in computation, resulting in state-transition model (Lovreglio, Ronchi and Nilsson, 2015; Adam and Gaudou, 2017) or sequential flow to describe human response to environmental stimuli (e.g. fire alarm). When evacuees confirm the potential risk in the environment, their states transit from the pre-movement phase to the movement phase and they start to move to an exit for safety.

In this article we will integrate a simple version of state transition model with opinion dynamics to calculate the pre-movement time of many individuals, and this model mainly describes social interaction in this phase, facilitating to investigate how collective pre-movement time emerges from individual interactions in a given social topology. The model critically captures the social structure of many individuals including the group size and the possible leader-and-follower effect, and the computational method has been partly tested in our simulation platform crowdEgress (Wang et al., 2023), not yet in FDS+Evac. In the following discussion we assume that all the individual evacuees receive fire alarm at $t=0$, and simulation also starts at $t=0$.

Suppose the pre-movement time for individual i is denoted by $tpre_i > 0$ as a continuous real number and the opinion model in Equation (4) is applied. The pre-movement time for crowd is computed by interactive opinion dynamics, and is given by Equation (7).

$$tpre_i(t+1) = (1-p_i)tpre_i(t) + p_i tpre_{others}(t) \quad tpre_{others}(t) = \sum_{R_i} c_{j \rightarrow i} tpre_j(t) \quad \sum_j c_{j \rightarrow i} = 1 \quad (7)$$

The pre-movement time of an individual $tpre_i$ is updated by mixing itself with a weighted average of others in certain social relationship. Both options are weighted with some parameter $(1-p_i)$ and p_i , and $tpre_i$ is updated by the statistical average. Here $c_{j \rightarrow i}$ is non-zero if individual i has access to acquire individual j 's pre-movement time $tpre_j$. As mentioned before, matrix $C=[c_{ij}]_{n \times n}$ defines how individuals are socially related to each other and whether they are able to perceive others' opinion in collective motion. The matrix F is deduced by using Equation (5), with its elements $p_{ij} = p_i c_{j \rightarrow i}$ if i is not equal to j . If $i=j$, we have $p_{ii} = (1-p_i) + p_i \cdot c_{ii}$ such that $p_{ii} = (1-p_i)$ for an individual with input social arc from others and $p_{ii} = (1-p_i) + p_i c_{ii} = 1$ for an isolated individual. Obviously, $F=[p_{ij}]_{n \times n}$ is also a stochastic matrix, i.e., a non-negative matrix with all its rows summing up to 1.

In practical computing p_i is updated relatively slower than $c_{j \rightarrow i}$ because p_i reflects one's tendency to adhering to his or her own opinion or following others, and it is an inherent characteristic in one's personality. In contrast $c_{j \rightarrow i}$ is more timely updated for it describes whether individual i is able to acquire opinion of individual j , or in other words, whether individual j 's opinion has impact to individual i . For practical computing, it is feasible to further decompose $c_{j \rightarrow i}$ as two elements, namely $b_{j \rightarrow i}$ and $s_{j \rightarrow i}$.

$$c_{j \rightarrow i} = \frac{b_{j \rightarrow i} s_{j \rightarrow i}}{\sum_k b_{k \rightarrow i} s_{k \rightarrow i}} \quad b_{j \rightarrow i} = 1 \text{ if individual } i \text{ has access to acquire individual } j \text{'s opinion} \quad (8)$$

In Equation (8) $b_{j \rightarrow i}$ is a kind of Boolean variables, timely updated based on people's locations and behavior, describing if people are able to exchange opinions (e.g., observing others' choice or talking with others to exchange opinions). In our simulation program crowdEgress (Wang et al., 2023) there are several lists attached to each individual, including see list and attention list, which describe whether individuals could see each other or whether an individual pays attention to others. Such lists facilitates timely update of the value of $b_{j \rightarrow i}$. Based on $b_{j \rightarrow i}$ we have a time-varying a directed graph (DAG). As below we illustrate such a time-varying graph as individual agents interact in social context, and this algorithm is being tested in our simulation platform crowdEgress (Wang et al., 2023).

In contrast $s_{j \rightarrow i}$ defines a quantitative measure of social relationship, and it is a real number which weighs how much one's opinion is possibly impacted by others. Such social relationship is relatively stable, and it is not easily changed as the same as parameter p_i . As for opinion model of pre-movement time, it may also be feasible to extend the effective range of $s_{j \rightarrow i}$ such as $s_{j \rightarrow i} \in [-1, 1]$. As a result, $s_{j \rightarrow i}$ is not a probability measure, but generally a weight parameter which causes the individuals' opinions to either converge or diverge. $0 < s_{j \rightarrow i} < 1$ means that individual i intends to stay in somewhere between his or her own opinion and opinion of individual j , and this moderate strategy often brings opinions to converge in a sense. In contrast, $-1 < s_{j \rightarrow i} < 0$ means that one is against others' opinion. In other words, the negative value of $s_{j \rightarrow i}$ implies that others' standpoint has an inverse impact on one's opinion. As a result, the more individual i acquires opinion of individual j , the more he or she will reject it and hold more firmly on his or her own standpoint. This strategy brings the individuals' opinions not to converge, but diverge to some extent. In sum, $s_{j \rightarrow i} \in [-1, 1]$ implies a more complex social connection among individuals, where interactions bring opinions either closer to each other, or more apart from each other. This model is complex in theoretical analysis, but it is more comparable to real-world situations because people do not always agree with each other. Disagreement exists in many scenarios. For example, it has been reported in many forest fires that the younger generation would like to follow the public instructions and leave their house for safety, but the elderly in the family insist to stay at home (Adam and Gaudou, 2017).

Table 2. Range of Parameter p in Opinion Dynamics

$-1 < s_{j \rightarrow i} < 0$	$s_{j \rightarrow i} = 0$	$0 < s_{j \rightarrow i} < 1$
Against others' opinions	Hold his own opinion and do not care about others' opinions	Support others' opinions

Now consider the negative value of $s_{j \rightarrow i}$, Equation (8) should be modified as below such that $c_{j \rightarrow i} \in [-1, 1]$, and it gives $C^* = [c_{ij}]_{n \times n}$ as a stochastic matrix, but $C = [c_{ij}]_{n \times n}$ is not. The random gossip algorithm could be applied where the probability of interacting agents is given by $C^* = [c_{ij}]_{n \times n}$.

$$c_{j \rightarrow i} = \text{sign}(s_{j \rightarrow i}) \frac{b_{j \rightarrow i} |s_{j \rightarrow i}|}{\sum_k b_{k \rightarrow i} |s_{k \rightarrow i}|} \quad b_{j \rightarrow i} = 1 \text{ if individual } i \text{ has access to acquire individual } j \text{'s opinion} \quad (9)$$

A list of $s_{j \rightarrow i}$ is given as an example in Table 3, where its values are normalized within the range of $[0, 1]$. In this table we can generally identify a group which consists of individual 0, 1, 2. In this group individual 0 is completely a follower to individual 1 with a weight of 1.0, and individual 1 also cares about individual 0 with a weight of 0.2. Such valuation represents a kind of leader-and-follower or child-and-parents relationship in social topology. Individual 1 and 2 care about each other significantly (with weight of 0.5 and 0.71), and both of them are also socially bonded with individual 3 (with weight 0.3 and 0.29).

In contrast individual 5, 6, 7 are considered as another social group. In particular individual 5 and 6 compose a stable pair because they care about each other mutually (e.g., couples). Individual 7 also follow individual 5 exclusively, but individual 5 does not care individual 7, and thus they are in leader-and-follower pattern.

Individual 3 and 4 connect the above two groups. However, such connection are directional in a sense that individual 3 and 4 are widely impacted by individual 5, 6, 7 (second group) and individual 0, 1, 2 (first group) are moderately impacted by them.

In this social topology individual 5 and 6 seems to have leadership, who directly impact individual 3, 4, 6 and 7 and also relies on individual 3 and 4 to indirectly affect individual 0, 1, 2. In additional individual 3 is the critical bridge who connects these two groups. If individual 3 moves out of this crowd, the two groups become isolated completely.

Table 3. Elements C_{ij} in Social Relationship Matrix C

&groupC	Person0	Person1	Person2	Person3	Person4	Person5	Person6	Person7
Person0	0	1.0	0	0	0	0	0	0
Person1	0.2	0	0.5	0.3	0	0	0	0
Person2	0	0.71	0	0.29	0	0	0	0
Person3	0	0	0.3	0	0	0.3	0.2	0.2
Person4	0	0	0	0.2	0	0.3	0	0.5
Person5	0	0	0	0	0	0	1.0	0
Person6	0	0	0	0	0	1.0	0	0
Person7	0	0	0	0	0	1.0	0	0

Given the above social relationship weights, and the above eight individuals are simulated in a multi-compartment layout to determine their pre-movement time (See Figure 10). There are three other factors of significant importance:

(1) Parameter p_i is also critical to determine how an individual keeps balance between his or her own opinion and others' opinions in the given social topology. The value of p_i for these individuals is given as 0.6, 0.6, 0.3, 0.3, 0.4, 0.36, 0.63, 0.66. Small value of p_i implies that the individual is self-centered such that he or she would not like to hear or learn from others. In contract large value of p_i indicates that the individual has an open mind and tends to hear or follow others' suggestions. This feature probably refers to one's personality, and it is feasible to infer p_i from existing psychological research.

(2) Initial positions of individuals are illustrated in Figure 10(a). The initial positions partly affects how individual move with others in the simulation process. As mentioned previously, opinion exchange as described by Equation (4) only becomes effective within the range of R_i , namely, $a_{ij} < R_i$. Thus, such physical positions really matter in the context of social interaction.

(3) Initial pre-movement time t_{pre} , is given by 3, 2, 10, 6, 22, 6, 16, 23 (seconds) sequentially. In existing research it is often assumed that pre-movement time follow certain probability distributions, such as Lognormal, Loglogistic or Weibull distribution (Forssberg et. al., 2019). Such statistical study is useful to explore whether or to what extent the pre-movement time is related to other factors such as the crowd size, or the location of evacuation (e.g., shopping mall or in office buildings). Thus, it may provides a guideline to initialize the pre-movement time in our simulation. However, the major difficulty exists in acquiring reliable data in real-world events, and how to initialize t_{pre} for each individual is not an easy task.

