Ph 20 Assignment 1

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1 Introduction

The focus of this assignment is Lissajous figures and beats. Lissajous figures are used extensively to tune circuits and frequencies, allowing for a comparison between the frequency of two sine waves.

The relationships that generate these figures are given as follows:

$$X(t) = A_x cos(2\pi f_X t) \tag{1}$$

$$Y(t) = A_Y sin(2\pi f_Y t + \phi) \tag{2}$$

$$Z(t) = X(t) + Y(t) \tag{3}$$

2 Examining the Ratio f_X/f_Y

By graphing $X(t)$ on the x axis against $Y(t)$ on the Y axis, I first investigated the relationship between the ratio of the frequencies f_X/f_Y.

I noticed that with a ratio of 1:1, I had an elliptical shape and when I varied the ratio to 1:2, this shape had an intersection and 2 intersections for a ratio of 1:3. Thus, the pattern here is that the number of intersections of the function $y(t)$ plotted against $x(t)$ increases as the ratio decreases. So far, it has been proportional to the denominator + 1.

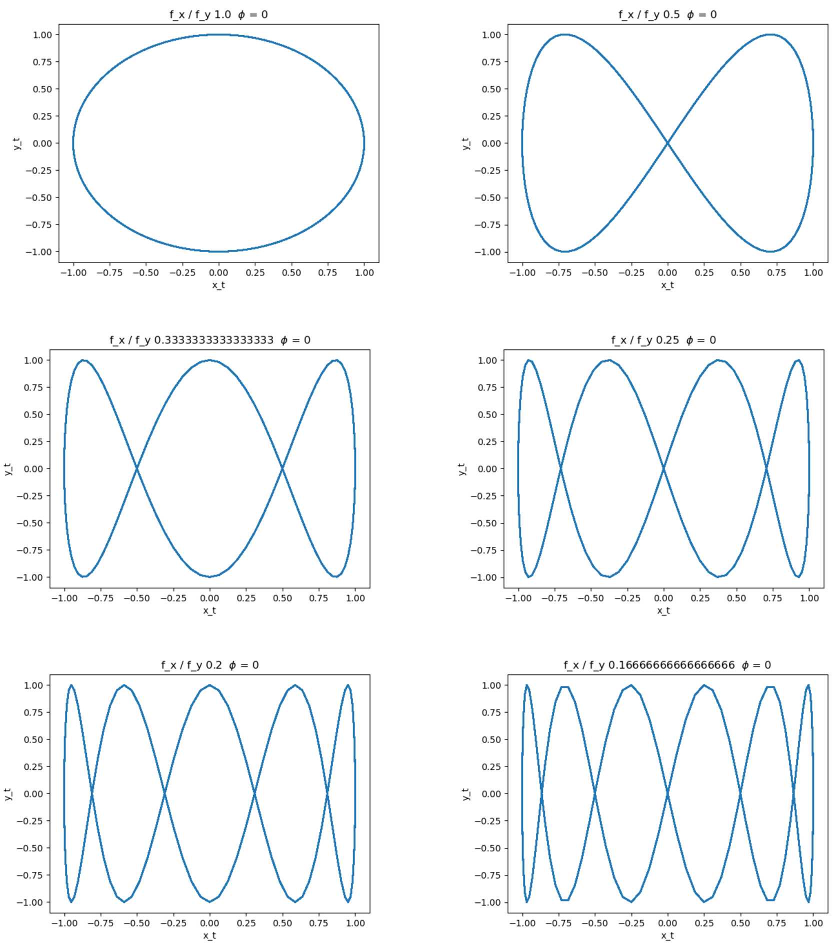


Figure 1: Decreasing the ratio f_x/f_y by integers in denominator

2.1 Closed Curves

Changing the ratio f_X/f_Y, I noticed that curves were closed if this ratio was rational and not closed if they were irrational. This is to say that when we have an irrational ratio, the pattern is mapped over and over again. For the plots on column 1 of Figure 2, notice that the lower right hand side has a line that stretches out of the figure, indicating that it is not closed. Further, compared to the same plots generate instead with 1 in the numerator instead of π (the second column), we notice that there are not as many repetitions and the plots all connect the start and end points.

