**A Cost Minimizing at Laser Cutting on CNC Machines**

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**Abstract.** The problem of cutting path optimization for CNC laser cutting machines is considered in this paper. As an objective function the cost function of cutting process is used. The model of exact cost function calculation is presented depending on the number of frames (commands) in the NC program. The each command is written using G-code. In order to most correctly to construct optimal cutting path the accurate value of objective function basic parameters should be calculated. To this end, the accurate calculation methodologies of basic parameters values are presented. The methodologies relate to calculation of cost parameters and cutting speed. Based on proposed methodology the subsystem of cutting cost calculation was developed by using .Net Framework technology. In order to solve optimization problem the special cutting techniques are used. There are some multi-contour and multi-segment cutting techniques. In this paper special cutting techniques for common geometrical types of contours widely used in blank production are presented. In order to verify the proposed methodologies on practice the computational experiments which show a statistically significant improvement of the objective function value compared with using standard cutting techniques are presented.

**Keywords: CNC laser cutting machines; thermal cutting; tool path optimization; cost of cutting process; cutting techniques.**

1. **Introduction**

One of the complex optimization problems arising in technical applications is the cutting path optimization problem for CNC sheet cutting machines. This problem belongs to the class of NP-hard problems of continuous-discrete optimization equivalent to some types of traveling salesman problem with additional restrictions that do not allow the use of known algorithms to solve them. As an objective function of the problem, the cost of parts cutting process for the resulting cutting path is considered.

Recently the CNC sheet cutting machines are widely used in order to manufacture sheet metal products. In particular, such machines include thermal cutting machines (laser, flame and plasma cutting). During development of NC programs need to take into account some important technological features and constraints arising in the process of part sheet cutting on CNC equipment.

Before cutting of part contour the piercings must be selected (Fig.1). Piercings are operations where the laser cutter penetrates the material. They are required when the laser starts to cut in an area of the plate where it has not cut before. The place of piercings is selected according to material type, its thickness and cutting parameters. In order to avoid material beading and part deformation the piercings must be selected by some distance from contour.

During thermal cutting the “burning out” of material is occurred. Due to the fact the contour of parts and cutting tool path do not matched. The cutting tool is moved by equidistant curve of contour (Fig.1).

The precedence constraint is taken into account which is due to the features of portal type machine [13, 19]. If the contour is fully cut, one detaches from the rest of the material and can possibly shift its position, and thus it will be impossible to continue cutting in this area [3]. The constraint ties to inner-outer contour relation which means that an inner contour needs to be completely cut before the outer contour is cut. Fig. 1 presents example of cutting precedence of contour by number 1-3.

The objective function (cutting cost ) is calculated by [15]:

 (1.1)

 is length of idle tool move;  is length of working tool move;  is cost of idle tool path unit;  is cost of working tool path unit;  is numbers of piercing;  is time of one piercing;  is cost of one piercing.

In general when using different types ( ) of piercing the  is calculated by:

 (1.2)

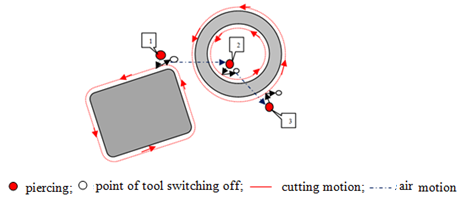


Fig.1. The cutting scheme example of two parts using standard cutting technique

The problem of cost function optimization is considered as *generalized travelling salesman problem (GTSP) with restrictions* [15, 16]. The formalization of minimization problem of cutting time and cost for CNC sheet cutting machines is presented in [15].

In [4, 7, 17] the following classes of cutting tool routing problems for CNC sheet cutting machines are allocated:

* The Traveling Salesman Problem – TSP;
* The Generalized Traveling Salesman Problem – GTSP;
* The Continuous Cutting Problem – CCP;
* The End Point Cutting Problem – ECP;
* Intermittent Cutting Problem – ICP;
* Based on conception of contours cutting by segment [17] the new class of optimization tool routing problem is presented [17] – Segment Continious Cutting Problem (SCCP).

The detailed analysis of existing methods, which are solving the optimization problem of cutting tool route, was presented in [4]. A few algorithms of tool routing for other technological equipments are particularly described in [14, 28].In these articles there are questions relating to cutting time optimization. The present methods of cutting time optimization relate to minimization of idle moves time and slightly to minimization of cutting time. The analysis of current methods is provided below.