In Figure 10(b) each colorful line represents an individual's t_{pre} , and the value of t_{pre} dynamically changes as people interact with each other. When the colorful line drops to x axis, it means that the individual reaches an exit and thus is removed from computational loop. The figure is plotted in our simulation platform, CrowdEgress. In particular there is a gray slash line which has angle of $\pi/4$ to the x axis, and this gray line divides the x-y plane equally into two regions. The upper left region indicates the pre-movement phase because t_{pre} is larger than simulation time ($t_{pre} > t$). The lower right region represents the movement phase since the simulation time t exceeds t_{pre} , and thus individuals start to move to exits.

In this article we are more interested in the upper left region, where we could identify how social interaction happens for such eight individuals. The pre-movement time converges as illustrated in Figure 10(b). Because individual 5 and 6 exclusively care about each other and do not follow any others, they compose a group and their pre-movement time converge to 14 seconds in Figure 10(b). In particular individual 2 (red line) sways the opinion between individual 0 and individual 3. He or she initially communicates with individual 1, and then with individual 3, and then back with individual 1 and 0, and again with individual 3. Finally, his or her opinion converges with individual 3, and the pre-movement time converges around 9 seconds with individual 3.

In this simulation result individual 4 directly converge with individual 7 with pre-movement time of 23 seconds. One important issue to be emphasized is that the social groups are not static features, but changing dynamically in the simulation process. Although the eight individuals are socially bonded with the value of $s_{j \rightarrow i}$ as listed in Table 3, other factors also matters. As a result, although individual 3 and 4 play the roles of connection between two groups, it is quite reasonable for him or her join one group. In the above simulation result, individual 3 join the first group and individual 4 join the other one only with individual 7.

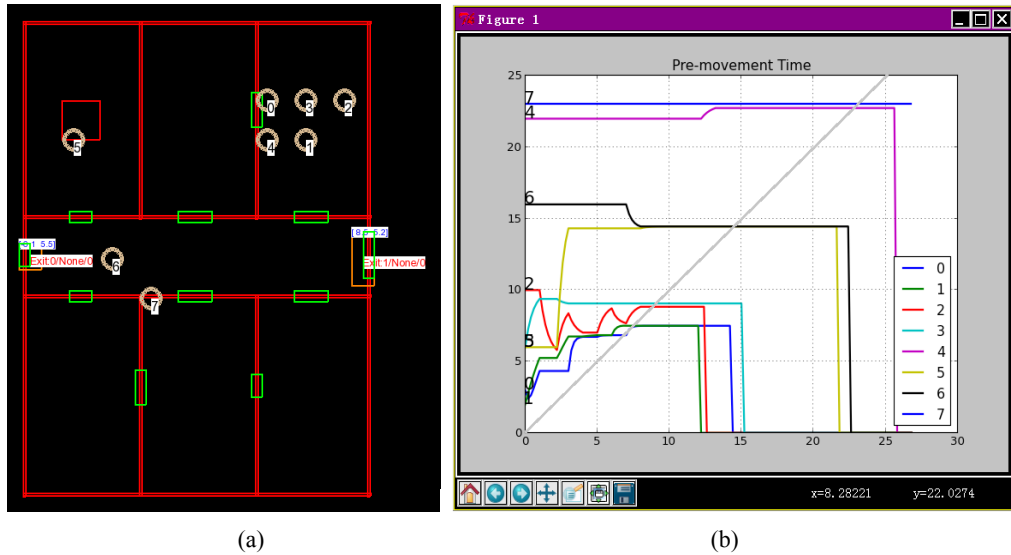


Figure 10. Opinion Dynamics and Group Formation in Pre-Movement Phase (CrowdEgress)

In a sense the matrix $\mathbf{P}=[p_{ij}]_{n \times n}$ induced from $c_{j \rightarrow i}$, $b_{j \rightarrow i}$ and $s_{j \rightarrow i}$ provides a whole picture to understand how individuals make decisions. In a mathematical sense $\mathbf{P}=[p_{ij}]_{n \times n}$ defines a time-varying linear structure to interpret how individuals' opinions evolve in social interactions, and such a linear system dynamics is formulated by $OPIN(t+1)=F \cdot OPIN(t)$. Matrix $\mathbf{P}=[p_{ij}]_{n \times n}$ is timely updated as individuals change their physical positions. Thus, in above example individual 4 can also join the second group, or individual 3 may also join the first group. Such simulation results are both possible and reasonable.

3. Social Group Behavior

People create group-level behavior beyond the ken of any single person, and in the past 20 years there has been growing realization in social science that such group-level organizations sometimes emerge spontaneously without any central design. Thus, it is reasonable to study such group phenomena in a bottom-up rather than a top-down manner. This approach creates computational units of individuals and their interactions, and to observe how the global structures are formed dynamically with their interactions. In evacuation process such group-level behavior widely exist in both pre-movement phase and movement-phase, and it significantly affects how people respond to environmental stressors such as fire alarms. For example, in emergencies people usually first seek for familiar ones (e.g., friends or parents) to exchange information and then respond in a collective sense. Thus, it is meaningful to develop a social group model to characterize how an evacuee is affiliated with other familiar and trust ones, especially based on their social relationship. In a psychological sense, social group behavior agrees with the flight-or-affiliation effect (Bañgate et al., 2017), which is different from the fight-or-flight response (Cannon, 1932) as presented in Section 2. In this section we will elaborate this issue and introduce a force-based method to describe such social groups.

(a) Group Social Force

In a group individuals exhibit some degree of social cohesion based on their relationship and they are more than a simple collection or aggregate of individuals. To model group dynamics, attraction is necessarily taken into account in interaction of individuals. For example, attraction will make acquainted people to join together and possibly form a group. In Helbing and Molnar, 1995 and Helbing et al., 2002 attraction was considered, but separate from the social force. In this paper attraction and repulsion are put in the same social context: repulsion makes people to keep proper distance while attraction makes them cohesive and form social groups. Thus, this subsection integrates attraction into the social force based on the concept of desired interpersonal distance. The resulting force is either repulsive or attractive, and it is especially useful to model group behavior in pedestrian crowd. The group social force is defined as below.

$$\mathbf{f}_{ij}^{soc} = \frac{A_{ij}}{B_{ij}} (d_{ij}^0 - d_{ij}) \exp\left[\frac{(d_{ij}^0 - d_{ij})}{B_{ij}}\right] \mathbf{n}_{ij} \quad \text{or} \quad \mathbf{f}_{ij}^{soc} = \left(\lambda_i + (1 - \lambda_i) \frac{1 + \cos \varphi_{ij}}{2}\right) \frac{A_{ij}}{B_{ij}} (d_{ij}^0 - d_{ij}) \exp\left[\frac{(d_{ij}^0 - d_{ij})}{B_{ij}}\right] \mathbf{n}_{ij} \quad (9)$$

Here A_{ij} and B_{ij} are similar parameters as introduced before, and \mathbf{n}_{ij} is the normalized vector which points from individual j to i . The group social force also functions in a feedback manner to make the realistic distance d_{ij} approaching towards the desired distance d_{ij}^0 . A difference is that \mathbf{v}_i and \mathbf{v}_j are vectors while d_{ij}^0 and d_{ij} are scalars.

When d_{ij} is sufficiently large, the group social force tends to be zero so that individual i and j have almost no interaction. This trend is the same as the traditional social force (Helbing and Molnar, 1995 and Helbing et al., 2002). If d_{ij} is comparable to d_{ij}^0 , interaction of individual i and j comes into existence. If $d_{ij}^0 < d_{ij}$, the group social force is attraction whereas it is repulsion if $d_{ij}^0 > d_{ij}$. The attraction reaches the extreme value when $d_{ij} = d_{ij}^0 + B_{ij}$, and the extreme value is $\mathbf{f}_{ij}^{soc} = -A_{ij} \exp(-1)$. The desired distance d_{ij}^0 makes the curve move horizontally with a certain interval, and it is the equilibrium position when an individual interacts with another one in pair. The curve shape is affected by parameter A_{ij} and B_{ij} . A_{ij} is a linear scaling factor which affects the strength of the force whereas B_{ij} determines the effective range of the interaction.

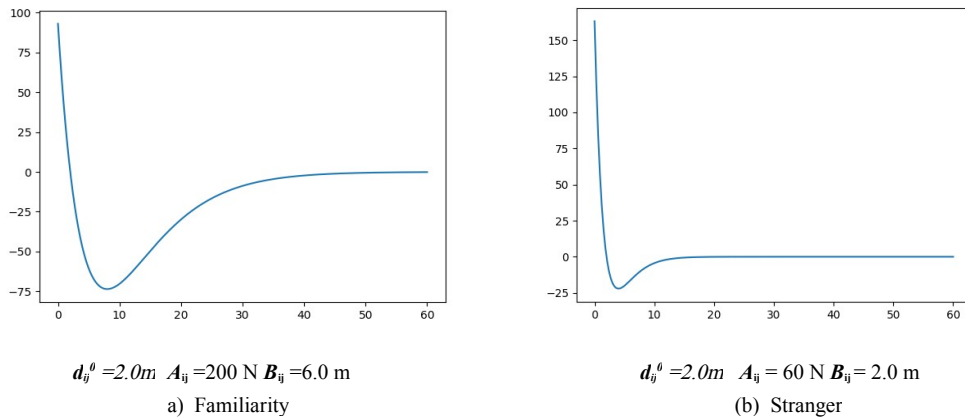


Figure 11. Extended social force from individual j to individual i (non-anisotropic formula): (a) To characterize two individuals who know each other, the force includes a negative segment representing attraction as well as a positive segment representing repulsion; (b) When two individuals are strangers, attraction significantly decreases in both strength and the effective range.

Two plots of Equation (9) are given as above: Figure 11(a) shows that individual i is attracted by individual j when they are sufficiently close, and this suggests that individual i and j are probably familiar with each other. Figure 11(b) does not show such relationship because their interaction range and magnitude both reduce remarkably.

In the above curve the negative segment represents attraction (See Equation 9), and it represents a kind of social cohesion which facilitates to form groups. In contrast the positive segment denotes repulsion and it functions like the traditional social force.