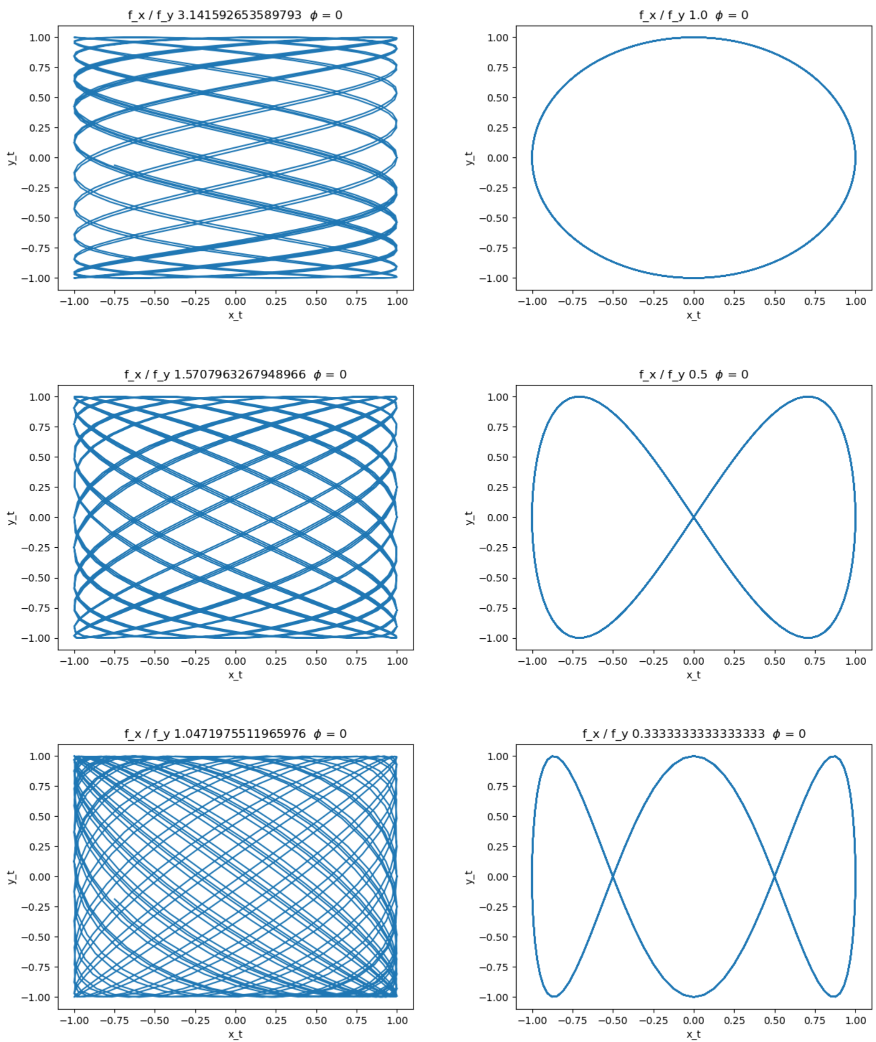


Figure 2: Irrational vs Rational Ratios

3 Phase shift

For this section, I kept the ratio of the frequencies f_X/f_Y constant and changed the phase shift (ϕ). The first column of Figure 1 is populated by a ratio of 1:1, the second column by 1:2, and the third column by 1:3.

I noticed that as we increased the phase shift for the 1:1 ratio, the functions combined to make a line with a positive slope of 1 at $\pi/2$. Thus, when the frequencies are out of phase by $\pi/2$, they are in tune or aligned peak to peak. The opposite would be true if we were considering a sine function for $x(t)$ and a sine function for $y(t)$ The same pattern repeats for the ratios of 1:2 and 1:3, getting a quadratic and a third degree function at a phase shift of $\pi/2$.

π equivalent	Numeric Value
π	0
$\pi/6$	0.5235987755982988
$\pi/4$	0.7853981633974483
$\pi/2$	3.141592653589793
0	1.5707963267948966