Analysis of existing methods shows that airtime and length of idle moves are usually minimized during cutting path optimizing. The following researchers present algorithms for idle moves optimization. Yang et al. [27] describe the airtime optimization problem in leather cutting. They proposed the hybrid intelligence algorithm. Castelino et al. [1] describe an algorithm for airtime minimizing by optimally connecting the tool path. They consider heuristic methods that are used in order to obtain the optimal or near optimal solutions. Murzakaev et al [12] consider problem of idle moves length minimization. The model is presented for standard cutting technique. In order to minimize cutter idle moves length the three metaheuristics (Simulated Annealing, Threshold Accepting, Great Deluge Algorithm) were chosen. They propose the generalized scheme of problem solving. The algorithm of idle moves minimizing is proposed by Chen et al [2]. They divide into two sub-optimal problems (pattern cutting order and entry/exit cutting point) and solve ones using an ant colony optimization algorithm (min-max ant system). Lee and Kwon [9] consider tool path problem and proposed two-step genetic algorithm. The aim is to minimize the idle moving of cutting tool. They combine global search for piercing optimization and local search for part sequencing. The two stage of sequential optimization procedure for nesting and cutting sequence are presented by Sherif et al [21]. The objectives are maximizing the material utilization and minimizing the cutter idle moves. They consider simulated annealing algorithm in order to find the near optimal cutting tool path.

Analysis of existing methods shows the deficiency of research in the field of piercings numbers  reducing and length of cutting moves  in solving the optimization problem (1.2). In [10] authors consider problem of  reducing in thermal cutting of sheet material in terms of graph theory. It should be noted that precedence constrain was not taken into account and the intersections of existing cuts are allowed. The problem of cutting path optimization in terms of parts group cutting with piercing is considered in [6]. The last stage of solving the problem (cutter routing optimization at idle moves) is reduced to the TSP. The problem of cutter routing optimization at CNC machines is formulated and the mathematical model of total cutting time minimization is proposed by using standard and special cutting techniques by Faizrahmanov et al. [5]. Verhoturov et al. [26] present “chained” cutting technique in order to minimize the numbers of pierce points.

The one of cutting time and cost minimization methods is application of special cutting techniques. In order to optimize the cutting parameters and to observe the necessary cutting requirements the some special cutting techniques are used. There are “chained” cutting [26], common cut , partial cutting of contour with the subsequent completion of the contour cutting after cutting the contour of another part, et al. The classification of various cutting techniques used to form the cutting tool path is proposed by Petunin and Krotov [18]. The cutting techniques are classified into three main classes: standard cutting, multi-contour cutting and multi-segment cutting technique. The every contour is cut with piercing by using *standard cutting technique*. The piercing numbers equal number of contours. The several contours are cut in one cutting segment with piercing by using *multi-contour cutting technique*. For example, the multi-contour cutting includes “chained” cutting, common cut. The several cutting segments are cut with some piercings by using *multi-segment technique*.

Analysis of optimization problem solutions shows that there are no or are negligible considered the questions about cutting cost optimization in addressed articles. To this end the methodology of exact calculation of cutting cost objective function is conducted in this article. In optimization problem (1.2) there are difficulties in calculating the basic parameters,  depending on many factors in order to exact computation of  and construct the exact optimal cutting path. For CNC laser cutting machines  and  depend on the type of laser used in CNC machine, type and thickness of treatment material. The selected factors depend on analytically or tabular functions. The formulas of ,  calculation and their values may significantly differ for various CNC sheet cutting machines. The analysis of existing methods for cutting cost calculation shown the insufficiency of research in the field. The time per pierce in laser cutting process is calculated in [8]. In [11] the laser cutting cost is compared with water jet, plasma and oxygen cutting costs when treatment sheet material of 1.0114 (thicknesses ∆=3-10 mm). In [20] the assessment of plasma and CO2 laser cutting machines operating cost is performed. But it should be noted that the calculation of cost parameters remains outside the scope of present researches, hence the calculation of , values and consequently exact calculation of cost function  value are actual problems today, which are solved in this article. The methodology of cost parameters calculation in objective function  is developed.