Moreover, the traditional formula of social force is compared with the group social force given by Equation (9), and it is noticed that the desired distance a_{ij}^0 is usually larger than r_{ij} , and parameter A_{ij} and B_{ij} are also in different values. In general, the traditional social force is usually considered as short-range interaction, and it plays a role of collision avoidance because it is calculated by using the physical size of individual agents (i.e., r_{ij}). In other words, the social force is effective only when people are very close to each other ($A_i=2000$; $B_i=0.8$), and it is fit to model high-density crowd. As for the new formula, it is relatively a long-range interaction where the desired distance a_{ij}^0 is commonly larger than r_{ij} , and parameter B_{ij} of group social force is often larger than B_i in the traditional formula. In our numerical testing, it is found that B_{ij} is usually in the range of $10^1 \sim 10^{-1}$ while A_{ij} is commonly in the range of $10^2 \sim 10^0$. This issue will be further discussed in detail in numerical testing results.

Equally importantly, the gap between a_{ij}^0 and a_{ij} is expressed in Equation (9), and the interpersonal stress is characterized in consistency with our previous discussion. The gap of a_{ij}^0 and a_{ij} is either negative or positive, meaning that being too far away or too close to someone result in stress in proximity. Keeping proper distance with others is the way to protect us from too much arousal, and this is evident in psychological study because being isolated or overcrowded can both lead to stressful conditions.

In order to well apply the group social force in pedestrian modeling, there is another important issue referring to oscillation phenomenon when an individual pedestrian get close to the equilibrium position a_{ij}^0 when interacting with another one. A common method is to integrate the relative velocity $v_{ji}=v_j-v_i$ as a force component in Equation (9). This term is an estimate of the future trend of motion based on its current rate of change. It is sometimes called "anticipatory control," which characterizes a pedestrian is able to anticipate the changing situation for a short time, and thus adjust the current state of motion to avoid future collision or oscillation. This force component contributes to reducing oscillation when two agents get closely enough, and it has been applied in several other pedestrian models (Stenffen, 2008, Gao et. al., 2013). Thus, by tuning a force component that is a function of relative velocity v_{ji} , the oscillation phenomenon will be significantly mitigated.

As above we briefly introduce $v_{ji}=v_j-v_i$ to offset oscillation. In practical computing even if the anticipatory force of v_{ji} is not used, we seldom observe oscillation. In fact the method of using relative velocity v_{ji} to adjust acceleration origins from vehicle traffic problems, where a vehicle adjusts the moving velocity based on the relative velocity to the front vehicle (Treiber, Hennecke, and Helbing, 2000). A major assumption is that vehicles are moving sequentially in lanes. However, human pedestrians are not often required to keep lanes in many occasions. In open areas such as in shopping malls or airports, pedestrians usually do not need to pay much attention to others' velocity unless they are very close to each other. Even if they are moving close, it is easy for a pedestrian to adjust moving direction in 2D space to avoid collision. This is a fundamental difference between pedestrian movement in 2D space and vehicle movement in 1D space. Thus, although the force term of v_{ji} is often used in pedestrian simulation, it is not theoretically justified to borrow this force term from vehicle traffic to pedestrian traffic.

$$d_{ij}^0(t+1)=(1-p_i)d_{ij}^0(t)+p_i d_{ji}^0(t) \quad d_{ji}^0(t+1)=(1-p_j)d_{ji}^0(t)+p_j d_{ij}^0(t) \quad (10)$$

In real-world scenario people usually have a target interpersonal distance and get there without oscillation in physical positions. Thus, an important question is whether we should assume v^0 and d^0 to be constant in the above analysis. As mentioned before, v^0 and d^0 are not physical entities, but reflect pedestrians' opinions. If v^0 and d^0 are not constant, pedestrians may adapt v^0 and d^0 dynamically, and this feature may provide another method to avoid oscillation. Now we consider the group social force as given in Equation (9), and a typical example is that a_{ij}^0 and a_{ji}^0 interact such that two individuals become a stable pair in physical positions when interacting. In particular based on Equation (4) we write the dynamics of a_{ij}^0 by Equation (10). Here it is assumed that $p_i = p_{ij}$ and $c_{j \rightarrow i} = 1$ when individual i is only talking to individual j . In Figure 12 we illustrate how a_{ij}^0 and a_{ji}^0 converge to the same value for two individuals.

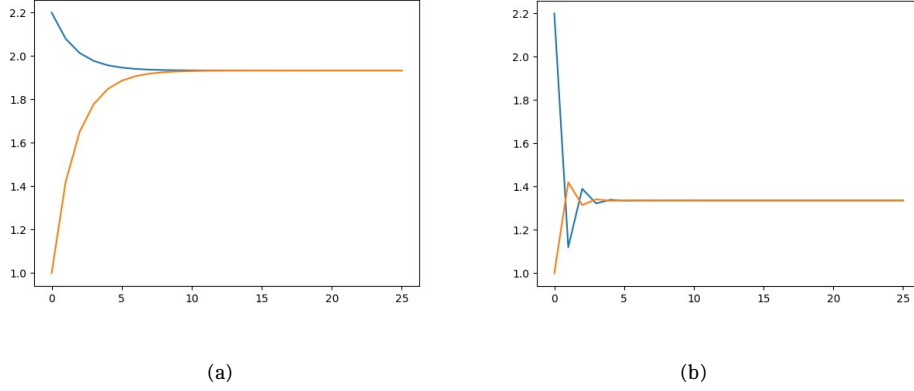


Figure 12. Interaction of d_{ij}^0 and d_{ji}^0 by using opinion dynamics: d_{ij}^0 and d_{ji}^0 converge to a common value. The blue line indicates dynamics of individual i while the red line is for individual j . Initially $d_{ij}^0 = 2.2$ while $d_{ji}^0 = 1.0$. In plot (a) $p_i = 0.1$ and $p_j = 0.35$ while in plot (b) $p_i = 0.9$ and $p_j = 0.35$.

(b) Grouping Dynamics

Based on the group social force by Equation (9), we are able to define a kind of matrix-based structure to describe social group behavior in crowd movement. Considering a group composed of n individuals, the social relationship of the group members is described by a $n \times n$ matrix D^0 , of which the element is defined by a_{ij}^0 in Equation (9). In a similar manner we define $n \times n$ matrices A and B , of which the elements are A_{ij} and B_{ij} as specified in Equation (9), respectively. Generally speaking, D^0 , A and B are asymmetrical, and the matrix-based structure is given by Equation (11).

$$\mathbf{A} = [A_{ij}]_{n \times n} \quad \mathbf{B} = [B_{ij}]_{n \times n} \quad \mathbf{D}^0 = [d_{ij}^0]_{n \times n} \quad (11)$$

The group social force among n individuals is described by the matrices D^0 , A and B , and the method has been tested in FDS+Evac as well as crowdEgress, another egress simulation platform (Wang et. al., 2023). To calculate the group social force exerted on individual i by individual j , we pick up the element a_{ij}^0 , A_{ij} and B_{ij} in the corresponding $n \times n$ matrix D^0 , A and B , and apply mathematical formula of Equation (9) to compute f_{ij}^{soc} . Thus, the social structure of n individuals is characterized by matrices D^0 , A and B .

A testing result of FDS+Evac is illustrated in Figure 13, where the multi-compartment layout is given based on an example in Pan et. al., 2007. Several groups are well observed in the simulation result. Some small groups merge into a large group and regrouping occurs at intersections or bottlenecks. In general, the group concept in the matrix-based structure is not static, and we do not need to define the group members in a static approach. Rather, we only define the social topology of n individuals, and their interaction forces will determine how such n individuals are self-organized into groups. Furthermore, the elements in matrices D^0 , A and B could be time-varying as exemplified by Equation (10), and the group could be restructured dynamically through the simulation process, and thus it inherently becomes a time-varying feature.

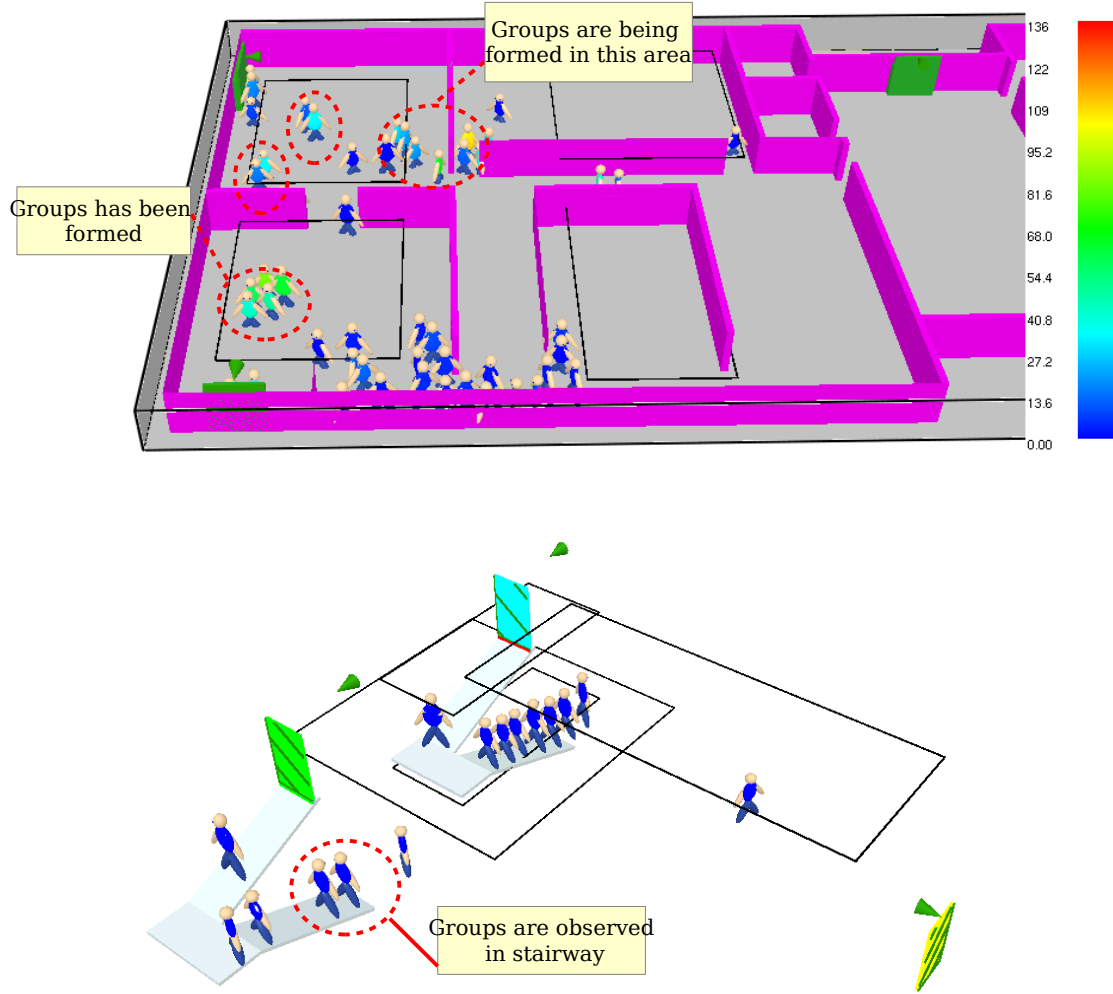


Figure 13. Simulation of Group Dynamics in FDS+Evac: The color bar indicates the intensity of the group social force given by Equation (10). An individual in active motion usually moves in the front of a group. Individuals in passive motion are followers in the group and they usually move behind the leaders.