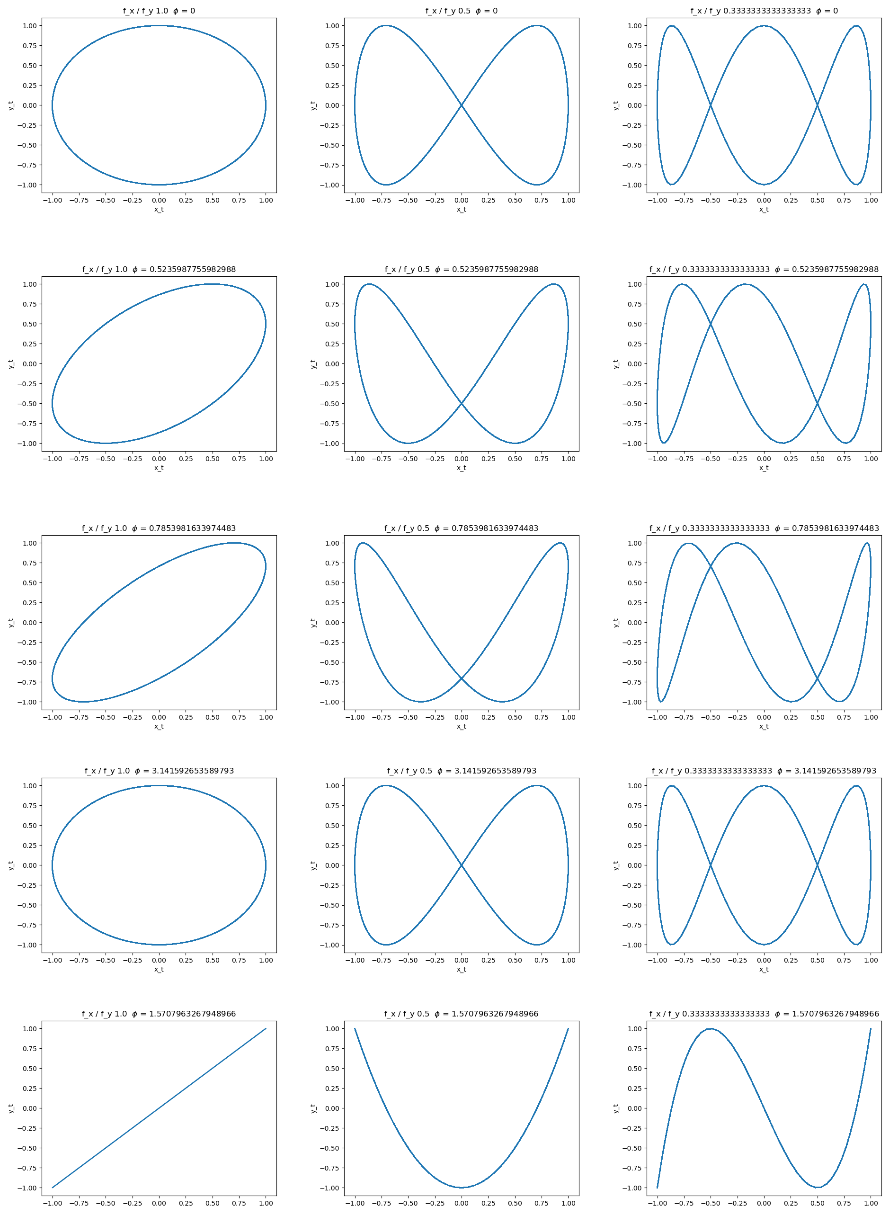
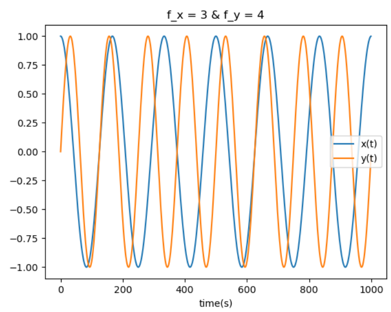


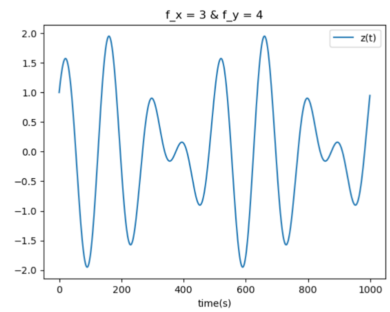
Figure 3: The conversion table for the phase shift angles is given above