As seen from (1.2)  also depends on  and . In turn  depends on value of cutting speed. The value of  is usually constant parameter which is programmed in the CNC controller of the laser cutting machine tool, but as the practice shows [19, 22, 24] that the value of actual cutting speed is varied with various technological factors and NC programs parameters. Consequently problem of accurate calculation of cost function (1.2) in optimization problem of tool path routing is arisen. In order to solve this problem the correction parameters for  values must be calculated. It should be noted, in particular, that the question of exact calculation of  values is remained open, then there is a need of research in order to calculate the correction parameters of  values and consequently cutting cost in this article according to example of CNC laser cutting machines.

It is observed that , are interdependent. In some cases the reduction of  leads to some increase of total cutting tool path length value  due to cutting moves between contours. Wherein the length of idle path  is reduced either.

The problem of cost function minimization (1.2) during parts treatment from sheet material at CNC cutting machines is solved with optimization of parameters  and . As the practice shows the length of cutting moves  and number of piercings impact on the cutting cost compared with the length of idle moves . Depending on the thickness and type of material the  can reach up to 33% from  and at the same time can exceed the  by three orders [25]. Consequently the most interest represents the solving methods aimed at the minimizing  and .

The article is organized as follows. The model of  calculation and basic parameters,  are presented in Section 2.1. Exact calculation of  values and values of correction coefficients for  is given in Section 2.2. Based on the exact computation of objective functions the cutting path and cutting cost are evaluated as a true. The results of the computational experiment are presented in Section 3.

1. **The exact calculation of cost function**  **in the cutter path optimization problem** 
   1. **The model of basic cost parameters **,  **calculation**

The most important economic characteristic of the developed NC program quality is the cost  of cutting parts on CNC machine.  includes the costs of electricity and expendable materials, maintenance of a CNC machine and other operating costs arisen during cutting process. The problem of exact calculation of cost function  in optimizing of cutting path related with search of adequate value which depends on basic parameters, . The allotted parameters in turn depend on values of **, ** и **. The functional dependence ,  on type and thickness of material, laser type in CNC machine, cost of expendable materials, cost of laser and technological gases, depreciation of equipment can be set either table functions or analytically. Frequently the cutting cost is not often considered in blank production or is calculated based on normative is not dependent on values of , . Obviously that necessity of cutting cost calculation arises on manufacturer which provides cutting material service for the third-party firm. As a rule during cutting cost calculation only ** is taken into account which usually equals compound perimeter of cutting parts edge contours that leads to inaccurate cutting cost calculation. Subsequently the calculation methodology of cutting cost parameters is actual problem today.

The calculation methodology of cutting cost parameters in optimization problem of cutting path applied to CNC laser cutting machine (laser type is CO2) is considered. In order to calculate  the following notations for cost parameters calculation on 1 m of cutting tool motion are entered:  - the cost of expendable materials (for example, adjudge, protective glass, gas tubes); * -* the cost of technological gas (nitrogen or oxygen depending on processed material); * -* the cost of cutting gas (when working on a gas flow laser);  the cost of electricity;  - the cost related with salary of accompanying personnel;  - amortization of equipment. In general the  is calculated by:

 (1.3)

In order to calculate  the additional notations are entered: * -* time spent on 1 m of cutting tool motion, h.; * -* the electricity costs for 1 hour of CNC laser machine work on cutting moves, kWh; * -* technological gas consumption, m3/h *;  -* laser gas consumption, m3/h;  - electricity cost per 1 kW;  - the cost of 1 m3 laser gas;  - the cost of 1 m3 technological gas;  - the cost of expendable materials unit;  - serviceable life of expendable materials; - cost of 1 h work of accompanying personnel; *A –* depreciation of 1 h work of CNC laser cutting machine; *N* - useful life of equipment;  - .initial cost of the CNC laser cutting machine.

The  are calculated by:

. (1.4)

Similarly ; (1.5)

; (1.6)

**. (1.7)

** (1.8)

** (1.9)

In order to calculate  the following notations are entered: – the electricity costs for 1 hour of CNC laser machine work on idle motion, kW/h; ** – the time spent on 1 m of idle tool motion, h. Consequently the  is calculated by:

. (1.10)

In order to calculate  the following notations for cost parameters calculation on 1 pierce point are entered:  - the cost of expendable materials (for example, adjudge, protective glass, gas tubes); * -* the cost of technological gas (nitrogen or oxygen depending on processed material); * -* the cost of cutting gas (when working on a gas flow laser);  - the cost of electricity;  - the cost related with salary of accompanying personnel;  - depreciation of equipment. In general the  is calculated by:

**. (1.11)

In order to calculate  the additional notations are entered: – the electricity costs for 1 pierce point, kW/h; ** – the time spent on 1 pierce point, h. Consequently the  is calculated by:

**; (1.12)

; (1.13)

;  (1.14)

. (1.15)

** (1.16)

** (1.17)

During calculation of  and  the following parameters  and  must be taken into account when processing of material at flow-through gas laser machines. The parameter  must be considered during calculation of  when technological gas is applied.