Based on Equation (10), a typical pattern of crowd movement is described as the leader-and-follower group. In this group pattern there is a kind of individuals whose behavior is mainly motivated by himself or herself. If others would like to follow them, they become leaders in group behavior. Thus, if individual i is the leader in a group, his or her motion is primarily motivated by self-driving force. In contrast followers are those whose behavior is mainly motivated by others, and their motions are thus mainly governed by group social force, and the self-driving force is secondary. As mentioned before imbalance (asymmetry) of a_{ji}^0 and a_{ij}^0 will contribute to model leadership in crowd behavior. In a similar manner, if i is a leader, B_{ji} is usually larger than B_{ij}^0 such that the leader will attract his surrounding people, but not easily be attracted by them. In brief an individual's motion can be classified into two types. One type of motion is primarily motivated by the self-driving force, and is called active motion. The other type of motion is largely motivated by surrounding other individuals, which is called passive motion. Usually an individual's motion is a combination of both types, but we can differentiate such two types in simulation results, and identify whether an individual's motion is dominated by either active or passive type.

As shown in Figure 13 the color bar is used to observe the magnitude of the group social force in simulation (Korhonen 2017; Forney, 2018; McGrattan et al., 2018). An individual in active motion often moves in the front of a group. Individuals in passive motion are followers in the group and they usually move behind the leader. The leader is commonly under smaller group social force than the followers.

(c) Social Groups and Opinion Dynamics

Social groups and opinion dynamics are related but different concepts. The group social force describes the social relationship of individuals and combine them together in physical positions. When people are sufficiently close to each other, herding effect and opinion dynamics becomes effective such that their opinion interact to form a common motive or move towards a common destination. This is due to the criterion of $a_{ij} < R_i$ as introduced above, namely individual i will be able to know surrounding people's opinions within the range of R_i by talking or observing their behavior (See Equation (8)). Furthermore, one's opinion is more affected by those who have close social relationship, such as family members or friends, and thus the interaction range is not only determined by physical distance a_{ij} , but may also refer to the social distance a_{ij}^0 , and social typology of individuals becomes an important factor. For example we may assume that opinion of individual i is impacted by individual j if $a_{ij} < a_{ij}^0 + 2B_i$.

As social groups are jointly modeled with opinion dynamics, a kind of convergent pattern is supposed to emerge in a crowd. A common example is evacuation of a stadium where people follow the crowd flow to move to an exit. There are a multitude of small groups composed of friends or family members, and they keep together in egress because of their social relationship. These small groups compose a large group of evacuees, and herding behavior widely exists among these groups, contributing to form a common motive of motion. In sum, the group social force makes individuals socially bonded with each other, and it primarily characterizes a kind of social relationship of individuals. Herding effect emphasizes how much an individual tends to follow others' opinion or behavior, and the effect is also related to their social relationship, and it is common to assume that an individual is more inclined to follow those who are in closer relationship. Because both social groups and herding effect considers social relationship and such two features are inter-related, and it is important to define them in consistency. In other words, the matrices D^0 , A and B should be defined in consistency with matrices F and C . In practical computing, it is important to first define matrix C and F as a quantitative measure of social relationship, and then specify D^0 , A and B in consistency.

Moreover, the group social force as presented above is also useful to model crowd behavior in pre-movement phase in crowd evacuation (Sorensen, 1991, Kuligowski, 2009). In brief, when the alarm or hazard is detected, people usually do not head to exits immediately, but go to find trust ones (e.g., family members or friends) for information exchange and collective decision. Such social group behavior delays their movement to exits (Lovreglio and Kuligowski, 2022). Thus, the above model of social groups contributes to modeling the crowd behavior in pre-movement phase and will be useful to investigate how the initial delay is formed and influenced by the group dynamics. In particular Equation (10) can be better explained by flight-or-affiliation effect in pre-movement phase of evacuation. The self-driving force motivates one to flee while the group social force makes one affiliated with others. This effect agrees with social attachment theory in psychological study (Mawson, 2007; Bañgate et al., 2017). The social attachment theory suggests that people usually seek for familiar ones (e.g., friends or parents) to relieve stress in an emergency situation, and this is rooted from our instinctive response to danger in childhood when a child seek for the parents for shelter. However, such social attachment could take some time and it might delay the movement towards exits. Thus, different from the fight-or-flight response (Cannon, 1932), the group social force agrees with the flight-or-affiliation effect, especially in the pre-movement phase.

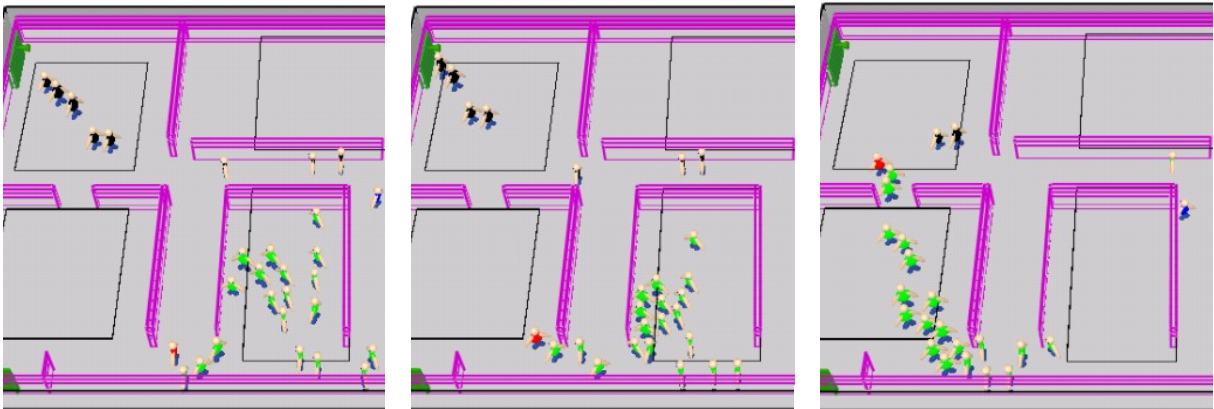


Figure 14. Herding Effect in Pre-Movement Phase and Movement Phase in Multi-Compartment Layout.

As above we illustrate a simulation result of FDS+Evac where herding effect is jointly used with social groups in the pre-movement phase. The group social force is applied as introduced in our previous work (Wang, 2016) so that agents are attracted to each other if they are in close relationship, and their opinions interact when they are physically close

enough, namely $a_{ij} < R_i$. Here the red agent has $tpre$ earlier than the green ones, and Equation (7) is effective to reduce the green agents' $tpre$. As a result, the pre-movement time converges for agents in both colors. Moreover, the group social force combine them together in physical positions. When the simulation time goes beyond their common $tpre$, the entire group of green agents tends to move with the red one towards an exit, and the leader-and-follower pattern appears due to the imbalance (asymmetry) of $a_{ij}^0 \neq a_{ji}^0$ between agents in two colors.

4. Way-Finding Activities

Way finding refers to how people orientate themselves towards exits within a multi-compartment building, and it involves perception and integration of various information acquired from surrounding facilities and individuals (Tong and Bode, 2022). The problem could be formalized in two steps: (a) How to choose a destination exit; (b) How to select a proper route to reach the exit selected. There are two major stressors that influence their decision making in way-finding activities. A major stressor is the guidance information received such as exit signs or voice information acquired from surroundings, and they usually helps to reduce the safe egress time. Another stressor is perception of hazard threat as discussed in the previous section. For example, if evacuees perceive smoke or heat on their ways to an exit, they probably will consider adjusting their escape route or simply move to another exit.

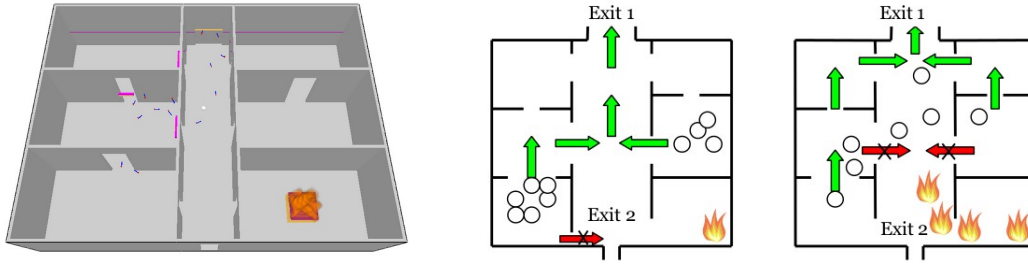


Figure 15. Way-Finding Activities and Exit-Selection Behavior: In this case the exit and route are dynamically updated in the time line of evacuation process, and the route is recalculated when the fire and smoke spread into the corridor area.