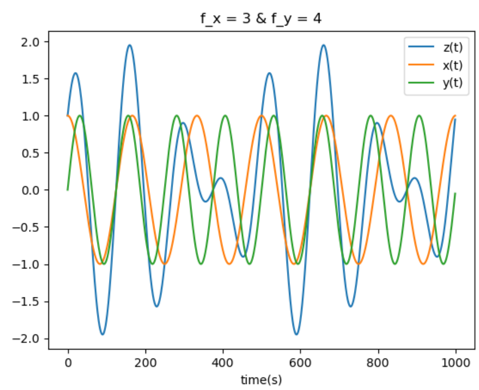
4 Beats

In this section of the project, I tweaked the $X(t)$ and $Y(t)$ values slightly while keeping the ratio $X(t)/Y(t)$ close to 1. I plotted $Z(t)$ as a function of time. Mathematically, the phenomenon of beats comes from the frequency mismatch of the waves. With slight differences in frequencies, we see the crests of the waves align and then go out of phase until reaching an almost destructive interference which results in the dips for the beats. Further, the interval for the beats can be derived from this mismatch of frequencies.

Taking the difference in frequencies $\omega_1 - \omega_2$, we get how much out of phase our two waves are. We can similarly obtain the carrier frequency: $\frac{\omega_1+\omega_2}{2}$ Below, I have given an example of how destructive and constructive interference combine to make the beat frequency.







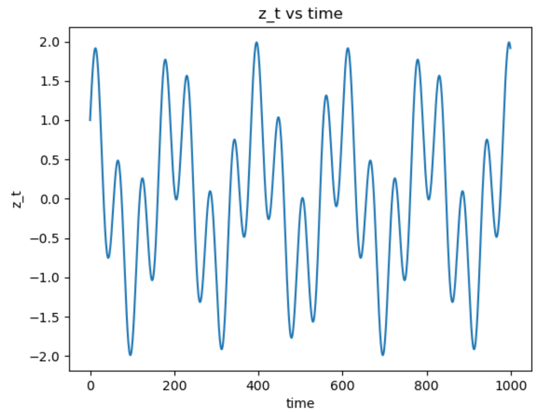


Figure 4: $f_x = 1$ and $f_y = 2$

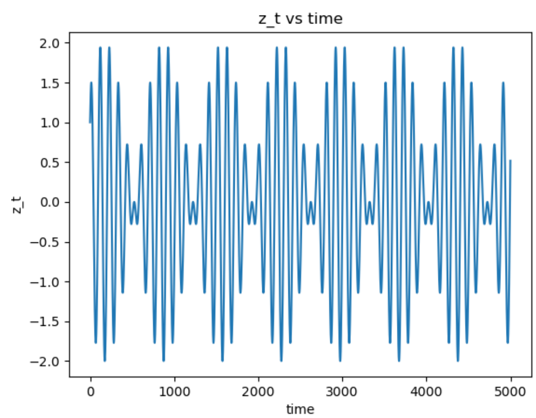


Figure 5: $f_x = 6$ and $f_y = 7$

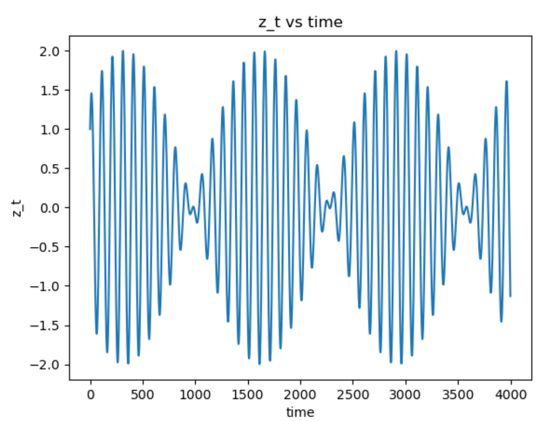


Figure 6: $f_x = 25$ and $f_y = 26$

5 Conclusion

This concludes the first assignment for this class. I can say I have enjoyed the process of learning more about Lissajous figures and coding with python. I had already been using Jupyter Notebook for a while to do astrophysical research with galaxy simulations, so I chose to use the same platform when generating my graphs. One new thing I had to learn for this assignment was using latex, and I think I am much more familiar with overleaf now. I look forward to future labs where I can work with more tools in the context of analyzing physical phenomena.

References

[1] Khan Academy Physics, *Beat frequency derivation*, Youtube, $https://youtu.be/pI6iJg_W1ug$, 2016

[2] Physics High, *Lissajous figures demonstrated and explained, and why they are used* Youtube, $https://youtu.be/DObrU30VZBg$, 2021