Consequently,  can be written as follows:

 (1.18)

The main expendable materials for gas laser include: swivel mirrors, focusing lenses, protective glasses, nozzles, adjusting units, gas tubes. The main expendable materials for fiber laser are nozzles, protective glasses, focusing lenses. And for the case of using solid-state lasers, the expendable materials are optical pumping lamps, protective glasses, mirrors, a quantron, an active element. In [25] the values of cost parameters  and  is presented by taken into account the above parameters (1.3)-(1.17) for CNC laser cutting machine according to example ByStar 3015. For each type of material the parameters  and  is calculated taken into account that  As the practice shows [19, 22] the value of actual cutting tool speed is varied with various technological factors and parameters of NC programs. Consequently problem of accurate calculation of cost function (1.2) in optimization problem of tool path construction is arisen. In order to solve the encountered problem the need of correction parameters calculation for  values is emerged.

Based on proposed methodology the subsystem «Сutting cost calculation» was developed by using Net. Framework technology. A subsystem may be integrated with existing CAM software. Fig.2 presents interface of developed subsystem. In order to calculate  the values of basic cost parameters  and  are added into database in XML format.

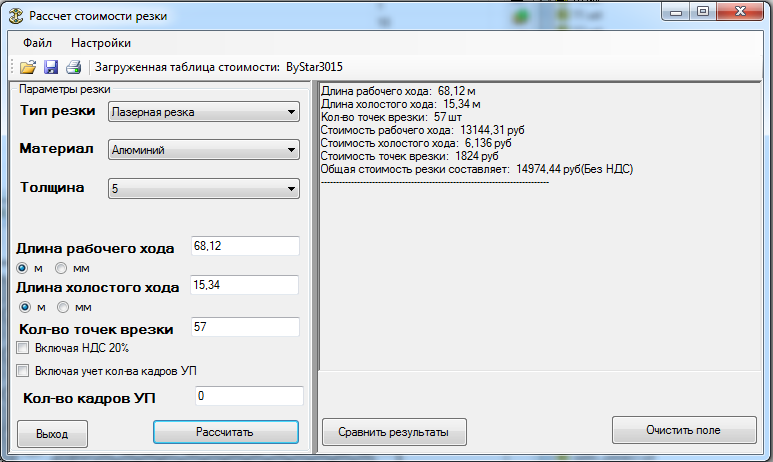


Fig.2 The interface of «Сutting cost calculation» subsystem

* 1. **The accurate calculation of  in objective function from the example of the CNC laser cutting machine ByStar 3015**

The inaccuracy of the actual cutting time and cost calculation is due to the fact that **,** which is controlledas constant value in NC program, is actually varied with various technological factors. It was found that increasing number of commands in NC program for various sets of parts, which have the same total perimeter of the contours, the actual **** is decreased [19,22]. The reasons why NC program can contain a large number of commands is mainly due to the contours of complex geometry (for example, splines) when converting from CAD systems to a CAM are divided into a large numbers of geometric primitives by reason of the difference of geometric file formats (for example, on segments of straight lines and circular arcs), i.e. approximated by simple geometric primitives. The difference in formats is due to the fact that almost all CNC systems are equipped with only linear and circular interpolators. As a rule the approximation of a complex geometry reduces to a linear approximation.

The functional dependence of **** should be determinedthe table functions or analytically. However in practiceand in this case the accuracy of objective function calculation during cutting path optimization is not provided. The algorithmization of objective function (1.2) calculation based on science – based determination of function parameters is requirement for the development of cutting path optimization algorithms. The one is optimal only if the objective function is adequate calculated. For this reason the exact parameters values of objective function must be calculated.

The some practical results on determining dependence of cutting speed on number of NC program commands are given below [9]. Based on received results the objective function (1.2) can exactly calculated and exact optimal cutting path can constructed.