Mathematically, it is reasonable to use a probabilistic model to describe such way-finding activities by integrating all the information available for evacuees (Wang et. al., 2008, Zheng et. al., 2015). Take the above figure for example, and there are two choices, and each evacuee is assigned with probability $[p1, p2]$ to select either exit 1 or exit 2. The prior probability distribution $[p1, p2]$ is given as the normal usage of the exits, and it can be given as $[0.5, 0.5]$ if there is no bias preset in people's mind, assuming the two exits are equally used in normal life. In principle the prior indicates the historical usage frequency of different ways on daily basis and it reflects a kind of way-selection habit of people. Such frequency-based data provides a reliable estimate of $[p1, p2]$ based on the law of large numbers in probability theory, and it well carries the information that people are inclined to use the familiar exits rather than unfamiliar ones (Proulx, 1993). The effect of fire drills or other training events may also be integrated in this prior probability distribution. For instance, evacuees ever participated in a fire drill where Exit 2 was used as fire escape route, and such past experience will dramatically increase $p2$ in initial estimate of the probability distribution.

In the simulation process the prior probability distribution of using multiple exits will be dynamically updated when new information received. In Figure 15, when the evacuees observe that the fire and smoke spread into the corridor, it will be reasonable to update the probability distribution, for example, to be $[0.9, 0.1]$ such that exit 1 is mainly used for safe egress. Consequently, it is feasible to build up a probabilistic graph model to link different factors in a statistical sense (Wang et. al., 2008). In addition, if evacuees find an exit sign at any intersections, they will balance it with their own knowledge or habit, and choose to follow the sign or not. This process refers to how an individual deals with the new information received from surroundings and make decisions. In existing social-economic systems utility theory is very useful to integrate various factors into one's decision making process (Tong and Bode, 2022), and it provides a quantitative measure on how useful a choice is to an individual. In the above exit-selection problem the utility of exit q to individual i consists of a deterministic component V_{iq} and a random component ε_{iq} . The deterministic component, for example, could be expressed as below.

$$V_{iq} = \pi_1 \text{DIST} + \pi_2 \text{SM} + \pi_3 \text{EL} \quad (12)$$

In Equation (12) DIST is the distance of the individual agent i to exit q (e.g., Eularian Distance or Manhattan Distance). SM is a boolean variable indicating if there is any smoke or hazardous materials perceived by agent i on the path to exit q , and EL is another boolean variable indicating if there is any evacuation lights or exit signs detected by agent i on the path to exit q . The sub-level parameters in Equation (12) include π_1 , π_2 and π_3 . To calibrate the sub-level parameters, random utility theory (RUT) is used based on data collected from real-world experiments, and according to Lovreglio, Fonzone and dell'Olio, 2016 and Edrisi, Lahoorpoor and Lovreglio, 2021 the above sub-level parameter are given by $\pi_1 = -0.011$, $\pi_2 = -0.985$ and $\pi_3 = 0.175$. Combining prior information with a utility function we formalize a probability function of discrete choices as below.

$$\text{Prob}(\text{individual } i \text{ select exit } q) = a_1 \text{prior}_{iq} + a_2 \frac{\exp(V_{iq})kw_{iq}}{\sum_k \exp(V_{ik})kw_{ik}} + a_3 \text{others}(q) \quad \text{and } a_1 + a_2 + a_3 = 1 \quad (13)$$

where V_{iq} is the utility function of exit q for individual i . Here $kw_{iq} = 1$ if individual i knows exit q , and $kw_{iq} = 0$ if not. It is also necessary to ensure the model to satisfy that $\text{prior}_{iq} = 0$ if $kw_{iq} = 0$, i.e., $\text{prior}_{iq} = 0$ if individual i does not know exit q . In other words, there is a consistency problem in updating the related terms in the right side of Equation (13). For example, kw_{iq} should be consistent with prior_{iq} such that prior_{iq} is non-zero only if $kw_{iq} = 1$. If others tell individual i that there is exit q , then kw_{iq} becomes non-zero, but prior_{iq} remains zero because individual i never go there before, and thus has no past experience of using the exit. In brief the above utility model reflects the idea of bounded rationality, which comes to dominate in most theories of individual decision making, especially in social-economic process. Such a decision process is “bounded,” because usually not all options are known to the decision-maker.

Next, we will expand the above model in timeline by introducing the memory effect. As a result, prior_{iq} , $\text{others}(q)$, V_{iq} and kw_{iq} are all functions of time t , which are timely updated in a consistent manner through the simulation process. In specific $\text{prior}_{iq}(t)$ represents the preference of individual i to follow past memory on selecting the alternatives q , and the memory effect is formalized by $\text{prior}_{iq}(t) = \sum_z \text{Prob}_{iq}(z)$ with $z = 0, 1, \dots, t-1$. Here $\text{Prob}_{iq}(t)$ denote the probability of individual i to select exit q at time t and $\text{prior}_{iq}(t)$ becomes a linear combination of $\text{Prob}_{iq}(z)$ from $z = 0$ to $z = t-1$. A most simple method is using Markovian property where prior_{iq} is iterated by $\text{prior}_{iq}(t) = \text{Prob}_{iq}(t-1)$.

More importantly, Equation (13) is well consistent with the opinion dynamic model as introduced in Section 2, where $a_3 = p_i$ and it indicates how much an individual tends to follow others' characteristics. The $\text{opinion}_i(t)$ in Equation (4) is replaced by a probability distribution in Equation (13), which represents an individual's selective preference of using different exits, and it is timely updated as the individual interacts with others or receives new information from egress facilities. Based on Equation (13) a collective decision-making problem is formulated, where exchange of individual opinions could lead to either one large group (consensus) or several small groups (clustering).

Based on the probability chain rule and Equation (4), each individual's decision is made by integrating three factors: (1) historical knowledge of a structural layout or prior information such as habit of using certain path; (2) collecting timely information and judgment on current situation; (3) learning from opinions of surrounding others. The above three factors are balanced by tuning parameter a_1 , a_2 , a_3 .

In a mathematical sense parameter a_1 , a_2 , a_3 are real numbers normalized in range of $[0, 1]$, indicating one's preference of either using prior experience or timely information for decisions, or simply following others' decisions. They could also be understood as probability measure based on the total probability formula. In a psychological sense the parameter a_1 , a_2 , a_3 critically refers to one's personality, such as whether an individual is stubborn or open-minded. We will elaborate this issue in detail as follows.

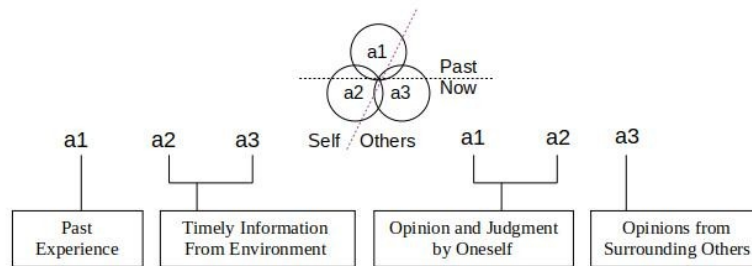


Figure 16. Meaning of parameter a_1 , a_2 , a_3 .

Increase of a_1 means that an individual relies heavily on his/her past experiences to make decisions while current situation is not specifically investigated. Because many life events are highly repetitive, it is common for an individual to compare current situation with their past experience and follow a set of routines to deal with the new situation. This behavioral pattern is straightforward and quick, and it does save time. In emergency egress, it is widely recognized that an individual's experiences can significantly impact his/her behavior, such as the familiarity of the surroundings, safety procedures, and fire drills. For example, evacuees are normally inclined to use their familiar path in egress, and such familiarity is integrated in the prior information as historical frequency of using different exits in normal situations, and parameter a_1 could be understood as one's probability of using such familiar path in egress. In a psychological sense a_1 increases if one is stubborn to their daily routine and not open-minded to new choices.

However, using prior evacuation experience to guide evacuation decisions, may or may not produce better outcomes, and a typical example is that most people tend to follow the route that they are most familiar with, and ignore alternate routes. The possible result is that they create a bottleneck path collectively as mentioned in Section 2. Thus, we also need parameter a_2 to describe a kind of bounded rationality by investigating timely situation. Increase of a_2 implies that an individual relies relatively less on their past experience, but tend to investigate current situation to make a decision. For example, an evacuee tends to use the nearest exit rather than normally-used path to reach the safety, and this decision process requires comparing the distance from the individual to all known candidate exits, and selecting of the optimal solution. The rational decision is widely studied in many social-economic systems, and it is mathematically described by discrete choice process with utility function V_{iq} . In our modeling framework parameter a_2 indicates how much one prefers to use such rational choice. Such rationality is "bounded" because usually not all options are known, and thus we have parameter kw_{iq} to describe if option q is known to individual i .

The parameter a_3 is of significant importance because it characterizes how people will interact with each other to reach a collective decision instead of an individual decision. This is especially useful to form a social group in crowd as introduced in Section 2, and also it is evident in most of egress events that people usually tend to stay together to gain a sense of safety. Thus, to reach a collective decision is more widely observed and meaningful than individual decision making, and people are thus self-organized into social groups to respond to the outside environment stressors.

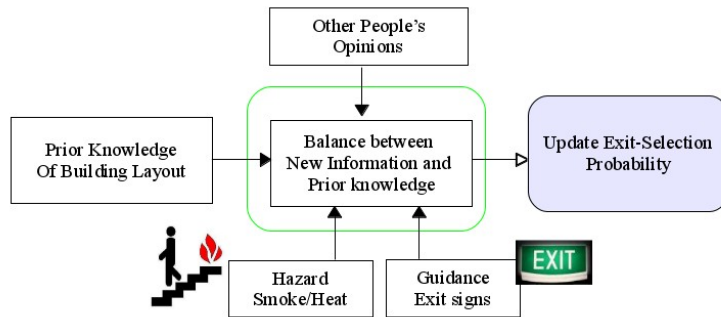


Figure 17. Probabilistic Graphic Model of Way-Finding Activities: The exit-selection probability integrates various information including people's prior knowledge on building structure and location of exits. The probability distribution is updated if new guidance received or smoke detected.

The value of a_1 , a_2 , a_3 could be formalized by using Analytic Hierarchy Process (AHP), which is a method for organizing and analyzing complex decisions, using math and psychology (See Appendix for more details). In sum the probabilistic model is useful to describe psychological findings on people's way-finding behavior, and a graphic structure is shown in Figure 17 to describe people's way-finding behavior. In fact analytic hierarchy process (AHP) and utility theory were initially learned from economic models. Different from the original use of such models in social-economic field, these models are partly renewed to capture various social-psychological findings in evacuation study. This study topics refers to an interdisciplinary field called social-physics or economic-physics (Quang et. al., 2018).

The above model is developed based on the assumption that crowd are mainly composed of rational individuals who require a sense of safety. Such a sense of safety is obtained by either simply following others or confidence on one's own judgment. In fact a small amount of panic individuals, or as called irrational behavior, could be also added as noise

in the right side of Equation (13), but we will not further discuss this issue in this article. In brief Equation (13) enables us to formulate a collective decision making process, and the problem is interesting because we discuss how individual decisions interact with each other and whether or in what condition they converge to form the group opinion.

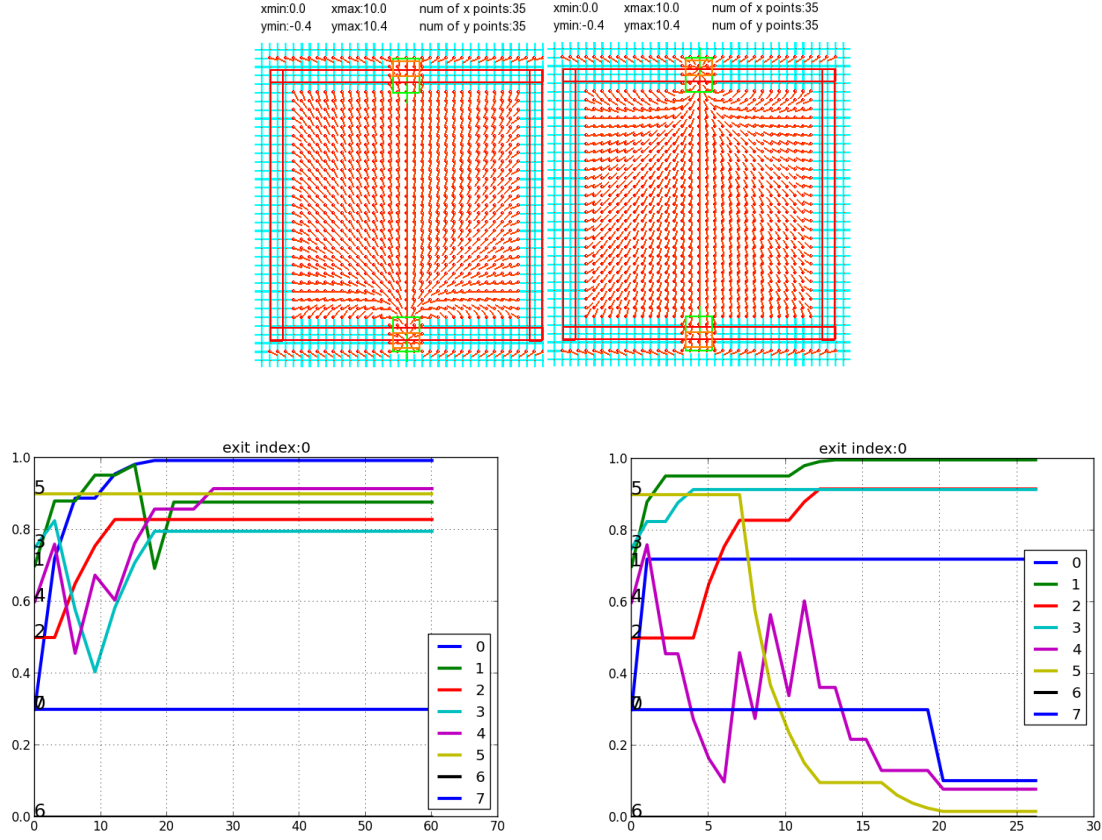


Figure 18. Simulation of crowd egress with exit-selection algorithm: A binary decision problem is mainly investigated for exit selection algorithm, and probabilistic model are applied to describe how evacuees select an exit from two alternatives.

It is shown in Figure 18 that exit selection probability could be either converged or polarized as many individual interact in given social topology. Take agent 4 for example. The probability for agent 4 to select the exit in the figure is initialized with 0.6. Through the simulation process the probability evolves as the agent preference shifts from one alternative to another.

In practical computing the way-finding activities mainly affect desired velocity v_i^0 in our model, and it is specified as shown in Figure 16. There are normally two steps involved: exit selection and route calculation.

The first step is to select an exit known by evacuees. This step is based on a probability model as mentioned above, integrating various information available for evacuees, including exit-selection habit as well as the guidance received and hazard perceived in emergency egress. In CrowdEgress and FDS+Evac all the exits are classified in two types: known exits and unknown exits. It is natural to assign each unknown exit with zero probability, which means that people never select an exit that they do not know. However, a known exit can also be assigned with zero probability, implying that people are told there is an exit, but they never use it previously. This case usually fits in any special passageway for fire egress or emergency use. Usually, people will not first select such an exit unless receive trustful guidance to update the probability of using it.

The second step is to generate the evacuation route based on the exit selected. There are a number of methods used in existing egress simulators. However, some algorithms were learned from route planning of autonomous robots, and they do not differentiate living bodies with conscious mind from non-living things like robots, and this seems a major drawback (Santos and Aguirre, 2004). In FDS+Evac the route is calculated by a two-dimensional flow solver. The

computation result is a 2D flow field that guides evacuees to the exit selected. The flow field can be better explained as a social field related to social norms or other behavioral characteristics (Helbing et. al., 2005; Lewin, 1951), and we will further elaborate the idea in future. In this algorithm each exit is a sink point, and solver calculates the route as the crowd flow move to the sink (Korhonen, 2018). The detailed discussion of the flow solver will not be included in this article, but we emphasize that this method is more suitable to describe human collective behavior on the background of social science and psychological theory.

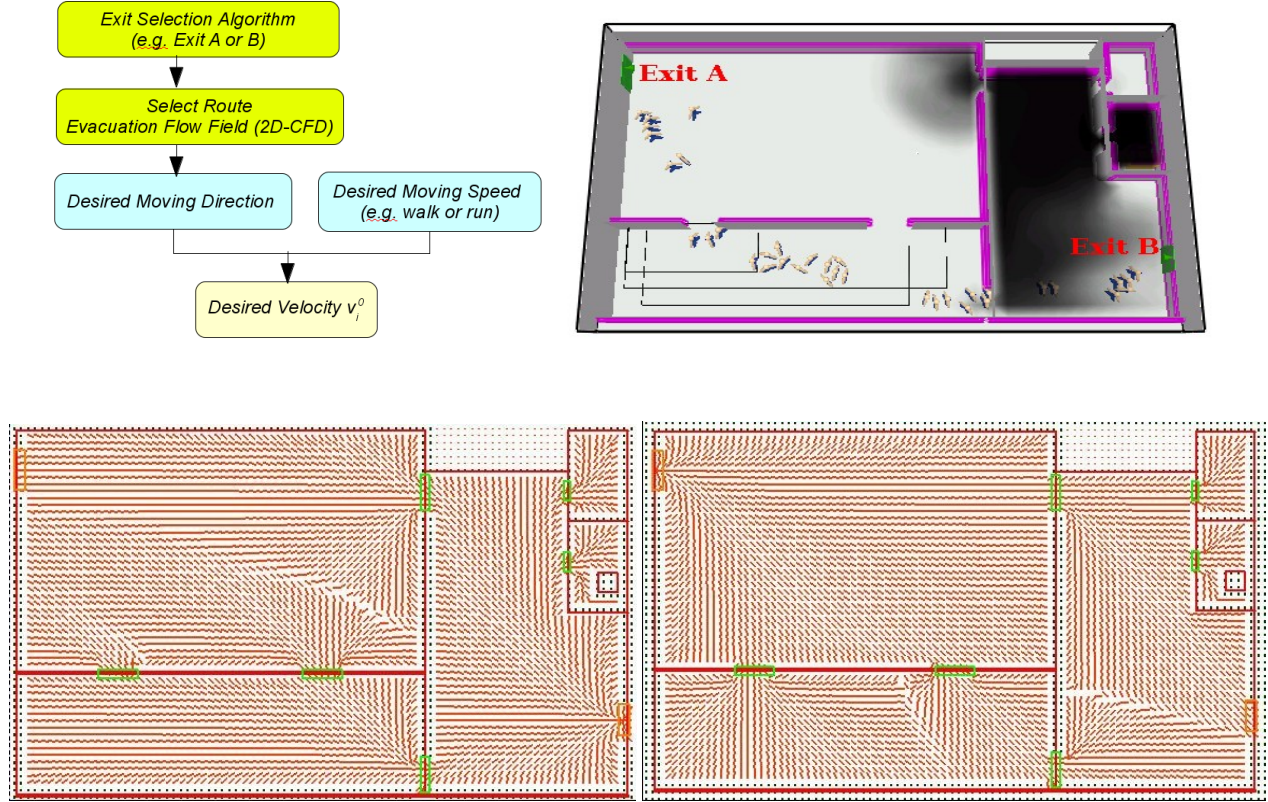


Figure 19. Simulation of Crowd Evacuation with Smoke: Evacuees change their destination exit and head for the left exit. Here the desired velocity of each agent is determined by exit-selection algorithm first and their escape routes are calculated by using a 2D-computational fluid solver in FDS+Evac.

When the target exit and route are both determined, evacuees start to move to the exit. In this process the exit and route may also be updated when new information received. For example smoke may block certain ways, and if such smoke is detected by evacuee agents, they may choose to update v_i^0 to bypass smoke or select another exit or route. In case of heavy smokiness agents may directly select another exit among all the known exits and recalculate the route. A simple logic is given as below.

If Hazard_Intensity > Threshold, target exit is reselected and egress routes is recalculated.