The research was conducted for following materials: 1.0114 (thickness ∆=1-10 mm) and AWAIMg3 (thickness ∆=1-5 mm). In order to conduct experiments the 150 NC programs for cutting of various types of parts with numbers of frames ** () for 1.0114 and 150 NC programs for cutting of various types parts with number of commands **  for AWAIMg3 were developed.

The statistical materials were processed by using “Mathcad”. Based on received results the following upshots were made:

1. The actual average value of cutting speed **** is monotonically decreasing function depending on number of NC program commands (fig.3);
2. The predetermined cutting speed **** coincides with the actual average speed when the numbers of frames reaches a certain threshold value *N*. When the frames numbers , then the actual speed is greater than predetermined cutting tool speed, if the number of NC program commands is arisen () then the actual speed is less than predetermined cutting speed of NC program (in the experiments the reduction of average actual cutting speed value compared with predetermined cutting tool speed in NC program is 70%);
3. The threshold value *N* is variedfor different thickness and grade of material.

In order to present the results of computational experiments the following notations are introduced: *n* – number of NC program commands; **** - the actual speed of cutting tool; *N* – the number of commands when;  - the deviation squares sum of the original data from the values of the approximation functions at these points.

When approximation the actual speed dependence presented on point chart on the number of commands with approximating curves in “Mathcad”  for all values of studied grade materials and thickness are achieved using logarithmic approximation function. Fig.3 presents following results for material of AWAIMg3 with ∆=1 mm. Similar results were obtained for AWAIMg3 with ∆=1-5 mm and 1.0114 with ∆=1-10 mm. The generalized formulas for calculating of cutting tool speed by example CNC laser cutting machine ByStar3015 are presented in table 1.

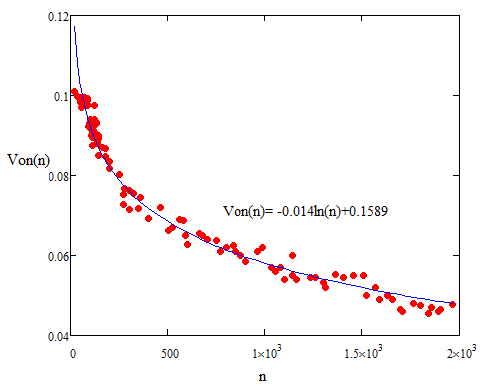


Fig.3. Change of the real cutting speed for AWAIMg3, ∆=1 mm

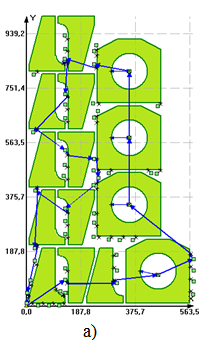
For subsystem «Сutting cost calculation» the module, in which the complexity of processed contours and consequently developed functional dependences (presented in table 1) may be taken into account, was developed. This enable to exact calculate an objective function . Based on practice obtained results of  calculation considering developed formulas the values of cutting cost is significantly differ compared with values of  calculated with above developed methodology.

In turn application of developed formulas during nesting and cutting path construction leads to modification of cutting path. The obtained path is accurate compared with path constrained with . For example, fig.4 presents the cutting path optimization for nesting of 15 parts (material of AWAIMg3, ∆=1 mm). Each contour is cut used standard cutting technique (when number of piercing equals number of parts contours). In order to reduce acceptable solutions set the acceptable piercings set is limited with finite aggregate consists of 55 points (these points are green squares in fig.4, in turn points of tool switching off are X). The blue arrows are idle moves of cutter. The number of NC program commands for this nesting is . For the case of fig.4a the, for the case of fig.4b the .