If evacuees choose to bypass the smoke-filled area, the hazard intensity may play a major role, which is abstracted as the hazard force as mentioned in Section 2, and it determines whether people are able to move fast in smokiness. Both of the driving force and smoke resistance are increased. If the driving force is larger than the resistance, evacuees will accelerate, otherwise people have to slow down (See Figure 3). This algorithm has only been tested in FDS+Evac, not in CrowdEgress. Please refer to the appendix for more details.

CONCLUSION

The social-force model was a physics-based model, and it was essentially developed to model either many particles or pedestrian crowd. In the past 20 years this model has been widely applied in crowd simulation in various scenarios. This article reexplains and renews the model to establish a general framework of using the model in crowd evacuation simulation. Very importantly, we introduce new concepts such as desired interpersonal distance and hazard force, and the model is thus extended in consistency of the stress theory in psychological studies, describing how environmental stressors (i.e., guidance and hazard) affect evacuee response. Such response mainly includes walking behavior at bottlenecks and smoke-filled areas. Furthermore, the pre-movement behavior and way-finding activities are discussed based on opinion dynamics and probabilistic model, and various environmental factors are integrated in this modeling framework. This simulation-based approach provides a useful tool to observe various scenarios in crowd egress, and thus helpful to identify potential risk in emergency evacuation.

APPENDIX

(a) Hazardous Effect on Pedestrian Movement

In building egress whether people can move fast critically depends on hazard intensity. In thin smoke people are able to speed up while heavy smokiness impede their motion significantly. In other words, the hazard condition plays an important role. An example is that f_{ih} increases in a square form of smoke density while the driving force increases in a linear form of smoke density (See Figure 3).

In the original setting of FDS+Evac an evacuation process is stimulated by using a pedestrian model extended from social-force model, where the psychological desire of motion is described by desired velocity v^0 . The desired velocity v^0 is next coupled with the fire/smoke condition: In a non-smoke area v^0 is equal to a preset value called the unimpeded walking speed, which is human walking speed in normal situation (e.g., 0.8m/s - 1.6m/s). If smoke density increases, v^0 will increase based on our argument in this article, i.e., $v_i^0(t) = v_i^{0-\text{preset}}(t) + K_1 \text{SOOT_DENS}$. As a result, larger v^0 is specified for motion such that people desire moving faster in smoke areas. This setting leads to an increase of driving force with smoke density. Other mathematical description of f^{drv} will be explored in the future.

In contrast the hazard force is specified by $|f_{ih}| = K_2 * \text{SOOT_DENS}^2$ to impede their physical motion even if they desire moving faster. Here the formula of hazard force is selected mainly due to Stevens's power function on human perception of stimuli (Stevens, 1971), where SOOT_DENS represents the physics measure of smoke density, and human perception of smoke is described by $K_2 * \text{SOOT_DENS}^2$. In a general sense $|f_{ih}|$ is denoted by high-order power function of SOOT_DENS or other hazard intensity, and this formula describes that people become sensitive to hazard intensity if it exceeds a certain threshold.

HR%FX_Hazard and HR%FY_Hazard are the force elements added to HUMAN_TYPE. SMOKE_BLK_FAC is a damping coefficient which slows down agents' movement when agents walk in smoke condition. HEAT_GRAD_FAC is a parameter which tunes smoke resistance with respect to gradient of gas temperature TMP_G.

$$\text{HR\%FX_Hazard} = -\text{HEAT_GRAD_FAC} * (\text{HUMAN_GRID}(\text{II}, \text{JJ}) \% \text{TMP_G} - \text{HUMAN_GRID}(\text{II}-1, \text{JJ}) \% \text{TMP_G}) * \text{HUMAN_GRID}(\text{II}, \text{JJ}) \% \text{TMP_G} ** 2 - \text{SMOKE_BLK_FAC} * \text{HR\%U} * \text{HUMAN_GRID}(\text{II}, \text{JJ}) \% \text{SOOT_DENS} ** 2 / \text{SQRT}(\text{HR\%U} ** 2 + \text{HR\%V} ** 2)$$

$$\text{HR\%FY_Hazard} = -\text{HEAT_GRAD_FAC} * (\text{HUMAN_GRID}(\text{II}, \text{JJ}) \% \text{TMP_G} - \text{HUMAN_GRID}(\text{II}, \text{JJ}-1) \% \text{TMP_G}) * \text{HUMAN_GRID}(\text{II}, \text{JJ}) \% \text{TMP_G} ** 2 - \text{SMOKE_BLK_FAC} * \text{HR\%V} * \text{HUMAN_GRID}(\text{II}, \text{JJ}) \% \text{SOOT_DENS} ** 2 / \text{SQRT}(\text{HR\%U} ** 2 + \text{HR\%V} ** 2)$$

$$\text{HR\%FX_Hazard} = \min(\text{HR\%FX_Hazard}, \text{HR\%Mass} * 2.0_EB) \quad ! \text{ Give a lower bound of hazard force}$$

$$\text{HR\%FY_Hazard} = \min(\text{HR\%FY_Hazard}, \text{HR\%Mass} * 2.0_EB) \quad ! \text{ Give a lower bound of hazard force}$$

(b) Group Social Force by Fortran Code in FDS+Evac

!! Group social force is added. Here I declare a 2D matrix when the number of EVAC lines is determined. Users may initialize or modify the matrix by using EVAC Namelist, and this matrix characterizes the social relationship of agents.
 !! The total number of groups is NPC_EVAC, then DFAC AFAC and BFAC are in dimension of NPC_EVAC*NPC_EVAC.
 !! DFAC: DFactor(I, IE) AFAC: AFactor(I, IE) BFAC: BFactor(I, IE)
 !! I: Index of the current agent (outer loop) IE: Index of the other agent (inner loop)
 !! The following Fortran code is in the loop where social force is computed.
 !! GROUP_FORCE is a boolean variable which enables the group dynamics in computation.

```

FCG_X = 0.0_EB
FCG_Y = 0.0_EB
IF (GROUP_FORCE) THEN
  TIM_DIST = MAX(0.001_EB, SQRT((X_TMP(2)-X_TMP(5))**2 + (Y_TMP(2)-Y_TMP(5))**2))
  FCG_X = (X_TMP(2)-X_TMP(5))*HR_A*AFAC*COSPHIFAC*EXP(-(TIM_DIST-
(R_TMP(2)+R_TMP(5))
*DFAC)/HR_B/BFAC)/TIM_DIST*((R_TMP(2)+R_TMP(5))*DFAC-TIM_DIST)
  FCG_Y = (Y_TMP(2)-Y_TMP(5))*HR_A*AFAC*COSPHIFAC*EXP(-(TIM_DIST-
(R_TMP(2)+R_TMP(5))
*DFAC)/HR_B/BFAC)/TIM_DIST*((R_TMP(2)+R_TMP(5))*DFAC-TIM_DIST)
END IF

!HR_A_CF
!HR_B_CF
HR%FX_Group = HR%FX_Group + FCG_X
HR%FY_Group = HR%FY_Group + FCG_Y

```

(c) Analytic Hierarchy Process (AHP) to determine the parameter a_1 , a_2 , a_3

Basically we investigate how people weigh different factors in their decision making processes. Suppose a person considers other people's decision weighs 3 times of importance as their prior knowledge, and their prior knowledge is 2 times important as their current investigation result. The following AHP matrix is given to describe the case. As commonly known, improper user input at <?> positions in AHP matrix could cause inconsistency problem.

AHP	a_1	a_2	a_3
a_1	1	1/2	3
a_2	2	1	?
a_3	3	?	1

Here we will use a kind of hierarchical approach rather than the standard AHP approach to avoid inconsistency problem. That is we will firstly determine $a_{12}=(a_1+a_2)$ and a_3 , where (a_1+a_2) represents the tendency of people to make a decision by themselves and w_3 denotes the tendency of following others decision. Based on AHP techniques the weigh matrix is computed in two steps as introduced below.

Step1: Suppose one think others' decision is three times of importance as his or her individual decision. The AHP array is specified as below.

$$\begin{array}{ccc}
 \text{AHP} & a_{12} & a_3 \\
 a_{12} & 1 & 3 \\
 a_3 & 1/3 & 1
 \end{array}
 \begin{bmatrix} 1 & 3 \\ 1/3 & 1 \end{bmatrix}
 \xrightarrow{\text{--Normalize--}}
 \begin{bmatrix} 0.75 & 0.75 \\ 0.25 & 0.25 \end{bmatrix}$$

Step2: Suppose one consider that his or her prior experience is two times of importance as investigating current situation. The AHP array is thus given by

$$\begin{array}{ccc}
AHP & a1 & a2 \\
a1 & 1 & 1/2 \\
a2 & 2 & 1
\end{array}
\begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}
\text{--Normalize--}\rightarrow
\begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 2/3 \end{bmatrix}$$

SUPPLEMENTARY DATA

The supplementary data to this article are available online at <https://github.com/godisreal/test-crowd-dynamics>. The output data of FDS+Evac examples are uploaded in the repository. Other numerical testing cases are mainly included online at <https://github.com/godisreal/crowdEgress> and <https://github.com/godisreal/group-social-force>. If you have any comment or inquiry about the testing result, please feel free to contact me at wp2204@gmail.com or start an issue on the repository.

ACKNOWLEDGMENTS

The author is thankful to Peter Luh, Kerry Marsh and Vivek Kant for helpful comments on earlier work in University of Connecticut. The author appreciates the research program funded by NSF Grant # CMMI-1000495 (NSF Program Name: Building Emergency Evacuation - Innovative Modeling and Optimization).

REFERENCES

- [1] F. Amblard, G. Deffuant, "The role of network topology on extremism propagation with the relative agreement opinion dynamics," *Physica A*, vol. 343, pp. 725-738, 2004.
- [2] C. Adam, B. Gaudou, Modelling Human Behaviours in Disasters from Interviews: Application to Melbourne Bushfires, Article in *Journal of Artificial Societies and Social Simulation*, The January 2017, DOI: 10.18564/jasss.3395.
- [3] J. Bañgate, J. Dugdale, C. Adam, E. Beck, "A Review on the Influence of Social Attachment on Human Mobility During Crises," *T2-Analytical Modelling and Simulation Proceedings of the 14th ISCRAM Conference*, Albi, France, May 2017.
- [4] W. B. Cannon, *Wisdom of the Body*. United States: W.W. Norton & Company, 1932.
- [5] S. Curtis A. Best and D. Manocha, Menge: A Modular Framework for Simulating Crowd Movement, Technical Report, University of North Carolina at Chapel Hill, <http://gamma.cs.unc.edu/Menge/>
- [6] G. Deffuant, D. Neau, F. Amblard, G. Weisbuch, "Mixing Beliefs Among Interacting Agents," *Advances in Complex Systems*, Vol. 3, pp. 87-98, 2000.
- [7] A. Edrisi and B. Lahoorpoor and R. Lovreglio, Simulating Metro Station Evacuation using Three Agent-based Exit Choice Models, Article in *Case Studies on Transport Policy*, July 2021.
- [8] W. Ebeling, F. Schweitzer, "Active Motion in Systems with Energy Supply," *Integrative Systems Approaches to Natural and Social Dynamics – Systems Sciences 2000*, Springer, Berlin, pp. 119–142, 2001.
- [9] G. P. Forney, "Smokeview, A Tool for Visualizing Fire Dynamics Simulation Data, Volume I: User's Guide", NIST Special Publication 1017-1 6th Edition, National Institute of Standards and Technology, Gaithersburg, MA, 2017, 212 p.
- [10] N. E. Friedkin and E. C. Johnsen, Social influence and opinions. *J. Math. Soc.* 15. pp. 193– 206, 1990.
- [11] N. E. Friedkin and E. C. Johnsen, Social influence networks and opinion change. *Advances in Group Processes* 16. pp. 1 – 29, 1999.
- [12] J. R. P. French. A formal theory of social power. *Psychological Review*, Vol. 63, pp. 181–94, 1956
- [13] K. Fridolf, E. Ronchi, D. Nilsson, H. Frantzich, Movement speed and exit choice in smoke-filled rail tunnels, *Fire Safety Journal*, Vol. 59, pp. 8-21, 2013.
- [14] Y. Gao, P. Luh, H. Zhang and T. Chen, "A modified social force model considering relative velocity of pedestrians," *IEEE International Conference on Automation Science and Engineering (CASE)*, 2013.
- [15] S. Gwynne, E. R. Galea, P. J. Lawrence, L. Filippidis, Modelling occupant interaction with fire conditions using the buildingEXODUS evacuation model, *Fire Safety Journal* Vol. 36, pp. 327-357, 2001.

- [16] E. T. Hall. "A System for the Notation of Proxemic Behavior," *American Anthropologist*. Vol. 65, No. 5, pp. 1003–1026, 1963.
- [17] D. Helbing, L. Buzna, A. Johansson, and T. Werner, "Self-organized pedestrian crowd dynamics: Experiments, simulations, and design solutions." *Transportation Science*, 39(1): 1-24, 2005.
- [18] D. Helbing, I. Farkas, P. Molnar, T. Vicsek, "Simulation of pedestrian crowds in normal and evacuation situations," in: Schreckenberg, M., Sharma, S.D. (Eds.), *Pedestrian and Evacuation Dynamics*, pp. 21–58. 2002.
- [19] S. Hostikka, T. Korhonen, T. Paloposki, T. Rinne, S. Heliovaara and K. Matikainen, "Development and Validation of FDS+Evac for Evacuation Simulations, Project Summary Report." VTT Research Notes 2421, VTT Technical Research Centre of Finland, 2007, 64 p..
- [20] IMO, "Guidelines for Evacuation Analyses for New and Existing Passenger Ships", MSC/Circ.1238, International Maritime Organization, London, UK, 30 October 2007.
- [21] T. Jin and T. Yamada, "Experimental Study of Human Behavior in Smoke Filled Corridors," *Fire Safety Science*, Vol. 2, pp. 511 – 519, 1989.
- [22] T. Korhonen, "Technical Reference and User's Guide for Fire Dynamics Simulator with Evacuation," (FDS+Evac, FDS 6.5.3, Evac 2.5.2), VTT Technical Research Center of Finland, 2018.
- [23] T. Korhonen, S. Hostikka, S. Heliovaara, H. Ehtamo, "FDS+Evac: Modelling Social Interactions in Fire Evacuation," *Proceedings of the 4th International Conference on Pedestrian and Evacuation Dynamics*, February 27-29m 2008, Wuppertal, Germany.
- [24] E. D. Kuligowski, "The Process of Human Behavior in Fires," *National Institute of Standards and Technology Technical Note 1632*, 2009.
- [25] T. I. Lakoba, D. J. Kaup, N. M. Finkelstein, "Modifications of the Helbing-Molnár-Farkas-Vicsek Social Force Model for Pedestrian Evolution," *Simulation*, Vol. 81, Issue 5, pp. 339-352, May 2005.
- [26] K. Lehrer, *Social consensus and rational agnology*. Synthese 31. pp. 141 – 160, 1975.
- [27] K. Lewin, *Field Theory in Social Science*, New York, Harper, 1951.
- [28] R. Lovreglio, E. Ronchi, and D. Nilsson, "A model of the decision-making process during pre-evacuation." *Fire Safety Journal*, Vol. 78, pp.168–179, 2015.
- [29] R. Lovreglio, A. Fonzone, L. dell'Olio, "A Mixed Logit Model for Predicting Exit Choice during Building Evacuations," *Transportation Research Part A*, 2016. DOI: 10.1016/j.tra.2016.06.018.
- [30] R. Lovreglio, E. Kuligowski, *A Pre-evacuation Study using Data from Evacuation Drills and False Alarm Evacuations in a University Library*, *Fire Safety Journal*, April 2022.
- [31] D. Low, "Following the Crowd," *Nature*, Vol. 407, pp. 465-466, September 2000.
- [32] M. Forssberg and A. Mossberg, J. Kjellström, H. Frantzich, D. Nilsson, *The Variation of Pre-movement Time in Building Evacuation*, *Fire Technology*, 2019.
- [33] A. R. Mawson, "Mass Panic and Social Attachment, *The Dynamics of Human Behavior*," Ashgate. Chap. 10, pp. 113–119, 2007.
- [34] K. McGrattan, S. Hostikka, R. McDermott, J. Floyd, C. Weinschenk and K. Overholt, "Fire Dynamics Simulator, User's Guide", NIST Special Publication 1019 6th Ed., National Institute of Standards and Technology, Gaithersburg, MA, 2018, 367 p.
- [35] F. Ozel, "Time Pressure and Stress as a Factor During Emergency Egress," *Safety Science*, Vol. 38, pp. 95-107, 2001.
- [36] H. J. Payne, *Models of freeway traffic and control*, in *Mathematical Models of Public Systems*, Vol. 1 of *Simulation Councils Proc. Ser.*, pp. 51-60, 1971.
- [37] G. Proulx, "A Stress Model for People Facing a Fire," *Journal of Environmental Psychology*, Vol. 13, No. 2, pp. 137-147, 1993.
- [38] L. A. Quang, N. Jung, E. S. Cho, J. H. Choi and J. W. Lee, *Agent-Based Models in Social Physics*, *Journal-Korean Physical Society*, 2018. DOI: 10:3938/jkps.72.1272.
- [39] G. Santos and B. E. Aguirre, *A Critical Review of Emergency Evacuation Simulation Models*, NIST Workshop on Building Occupant Movement during Fire Emergencies, June 9-10, 2004.
- [40] H. Selye. "Confusion and controversy in the stress field". *Journal of Human Stress*. Vol. 1, No. 2, pp. 37–44, 1975.
- [41] J. D. Sime, "An Occupant Response Shelter Escape Time (OSET) Model." *Safety Science*, Vol. 38, pp. 109-125. 2001.
- [42] J. H. Sorensen, "When Shall We Leave? Factors Affecting the timing of evacuation departures". *International Journal of Mass Emergencies and Disasters* 9.2, pp. 153–165. 1991. M. A. Staal, "Stress, Cognition, and Human Performance: A Literature Review and Conceptual Framework (NASA/TM – 204-212824)," August 2004, Hanover, MD: NASA Scientific and Technical Information Program Office.
- [43] B. Stenffen, "A modification of the social force model by foresight," *Proceedings of the 4th International Conference on Pedestrian and Evacuation Dynamics*, February 27–29, 2008, Wuppertal, Germany.
- [44] S. S. Stevens, *Issues in psychophysical measurement*. *Psychological Review*. Vol. 78, pp. 426-450, 1971.
- [45] Y. Tong and N. W. F. Bode, *The principles of pedestrian route choice*, *Journal of The Royal Society Interface*, Vol. 19: 20220061. 2022.
- [46] M. Treiber, A. Hennecke, and D. Helbing, *Congested Traffic States in Empirical Observations and Microscopic Simulations*, *Physical Review E*, 2000. DOI: 10.1103/PhysRevE.62.1805
- [47] T. Vicsek, A. Czirok, E. Ben-Jacob, I. Cohen, O. Shochet, "Novel type of phase transition in a system of self-driven particles," *Physical Review Letters* 75. pp. 1226-1229. 1995.
- [48] P. Wang, "Understanding Social Force Model in Psychological Principle of Collective Behavior," [arXiv:1605.05146v10](https://arxiv.org/abs/1605.05146v10), Master Thesis, 2016.

- [49] P. Wang, P. B. Luh, S. C. Chang and J. Sun, "Modeling and Optimization of Crowd Guidance for Building Emergency Evacuation," Proceedings of the 2008 IEEE International Conference on Automation Science and Engineering (CASE 2008), Washington, D.C., pp. 328 – 334, August 2008.
- [50] P. Wang, X. Wang, P. B. Luh, N. Olderman, C. Wilkie, T. Korhonen, "CrowdEgress: A Multi-Agent Simulation Platform for Pedestrian Crowd," Technical Report (User Manual), 2023.
- [51] J. Wąs, J. Porzycki¹, N. Schmidt-Polończyk², Grouping behaviour and decision making in road tunnels on evacuation in smoke conditions, Experimental approach Proceedings from the 9th International Conference on Pedestrian and Evacuation Dynamics (PED2018), Lund, Sweden – August 21-23, 2018.
- [52] Weisbuch G, Deffuant G, Amblard F and Nadal J P (2001), Interacting agents and continuous opinion dynamics. <<http://arXiv.org/pdf/cond-mat/0111494>>.