Table 1

The generalized table of formulas for calculating of cutting tool speed by example CNC laser cutting machine ByStar3015

|  |  |
| --- | --- |
| Material and thickness of material ∆ | The formulas for cutting tool speed calculation |
| 1.0114, ∆=1 mm |  |
| 1.0114, ∆=2 mm |  |
| 1.0114, ∆=3 mm |  |
| 1.0114, ∆=3.5 mm |  |
| 1.0114, ∆=4 mm |  |
| 1.0114, ∆=8 mm |  |
| 1.0114, ∆=10 mm |  |
| AWAIMg3, ∆=1 mm |  |
| AWAIMg3, ∆=2 mm |  |
| AWAIMg3, ∆=3 mm |  |
| AWAIMg3, ∆=5 mm |  |



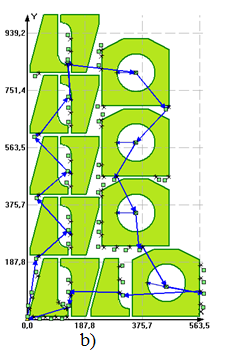


Fig.4. Optimal cutting path in the case  (a) and  (b)

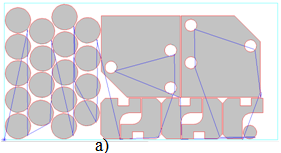
Based on proposed results (fig.4) the accurate calculation of objective function ensures not only the exact computation of function extremum value but also the correct results of optimal cutting path search taking into account parts complexity.

1. **Computational experiments**

The proposed methodology of  calculation taking into account dependence of cutting speed  on parts complexity is useful during practice technological problems solving in terms of optimal cutting route planning on CNC thermal machines. There is example of cutting route planning below with reduction of  for shaped parts taken into account application of special cutting techniques with thermal deformation reduction developed in [23]. The conditions of thermal deformation reduction during nesting and cutting route planning are considered in [13].

Based on algorithms presented on [23] the cutting tool route for nesting is automatically built at CAD/CAM “SIRIUS”. In order to evaluate the effectiveness of developed special cutting techniques the two nesting is obtained by using standard (fig.5a) and special (fig.5b) cutting techniques for various types of parts.

Fig.5a presents the cutting tool path built for various geometrical types of parts including circles using standard cutting techniques. Fig.5b presents the cutting tool path built for various geometrical types of parts including circles using special cutting techniques.



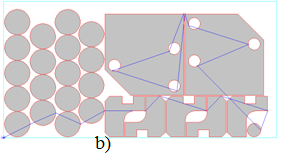


Fig.5. The cutting scheme example of parts using standard (a) and special (b) cutting techniques

Table 2 presents computational results of basic cutting parameters and values of  for obtained NC programs. The calculation of  was carried out by using «Сutting cost calculation. The results are calculated for AWAIMg3 ∆= 1 and 5 mm.

Table 2

The results of  calculation

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Material and thickness | Cutting techniques type | Basic cutting parameters | | | n | , rub | , rub | % |
|  | , m | , m |
| AWAIMg3, ∆=1 | Standard (fig.5a) | 32 | 16,43 | 36,82 | 130 | 809,5 | 866,5 | 6,6 |
| Special (fig.5b) | 18 | 8,6 | 36,97 | 757,1 | 814,3 | 7 |
| AWAIMg3,  ∆=5 | Standard (fig.5a) | 32 | 16,43 | 36,82 | 13121,1 | 16062,2 | 18,3 |
| Special (fig.5b) | 18 | 8,6 | 36,97 | 12748,1 | 15701,2 | 18,8 |

The following notations in table 2 are used: *n –* numbers of frames in NC program;  - cutting cost calculated taking into account that ;  - cutting cost calculated taking into account that  and depends on the frames numbers of NC program; % - value of difference between  and.

The results presented in table 2 indicate that the basis cutting parameters are reduced by using special cutting techniques compared with standard cutting. In turn the difference between  andreaches to 18%.

1. **Conclusions**

In this paper the following results were obtained:

1. In order to exact calculate objective function and consequently to construct exact tool path the methodology of objective function  and basic cost parameters calculation is presented for CNC laser cutting machines. Due to the problem of exact  values calculation have arisen on many production factories and based on proposed methodology the subsystem «Сutting cost calculation» were developed used Net. Framework technology, which may be integrated with exciting CAM software;
2. The functional dependencies on number of NC program commands for  are developed. These dependencies ensure exact calculation of objective function  and exact tool optimization path construction. For subsystem «Сutting cost calculation» the module of exact cutting cost computation of cutting cost was developed used functional dependencies for ;
3. In order to evaluate the developed results the computational experiments have been conducted taking into account previously proposed special cutting techniques compared with standard cutting. The cutting route is constructed with taking into account the thermal deformation reduction. The correct  values calculation is carried out with developed above methodology given the () and  ().The results shown that the cutting cost is reduced by using special cutting techniques compared with standard cutting. In turn the difference between  andreaches to 18%.

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